Date :- 3-3

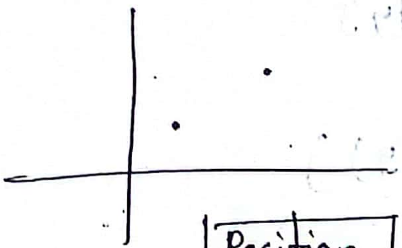
Chap-4

Transformation

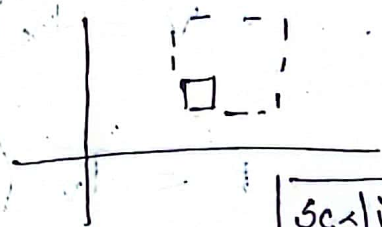
2D Transformation

3 types
- Geometric
- Co-ordinate

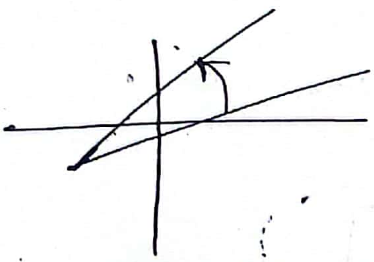
transformation can change



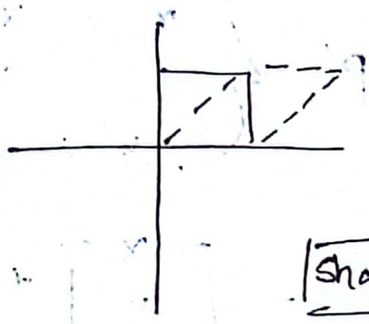
Position



Scaling



orientation

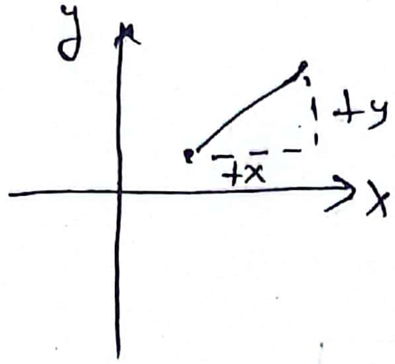


Shape

Vector, $p = \begin{bmatrix} x \\ y \end{bmatrix}$

translation

→ How to translate an object with multiple ventices ?



$$\vec{V} = t_x \hat{i} + t_y \hat{j}$$

$$P' = T_{\vec{v}}(P(x, y))$$

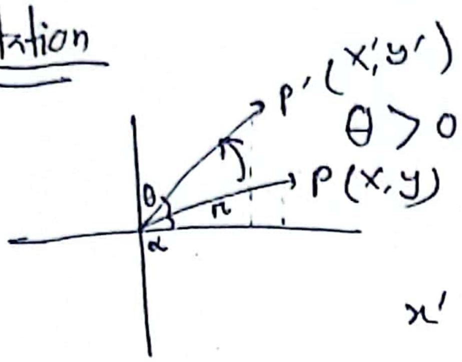
$$P' \begin{cases} x' = x + t_x \\ y' = y + t_y \end{cases}$$

$p' = p + t$

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

translation

Rotation



$P' = R(P)$

$x = r\cos\alpha$

$y = r\sin\alpha$

$x' = r\cos(\alpha + \theta)$

$= r(\cos\alpha \cdot \cos\theta - \sin\alpha\sin\theta)$

$$\boxed{x' = x\cos\theta - y\sin\theta}$$

$y' = r(\sin(\alpha + \theta))$

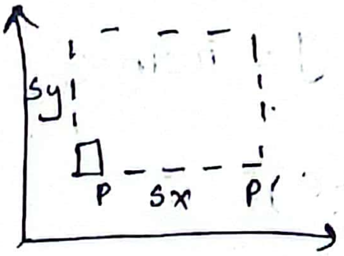
$= r(\sin\alpha\cos\theta + \cos\alpha\sin\theta)$

$\cancel{= y\sin\theta + x\cos\theta} \quad = y\cos\theta + x\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling



$$x' = x * s_x$$

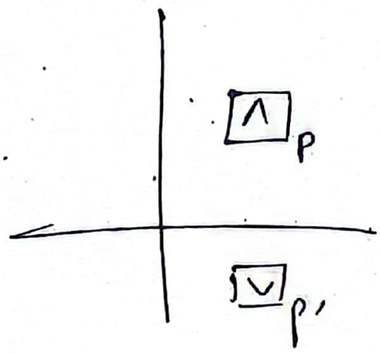
$$y' = y * s_y$$

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

composite transformation matrix

Mirror Reflection

$$P' = M(P)$$



X axis MR

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

6

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

3X3 Rotation matrix

3X3 translation

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

why use 3X3 matrices? p-32

→ matrix Concatenation Benifits?

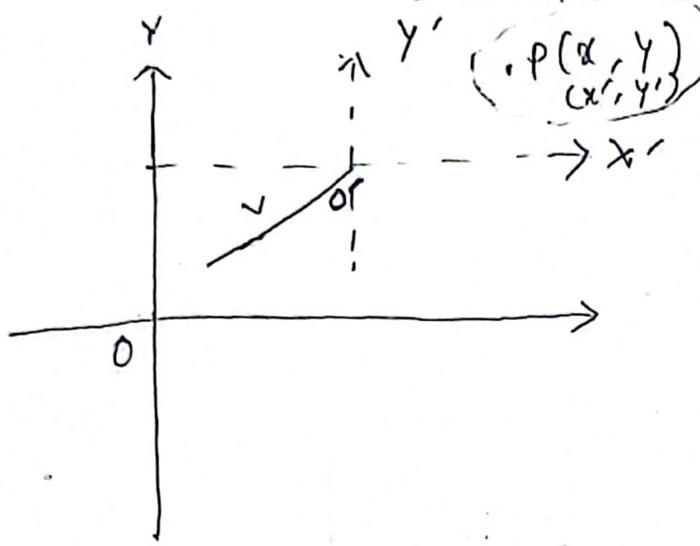
P(1-33)

Shearing Not important

* Wizard of Oz

cas

4.3.24



$\boxed{\text{Coordinate}}$ Transformation

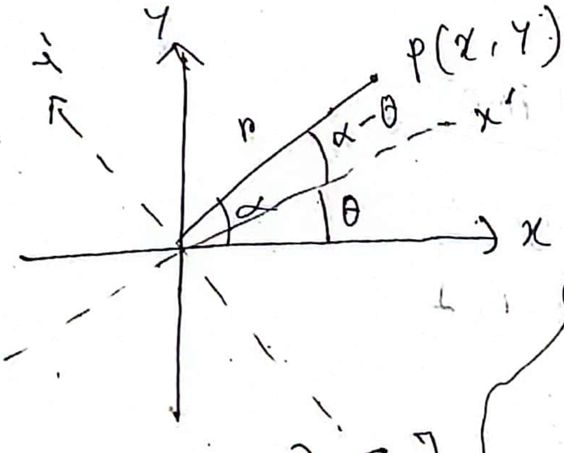
(i) Translation

$$\boxed{\begin{aligned} \vec{v} &= h\hat{i} + k\hat{j} \\ x' &= x - h \\ y' &= y - k \end{aligned}}$$

Matrix representation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(ii) Rotation:



$$\boxed{\begin{aligned} x &= r\cos\alpha \\ y &= r\sin\alpha \end{aligned}}$$

$\boxed{x'} = r\cos(\alpha - \theta)$
$= r(\cos\alpha\cos\theta + \sin\alpha\sin\theta)$

$$\boxed{= x\cos\theta + y\sin\theta}$$

$\boxed{y'} = r\sin(\alpha - \theta)$
$= r(\sin\alpha\cos\theta - \cos\alpha\sin\theta)$

$$\boxed{= y\cos\theta - x\sin\theta}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

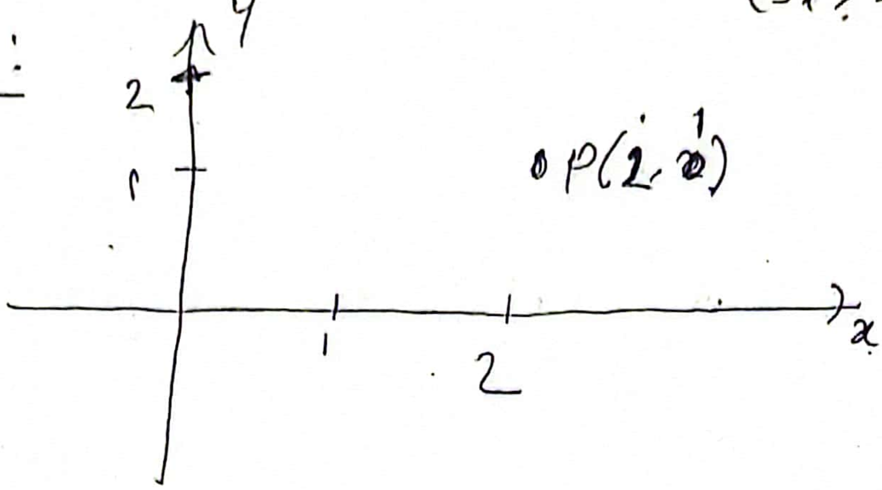
↳ matrix representation

(S_x, S_y)

(iii) Scaling:

$\bar{S}_x = 2$

$\bar{S}_y = 1/2$



Old unit	New unit
S_x	1
1	$1/S_x$

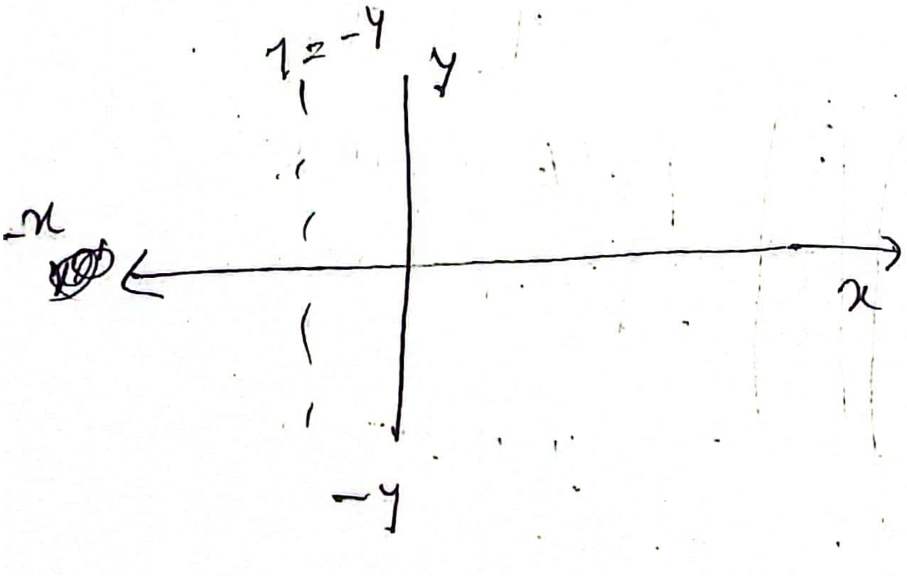
$$\boxed{x' = x \times \frac{1}{S_x}, \quad y' = y \times 1/S_y}$$

matrix: $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{S_x} & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

(iv) Mirror reflection:

$\bar{M}_x = \bar{S}_{1,-1}$

$\bar{M}_y = \bar{S}_{-1,1}$

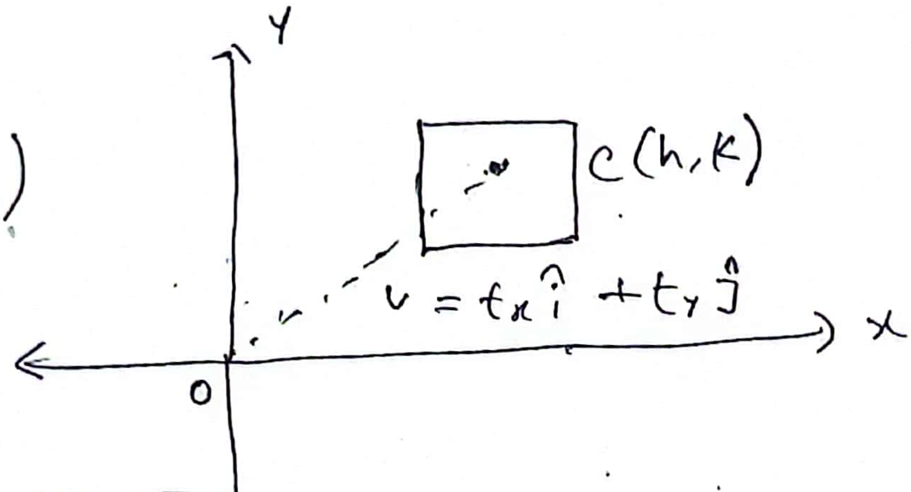


* Diff between geometric and co-ordinate transformation:

Geo	Coord
$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	$\bar{R}_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
$S_{sx,sy} =$	$\bar{S}_{sx,sy} =$

$\frac{CT}{PO1}$

* $M(s_x, s_y)$

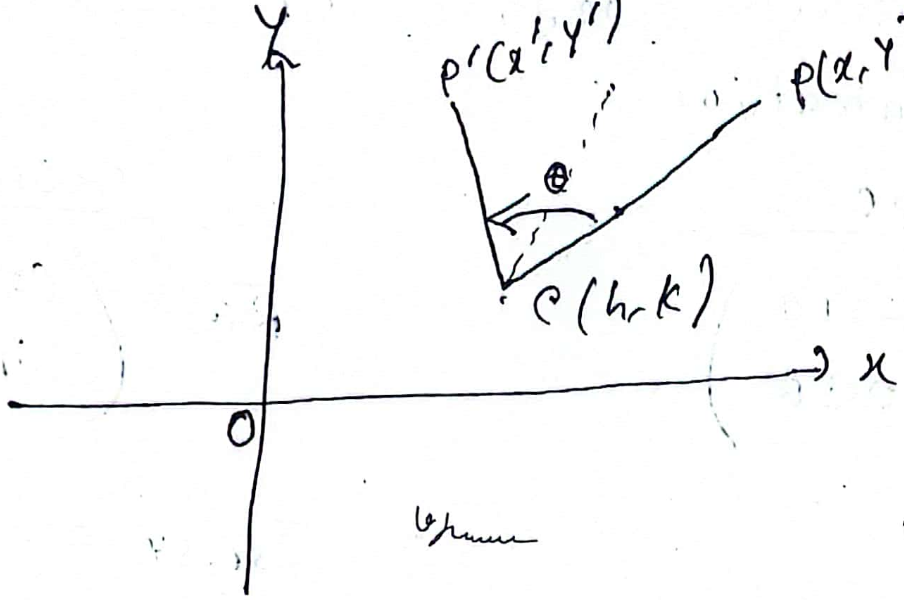


$$\boxed{T_v \cdot S_{sx,sy} \cdot T_{-v}}$$

$$= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

* Composite Transformation Matrix

P02

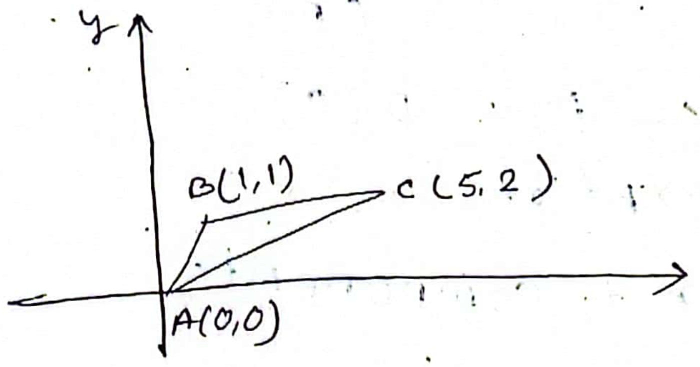


$T_v \cdot R_\theta \cdot T_{-v}$

CGI

Problem Solving

$$ABC = \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$



① 45° rotation about the O(0,0)

② 45° rotation about a fix point P(-1,-1)

Ans:-

$\theta = 45°$

$$CTM = R_\theta = R_{45} = \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[A', B', C']$

$$\cancel{\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix}} = \cancel{CTM} * ABC$$

CTM

(11) (-1, -1)

$V = -\hat{i} - \hat{j}$

$CTM = T_V \cdot R_\theta \cdot T_{-V}$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) Magnify the triangle vertices twice its size keeping c point fix

$S_x = 2$
$S_x = 2$

$CTM = T_V \cdot S \cdot T_{-V}$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Geometric scaling}$$