Chapter 1-2

(*) what is Computer Graphics ?

→ refers to the visual field that focus on generating, manipulating, displaying visual content using computers.

(*) what is Resolution of an Image ?

Final

→ Resolution refers to the number of pixels per unit length

(*) what is pixels ?

→ Pixel refers to picture element, is the smallest controllable element of a digital Image or display device

(*) what is Image ?

→ 2D array of pixels.

CamScanner

~~(*) Compute the resolution~~

Final

(*) if we use direct coding with 8 bits per primary color. How many possible colors do we have for each pixel?

$\rightarrow \quad 2^8 \times 2^8 \times 2^8 \Rightarrow 2^{24}$

$= 16777216$

if (bit = 2) :- $2^2 \times 2^2 \times 2^2 = 2^6 = 32$

Definitation

Direct color coding

Final

(*) The direct coding method is flexible in that it allows the allocation of a different number of bits to each primary color. if we use 5 bits each for red, blue and 6 for green for total 16 bits per pixel. How many possible simultaneous color do we have?

$\rightarrow \quad 2^5 \times 2^5 \times 2^6 = 2^{16}$

$= 65{,}536$

CamScanner

(*) a 3X2 inch image at a resolution of 300 pixels per inch would have a total pixels of?

class

⇒ width = 3, height = 2

Resolution = 300 pixels per inch

∴ Total pixels = (3x300) * (2x300)

= 540,000 pixels

$$\boxed{\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} c \\ m \\ y \end{bmatrix}}$$

RGB = 1 - cmy

cmy = 1 - RGB

cyan - magenta - yellow

(*) Find the cmy condinates of colors at (.2, 1, 0.5) a tRGB colorspace?

= (1 - .2), (1 - 1), (1 - 0.5)

= (.8, 0, .5) → cmy

CS CamScanner

Final

(*) Compute the size of a 640x480 image at 240 pixels peninch?

$= \frac{640}{240}$ by $\frac{480}{240}$ inches

$= 2\frac{2}{3}$ by 2 inches

(*) what is aspect ratio?

→ The ratio of its width to height measured in unit length on number of pixels.

(*) if image height = 2 and aspect ratio = 1.5 what is width?

⇒ width = height * aspect ratio

$= 2 \times 1.5$

$= 3$ inch

CS CamScanner

Class

How Lookup Table reduce Image size?

→ Assume

Image = 1000 × 1000

Direct coding

R = 8bit, G = 8bit, B = 8

Total = 24bit color

color size = 24bit

= 3 byte

(1 byte = 8bit)

∴ Total size = 1000×1000 ×3

= 3000000

= 3MB

Image size = 3mB

VS

Lookup table

Image :- 8 bit per pixel

∴ Image size = 1000×1000 ×8(1)/8

= 1MB

∴ Lookup table size

$2^8 = 256$

= 256 × 3byte [24bit RGB]

1byte = 256 (2^8)

= 0.00076 MB

Total size = 1 + 0.76

= 1.76 MB

= 1.000768

Image size = 1.76MB / 1.00076

(*) if we use 12 bit pixels in a lookup table representation, how many entries does the lookup table have?

$2^{12} = 4096$ entries

(*) if we use $\boxed{\text{2byte}}$ pixels values in a 24bit lookup table representation. How many bytes does the lookup table occupy?

$= 2^{16} * \frac{24}{8} = 196,608$

1byte = 256

2 " = $256 \times 256 = 2^{16}$

2byte = 16 bit

chapter-3

(*) what is scan Convension / Rasterization?

→ It is a process of converting 2D or 3D graphics primitive (point, circle, filled and polygon) from their geometric definitions into set of pixels in the image space.

Mathmatica property → actual pixels

(*) what are the step of scan converting a point?

① Input (x y) point

② as the pixels grid is discrete, round the coordinate to the nearest integer value

$$\boxed{\begin{aligned} x' &= \text{floor}(x + 0.5) \\ y' &= \text{floor}(y + 0.5) \end{aligned}}$$ → point → pixels

∴ if $(x, y) = (2.3, 4.6)$ → $x' = \text{floor}(2.3 + 0.5) = 2$
$y' = \text{floor}(4.6 + 0.5) = 5$

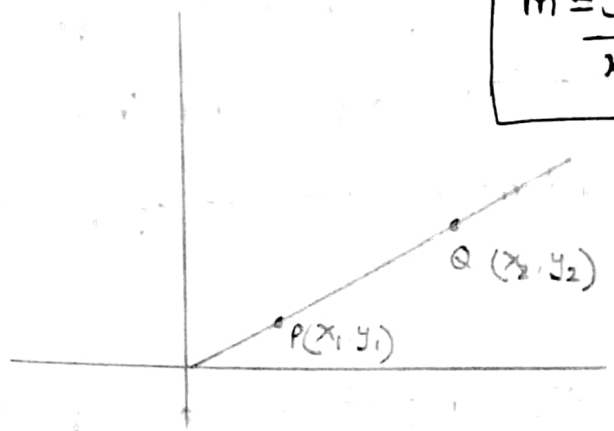
③ Plot pixels $(2, 5)$ / (x', y')

CamScanner

Direct Line Equation

case-1

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$|m| <= 1$ → condition

To draw line PQ

we will increase x everytime by 1 from and fill each pixels based on (x, y)

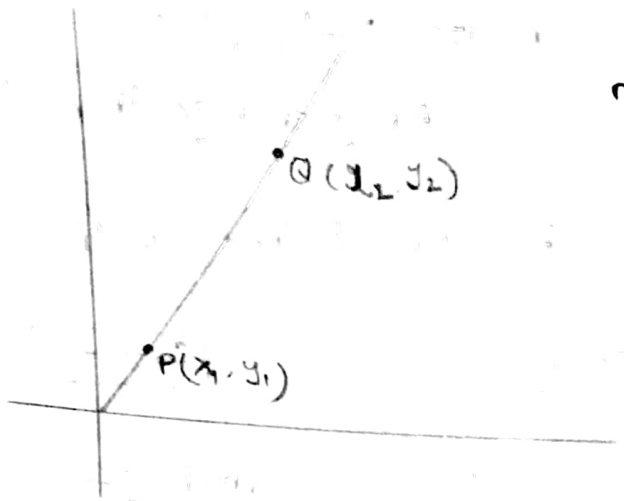
```
x = x++
y = mx + b
y = round(y)
```

→ when $|m| <= 1$

do untill $(x_1 < x_2)$

CamScanner

case-2



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$\boxed{|m| > 1}$ → condition

To draw the line PQ

```
y = y++
x = (y-b)/m
x = round(x)
```

when $|m| > 1$

Do until $y_1 < y_2$

Digital Differential Analyzer Algorithm (DDA)

speeding the direct line equation

Case-1

$|m| \leq 1$ condition

$\Delta x = 1$

$x(i+1) = x_i + 1$

$y(i+1) = y_i + m$

∴ while $(x_i \leq x_f)$ {

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i + m$

$y_{i+1} = \text{floor}(y_{i+1} + 0.5)$

}

Case-2

$|m| > 1$ condition

$\Delta y = 1$

$y(i+1) = y_i + 1$

$x(i+1) = x_i + \frac{1}{m}$

while $(y_i \leq y_f)$ {

$y_{i+1} = y_i + 1$

$x_{i+1} = x_i + (1/m)$

$x_{i+1} = \text{floor}(x_{i+1} + 0.5)$

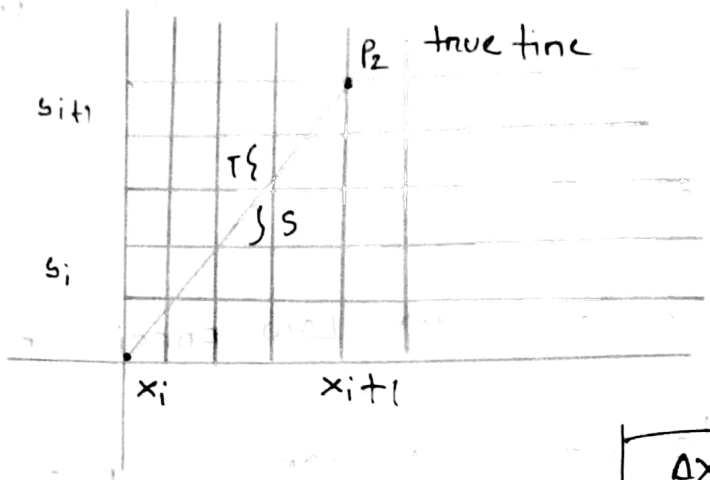
}

CamScanner

DDA Algorithm	Direct Line Equation
① Best For :- computation	② Best for :- Precision
② Speed :- Fast	③ Slow
③ High precision Loss	③ Low precision Loss
④ Less Accurate end point	④ more accurate end point
⑤ Sometimes may never reach end point	⑤
	⑥ Easy to implement
⑥ as we round the value errors add in each step which makes line slightly off from real.	

CamScanner

Bresenham's Line drawing Algorithm.



$$\Delta x = x_2 - x_1$$
$$\Delta y = y_2 - y_1$$

$P(x_1, y_1)$ and $Q(x_2, y_2)$

To draw PQ

$$\boxed{\text{Base case :- } d_1 = 2\Delta y - \Delta x}$$

Then

$\boxed{\text{if } (d_i \geq 0)}$ condition

$$\boxed{d_{i+1} = d_i + 2\Delta y - 2\Delta x} \quad (x_{i+1}, y_{i+1})$$

$\boxed{\text{if } (d_i < 0)}$

$$\boxed{d_{i+1} = d_i + 2\Delta y} \quad (x_{i+1}, y_i)$$

CamScanner

Final

DDA Algo	Bresenham's
① only multipication and division are used	① addition & subtraction
② less efficient	② more efficient
③ not enough fast than Bresenham's	③ fast
④ less accuracy and presion	④ more accurate and precise
⑤ less optimized	⑤ more optimized
⑥ High cost	⑥ less cost
⑦ floating point Real Arithmatic	⑦ fixed point integer Arithmatic
⑧ round off coordinate to integer value that is nearest to line	⑧ doesnot round off (incremental value operation)

CamScanner

Q. Final

if $d_i = -3$, $x_i = 3$, $y_i = 2$; x_{i+1} ?
$y_{i+1} = ?$

use Bresenham's

Ans:-

$d_i = -3 < 0$

$x_{i+1} = x_i + 1 = 3 + 1 = 4$

$y_{i+1} = y_i = 2$

$x_{i+1}, y_{i+1} = (4, 2)$

Final

Use DDA vs Bresenham's

Bresenham's	Direct Line
① use simple integer Arithmatic	① use floating point arithmatic
② very efficient	② less efficient
③ fast	③ slow

CamScanner

Final

(*) The end points of a line (0,0) and (6,18) compute each value of y as x steps from 0 to 6 and plot the result

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18-0}{6-0} = 3$$

$$b = y - mx$$

$$= 0 - 3\,0 = 0$$

$$\boxed{y = mx + b = 3x}$$

$x_0 = 0 \rightarrow y_0 = 0$

$x_1 = 1 \rightarrow y_1 = 3$

$x_2 = 2 \rightarrow y_2 = 6$

$x_3 = 3 \rightarrow y_3 = 9$

$x_4 = 4 \rightarrow y_4 = 12$

$x_5 = 5 \rightarrow y_5 = 15$

$x_6 = 6 \rightarrow y_6 = 18$

CS CamScanner

Q

Apply Bresenham's to draw a line from (2,3) to (8,12) show steps and Resulting pixels

Ans :-

$\Delta x = 8-2 = 6$

$\Delta y = 12-3 = 9$

Intial decision variable

$d_1 = 2\Delta y - \Delta x = 18-6 = 12$

Stanting pixel (2,3) (x_1, y_1)

Step-1

$d_1 = 12 \geq 0$; $d_{i+1} = d_i + 2\Delta y - 2\Delta x$

$\therefore d_2 = 18$

$x_2 = x_1 + 1 = 2+1 = 3$

$y_2 = y_1 + 1 = 3+1 = 4$

(3,4)

Step-2

$d_2 = 18 \geq 0$; $d_{i+1} = d_i + 2\Delta y - 2\Delta x$

$d_3 = 24$

$x_3 = 4$

$x_4 = 5$

(4,5)

CamScanner

Step-3 :- (5,6)

Step-4 :- (6,7)

step-5 :- (7,8)

step-6 :- (8,9) → $\boxed{\text{stop}}$

Step-7 :- (9,10)

Step-8 :- (10,11)

step-9 :- (11,12)

Step-10 :- (12,13)

do bresenline drawing with

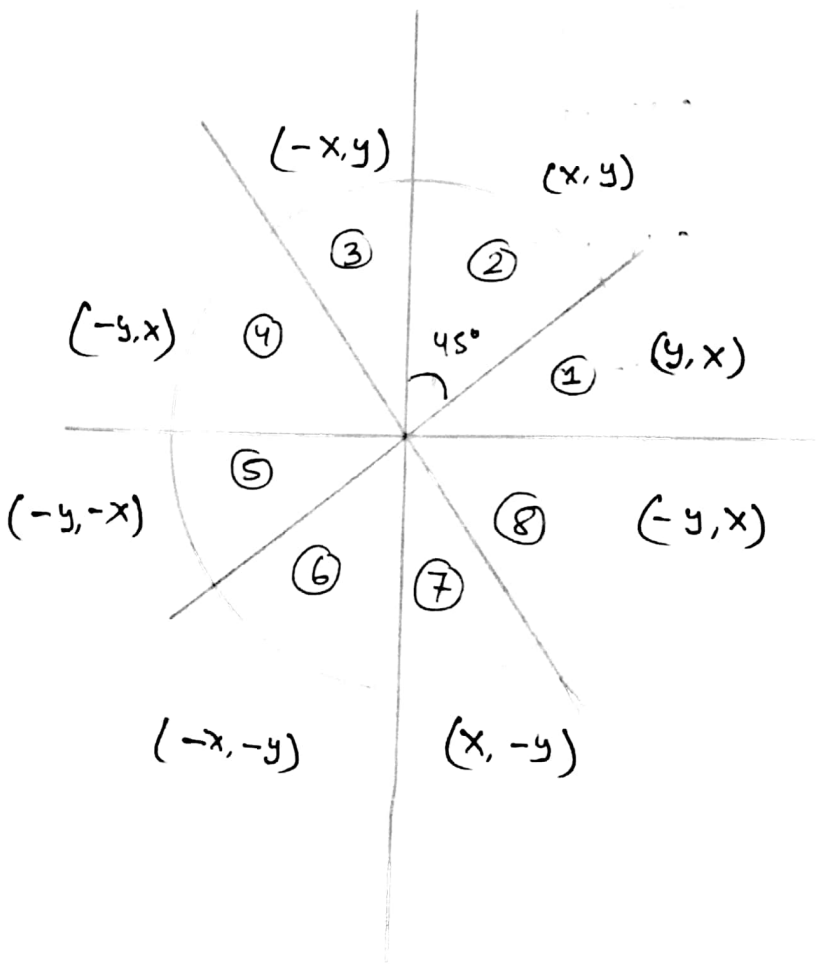
$\boxed{x_1 \le x_2}$

X no need

CamScanner

(*) what is a 8way symentry cincle?

→ cicle is 8 way symentry tiqune. The shape of cincle is the same in all quadrants In each quadrants thein are 2 octrants (1/8)



Bresenham's Circle Algorithm

$$x^2 + y^2 = r^2$$

Given, x_c, y_c, r

We know
$x = 0$
$y = r$

Intial decision parameter,

$$d = 3 - (2 * r)$$

```
while (x <= y) {
    setpixel(x, y)
```

→ function drawing 8 pixels from x, y each time

```
    if di < 0   condition
        di+1 = di + 4x + 6      xi+1 , yi

    if di ≥ 0 {   condition
        di+1 = di + 4(x-y) + 10      xi+1 , yi-1
        y--; y

    x++;
}
```

function setpixel (x, y) {

① putpixel ($x^c + x$, $y^c + y$, colon)

② " ($x^c - x$, $y^c + y$, colon)

③ " ($x^c + x$, $y^c - y$, ")

④ " ($x^c - x$, $y^c - y$, ")

⑤ " ($y^c + y$, $x^c + x$, ")

⑥ " ($y^c - y$, $x^c + x$, ")

⑧ " ($y^c + y$, $x^c - x$, ")

⑤ " ($y^c - y$, $x^c - x$, ")

}

Final

Q. Plot a dashed circle using Bresenham's circle Algorithm

```
int drawcircle = 1

functinon setpixel (x, y) {

    if (drawcircle)
        { ... 8way
              symentric
        }

    drawcircle = ! draw circle

}
```

(*) modify the midpoint circle Algorithm pseduocode to draw a dashed circle?

Final

Mid point Circle Algorithm

Given, x^c, y^c, r

Assume,

$$\boxed{\begin{aligned} x &= 0 \\ y &= r \end{aligned}}$$

Intial decision parameter

$$\boxed{d = 1 - r} \quad \frac{5}{4} \cong 1$$

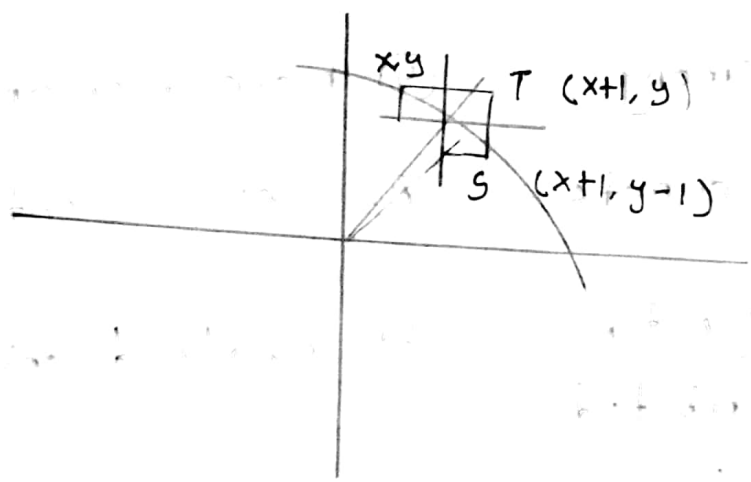
```
while (x <= y) {

        setpixel (x, y)

  if di < 0          condition
      di+1 = di + 2x + 3        xi+1, yi
if di >= 0 {         condition
      di+1 = di + 2(x-y) + 5    xi+1, yi-1
      y--; y

x++;
```

Bresenham's	Midpoint
① deals with Integer Value	① deals with floating value
② more optimized	② less optimize
③ less time	③ more time
④ less memory	④ more memory
⑤ more accurate	⑤ less accurate
⑥ pixels may not be evenly distributed but still form a circle	⑥ evenly distributed
⑦ complex	⑦ easy

CamScanner

Final

Q. In Brasenham's Circle drawing, we defined the decision variable $d_i = D(T)$ and $D(S)$ where T is the top pixel and S is the lower pixel. Is there any chance that $d_i = 0$ but pixels S & T are not equally far from true circle?



Yes; there's chance that T and S are not equally far from

D = not true measure of distance from center of a pixel to the true circle

d_S = actual distance from S to true circle

d_T = " " " T to "

$(x = x_i + 1,\ y = y_i)$

$$d_i = 2x^2 + y^2 + (y-1)^2 - 2r^2 = 0$$

Since

$$(r + d_T)^2 = x^2 + y^2$$

$$\Rightarrow 2r d_T + d_T^2 = x^2 + y^2 - r^2 \quad \text{— (I)}$$

again $(r - d_S)^2 = x^2 + (y-1)^2$

$$\Rightarrow -2r d_S + d_S^2 = x^2 + (y-1)^2 - r^2 \quad \text{— (II)}$$

(I) + (II)

$$2r d_T + d_T^2 - 2r d_S + d_S^2 = \overbrace{2x^2 + y^2 + (y-1)^2 - 2r^2}^{d=0}$$

$$= 0$$

$$d_T(2r - d_T) = d_S(2r - d_S)$$

$$\therefore d_T/d_S = \frac{2r - d_S}{2r - d_T} < 1$$

$d_T < d_S$ → if $d_i = 0$ T is closer to true circle than S

Final

(*) what are 4connected and 8connected pixels example and diagram.

→ Pixels are considered connected to each other to form continous boundary :-

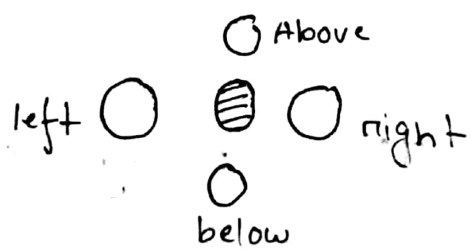
2 methods for determining pixel connectivity

(i) 4connected

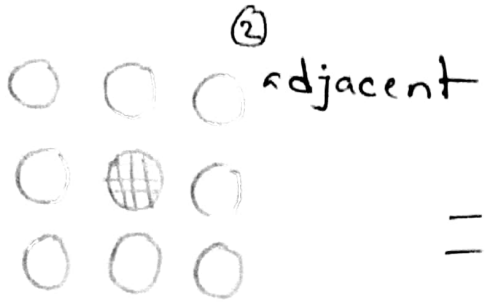
(ii) 8 connected

4 connected

A pixel may have up to 4 neighbours.



8 connected :-



- above
- below
- right
- left

Final

Circle symmetry property :-

- Circle looks same when you rotate / reflect it.
- same look from diff angel / directions

→ because of certain points

CamScanner

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

① Direct line equation

$|m| \leq 1$

```
while (xi < xj)
    x++;
    y = mx+b;
    y = round(y)
```

$|m| > 1$

```
while (yi < yj)
    y++
    x = (y-b)/m
    x = round(x)
```

② DDA Algorithm

$|m| \leq 1$

```
while (xi ≤ xj)
    xi+1 = xi + 1
    yi+1 = yi + m
    yi+1 = floor(yi+1 + 0.5)
```

$|m| > 1$

```
while (xi < yj)
    yi+1 = yi + 1
    xi+1 = xi + 1/m
    xi+1 = floor(xi+1 + .5)
```

③ Bresenham's Line drawing algorithm

Base = $d = 2\Delta y - \Delta x$

if $d_i \geq 0$

$d_{i+1} = d_i + 2\Delta y - 2\Delta x \longrightarrow x_{i+1} = x_i + 1$

$y_{i+1} = y_i + 1$

if $d_i < 0$

$d_{i+1} = d_i + 2\Delta y \longrightarrow x_{i+1} = x_i + 1$

$y_{i+1} = y_i$

CamScanner

Dashed

```
bool drawcircle = 1

function setpixel(x, y)

if (drawcircle)
    plotpixel (     xc+x, yc+y, colon)
    put "   (xc-x, yc+y,    "   )
    put "   (xc+x, yc -y,   "   )
    put "   (xc-x, yc-y,   "   )
    put "   (yc+y, xc+x,   "   )
    put "   (yc+y, xc-x,   "   )
    put "   (yc-y, xc+x,   "   )
    put "   (yc-y, xc-x,   "   )
```

$\boxed{\text{drawcircle} = !\ \text{drawcircle}}$

⊛ Bresenham's Circle drawing (x_c, y_c, r)

$x = 0, y = r$

base, $d = 3 - 2r$

while $(x < y)$

setpixels (x, y)

if $d_i < 0$

$d_{i+1} = d_i + 4x + 6 \rightarrow x_{i+1} = x_i + 1$, $y_{i+1} = y_i$

if $d_i > 0$

$d_{i+1} = d_i + 4(x-y) + 10 \rightarrow x_{i+1} = x_i + 1$, $y_{i+1} = y_i - 1$

⊛ Mid point circle Algo.

base, $d = 1 - r$

while $(x < y)$

setpixels (x, y)

if $d_i < 0$

$d_{i+1} = d_i + 2x + 3 \rightarrow x_{i+1}, y_i$

if $d_i > 0$

$d_{i+1} = d_i + 2(x-y) + 5 \rightarrow x_{i+1}, y_i - 1$

CS CamScanner

$D(T) = x^2 + y^2 - r^2 > 0$

$D(S) = x^2 + (y-1)^2 - r^2 < 0$

$\therefore$ decision variable, $d = d(T) + d(S)$

$= 2x^2 + y^2 + (y-1)^2 + (-2r^2)$

assume $d = 0$ — (i)

Since

$(r + dT)^2 = x^2 + y^2$

$\Rightarrow r^2 + 2r\,dT + dT^2 = x^2 + y^2$

$\Rightarrow 2r\,dT + dT^2 = x^2 + y^2 - r^2$ — (ii)

(ii) same

$(r - ds)^2 = x^2 + (y-1)^2$

$\Rightarrow -2r\,dT + ds^2 = x^2 + (y-1)^2 - r^2$ — (iii)

(ii) + (iii) $\rightarrow$

$2r\,dT + dT^2 - 2r\,ds + ds^2 = 2x^2 + y^2 + (y-1)^2 + (-2r^2)$

$\Rightarrow dT(2r - dT) - ds(2r - ds) = 0$

$\Rightarrow dT(2r - dT) = ds(2r - ds)$

$\Rightarrow \dfrac{dT}{ds} = \boxed{\dfrac{2r - ds}{2r - dT} < 0}$

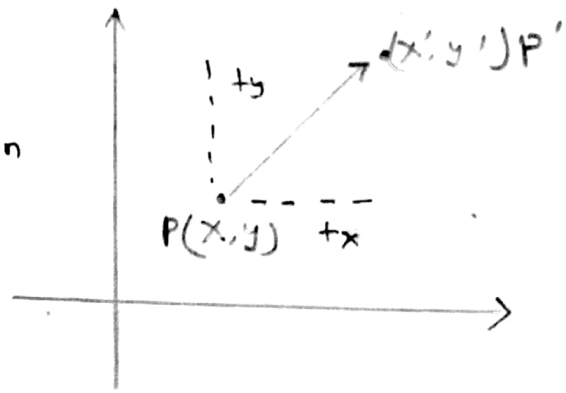
$\boxed{dT < ds}$ even if $d = 0$

CS CamScanner

(17) Q. Define translation

translation

A translation moves an object from one position to different position on screen.



(*) Geometric Transformations

$$P' = P + T$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

→ translation / Shift vector (given)

coordinate Translation

$$T_v = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \qquad \vec{v} = h\hat{i} + k\hat{j}$$

$$T_v^{-1} = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

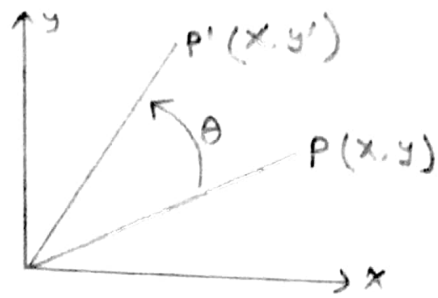
3x3 Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

CS CamScanner

Rotations

In Rotatation we rotate the object at panticular angel θ from the origin.



Geometric Transformation

$$P' = P \cdot R_\theta \longrightarrow \text{Rotation matrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$ Neg rotation

Cordinate Transformation

$$P' = P \cdot R_{-\theta}$$

Postive rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

CamScanner

$$T_V = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \qquad \bar{T}_V = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \bar{R}_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

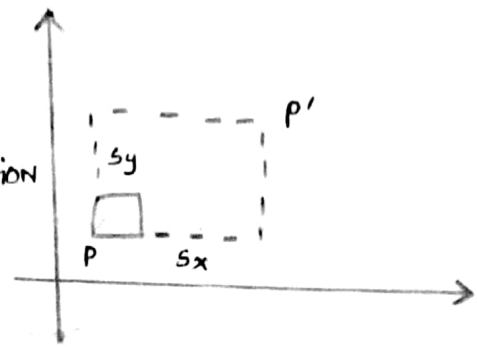
$$S_{x,y} = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \bar{S}_{x,y} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

CamScanner

Scaling

In scaling you either expanp/magnify ($S>1$) or compness/reduction ($S<1$) the dimmension of objects.



⊛ Geo-metric Transformation

$$P' = P \cdot S_{Sx, Sy}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

scaling factons

⊛ Coordinate Transformation

$$P' = P \cdot \bar{S}_{Sx, Sy}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{S_x} & 0 \\ 0 & \frac{1}{S_y} \end{bmatrix}$$

CS CamScanner

3x3 scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1/s_x & 0 & 0 \\ 0 & 1/s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

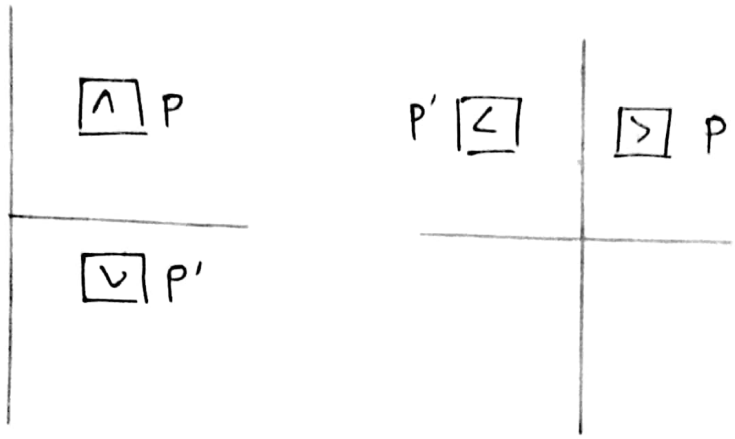
Q.17 Five two real life example of scaling

1. Image Resizing
2. Map zooming

CamScanner

Reflection

→ is the mirror image of original object



x axis mirror Reflection

y axis mirror Reflection

Geometric Transformation

$$P' = m_x \cdot P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = m_y \cdot P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transformation

$$P' = \bar{m}_x \cdot P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = \bar{m}_y \cdot P$$

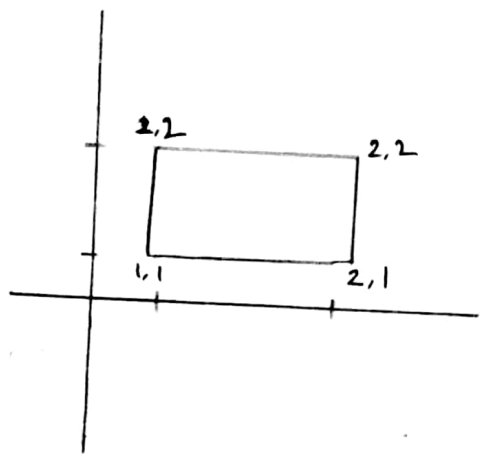
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

CamScanner

(17)

Q what is composite transformation?

A composite transformation is when two or more transformations are performed on a figure to produce a new figure



In this rectangle,

1. first we apply scaling ($S_x = S_y = 2$)
2. Rotation transformation in clockwise by 90^0
3. Then the Reflection transformation about origin

CamScanner

(*) Perform a 45° rotation of triangle A(0,0) B(1,1) C(5,2)

$$\boxed{\Delta' = R_\theta \cdot \Delta}$$

(a) about origin

// we represent traingle

$$\Delta = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \qquad \theta = 45°$$

// Rotation matrix

$$R_{45°} = \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A'B'C'] = R_{45°} \times [A, B, C]$$

$$\Rightarrow [A'B'C'] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 3/\sqrt{2} \\ 0 & \sqrt{2} & 7/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

Thus new condinates, $A' = 0, 0$ $\quad B = 0, \sqrt{2}$ $\quad C = 3/\sqrt{2}, 7/\sqrt{2}$

CamScanner

17 final

(b) about a point $P(-1,-1)$

$$\Delta' = R_{\theta.P}\,\Delta$$

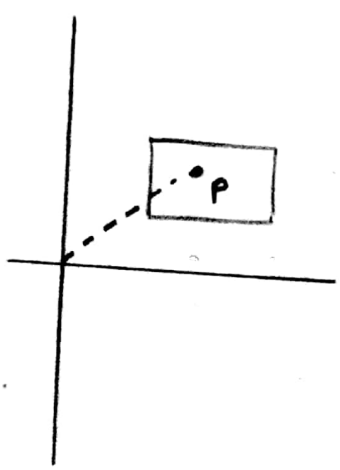
$$R_{\theta.P} = T_v \cdot R_\theta \cdot T_v^{-1}$$

$$R_{\theta,0} = T_v \cdot R_\theta \cdot T_{-v}$$

$$\Rightarrow \quad R_{\theta.0} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ here

$h = -1,\ k = -1$



$$R_{45^\circ.P} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & (\sqrt{2}-1) \\ 0 & 0 & 1 \end{bmatrix}$$

CamScanner

$$[A'B'C'] = R_{45^\circ p} \; [A.B.C]$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & \frac{3}{2}\sqrt{2}-1 \\ \sqrt{2}-1 & 2\sqrt{2}-1 & \frac{9}{2}\sqrt{2}-1 \\ 1 & 1 & 1 \end{bmatrix}$$

<u>New condinates</u>

$A' = -1, \; \sqrt{2}-1$

$B' = -1 \quad 2\sqrt{2}-1$

$C' = \frac{3}{2}\sqrt{2}-1, \; \frac{9}{2}\sqrt{2}-1$

— · —

<u>a</u> Real life example of coondinate transformation

Ans:

CamScanner

Q17: Magnify a triangle with vertices A(0,0) B(1,1) C(5,2) to twice its size while keeping C(5,2) fixed.

$$\frac{S_x}{a} = \frac{S_y}{b} = 2$$

$$\boxed{S_{a,b,P} = T_V \cdot S_{a,b} \cdot T_V^{-1}}$$

$$= \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

Here,

$$S_{2,2,C} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Representing a point P with condinates x, y with column vector $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$S_{2,2,C} \cdot A = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$$

CS CamScanner

$$S_{2.2c} \cdot B = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$S_{2.2.c} \cdot C = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

So,

$A' = -5, -2$

$B' = -3, 0$

$C' = 5, 2$

or

Applying $S_{2.2.c}$ to ABC

$$S_{2.2.c} \cdot [ABC] = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$A' = -5, -2$

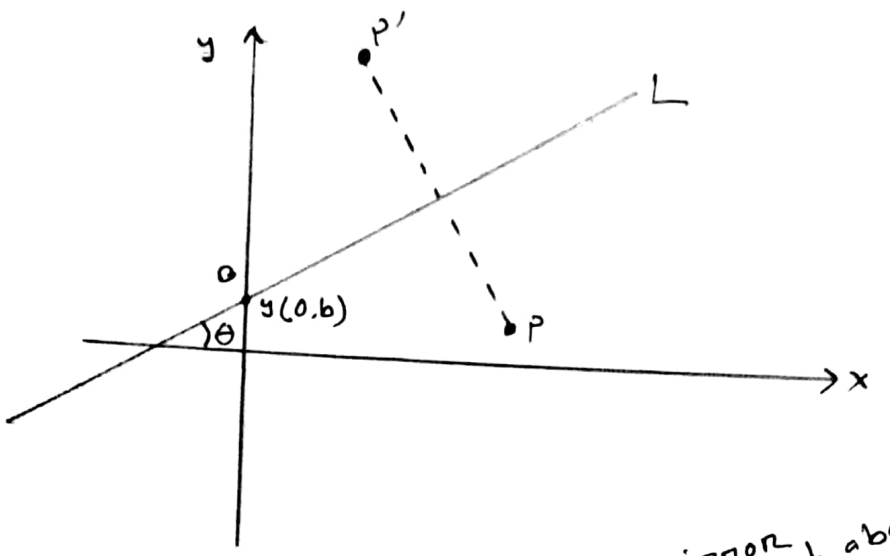
$B' = -3, 0$

$C' = 5, 2$

CamScanner

SWE

Q Describe the transformation m_L which reflects an objects a line



mirror reflect about x axis

$$\boxed{m_L = T_V \cdot R_\theta \cdot m_x \cdot R_{-\theta} \cdot T_{-V}}$$

Translate back to (0, b)

Rotate back θ

Rotate by −θ so that L aligns in x axis

translate the intersection point O to origin

18 Final

Q/ Reflect a dimond shaped polygon where vertices are A(-1,0) B(0,-2) C(1,0) D(0,2)

(a) a the horizontalline $y=2$ if the line

diamond shape $= V = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$y=2$

$\Rightarrow (0,b) = (0,2) \qquad \theta = 0^\circ$

$$\#m_L = T_V \cdot R_\theta \cdot M_x \cdot R_{-\theta} \cdot T_{-V}$$

$$= \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overset{0}{\cos\theta} & \overset{0}{-\sin\theta} & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

CamScanner

Reflection $= m_L V$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 1 & 0 \\ 4 & 6 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$A' = -1, 4$ $\quad$ $B' = 0, 6$

$C' = 1, 4$ $\quad$ $D' = 0, 2$

(b) if the line $y = x + 2$

if $x = 0$, $y = 2$

$(0, b) = (0, 2)$

$\therefore \quad m = 1$

$\tan\theta = m$

$\therefore \theta = \tan^{-1}(m)$

CamScanner

(e) if the line $x = 2$

$$(0, b) = (2, 0)$$

$$m_L = T_V \cdot m_x \cdot T_{-V}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

—·—

18 Final Question

Traingle ABC where the vertices of ΔABC $A(-1,-3)$ $B(-4,-1)$ and $C(-6,-4)$ undengoes a compostition of transformations described as translation 10 units to rights and then a reflection in x axis. After all the tranformation are applied what will be traingle coordinates.

CS CamScanner

$$\Delta ABC = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

translation to 10units right ( )

$A = -1, 0 = -1+10, 0 = 9, 0$

$B = 0, -2 = 0+10, -2 = 10, -2$

$C = 1, 0 = 1+10, 0 = 11, 0$

A reflection twonds x axis

$$A' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 10 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$C' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \end{bmatrix}$$

CS CamScanner

show that

$$S_{a,b} \cdot S_{c,d} = S_{c,d} \cdot S_{a,b} = S_{ac,\, bd}$$

$$S_{a,b} \cdot S_{c,d} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

$$S_{c,d} \cdot S_{a,b} = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

$$S_{ac,\, bd} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

Proved