模型：

$$h(x) = wx + b$$

代价函数：

$$J = \frac{1}{n}\sum_{i=1}^{n}(y_i - h(x_i))^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - wx_i - b)^2$$

J 分别对 w 和 b 求偏导：

$$\frac{\partial J}{\partial w} = -\frac{2}{n}\sum_{i=1}^{n}\left((y_i - wx_i - b)x_i\right) = -\frac{2}{n}\sum_{i=1}^{n}(x_iy_i - wx_i^2 - bx_i) = 0$$

$$\frac{\partial J}{\partial b} = -\frac{2}{n}\sum_{i=1}^{n}(y_i - wx_i - b) = -\frac{2}{n}\left(\sum_{i=1}^{n}y_i - \sum_{i=1}^{n}b - \sum_{i=1}^{n}wx_i\right) = -2(\bar{y} - b - w\bar{x}) = 0$$

解得

$$b = \bar{y} - w\bar{x}$$

带回原式，有

$$\sum_{i=1}^{n}(x_iy_i - wx_i^2 - \bar{y}x_i + w\bar{x}x_i) = 0$$

$$\sum_{i=1}^{n}x_iy_i - w\sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}\bar{y}x_i + w\sum_{i=1}^{n}\bar{x}x_i = 0$$

因为

$$\sum_{i=1}^{n}x_i = n\bar{x}$$

所以

$$\sum_{i=1}^{n}x_iy_i - w\sum_{i=1}^{n}x_i^2 - n\bar{x}\bar{y} + nw\bar{x}^2 = 0$$

所以

$$w = \frac{\sum_1^n x_iy_i - n\bar{x}\bar{y}}{\sum_1^n x_i^2 - n\bar{x}^2}$$

因为

$$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n}(x_iy_i - \bar{x}y_i - x_i\bar{y} + \bar{x}\bar{y}) = \sum_{i=1}^{n}x_iy_i - n\bar{x}\bar{y}$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum_{i=1}^{n}x_i^2 - n\bar{x}^2$$

所以

$$w = \frac{\sum_1^n\left((x_i - \bar{x})(y_i - \bar{y})\right)}{\sum_1^n(x_i - \bar{x})^2}$$

$$b = \bar{y} - \left(\sum_{i=1}^{n}x_i^2 - 2n\bar{x}^2 + n\bar{x}^2\right)\bar{x}$$

代价函数

$$J = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - h(x_i)\right)^2 = \frac{1}{n}\left(\left(y_1 - h(x_1)\right)^2 + \left(y_2 - h(x_2)\right)^2 + \cdots + \left(y_n - h(x_n)\right)^2\right)$$

$$h(X) = \left(h(x_1), h(x_2), \ldots, h(x_n)\right)$$

$$Y - h(X) = \left(y_1 - h(x_1), y_2 - h(x_2), \ldots, y_n - h(x_n)\right)$$

$$\left(Y - h(X)\right)^T\left(Y - h(X)\right) = \left(y_1 - h(x_1)\right)^2 + \left(y_2 - h(x_2)\right)^2 + \cdots + \left(y_n - h(x_n)\right)^2$$

所以

$$J = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - h(x_i)\right)^2 = \frac{1}{n}\left(Y - h(X)\right)^T\left(Y - h(X)\right)$$

设参数向量为 W,则

$$h(X) = WX$$

$$J = \frac{1}{n}(Y - WX)^T(Y - WX)$$

对 W 求导

$$\frac{\partial J}{\partial W} = \frac{2}{n}X^T(Y - WX) = 0$$

$$X^T Y = X^T X W$$

$$W = (X^T X)^{-1}X^T Y$$

$$J = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - h(x_i)\right)^2$$

$$h(x) = w_0 + w_1 x_1 + w_2 x_2$$

$$h(x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1 x_2 + w_4 x_1^2 + w_5 x_2^2$$

$$z = [z_1, z_2, z_3, z_4, z_5] = [x_1, x_2, x_1 x_2, x_1^2, x_2^2]$$

$$h(x) = w_0 + w_1 z_1 + w_2 z_2 + w_3 z_3 + w_4 z_4 + w_5 z_5$$

$$h(x) = w_0 + w_1 x_1 + w_2 x_1^2$$
$$h(x) = w_0 + w_1 z_1 + w_2 z_2$$