数理逻辑第四次作业

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第 1 题					
总分					
备注	1. 作业提交邮箱：hitsz_logic_2022@163.com。作业提交截止时间：2022-06-20-24:00，超过提交截止时间的作业视为无效。 2. 确因网络等特殊原因无法及时提交作业的学生，应至少提前 1 小时与助教联系沟通（徐朕燃，QQ：1319282215，电话：13713994811 许天骁，QQ：1140931320，电话：18800415868）。 3. 作业文件名命名方式：第 x 次-学号-姓名-x 班（例：第 4 次-180110504-张三-5 班.pdf）；邮件主题为：第 x 次-学号-姓名-x 班（例：第 4 次-180110504-张三-5 班）。缺少这些信息的作业将被酌情扣分。注意作业次数以阿拉伯数字命名。 4. 可手写拍照转为 PDF 格式。				

1. 在 ND 中证明：

(1) $\vdash_{ND} (\neg A \to A) \to A$

(3) $\vdash_{ND} ((A \vee B) \to C) \leftrightarrow (A \to C) \wedge (B \to C)$

(5) $\vdash_{ND} \neg(A \to B) \leftrightarrow A \wedge \neg B$

(7) $\vdash_{ND} (A \wedge B) \leftrightarrow A \wedge (\neg A \vee B)$

(1)

$(1). \neg A \to A, A \vdash_{ND} A \quad (\in)$
$(2). \neg A \to A, \neg A \vdash_{ND} \neg A \quad (\in)$
$(3). \neg A \to A, \neg A \vdash_{ND} \neg A \to A \quad (\in)$
$(4). \neg A \to A, \neg A \vdash_{ND} A \quad (3)(4)(\to -)$
$(5). \neg A \to A \vdash_{ND} A \quad (1)(4)(-)$
$(6). \vdash_{ND} (\neg A \to A) \to A \quad (5)(\to +)$

(2)

(1). $(A \lor B) \to C, A \vdash_{ND} A \quad (\in)$
(2). $(A \lor B) \to C, A \vdash_{ND} (A \lor B) \to C \quad (\in)$
(3). $(A \lor B) \to C, A \vdash_{ND} A \lor B \quad (1)(\lor +)$
(4). $(A \lor B) \to C, A \vdash_{ND} C \quad (2)(3)(\to -)$
(5). $(A \lor B) \to C \vdash_{ND} A \to C \quad (4)(\to +)$
(6). $(A \lor B) \to C, B \vdash_{ND} B \quad (\in)$
(7). $(A \lor B) \to C, B \vdash_{ND} (A \lor B) \to C \quad (\in)$
(8). $(A \lor B) \to C, B \vdash_{ND} A \lor B \quad (6)(\lor +)$
(9). $(A \lor B) \to C, B \vdash_{ND} C \quad (7)(8)(\to -)$
(10). $(A \lor B) \to C \vdash_{ND} B \to C \quad (9)(\to +)$
(11). $(A \lor B) \to C \vdash_{ND} (A \to C) \land (B \to C) \quad (5)(10)(\land +)$
(12). $\vdash_{ND} ((A \lor B) \to C) \to (A \to C) \land (B \to C) \quad (11)(\to +)$
(13). $(A \to C) \land (B \to C), A \lor B, A \vdash_{ND} (A \to C) \land (B \to C) \quad (\in)$
(14). $(A \to C) \land (B \to C), A \lor B, A \vdash_{ND} (A \to C) \quad (13)(\land -)$
(15). $(A \to C) \land (B \to C), A \lor B, A \vdash_{ND} A \quad (\in)$
(16). $(A \to C) \land (B \to C), A \lor B, A \vdash_{ND} C \quad (14)(15)(\to -)$
(17). $(A \to C) \land (B \to C), A \lor B, B \vdash_{ND} (A \to C) \land (B \to C) \quad (\in)$
(18). $(A \to C) \land (B \to C), A \lor B, B \vdash_{ND} (B \to C) \quad (17)(\land -)$
(19). $(A \to C) \land (B \to C), A \lor B, B \vdash_{ND} B \quad (\in)$
(20). $(A \to C) \land (B \to C), A \lor B, B \vdash_{ND} C \quad (18)(19)(\to -)$
(21). $(A \to C) \land (B \to C), A \lor B \vdash_{ND} A \lor B \quad (\in)$
(22). $(A \to C) \land (B \to C), A \lor B \vdash_{ND} C \quad (16)(20)(21)(\lor -)$
(23). $(A \to C) \land (B \to C) \vdash_{ND} (A \lor B) \to C \quad (22)(\to +)$
(24). $\vdash_{ND} (A \to C) \land (B \to C) \to ((A \lor B) \to C) \quad (23)(\to +)$
(25). $\vdash_{ND} ((A \lor B) \to C) \leftrightarrow (A \to C) \land (B \to C) \quad (11)(24)(\leftrightarrow +)$

(3)

$(1). \neg(A \to B), \neg A, A \vdash_{ND} A \quad (\in)$
$(2). \neg(A \to B), \neg A, A \vdash_{ND} \neg A \quad (\in)$
$(3). \neg(A \to B), \neg A, A \vdash_{ND} B \quad (1)(2)(\neg -)$
$(4). \neg(A \to B), \neg A \vdash_{ND} A \to B \quad (3)(\to +)$
$(5). \neg(A \to B), \neg A \vdash_{ND} \neg(A \to B) \quad (\in)$
$(6). \neg(A \to B) \vdash_{ND} \neg\neg A \quad (4)(5)(\neg +)$
$(7). \neg(A \to B) \vdash_{ND} A \quad (6)(\neg\neg -)$
$(8). \neg(A \to B), B, A \vdash_{ND} B \quad (\in)$
$(9). \neg(A \to B), B \vdash_{ND} A \to B \quad (8)(\to +)$
$(10). \neg(A \to B), B \vdash_{ND} \neg(A \to B) \quad (\in)$
$(11). \neg(A \to B) \vdash_{ND} \neg B \quad (9)(10)(\neg +)$
$(12). \neg(A \to B) \vdash_{ND} A \wedge \neg B \quad (7)(11)(\wedge +)$
$(13). \vdash_{ND} \neg(A \to B) \to A \wedge \neg B \quad (12)(\to +)$
$(14). A \wedge \neg B, A \to B \vdash_{ND} A \wedge \neg B \quad (\in)$
$(15). A \wedge \neg B, A \to B \vdash_{ND} \neg B \quad (14)(\wedge -)$
$(16). A \wedge \neg B, A \to B \vdash_{ND} A \quad (14)(\wedge -)$
$(17). A \wedge \neg B, A \to B \vdash_{ND} A \to B \quad (\in)$
$(18). A \wedge \neg B, A \to B \vdash_{ND} B \quad (16)(17)(\to -)$
$(19). A \wedge \neg B \vdash_{ND} \neg(A \to B) \quad (15)(18)(\neg +)$
$(20). \vdash_{ND} A \wedge \neg B \to \neg(A \to B) \quad (19)(\to +)$
$(21). \vdash_{ND} \neg(A \to B) \leftrightarrow A \wedge \neg B \quad (13)(20)(\leftrightarrow +)$

(4)

$(1). A \wedge B \vdash_{ND} A \wedge B \quad (\in)$
$(2). A \wedge B \vdash_{ND} A \quad (1)(\wedge -)$
$(3). A \wedge B \vdash_{ND} B \quad (1)(\wedge -)$
$(4). A \wedge B \vdash_{ND} \neg A \vee B \quad (3)(\vee +)$
$(5). A \wedge B \vdash_{ND} A \wedge (\neg A \vee B) \quad (3)(4)(\wedge +)$
$(6). \vdash_{ND} (A \wedge B) \to A \wedge (\neg A \vee B) \quad (5)(\to +)$
$(7). A \wedge (\neg A \vee B) \vdash_{ND} A \wedge (\neg A \vee B) \quad (\in)$
$(8). A \wedge (\neg A \vee B) \vdash_{ND} A \quad (7)(\wedge -)$
$(9). A \wedge (\neg A \vee B) \vdash_{ND} \neg A \vee B \quad (7)(\wedge -)$
$(10). A \wedge (\neg A \vee B), B \vdash_{ND} B \quad (\in)$
$(11). A \wedge (\neg A \vee B), \neg A \vdash_{ND} A \quad (8)(+)$
$(12). A \wedge (\neg A \vee B), \neg A \vdash_{ND} \neg A \quad (\in)$
$(13). A \wedge (\neg A \vee B), \neg A \vdash_{ND} B \quad (11)(12)(\neg -)$
$(14). A \wedge (\neg A \vee B) \vdash_{ND} B \quad (9)(10)(13)(\vee -)$
$(15). A \wedge (\neg A \vee B) \vdash_{ND} A \wedge B \quad (8)(14)(\wedge +)$
$(16). \vdash_{ND} A \wedge (\neg A \vee B) \to (A \wedge B) \quad (15)(\to +)$
$(17). \vdash_{ND} (A \wedge B) \leftrightarrow A \wedge (\neg A \vee B) \quad (6)(16)(\leftrightarrow +)$