UPPSALA UNIVERSITET
DEPARTMENT OF MATHEMATICS
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Modeling Complex Systems (1MA256)
Spring 2023

LAB 2: Greenberg-Hasting's CA (GHCA)

The deadline for this assignment is April 30th.

Please submit by email, upload to Studium, GitHub or any other "box", if the files/data are too large (provide a link). All code should be included. Feel free to submit videos illustrating your results where appropriate, via Studium or uploaded elsewhere such as vimeo or youtube. You may work in groups of size 1-7, and only one group member needs to submit the assignment. State clearly the members of the group.

Task 1: Write a code which

1) Randomly generates some initial configuration for GHCA, with some prescribed N and e, on an $n \times m$ grid. Or/and reads values for the cells in the initial configuration from a file.
2) Advances the configuration by an arbitrary number of steps, and saves the result in a file.
3) Draws the resulting configuration in color.

Question 2: Now, GHCA became a map acting on a finite set of configurations. What is the cardinality of this set (=number of all possible configurations)?

Task 3: Any initial configuration $\mathcal{C}$ on a finite grid is pre-periodic (or "eventually periodic"). That is, after some transient time k, we have that the configuration $F^k(\mathcal{C})$ repeats with some period m:

$$F^{m+k}(\mathcal{C}) = F^k(\mathcal{C}).$$

Write a code which finds the eventual period for any initial configuration.

Task 4: Pick some periodic configuration $\mathcal{C}_p$, and study if it is attracting or repelling experimentally: take another configuration $\mathcal{C}$, close to $\mathcal{C}_p$ (for example it may differ from $\mathcal{C}_p$ only a several cells), iterate it, and see if its orbit eventually begins to coincide with that of $\mathcal{C}_p$ or not.

Task 5: Check experimentally, or analytically, if GHCA is additive, that is $F(\mathcal{C}_1 + \mathcal{C}_2) = F(\mathcal{C}_1) + F(\mathcal{C}_2)$ for any to configurations $\mathcal{C}_k$, $k = 1, 2$. Here, the sum of two configurations may be defined cell-wise, for example, as $v_1^{i,j} + v_2^{i,j} \mod N$, where $v_k^{i,j} \in \{0, 1, \ldots, N-1\}$ are values of (i,j)-th cells in configuration $\mathcal{C}_k$, $k = 1, 2$. (Ex: if $N = 5$, and cell $(3,4)$ in configuration $\mathcal{C}_1$ has value 2, and cell $(3,4)$ in configuration $\mathcal{C}_2$ has value 4, then cell $(3,4)$ in configuration $\mathcal{C}_1 + \mathcal{C}_2$ would have value $2 + 4 \mod 5 = 6 \mod 5 = 1$).