Baye's Theorem

Find Probability of event A Given that B even has already occured,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{— (I)}$$

Given that

$$\therefore \quad P(B|A) = \frac{P(B \cap A)}{P(A)} \quad \text{— (II)}$$

From (I) and (II)

$P(A \cap B) = P(m\, B \cap A)$

$= P(A|B)\, P(B) = P(B|A)\, P(A)$

Bayes Theorem

$$\therefore \quad \boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

Probability of evidance given that hypothesis true

Probability of Hypothesis after evidance observed

Likelihood

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Posterior (Hypothisis) — P(A|B)

Prior — P(A)

evidance / Givendata — B

marginal probability of givendata on evidance — P(B)

Probabibility of hypothesis before evidance observed

Face card :- 4 (K, Q, J)

In card deck, Find P(King | Face) = ?

$$P(\text{King} \mid \text{Face}) = \frac{P(\text{Face} \mid \text{King}) \cdot P(\text{King})}{P(\text{Face})}$$

$$= \frac{\frac{4}{4} \cdot \frac{4}{52}}{\frac{12}{52}}$$

$$= \frac{1}{3}$$

Naive Bayes

$$P(y \mid x_1, \ldots, x_n)$$

$$= \frac{P(x_1 \mid y)\, P(x_2 \mid y) \ldots P(x_n \mid y) \times P_y}{P(x_1)\, P(x_2) \ldots P(x_n)}$$

Q

Red = R
Sports = Sp

SU = SUV
D = Domestic
Y = Yellow

I = Imported

No	Clolor x_1	Type x_2	Origin x_3	Stolen? y
1	Red	Sports	D	Yes
2	R	Sp	D	No
3	R	Sp	D	Yes
4	Yellow	Sp	D	No
5	Y	Sp	I	Yes
6	Y	SUV	I	No
7	Y	SU	I	Yes
8	Y	SU	D	No
9	R	SU	I	No
10	R	Sp	I	Yes

Prequist

∵ Each featune must Independent

∴ all featune must contribote Equally

Hene

Noot featune = 3 (x_1, x_2, x_3)

x_1 = color

x_2 = Type

x_3 = origin

y = stolen?

Now find if can has featone Red, SUV

and domenstic. Is it stolen on not?

Ans:-

$\boxed{\text{Step-1}}$ Frequency and likelihood table's of color

Frequency Table

		Stolen	
		Yes	NO
color	Red	3	2
	Yellow	2	3

→

Likelihood Table

		stolen	
		P(Yes)	P(NO)
color	Red	$\frac{3}{5}$	$\frac{2}{5}$
	Yellow	$\frac{2}{5}$	$\frac{3}{5}$

Yes = 5
NO = 5 } → cane
we ane dividing

step-3 Fnequency Table + Likelihood Table

For Type

stolen

Type	Yes	NO
Sponts	4	2
SUV	1	3

stolen

	P(Yes)	P(NO)
Sponts	4/5	2/5
SUV	$\frac{1}{5}$	$\frac{3}{5}$

Step-3 Fnequency + Liklihood Table onigin

stolen

onigin	Yes	No
Domestic	2	3
Imponted	3	2

stolen

	P(Yes)	P(NO)
Domenstic	2/5	3/5
Imponted	3/5	2/5

From Table's Found

$P(Red \mid Yes) = 3/5$

$P(SUV \mid Yes) = 1/5$

$P(Domestic \mid Yes) = 2/5$

$P(yes) = \frac{5}{10} = .5$

Naive Bayes Formula

$V_{NB}(y) = \boxed{V_{NB} = \arg\max_y P(y) \prod_{i=1}^{n} P(x_i \mid y)}$

$P(y \mid \underline{x_1 \cdots x_n}) =$
$\quad\quad x$

$P(yes \mid X) = P(yes) \times P(Red \mid yes] \times P(SUV \mid yes)$
$\times P(Domenstic \mid yes)$

$= 0.5 \times 3/5 \times 1/5 \times 2/5$

$= .024$

$\because$ Probability of stolen yes if car is Red, SUV and domenstic $= 0.24$

NOW,

$$P(\text{Red} | \text{NO}) = 2/5$$

$$P(\text{SUV} | \text{NO}) = 3/5$$

$$P(\text{Domenstic} | \text{NO}) = 3/5$$

$$P(\text{NO}) = 5/10 = .5$$

$P(\text{NO} | \text{Red}, \text{SUV}, \text{Domenstic})$

$$= P(\text{NO}) \cdot P(\text{Red} | \text{NO})\ P(\text{SUV} | \text{NO})\ P(\text{Domenstic} | \text{NO})$$

$$= .5 \times 2/5 \times 3/5 \times 3/5$$

$$= 0.072$$

Probabibility of car is not stolen if Red, SUV, Domenstic = 0.072

P(No | Red, SUV, Domenstic) > P(yes | Red, SUV, Domenstic)

∴ if car is Red, SUV, Domenstic

∴ car is not stolen

Ans

— o —

Problem of Naive Bayes

Zero frequency problem

suppose if $P(\text{Red} \mid \text{No}) = 0$

then $P(\text{No} \mid \text{Red, SUV, Domenstic})$

$= 0$

How to solve Zero frequency Problem

if we get this situation like in frequency Table :

	Yes	No
Red	0 ***	3
Yellow	2	3

Solution :- increase all value by 1 and get rid of 0

	Yes	No
Red	0+1 = 1	3+1 = 4
Yellow	2+1 = 3	3+1 = 4