

NATIONAL TAIWAN NORMAL UNIVERSITY
Department of Computer Science and Information Engineering

Numerical Methods

Midterm Examination

Friday 10/28/2022

Instructions:

- This exam contains two parts: computing problems and written problems. You have 100 minutes.
- This exam is closed book. No hand-held devices are permitted.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it.
- **Good luck!**

Part 1: Computing problems (55)

Please use your codes to solve the following mathematical problems. If the answer is not an integer, please round off to the **4th** decimal place.

(a) $x^3 + 4x^2 - 10 = 0$ for $1 \leq x \leq 2$ (5)

(b) $3.06 = \frac{(1-x)(3+x)}{x(1+x)}$ (10)

(c) Solve the following linear system. (10)

$$a_{ii} = 3$$

$$a_{ij} = -1 \text{ if } |i - j| = 1$$

$$a_{ij} = 0 \text{ if } |i - j| > 1 \text{ where } i, j = 1, 2, \dots, 10$$

$$\underline{b}^T = [2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 2]$$

(d) Find the PA=LU factorization of the system and solve \underline{x} . (20)

$$1.19x_1 + 2.11x_2 - 100x_3 + x_4 = 1.12,$$

$$14.2x_1 - 0.122x_2 + 12.2x_3 - x_4 = 3.44,$$

$$100x_2 - 99.9x_3 + x_4 = 2.15,$$

$$15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16.$$

(e) Find a solution to the following nonlinear system:

$$\sin x + y^2 + \log_e z = 7$$

$$3x + 2y - z^3 = -1$$

$$x + y + z = 5$$

Use the initial guess $[0, 2, 2]^T$. (10)

Part2: Written problems (45)

Problem #1 (5). Nested multiplication (or Horner's method) is an efficient method for evaluating polynomials. Please write down the nested form of the following polynomial:

$$f(x) = x^{14} - 4x^{11} + 5x^8 + 3x^5$$

Problem #2 (10). Consider the equation $x^3 + x - 2 = 0$, with root $r = 1$. Add the term cx to both sides and divide by c to obtain $g(x)$.

- (a) For what condition of c would FPI be linearly convergent to $r = 1$? **(5)**
- (b) For what condition of c would FPI be faster than bisection? **(5)**

Problem #3 (10). A *permutation matrix* is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere. It can be used to reorder the rows of another matrix.

- (a) How many permutation matrices of size 10×10 exist? **(5)**
- (b) Please show how do we use a permutation matrix to make the following matrix strictly diagonally dominant. **(5)**

$$A = \begin{bmatrix} 1 & -8 & -2 \\ 1 & 1 & 5 \\ 3 & -1 & 1 \end{bmatrix}$$

[Please turn over for Problem #4.]

Problem #4 (20). Given three circles with centers (x_1, y_1) , (x_2, y_2) , (x_3, y_3) and radii R_1 , R_2 and R_3 . We expand the radii of each circle **by the same amount** k to get an intersection point (the black dot in the figure). The goal is to compute the coordinate of the intersection point and k .

This problem can be addressed by formulating a nonlinear system and use multivariate Newton's method to solve it. Please write down the $F(\cdot)$ and $D_F(\cdot)$ needed for applying this method.

$$\begin{aligned} \underline{x}_0 &= \text{initial vector} \\ \text{solve } D_F(\underline{x}_k)\underline{s} &= -F(\underline{x}_k) \\ \underline{x}_{k+1} &= \underline{x}_k + \underline{s} \text{ for } k = 0, 1, 2, \dots \end{aligned}$$

