**Problem A:** **Moving North**

Global ocean temperatures affect the quality of **habitats** for certain ocean-dwelling species. When temperature changes are too great for their continued thriving, these species move to seek other habitats better suited to their present and future living and reproductive success. One example of this is seen in the lobster population of Maine, USA that is slowly migrating north to Canada where the lower ocean temperatures provide a more suitable habitat. This geographic population shift can significantly disrupt the livelihood of companies who depend on the stability of ocean-dwelling species.

Your team has been hired as consultants by a Scottish North Atlantic **fishery** management consortium. The consortium wants to gain a better understanding of issues related to the potential migration of Scottish herring and mackerel from their current habitats near Scotland if and when global ocean temperatures increase. These two fish species represent a significant economic contribution to the Scottish fishing industry. Changes in population locations of herring and mackerel could make it economically impractical for smaller Scotland-based fishing companies, who use fishing vessels without on-board refrigeration, to harvest and deliver fresh fish to markets in Scotland fishing ports.

**Requirements**  
  
1. Build a mathematical model to identify the most likely locations for these two fish species over the next 50 years, assuming that water temperatures are going to change enough to cause the populations to move.

2. Based upon how rapidly the ocean water temperature change occurs, use your model to predict best case, worst case, and most likely elapsed time(s) until these populations will be too far away for **small fishing companies** to harvest if the small fishing companies continue to operate out of their current locations.

3. In light of your predictive analysis, should these small fishing companies make changes to their operations?

a. If yes, use your model to identify and assess practical and economically attractive strategies for small fishing companies. Your strategies should consider, but not be limited to, realistic options that include:

- Relocating some or all of a fishing company’s assets from a current location in a Scottish port to closer to where both fish populations are moving;  
- Using some proportion of small fishing vessels capable of operating without land-based support for a period of time while still ensuring the freshness and high quality of the catch.  
- Other options that your team may identify and model.

b. If your team rejects the need for any changes, justify reasons for your rejection based on your modeling results as they relate to the assumptions your team has made.

4. Use your model to address how your proposal is affected if some proportion of the fishery moves into the **territorial waters (sea)** of another country.

5. In addition to your technical report, prepare a one- to two-page article for *Hook Line and Sinker* magazine to help fishermen understand the seriousness of the problem and how your proposed solution(s) will improve their future business prospects.

Your submission should consist of:

* One-page Summary Sheet
* Table of Contents
* One- to Two-page Article
* Your solution of no more than 20 pages, for a maximum of 24 pages with your summary, table of contents, and article.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

**Glossary**

**Fishery**: The collection of fish of a given species and the area that they inhabit.

**Habitat**: The type of environment in which an organism or group normally lives or occurs.

**Small Fishing Company**: A company engaged in commercial fishing with limited or very limited financial resources to invest in new equipment/vessels.

**Territorial Waters (sea)**: "as defined by the 1982 United Nations Convention on the Law of the Sea, is a belt of coastal waters extending at most 12 nautical miles (22.2 km; 13.8 mi) from the baseline (usually the mean low-water mark) of a coastal state. The territorial sea is regarded as the sovereign territory of the state, although foreign ships (military and civilian) are allowed innocent passage through it, or transit passage for straits; this sovereignty also extends to the airspace over and seabed below." [Territorial Waters. (n.d.). In Wikipedia. Retrieved January 28, 2020, from https://en.wikipedia.org/wiki/Territorial\_waters.]

**Problem B:** **The Longest Lasting Sandcastle(s)**

Wherever there are recreational sandy ocean beaches in the world, there seem to be children (and adults) creating sandcastles on the seashore. Using tools, toys, and imagination, beach goers create sandcastles that range from simple mounds of sand to complicated replicas of actual castles with walls, towers, moats, and other features that mimic real castles. In all these, one typically forms an initial foundation consisting of a single, nondescript mound of wetted sand, and then proceeds to cut and shape this base into a recognizable 3-dimensional geometric shape upon which to build the more castle-defining features.

Inevitably, the inflow of ocean waves coupled with rising tides erodes sandcastles. It appears, however, that not all sandcastles react the same way to waves and tides, even if built roughly the same size and at roughly the same distance from the water on the same beach. Consequently, one wonders if there exists a best 3-dimensional geometric shape to use for a sandcastle foundation.

**Requirements**

1. Construct a mathematical model to identify the best 3-dimensional geometric shape to use as a sandcastle foundation that will last the longest period of time on a seashore that experiences waves and tides under the following conditions:

* built at roughly the same distance from the water on the same beach, and
* built using the same type of sand, roughly the same amount of sand, and the same water-to-sand proportion.

2. Using your model, determine an optimal sand-to-water mixture proportion for the castle foundation, assuming you use no other additives or materials (e.g. plastic or wooden supports, stones, etc.).

3. Adjust your model as needed to determine how the best 3-dimensional sandcastle foundation you identified in requirement 1 is affected by rain, and whether it remains the best 3-dimensional geometric shape to be used as a castle foundation when it is raining.

4. What other strategies, if any, might you use to make your sandcastle last longer?

5. Finally, write an informative, one- to two-page article describing your model and its results for publication in the vacation magazine: *Fun in the Sun,* whose readers are mainly non-technical.

Your submission should consist of:

* One-page Summary Sheet
* Table of Contents
* One- to Two-page Article
* Your solution of no more than 20 pages, for a maximum of 24 pages with your summary, table of contents, and article.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

**Problem C:** **A Wealth of Data**

In the online marketplace it created, Amazon provides customers with an opportunity to rate and review purchases. Individual ratings - called "**star ratings**" - allow purchasers to express their level of satisfaction with a product using a scale of 1 (low rated, low satisfaction) to 5 (highly rated, high satisfaction). Additionally, customers can submit text-based messages - called "**reviews**" - that express further opinions and information about the product. Other customers can submit ratings on these reviews as being helpful or not - called a "**helpfulness rating**" - towards assisting their own product purchasing decision. Companies use these data to gain insights into the markets in which they participate, the timing of that participation, and the potential success of product design feature choices.

Sunshine Company is planning to introduce and sell three new products in the online marketplace: a microwave oven, a baby **pacifier**, and a hair dryer. They have hired your team as consultants to identify key patterns, relationships, measures, and parameters in past customer-supplied ratings and reviews associated with other competing products to 1) inform their online sales strategy and 2) identify potentially important design features that would enhance product desirability. Sunshine Company has used data to inform sales strategies in the past, but they have not previously used this particular combination and type of data. Of particular interest to Sunshine Companyare time-based patterns in these data, and whether they interact in ways that will help the company craft successful products.

To assist you, Sunshine's data center has provided you with three data files for this project: **hair\_dryer.tsv**, **microwave.tsv**, and **pacifier.tsv**. These data represent customer-supplied ratings and reviews for microwave ovens, baby pacifiers, and hair dryers sold in the Amazon marketplace over the time period(s) indicated in the data. A glossary of data label definitions is provided as well. THE DATA FILES PROVIDED CONTAIN THE ONLY DATA YOU SHOULD USE FOR THIS PROBLEM.

**Requirements**  
  
1. Analyze the three product data sets provided to identify, describe, and support with mathematical evidence, meaningful quantitative and/or qualitative patterns, relationships, measures, and parameters within and between star ratings, reviews, and helpfulness ratings that will help Sunshine Company succeed in their three new online marketplace product offerings.

2. Use your analysis to address the following specific questions and requests from the Sunshine Company Marketing Director:

* Identify data measures based on ratings and reviews that are most informative for Sunshine Companyto track, once their three products are placed on sale in the online marketplace.
* Identify and discuss time-based measures and patterns within each data set that might suggest that a product's reputation is increasing or decreasing in the online marketplace.
* Determine combinations of text-based measure(s) and ratings-based measures that best indicate a potentially successful or failing product.
* Do specific star ratings incite more reviews? For example, are customers more likely to write some type of review after seeing a series of low star ratings?
* Are specific quality descriptors of text-based reviews such as 'enthusiastic', 'disappointed', and others, strongly associated with rating levels?

3. Write a one- to two-page letter to the Marketing Director of Sunshine Company summarizing your team's analysis and results. Include specific justification(s) for the result that your team most confidently recommends to the Marketing Director.

Your submission should consist of:

* One-page Summary Sheet
* Table of Contents
* One- to Two-page Letter
* Your solution of no more than 20 pages, for a maximum of 24 pages with your summary sheet, table of contents, and two-page letter.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

**Glossary**

**Helpfulness Rating:** an indication of how valuable a particular product review is when making a decision whether or not to purchase that product.

**Pacifier:** a rubber or plastic soothing device, often nipple shaped, given to a baby to suck or bite on.

**Review:** a written evaluation of a product.

**Star Rating:**a score given in a system that allows people to rate a product with a number of stars.

**Attachments: The Problem Datasets**

**[Problem\_C\_Data.zip](http://www.mathmodels.org/Problems/2020/MCM-C/Problem_C_Data.zip" \t "_blank)**  
The three data sets provided contain product user ratings and reviews extracted from the Amazon Customer Reviews Dataset thru Amazon Simple Storage Service (Amazon S3).  
hair\_dryer.tsv  
microwave.tsv  
pacifier.tsv

**Data Set Definitions:** Each row represents data partitioned into the following columns.

* marketplace (string): 2 letter country code of the marketplace where the review was written.
* customer\_id (string): Random identifier that can be used to aggregate reviews written by a single author.
* review\_id (string): The unique ID of the review.
* product\_id (string): The unique Product ID the review pertains to.
* product\_parent (string): Random identifier that can be used to aggregate reviews for the same product.
* product\_title (string): Title of the product.
* product\_category (string): The major consumer category for the product.
* star\_rating (int): The 1-5 star rating of the review.
* helpful\_votes (int): Number of helpful votes.
* total\_votes (int): Number of total votes the review received.
* vine (string): Customers are invited to become Amazon Vine Voices based on the trust that they have earned in the Amazon community for writing accurate and insightful reviews. Amazon provides Amazon Vine members with free copies of products that have been submitted to the program by vendors. Amazon doesn't influence the opinions of Amazon Vine members, nor do they modify or edit reviews.
* verified\_purchase (string): A "Y" indicates Amazon verified that the person writing the review purchased the product at Amazon and didn't receive the product at a deep discount.
* review\_headline (string): The title of the review.
* review\_body (string): The review text.
* review\_date (bigint): The date the review was written.

**Problem D:** **Teaming Strategies**

As societies become more interconnected, the set of challenges they face have become increasingly complex. We rely on interdisciplinary teams of people with diverse expertise and varied perspectives to address many of the most challenging problems. Our conceptual understanding of team success has advanced significantly over the past 50+ years allowing for better scientific, creative, or physical teams to address these complex issues. Researchers have reported on best strategies for assembling teams, optimal interactions among teammates, and ideal leadership styles. Strong teams across all sectors and domains are able to perform complex tasks unattainable through either individual efforts or a sequence of additive contributions of teammates.

One of the most informative settings to explore team processes is in competitive team sports. Team sports must conform to strict rules that may include, but are not limited to, the number of players, their roles, allowable contact between players, their location and movement, points earned, and consequences of violations. Team success is much more than the sum of the abilities of individual players. Rather, it is based on many other factors that involve how well the teammates play together. Such factors may include whether the team has a diversity of skills (one person may be fast, while another is precise), how well the team balances between individual versus collective performance (star players may help leverage the skills of all their teammates), and the team's ability to effectively coordinate over time (as one player steals the ball from an opponent, another player is poised for offense).

In light of your modeling skills, the coach of the Huskies, your home soccer (known in Europe and other places as football) team, has asked your company, **I**ntrepid **C**hampion **M**odeling (ICM), to help understand the team's dynamics. In particular, the coach has asked you to explore how the complex interactions among the players on the field impacts their success. The goal is not only to examine the interactions that lead directly to a score, but to explore team dynamics throughout the game and over the entire season, to help identify specific strategies that can improve teamwork next season. The coach has asked ICM to quantify and formalize the structural and dynamical features that have been successful (and unsuccessful) for the team. The Huskies have provided data**[1]** detailing information from last season, including all 38 games they played against their 19 opponents (they played each opposing team twice). Overall, the data covers 23,429 passes between 366 players (30 Huskies players, and 336 players from opposing teams), and 59,271 game events.

To respond to the Huskie coach's requests, your team from ICM should use the provided data to address the following:

* Create a network for the ball passing between players, where each player is a node and each pass constitutes a link between players. Use your passing network to identify network patterns, such as **dyadic** and **triadic configurations** and team formations. Also consider other structural indicators and network properties across the games. You should explore multiple scales such as, but not limited to, micro (pairwise) to macro (all players) when looking at interactions, and time such as short (minute-to-minute) to long (entire game or entire season).
* Identify performance indicators that reflect successful teamwork (in addition to points or wins) such as diversity in the types of plays, coordination among players or distribution of contributions. You also may consider other team level processes, such as adaptability, flexibility, tempo, or flow. It may be important to clarify whether strategies are universally effective or dependent on opponents' counter-strategies. Use the performance indicators and team level processes that you have identified to create a model that captures structural, configurational, and dynamical aspects of teamwork.
* Use the insights gained from your teamwork model to inform the coach about what kinds of structural strategies have been effective for the Huskies. Advise the coach on what changes the network analysis indicates that they should make next season to improve team success.
* Your analysis of the Huskies has allowed you to consider group dynamics in a controlled setting of a team sport. Understanding the complex set of factors that make some groups perform better than others is critical for how societies develop and innovate. As our societies increasingly solve problems involving teams, can you generalize your findings to say something about how to design more effective teams? What other aspects of teamwork would need to be captured to develop generalized models of team performance?

Your submission should consist of:

* One-page Summary Sheet
* Table of Contents
* Your solution of no more than 20 pages, for a maximum of 22 pages with your summary and table of contents.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

**Attachment**

**[2020\_Problem\_D\_DATA.zip](http://www.mathmodels.org/Problems/2020/ICM-D/2020_Problem_D_DATA.zip" \t "_blank)**  
fullevents.csv  
matches.csv  
passingevents.csv  
README.txt

This data set was processed from a much larger dataset covering nearly 2000 matches from five European national soccer competitions, as well as the 2018 World Cup.

**Glossary**

**Dyadic Configurations:** relationships involving pairs of players.

**Triadic Configurations:** relationships involving groups of three players.

**Cited Reference**

**[1]** Pappalardo, L., Cintia, P., Rossi, A. *et al.* *A public data set of spatio-temporal match events in soccer competitions*. *Sci Data* 6, 236 (2019).

**Optional Resources**

Research in football (soccer) networks has led to many articles that discuss related topics. A few articles are listed below. You are not required to use any of these sample articles in your solution, nor is it a comprehensive list.Â  We encourage teams to utilize any journal article that supports their approach to the problem.

BuldÃº, J.M., Busquets, J., Echegoyen, I. *et al.* (2019). Defining a historic football team: Using Network Science to analyze Guardiola's F.C. Barcelona. *Sci Rep*, 9, 13602.

Cintia, P., Giannotti, F., Pappalardo, L., Pedreschi, D., & Malvaldi, M. (2015). The harsh rule of the goals: Data-driven performance indicators for football teams*.* *2015 IEEE International Conference on Data Science and Advanced Analytics (DSAA)*, 1-10, 7344823.

Duch J., Waitzman J.S., Amaral L.A.N. (2010). Quantifying the performance of individual players in a team activity. *PLoS ONE*, 5: e10937.

GÃœRSAKAL, N., YILMAZ, F., Ã‡OBANOÄžLU, H., Ã‡AÄžLIYOR, S. (2018). Network Motifs in Football. *Turkish Journal of Sport and Exercise*, 20 (3), 263-272.

**Problem E: Drowning in Plastic**

Since the 1950s, the manufacturing of plastics has grown exponentially because of its variety of uses, such as food packaging, consumer products, medical devices, and construction. While there are significant benefits, the negative implications associated with increased production of plastics are concerning. Plastic products do not readily break down, are difficult to dispose of, and only about 9% of plastics are recycled**[1]**.聽 Effects can be seen by the approximately 4-12 million tons of **plastic waste** that enter the oceans each year**[1,2]**. Plastic waste has severe environmental consequences and it is predicted that if our current trends continue, the oceans will be filled with more plastic than fish by 2050**[2]**. The effect on marine life has been studied**[3]**, but the effects on human health are not yet completely understood**[4]**. The rise of **single-use** and **disposable** **plastic products** results in entire industries dedicated to creating plastic waste. It also suggests that the amount of time the product is useful is significantly shorter than the time it takes to properly **mitigate** the plastic waste. Consequently, to solve the plastic waste problem, we need to slow down the flow of plastic production and improve how we manage plastic waste.

Your team has been hired by the **I**nternational **C**ouncil of Plastic Waste **M**anagement (ICM) to address this escalating environmental crisis. You must develop a plan to significantly reduce, if not eliminate, single-use and disposable plastic product waste.

* Develop a model to estimate the maximum levels of single-use or disposable plastic product waste that can safely be mitigated without further environmental damage. You may need to consider, among many factors, the source of this waste, the extent of the current waste problem, and the availability of resources to process the waste.
* Discuss to what extent plastic waste can be reduced to reach an environmentally safe level. This may involve considering factors impacting the levels of plastic waste to include, but not limited to, sources and uses of single-use or disposable plastics, the availability of alternatives to plastics, the impact on the lives of citizens, or policies of cities, regions, countries, and continents to decrease single-use or disposable plastic and the effectiveness of such policies. These can vary between regions, so considering regional-specific constraints may make some policies more effective than others.
* Using your model and discussion, set a target for the minimal achievable level of global waste of single-use or disposable plastic products and discuss the impacts for achieving such levels. You may consider ways in which human life is altered, the environmental impacts, or the effects on the multi-trillion-dollar plastic industry.
* While this is a global problem, the causes and effects are not equally distributed across nations or regions. Discuss the equity issues that arise from the global crisis and your intended solutions. How do you suggest ICM address these issues?
* Write a two-page memo to the ICM describing a realistic global target minimum achievable level of global single-use or disposable plastic product waste, a timeline to reach this level, and any circumstances that may accelerate or hinder the achievement of your target and timeline.

Your submission should consist of:

* One-page Summary Sheet
* Table of Contents
* Two-page Memo
* Your solution of no more than 20 pages, for a maximum of 24 pages with your summary, table of contents, and two-page memo.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

**Glossary**

**Disposable Plastic Products:** plastic materials or products that are not recyclable and become trash.

**Mitigate:** To make less severe, to moderate, to alleviate.

**Plastic Waste:** plastic objects that have not been recycled properly or cannot be recycled; debris made of plastic.

**Single-Use Plastic Products:** products made of plastic intended for one time use before being discarded.

**Cited References**

**[1]** Geyer, R., Jambeck, J. R., & Law, K. L. (2017). Production, use, and fate of all plastics ever made. *Science Advances*, 3(7), e1700782.  
  
**[2]** Jambeck, J. R., Geyer, R., Wilcox, C., Siegler, T. R., Perryman, M., Andrady, A., 鈥� & Law, K. L. (2015). Plastic waste inputs from land into the ocean. Science, 347(6223), 768-771.  
  
**[3]** Li, W. C., Tse, H. F., & Fok, L. (2016). Plastic waste in the marine environment: A review of sources, occurrence and effects. *Science of the Total Environment*, 566, 333-349.  
  
**[4]** Galloway T.S. (2015) Micro- and Nano-plastics and Human Health. In: Bergmann M., Gutow L., Klages M. (eds) *Marine Anthropogenic Litter*.

**Problem F:** **The Place I Called Home...**

Researchers have identified several island nations, such as The Maldives, Tuvalu, Kiribati, and The Marshall Islands, as being at risk of completely disappearing due to rising sea levels. What happens, or what should happen, to an island's population when its nation's land disappears? Not only do these **environmentally displaced persons** (EDPs) need to relocate, but there is also risk of losing a unique culture, language, and way of life. In this problem, we ask you to look more closely at this issue, in terms of both the need to relocate people and the protection of culture. There are many considerations and questions to address, to include: Where will these EDPs go? What countries will take them? Given various nations' disproportionate contributions to the green-house gasses both historically and currently that have accelerated climate change linked to the rising seas, should the worst offenders have a higher obligation to address these issues? And, who gets a say in deciding where these nationless EDPs make a new home â€“ the individuals, an intergovernmental organization like the United Nations (UN), or the individual governments of the states absorbing these persons? A more detailed explanation of these issues is given in the [Issue Paper](http://www.mathmodels.org/Problems/2020/ICM-F/2020_ICM_Problem_F-ISSUE.pdf" \t "_blank).

As a result of a recent UN ruling that opened the door to the theoretical recognition of EDPs as refugees, the **I**nternational **C**limate **M**igration **F**oundation (ICM-F) has hired you to advise the UN by developing a model and using it to analyze this multifaceted issue of when, why, and how the UN should step into a role of addressing the increasing challenge of EDPs. The ICM-F plans to brief the UN on guidance for how the UN should generate a systemized response for EDPs, especially in consideration of the desire to preserve **cultural heritage**. Your assignment is to develop a model (or set of models) and use your model(s) to provide the analysis to support this briefing. The ICM-F is especially interested in understanding the scope of the issue of EDPs. For example, how many people are currently at risk of becoming EDPs**[1]**; what is the value of the cultures of at-risk nations; how are those answers likely to change over time? Furthermore, how should the world respond with an international policy that specifically focuses on protecting the rights of persons whose nations have disappeared in the face of climate change while also aiming to preserve culture? Based on your analysis, what recommendations can you offer on this matter, and what are the implications of accepting or rejecting your recommendations?

This problem is extremely complex. We understand that your submission will not be able to fully consider all of the aspects described in the Issue Paper beginning on page 3. However, considering the aspects that you address, synthesize your work into a cohesive answer to the ICM-F as they advise the UN. At a minimum, your team's paper should include:

* An analysis of the scope of the issue in terms of both the number of people at risk and the risk of loss of culture;
* Proposed policies to address EDPs in terms of both human rights (being able to resettle and participate fully in life in their new home) and cultural preservation;
* A description of the development of a model used to measure the potential impact of proposed policies;
* An explanation of how your model was used to design and/or improve your proposed policies;
* An explanation, backed by your analysis, of the importance of implementing your proposed policies.

The ICM-F consists of interdisciplinary judges including mathematicians, climate scientists, and experts in refugee migration to review your work. Therefore, your paper should be written for a scientifically literate yet diverse audience.

Your submission should consist of:

* One-page Summary Sheet
* Table of Contents
* Your solution of no more than 20 pages, for a maximum of 22 pages with your summary and table of contents.

NOTE: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

**Glossary**

**Environmentally displaced persons (EDPs):** people who must relocate as their homeland becomes uninhabitable due to climate change events

**Cultural heritage:** the ways of living of a group or society passed through generations to include customs, practices, art, and values.

**[1]** There are multiple estimates for the current and predicted number of climate refugees in the existing literature, but they are vastly different. Therefore, you need to support your conclusions with analysis based on your own model(s), either building off of existing analysis or with a new and independent analysis.