模型：

$$\mathrm{h}(x) = wx + b$$

代价函数：

$$J = \frac{1}{n}\sum_{i=1}^{n}(y_i - h(x_i))^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - wx_i - b)^2$$

J 分别对 w 和 b 求偏导：

$$\frac{\partial \mathrm{J}}{\partial \mathrm{w}} = -\frac{2}{n}\sum_{i=1}^{n}\big((y_i - wx_i - b)x_i\big) = -\frac{2}{n}\sum_{i=1}^{n}(x_i y_i - w{x_i}^2 - bx_i) = 0$$

$$\frac{\partial \mathrm{J}}{\partial \mathrm{b}} = -\frac{2}{n}\sum_{i=1}^{n}(y_i - wx_i - b) = -\frac{2}{n}\left(\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} b - \sum_{i=1}^{n} wx_i\right) = -2(\bar{y} - b - w\bar{x}) = 0$$

解得

$$\mathrm{b} = \bar{y} - w\bar{x}$$

带回原式，有

$$\sum_{i=1}^{n}(x_i y_i - w{x_i}^2 - \bar{y}x_i + w\bar{x}x_i) = 0$$

$$\sum_{i=1}^{n} x_i y_i - w\sum_{i=1}^{n} {x_i}^2 - \sum_{i=1}^{n} \bar{y}x_i + w\sum_{i=1}^{n} \bar{x}x_i = 0$$

因为

$$\sum_{i=1}^{n} x_i = n\bar{x}$$

所以

$$\sum_{i=1}^{n} x_i y_i - w\sum_{i=1}^{n} {x_i}^2 - n\bar{x}\bar{y} + nw\bar{x}^2 = 0$$

所以

$$\mathrm{w} = \frac{\sum_1^n x_i y_i - n\bar{x}\bar{y}}{\sum_1^n {x_i}^2 - n\bar{x}^2}$$

因为

$$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n}(x_i y_i - \bar{x}y_i - x_i\bar{y} + \bar{x}\bar{y}) = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} {x_i}^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum_{i=1}^{n} {x_i}^2 - n\bar{x}^2$$

所以

$$\mathrm{w} = \frac{\sum_1^n\big((x_i - \bar{x})(y_i - \bar{y})\big)}{\sum_1^n (x_i - \bar{x})^2}$$

$$b = \bar{y} - \left(\sum_{i=1}^{n} {x_i}^2 - 2n\bar{x}^2 + n\bar{x}^2\right)\bar{x}$$

代价函数

$$J=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-h(x_i)\right)^2=\frac{1}{n}\left(\left(y_1-h(x_1)\right)^2+\left(y_2-h(x_2)\right)^2+\cdots+\left(y_n-h(x_n)\right)^2\right)$$

$$h(X)=\left(h(x_1),h(x_2),\dots,h(x_n)\right)$$

$$Y-h(X)=\left(y_1-h(x_1),y_2-h(x_2),\dots,y_n-h(x_n)\right)$$

$$\left(Y-h(X)\right)^T\left(Y-h(X)\right)=\left(y_1-h(x_1)\right)^2+\left(y_2-h(x_2)\right)^2+\cdots+\left(y_n-h(x_n)\right)^2$$

所以

$$\mathrm{J}=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-h(x_i)\right)^2=\frac{1}{n}\left(Y-h(X)\right)^T\left(Y-h(X)\right)$$

设参数向量为 W,则

$$\mathrm{h}(X)=WX$$

$$\mathrm{J}=\frac{1}{n}(Y-WX)^T(Y-WX)$$

对 W 求导

$$\frac{\partial \mathrm{J}}{\partial \mathrm{W}}=\frac{2}{n}X^T(Y-WX)=0$$

$$X^TY=X^TXW$$

$$W=(X^TX)^{-1}X^TY$$

$$J=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-h(x_i)\right)^2$$

$$\mathrm{h}(x)=w_0+w_1x_1+w_2x_2$$

$$\mathrm{h}(x)=w_0+w_1x_1+w_2x_2+w_3x_1x_2+w_4x_1^2+w_5x_2^2$$

$$z=[z_1,z_2,z_3,z_4,z_5]=[x_1,x_2,x_1x_2,x_1^2,x_2^2]$$

$$\mathrm{h}(x)=w_0+w_1z_1+w_2z_2+w_3z_3+w_4z_4+w_5z_5$$

$$\mathrm{h}(x)=w_0+w_1x_1+w_2x_1^2$$

$$\mathrm{h}(x)=w_0+w_1z_1+w_2z_2$$

$$\mathrm{h}(x) = 119.6 - 54x + 28.5x^2$$

$$\mathrm{h}(x) = -22.8 + 116.99x$$

$$\mathrm{g(x)} = \frac{1}{1 + e^{-x}}$$

在此处键入公式。

h(x)= $W^T X$

h(x)= $g(W^T X)$

h(x)= $\frac{1}{1+e^{-W^T X}}$

$$h(x) = P(y = 1|x; W)$$

$$\mathrm{Cost}\left(\mathrm{y_i}, \mathrm{h}(\mathrm{x_i})\right) = (\mathrm{y_i} - \mathrm{h}(\mathrm{x_i}))^2$$

$$J = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Cost}\left(\mathrm{y_i}, \mathrm{h}(\mathrm{x_i})\right)$$

$$\mathrm{Cost}\left(\mathrm{y}, \mathrm{h(x)}\right) = -\mathrm{ylog}\left(\mathrm{h(x)}\right) - (1 - \mathrm{y})\log\left(1 - \mathrm{h(x)}\right)$$

$$J = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Cost}\left(y_i, \mathrm{h}(x_i)\right) = -\frac{1}{n}\sum_{i=1}^{n} \left(y_i \log\left(\mathrm{h}(x_i)\right) + (1 - y_i)\log\left(1 - \mathrm{h}(x_i)\right)\right)$$

$$P(y = 1|x; W) = h_w(x)$$

$$P(y = 0|x; W) = 1 - h_w(x)$$

$$P(y|x; W) = (h_w(x))^y (1 - h_w(x))^{1-y}$$

$$L(W) = \prod_{i=1}^{n} P(y_i|x_i; W) = \prod_{i=1}^{n} (h_w(x_i))^{y_i} (1 - h_w(x_i))^{1-y_i}$$

$$l(w) = \frac{1}{n}\sum_{i=1}^{n} (y_i \log\left(h_w(x_i)\right) + (1 - y_i)\log\left((1 - h_w(x_i))\right))$$

$$l(w) = -\frac{1}{n}\sum_{i=1}^{n}(y_i log\,(\frac{1}{1+e^{-W^T x_i}}) + (1-y_i)log\,(1-\frac{1}{1+e^{-W^T x_i}}))$$