Performance Analysis of RIS-assisted Cellular Networks Based on Matérn Cluster Processes[1]

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[1] https://arxiv.org/abs/2310.06754

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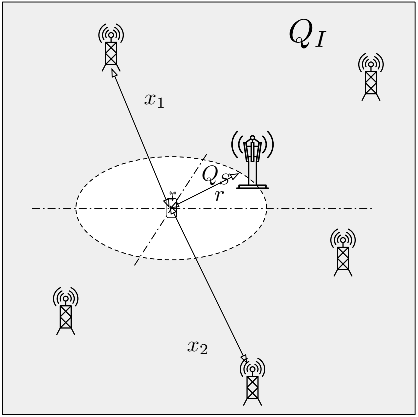


If all BSs send signals s_i, the UE receives

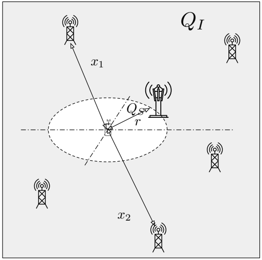
$$\bar{s}_o = \rho_o \sqrt{g(r)} s_o + \sum_{\mathbf{x}_i \in \phi \setminus \mathbf{x}_o} \rho_i \sqrt{g(x_i)} s_i + w,$$

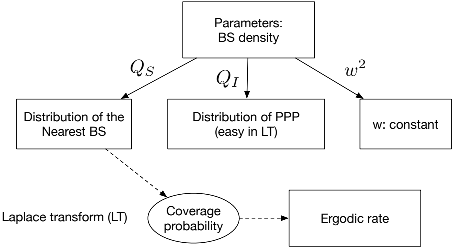
where

- $\mathbf{x} \in \phi$: point process for the location of BSs
- $r = \|\mathbf{x}_o\|$: distance to the nearest BS
- $x_i = \|\mathbf{x}_i\|$: distances to interfering BSs
- ρ: Rayleigh fading
- $g(\cdot)$: pathloss function
- $w \sim \mathcal{CN}(0, w^2)$: additional Gaussian noise
- Q_S: power of signal
- Q_I: power of interference

SINR: $\frac{Q_S}{Q_I + w^2}$
Coverage probability:

$$\mathbb{P}\left[\frac{Q_S}{Q_I + w^2} \geq T\right] = \mathbb{P}[Q_S \geq T(Q_I + w^2)]$$





Signal Q_S: used as a leverage

- Rayleigh fading: $|\rho_o|^2 \sim \mathrm{Exp}(1)$
- Signal attenuation: $g(r)$
- Signal power $\mathbb{E}[s^2] = P_0$

Interference Q_I: characterized by the Laplace transform:

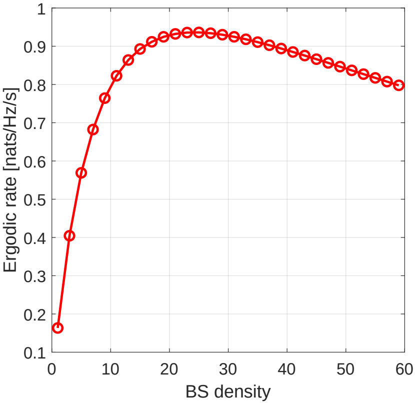
$$\mathcal{L}_{Q_I}(s) = \exp\left(-2\pi\lambda_{\mathrm{BS}} \int_r^{\infty} (1 - \mathcal{L}_{|\rho_i|^2}(sP_0 g(x)))\mathrm{d}x \right)$$

where $\mathcal{L}_{|\rho_i|^2}(sP_0 g(x)) = \frac{1}{1+sP_0 g(x)}$

Trick

$$\mathbb{P}\left[|\rho_{D_o}|^2 \geq \frac{T(I+w^2)}{P_0 g(r)} \middle| r \right] \overset{|\rho_{D_o}|^2 \sim \mathrm{Exp}(1)}{=} \int_0^{\infty} f_{T(I+w^2)}(x) e^{-\frac{x}{P_0 g(r)}} \mathrm{d}x = \mathcal{L}_{TI+w^2}\left(\frac{1}{P_0 g(r)} \right).$$

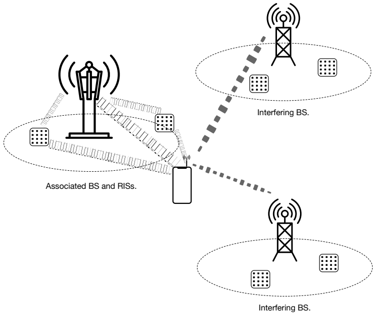
Performance of BS densification[2]

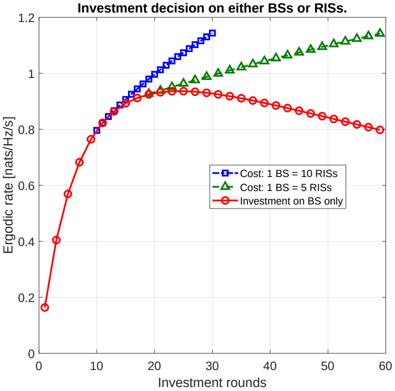


Analytical expression can compute the ergodic rate from the BS density

- Densification of BSs will have negative return
- UE is closer to a BS, thus better signal strength
- But it also suffers from a higher interference level

[2] AlAmmouri, Ahmad, Jeffrey G. Andrews, and François Baccelli. "SINR and throughput of dense cellular networks with stretched exponential path loss." IEEE Transactions on Wireless Communications 17.2 (2017): 1147-1160.





Since RISs are controlled by BSs, we model RISs as a Matérn Cluster Process (MCP)

- RISs can assist deployed BSs
- RIS deployment is cost- and energy-efficient

Problem Formulation

Section 2 What is a stochastic geometry model for the distributed RISs?

- How to characterize the interplay between RISs and existing nodes (BSs and UEs)?
- How is the signal processed?

Section 3 What are the difficulties associated with this model?

Section 4 What are the overall advantages of large-scale deployment of RISs?

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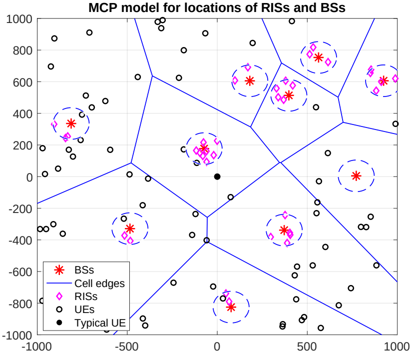
Modelling RISs as Matérn cluster processes



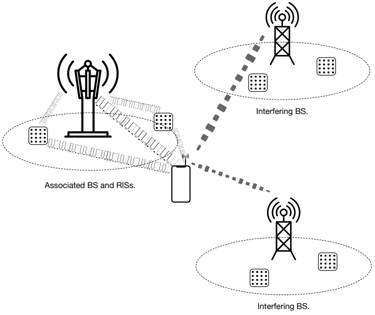
Poisson point process (PPP) $\phi_{\mathrm{BS}} \triangleq \mathbf{x}_i$
Matérn cluster process (MCP) $\Phi_{\mathrm{RIS}} \triangleq \mathbf{y}_{i,j} \in \cup \phi_i$

The cluster of RIS ϕ_i is a PPP defined on the ring $\mathbb{D}_{\mathbf{x}_i}(R_{\mathrm{in}}, R_{\mathrm{out}})$ with density λ_{RIS}.

- Each RIS has M reflective elements
- Each RIS allocates a batch $M_o \leq M$ to perform beamforming to the typical UE
- RISs serve UEs in the same Voronois cell

Possible extensions:

- MIMO
- Probabilistic blockage



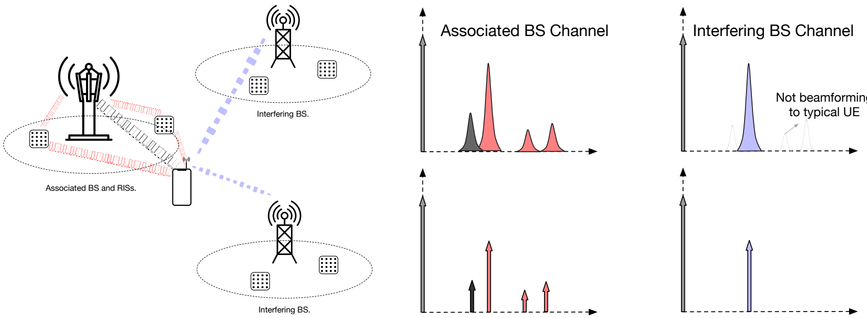
- UE is associated with the nearest BS and the RISs in its cluster
- RISs not in the associated cluster
  - Do not beamform to the typical UE
  - Probability of interfering beam overlap the typical UE is low
- RISs are passive preconfigured device
  - Sidelobes of reflected interference are considered in the fading
  - More energy-efficient than BSs

All signals pass through the composite channels $h_o[n]$ and $h_i[n]$ so that the UE receives

$$\bar{s}[n] = h_o \circledast s_o[n] + \sum_{i \neq o} \Big(h_i \circledast s_i[n] \Big) + w,$$

for $n \in [N_s]$, where $\circledast$ is the circular convolution operation. Here, $w \in \mathcal{CN}(0, w^2)$ represents the noise (AWGN).

Model the channel as tapped delay line filters



Channel for signaling

$$h_o[n] = \rho_{D_o}\sqrt{g(\|\mathbf{x}_o\|)}\delta[n-n_{D_o}] + \sum_{\mathbf{y}_{o,j}\in\phi_o} \alpha_{R,o,j}\sqrt{g(\|\mathbf{x}_o-\mathbf{y}_{o,j}\|)g(\|\mathbf{y}_{o,j}\|)}\delta[n-n_{R,o,j}]$$

where n_{D_i}, $n_{R,i,j}$ denote the time delays, α is the amplitude of the fading of the reflected beams.

Channel for interference

$$h_i[n] = \rho_{D_i}\sqrt{g(\|\mathbf{x}_i\|)}\delta[n-n_{D_i}]$$

OFDM Signal Processing

OFDM system applies matched filter in frequency domain

$$\bar{s}_o[n] = h_o[n] \circledast s_o[n] \xrightarrow{\text{DFT}} \tilde{\bar{\mathbf{s}}}_o[k] = \tilde{h}_o[k] \cdot \tilde{s}_o[k],$$

where $\tilde{h}_o[k]$ denotes the channel gain of the k^{th} subcarrier.

Post-processed signal level

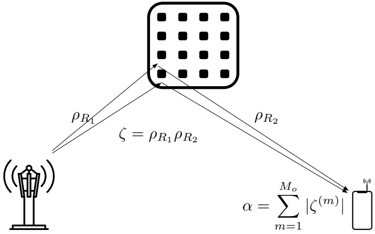
Thanks to Parseval's theorem, the power of the effective channel gain averaging over the subcarriers is the sum of power of taps

$$\left|\hat{h}_o\right|^2 = \left|\rho_{D_o}\right|^2 g(\|\mathbf{x}_i\|) + \sum_{j \in \mathcal{J}_o} \left|\alpha_{R,o,j}\right|^2 g(\|\mathbf{x}_o - \mathbf{y}_{o,j}\|) g(\|\mathbf{y}_{o,j}\|).$$

Post-processed Interference level

$$\sum_{i \in \mathcal{I} \backslash o} \left|\hat{h}_i\right|^2 = \sum_{i \in \mathcal{I} \backslash o} \left|\rho_{D_i}\right|^2 g(\|\mathbf{x}_i\|).$$

Fading for the reflected beams



Each RIS uses M_o elements to beamform, i.e., in-phase signal superposition. As a result, α is the sum of M_o i.i.d. $|\zeta| = |\rho_{R_1}\rho_{R_2}|$:

$$\alpha = \sum_{m=1}^{M_o} \left|\zeta^{(m)}\right|.$$

Since M_o is large, we apply the central limit theorem and approximate

$$\alpha \approx \mathcal{N}\left(M_o \mathbb{E}[|\zeta|], M_o \mathbb{V}[|\zeta|]\right)$$

.

Laplace transform of reflected signals

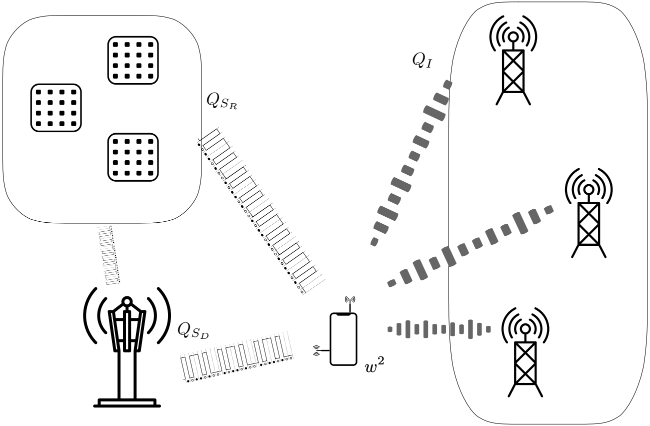
The Laplace transform of the aggregated power of the reflected signals

$$\mathcal{L}_{Q_{S_R}}(s) = \exp\left(-\lambda_{\mathrm{RIS}} \int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} \int_0^{2\pi} y\left(1 - \mathcal{L}_{|\alpha|^2}\left[-sP_0 G(r, y, \psi)\right]\right) \mathrm{d}\psi \mathrm{d}y\right),$$

where $G(r, y, \psi) = g(\|\mathbf{x}_o\|) g(\|\mathbf{x}_o - \mathbf{y}_{o,j}\|)$. For arbitrary beam,

$$\mathcal{L}_{|\alpha|^2}(s) = \frac{e^{-\frac{\mu^2 s}{1+2s\sigma^2}}}{(1+2s\sigma^2)^{1/2}}, \quad s > -\frac{1}{2\sigma^2}.$$

Takeaways from Modeling



Reflected signals Q_{S_R} from many RISs in the cluster are another stochastic geometry element

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Coverage probability and Manipulation of components

The SINR of the typical UE is larger than a target threshold $T > 0$, namely

$$\mathrm{P}_c(T) \triangleq \mathbb{P}(\mathrm{SINR} \geq T) = \mathbb{E}_r[\mathbb{P}(\mathrm{SINR} \geq T|r)] = \mathbb{E}_r\left[\mathbb{P}\left(\frac{Q_{S_D}(r) + Q_{S_R}(r)}{Q_I(r) + w^2} \geq T\middle|r\right)\right].$$

By manipulating the components,

$$\mathbb{P}\left[\frac{Q_{S_D}(r) + Q_{S_R}(r)}{Q_I(r) + w^2} \geq T\middle|r\right] = \mathbb{P}\big[Q_{S_D}(r) \geq TQ_I(r) + Tw^2 - Q_{S_R}(r)|r\big].$$

Introducing the random variable

For computing $\mathbb{P}\big[Q_{S_D}(r) \geq \Upsilon|r\big]$

$$\Upsilon = TQ_I(r) + Tw^2 - Q_{S_R}(r).$$

Laplace transform of Υ

Since Q_I and Q_{S_R} are built from independent random point processes, we have[3]

$$\Upsilon = TQ_I(r) + Tw^2 - Q_{S_R}(r) \Rightarrow \mathcal{B}_\Upsilon(s) = \mathcal{B}_{TQ_{I(r)}}(s)\mathcal{B}_{Tw^2}(s)\mathcal{B}_{-Q_{S_R}(r)}(s)$$

$\mathcal{B}_{TQ_{I(r)}}(s)$:

- LT of aggregated interference from PPP

$$e^{\left(-\lambda_{\mathrm{BS}} \int (1-\mathcal{L}_{|\rho|^2}(s)\mathrm{d}x\right)}$$

- LT of the Rayleigh fading

$$\mathcal{L}_{|\rho|^2}(s) = \frac{1}{1+s}$$

$\mathcal{B}_{Tw^2}(s)$

- The power of w^2 is constant, its LT is

$$e^{-sTw^2}$$

$\mathcal{B}_{-Q_{S_R}(r)}(s)$

- LT of aggregated signals from PPP

$$e^{\left(-\lambda_{\mathrm{RIS}} \int_{\mathbb{D}} (1-\mathcal{L}_{|\alpha|^2}(s)\mathrm{d}x\right)}$$

- LT of the beamformed fading

$$\mathcal{L}_{|\alpha|^2}(s) = \frac{e^{-\frac{\mu^2 s}{1+2s\sigma^2}}}{(1+2s\sigma^2)^{1/2}}$$

[3] We denote the bilateral Laplace transfrom as $\mathcal{B}(s)$ and unilateral Laplace transfrom as $\mathcal{L}(s)$

The coverage probability can be expressed in terms of a Laplace transform,

$$\mathbb{P}\left[|\rho_{D_o}|^2 P_0 g(r) \geq \Upsilon \middle| r\right] = \mathcal{L}_{\Upsilon^+}\left(\frac{1}{P_0 g(r)}\right) + \mathbb{P}\left[\Upsilon < 0\right],$$

Proof.

$$\mathbb{P}\left[|\rho_{D_o}|^2 \geq \frac{\Upsilon}{P_0 g(r)} \middle| r\right] \overset{|\rho_{D_o}|^2 \sim \mathrm{Exp}(1)}{=} \int_0^{\infty} f_\Upsilon(x) e^{-\frac{x}{P_0 g(r)}} \mathrm{d}x + \int_{-\infty}^{0} f_\Upsilon(x) \mathrm{d}x.$$

$\square$

Note: only the positive part of $\Upsilon^+ \triangleq \max(0, \Upsilon)$.

In the signal .vs interference case, the power of interference is always positive. The case with reflected signals from RISs defines $\Upsilon \in \mathbb{R}$. Nevertheless, coverage probability only needs $\mathcal{L}_{\Upsilon^+}(s)$.

Expressing $\mathcal{L}_{\Upsilon^+}(s)$ in function of $\mathcal{B}_{\Upsilon}(s)$

Lemma 1

The Laplace transform of the positive part of a random variable can be derived from its bilateral Laplace transform via the formula

$$\mathcal{L}_{\Upsilon^+}(s) = \frac{1}{2\pi\imath}\int_{-\infty}^{\infty}\frac{\mathcal{B}_{\Upsilon}(s-\imath u)-\mathcal{B}_{\Upsilon}(-\imath u)}{u}\mathrm{d}u + \frac{1}{2}\Big(1+\mathcal{B}_{\Upsilon}(s)\Big) - \mathcal{B}^{-1}_{\mathcal{B}_{\Upsilon}(s)/s}(0),$$

where $\int_{-\infty}^{\infty}\frac{\mathrm{d}u}{u}$ is understood in the sense of Cauchy principal-value[a] and $\mathcal{B}^{-1}(0)$ denotes the inverse Laplace transform evaluated at 0.

[a] $\int_{-\infty}^{\infty} = \lim_{\epsilon\downarrow 0^+}\int_{-\infty}^{-\epsilon} + \int_{\epsilon}^{\infty}$.

Coverage probability

The coverage probability for the communication threshold T is

$$\mathrm{P}_c(T) = \int_0^\infty \left(\mathcal{L}_{\Upsilon^+}\left(\frac{1}{P_0 g(r)}\right) + \mathbb{P}(\Upsilon < 0) \right) \underbrace{2\pi r \lambda_{\mathrm{BS}} e^{-\pi \lambda_{\mathrm{BS}} r^2}}_{(a)} \mathrm{d}r,$$

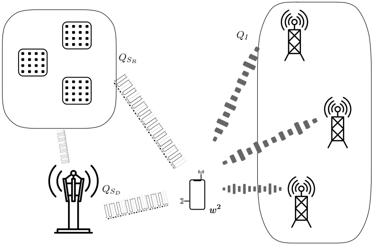
where (a) is the distribution of the distance to the associated BS.

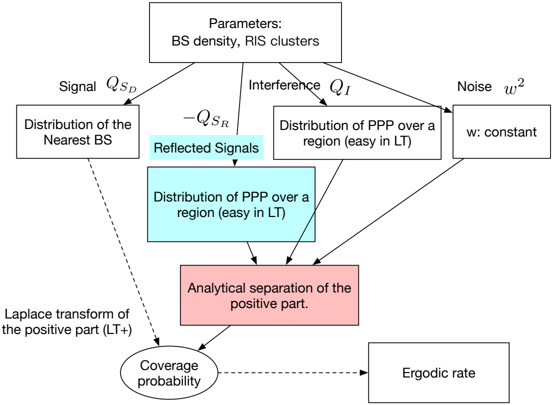
Ergodic rate

The ergodic rate is a function based on the distribution of the coverage probability $\mathrm{P}_c(T)$

$$\tau \triangleq \mathbb{E}[\log(1 + \mathrm{SINR})] = \int_0^\infty \frac{\mathrm{P}_c(t)}{t+1} \mathrm{d}t.$$

Analytical Takeaways





An analytical framework in stochastic geometry to analyze the role of additional signals sources.

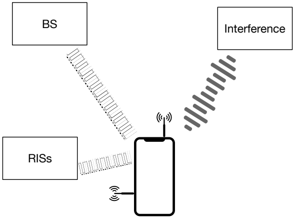
A receiver with multiple antenna N_r can apply maximal ratio combining (MRC) w.r.t. the channel information from the BS.

Direct signal:

- Fading power follows the Gamma distribution
- Coverage probability is the Laplace transform and its higher order derivatives

Reflected signals:

- Scale with N_r

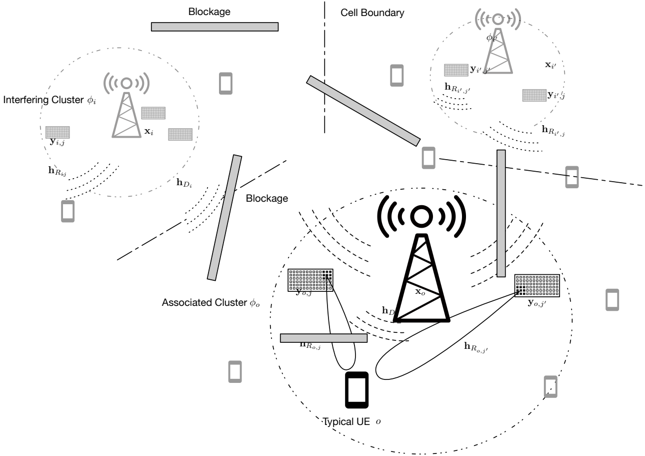


Interference

- Power of fading of the interference follows Exponential distribution

Extension: Blockage

Consider a simplified blockage model where each link can be blocked with a probability by flipping a coin



q_D: the blockage probability for direct links; q_R: the blockage probability for reflected links.

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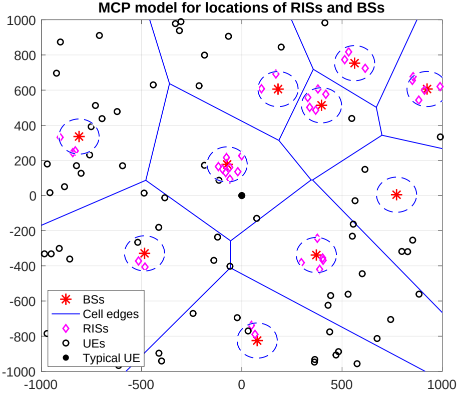
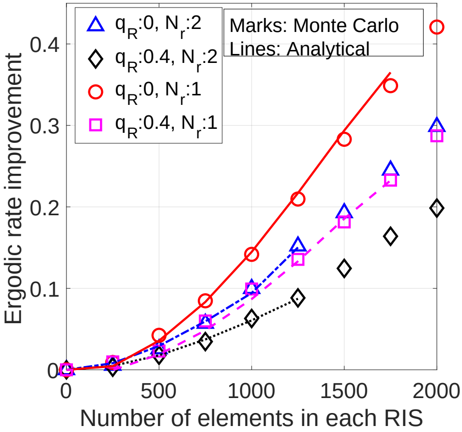


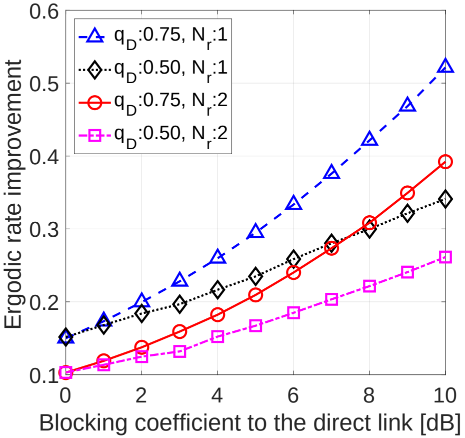
- Density of BSs: $10/\text{km}^2$
- Cluster ring
  - Inner radius: 10 meters
  - Outer radius: 25 meters
- Average number of RISs: 5/Voronoi cell
- Average number of UEs: 5/Voronoi cell
- Number of the receiver antenna $N_r \in \{1, 2\}$

Evaluating the Ergodic Rate for the typical UE

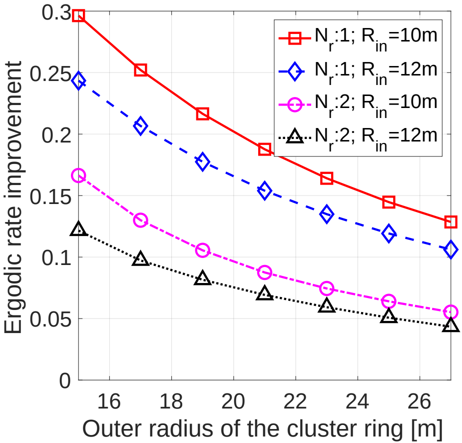


- Blockage probability for the reflected links $q_R \in \{0, 0.4\}$
- Number of receiver antennas $N_r \in \{1, 2\}$
- Ergodic rate can increase by about 45% when
  - reflected links are not blocked
  - each RIS employs 400 elements

Assessing the impact of the blockage on the ergodic rate

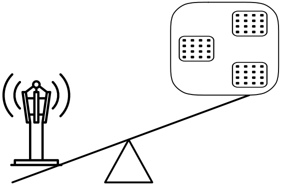


- Setup
  - Number of RIS element: 1000
  - Blocking probability for reflected links $q_R = 0$
  - Blocking probability for direct links $q_D \in \{0.5, 0.75\}$
- The *x*-axis is the constant blocking penalty C_b ranging from 0 to 10 dB
- The improvement of the ergodic rate caused by the RISs increases when the blocking coefficient is larger
- The scenario where RISs play a significant role when the direct link is severely blocked and the reflected signals provide an essential multipath diversity



- When the inner radius is set, the relative gains always decrease when the outer radius increases, implying that a smaller outer radius of the cluster is favorable
- Comparing the line sets for the inner radius being 10m against 12m, the curves show that a smaller inner radius always outperforms a larger radius, suggesting the superiority of a smaller inner radius
- We conclude that the RISs should be deployed as close to the BSs as possible

Which is the best investment strategy for operators: BSs or RISs?

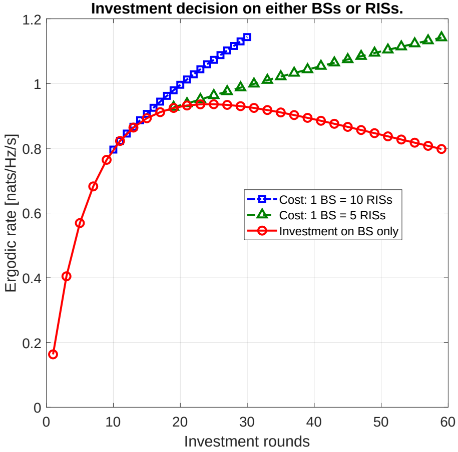


τ: ergodic rate as the return of investment (RoI) metric
$\bar{C}$: total cost of ownership (TCO) considering both CapEx and OpEx

Given $\bar{C}_{\mathrm{BS}}, \bar{C}_{\mathrm{RIS}}$.

while investing **do**
  Get $\lambda_{\mathrm{BS}}, \lambda_{\mathrm{RIS}}$
  Sensitivity of τ w.r.t. λ_{BS}: $E_{\mathrm{BS}} \triangleq \frac{\partial \tau}{\partial \lambda_{\mathrm{BS}}}$
  Sensitivity of τ w.r.t. λ_{RIS}: $E_{\mathrm{RIS}} \triangleq \frac{\partial \tau}{\partial \lambda_{\mathrm{RIS}}}$
  if $\frac{E_{\mathrm{BS}}}{\bar{C}_{\mathrm{BS}}} \geq \frac{E_{\mathrm{RIS}}}{\bar{C}_{\mathrm{RIS}}}$ **then**
    Allocating the newly investment to BS
  else
    Allocating to additional RIS
  end if
end while

Discussion on informed decisions



Strategy:

- Start with investing on BSs
- Network scenarios and the costs
- When the performance gain thanks to RIS can be achieved in a cheaper way, switch to investing on RISs

Discussion on the policy:

+ Sequential decision-making process is easy to understand and implement
+ Decision is made both qualitatively and quantitatively
- They may not capture all uncertainty in decision-making

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Key Points

RIS technology proves to be a cost-effective and energy-efficient solution that can bring significant system-level advantages, when

- direct links are blocked
- RISs are cheap compared with BSs
- BSs introduce too much interference

Analytical expressions are efficient and powerful in techno-economic prediction