PCA

Step-1 :- Find the mean of feature

$$\bar{x}_1 = \frac{(x_{11} + x_{12} + x_{13} + x_{14})}{4}$$

$$= \frac{1}{4}(1 + 4 + 5 + 7)$$

$$= 4.25$$

$$\bar{x}_2 = \frac{1}{4}(3 + 7 + 8 + 9) = 6.75$$

Step-2 :- Find the covariance matrix

$$S = \begin{bmatrix} cov(x_1, x_1) & cov(x_1, x_2) \\ cov(x_2, x_1) & cov(x_2, x_2) \end{bmatrix}$$

Use calculator

Alpha calc with A, B

$$\therefore cov(x_1, x_2) = \frac{1}{n-1}\left[\sum_{i=1}^{n} (\overset{A}{x} - \bar{x}_1)(\overset{B}{x} - \bar{x}_2)\right]$$

$$= \frac{1}{n-1}(12.1875 + (-.06) + .936 + 6.1875)$$

$\text{cov}(x_1, x_2) = 6.42$

$\text{cov}(x_2, x_1) = 6.42$

$$\text{cov}(x_1, x_1) = \frac{1}{n-1} \sum_{i=1}^{n} (x - \underline{\bar{x}_1}^{A})(x - \underline{\bar{x}_1}^{A})$$

use A calc

direct calculation

$$= \frac{1}{3} (\quad)$$

$$= 6.75$$

$$\text{cov}(x_2, x_2) = \frac{1}{n} \sum_{i=1}^{n} (x - \underset{A}{\bar{x}_2})(x - \underset{A}{\bar{x}_2})$$

$$= \frac{1}{3} (20.75)$$

$$= 6.92$$

∴ covariance matrix

$$S = \begin{bmatrix} 6.75 & 6.42 \\ 6.42 & 6.92 \end{bmatrix}.$$

Step-3:-

Find eigan Value

$$\boxed{\det(S - \lambda I) = 0}$$

$$\Rightarrow \det\left(\begin{bmatrix} 6.25 & 6.42 \\ 6.42 & 6.92 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} 6.25-\lambda & 6.42 \\ 6.42 & 6.92-\lambda \end{bmatrix}\right) = 0$$

matrix sum

$$\Rightarrow (6.25-\lambda)(6.92-\lambda) - 41.22 = 0$$

$$\Rightarrow 43.25 - \lambda 6.92 - 6.25\lambda + \lambda^2 - 41.22 = 0$$

$$\Rightarrow \lambda^2 - 13.17\lambda + 2.03 = 0$$

$$\therefore \lambda_1 = 13.01$$

use calculator

$$\lambda_2 = 0.156$$

Step-4 :- find the eigan vector

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\boxed{(S - \lambda I)U = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

$$\Rightarrow \underset{2\times 2}{\begin{bmatrix} 6.25-\lambda & 6.42 \\ 6.42 & 6.92-\lambda \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (6.25-\lambda)u_1 + 6.42u_2 \\ 6.42u_1 + (6.92-\lambda)u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

matrix multipication

$$\Rightarrow (6.25-\lambda)u_1 + 6.42u_2 = 0 \quad \text{—— (I)}$$

$$6.42u_1 + (6.92-\lambda)u_2 = 0 \quad \text{—— (II)}$$

① $\longrightarrow$

Important

$$(6.25-\lambda)u_1 + 6.42u_2 = 0$$

$$\Rightarrow \quad u_1(6.25-\lambda) = -6.42u_2$$

$$\Rightarrow \quad \frac{u_1}{-6.42} = \frac{u_2}{6.25-\lambda} = t$$

$$\therefore \quad u_1 = -6.42t$$

$$\therefore \quad u_2 = (6.25-\lambda)t$$

$\therefore$ Assuming $t = 1$

$$u_1 = -6.42$$

$$u_2 = 6.25-\lambda$$

$$\therefore \quad u = \begin{bmatrix} -6.42 \\ 6.25-\lambda \end{bmatrix}$$

$$\lambda = \lambda_1 = 13.01$$

$$\therefore \quad u_1 = \begin{bmatrix} -6.42 \\ 6.25-13.01 \end{bmatrix} = \begin{bmatrix} -6.42 \\ -6.76 \end{bmatrix}$$

$$\|u_1\| = \sqrt{(-6.42)^2 + (-6.76)^2}$$

$$= 9.32$$

step-5

First principle component

$$e_1 = \begin{bmatrix} \frac{-6.42}{\|u_1\|} \\ \frac{-6.76}{\|u_2\|} \end{bmatrix}$$

$$= \begin{bmatrix} -0.69 \\ -0.73 \end{bmatrix}$$

$$\therefore PC_1 = e_1^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix}$$

$$= \begin{bmatrix} -.69 & -.73 \end{bmatrix}_{1\times2} \begin{bmatrix} 1 - 4.25 \\ 3 - 6.75 \end{bmatrix}_{2\times1}$$

matrix mult

$$= (-.69)(\underset{A}{1} - 4.25) + (-.73)(\underset{A}{3} - 6.75)$$

$$= 4.98$$

↓ use this for all others

use calc A.B

$$PC_2 = (-.69)(\underset{\bar{A}}{4} - 4.25) + (-.73)(\underset{\bar{B}}{7} - 6.25)$$

$$= -.01$$

$$PC_3 = (-.69)(\underset{\bar{A}}{5} - 4.25) + (-.73)(\underset{\bar{B}}{8} - 6.25)$$

$$= -1.43$$

$$PC_4 = (-.69)(\underset{\bar{A}}{7} - 4.25) + (-.73)(\underset{\bar{B}}{9} - 6.275)$$

$$= -3.54$$

	PC1	PC2	PC3	PC4
PC	4.98	-.01	-1.43	-3.54