■ $\mathcal{L}_{\mathrm{g}}(\phi) = \int_{\Omega} g\delta(\phi)|\nabla\phi|dxdy$

$$\mathcal{L}_{\mathrm{g}}(\phi + th) = \int_{\Omega} g\delta(\phi + th)\sqrt{(\phi + th)_x^2 + (\phi + th)_y^2}dxdy,$$

where is t is a small variable and h is an arbitrary function which satisfies $h|_{\partial\Omega} = 0$.

$$\left.\frac{\partial\mathcal{L}_g(\phi + th)}{\partial t}\right|_{t\to 0} = \int_{\Omega} g\left.\frac{\partial\delta(\phi + th)\sqrt{(\phi + th)_x^2 + (\phi + th)_y^2}}{\partial t}\right|_{t\to 0} dxdy$$

$$= \int_{\Omega} gh\delta^{'}(\phi)|\nabla\phi| + g\delta(\phi)\frac{\nabla h\nabla\phi}{|\nabla\phi|}dxdy$$

$$= \int_{\Omega} gh\delta^{'}(\phi)|\nabla\phi| + \frac{\partial gh\delta(\phi)\frac{\phi_x}{|\nabla\phi|}}{\partial x} + \frac{\partial gh\delta(\phi)\frac{\phi_y}{|\nabla\phi|}}{\partial y}$$

$$-gh\delta'(\phi)\frac{|\nabla\phi|^2}{|\nabla\phi|} - gh\delta(\phi)\left[\frac{\partial\frac{\phi_x}{|\nabla\phi|}}{\partial x} + \frac{\partial\frac{\phi_y}{|\nabla\phi|}}{\partial y}\right]dxdy$$

Note that

$$\int_{\Omega}\frac{\partial gh\delta(\phi)\frac{\phi_x}{|\nabla\phi|}}{\partial x} + \frac{\partial gh\delta(\phi)\frac{\phi_y}{|\nabla\phi|}}{\partial y}dxdy = \oint_{\partial\Omega} gh\delta(\phi)\frac{\phi_y}{|\nabla\phi|}dy + gh\delta(\phi)\frac{\phi_x}{|\nabla\phi|}dx$$

$$= 0.$$

Then

$$\left.\frac{\partial\mathcal{L}_g(\phi + th)}{\partial t}\right|_{t\to 0} = \int_{\Omega} - gh\delta(\phi)div(\frac{\nabla\phi}{|\nabla\phi|})dxdy,$$

i.e.,

$$\phi_t = \delta(\phi)div(g\frac{\nabla\phi}{|\nabla\phi|})$$

■ $\mathcal{A}_g(\phi) = \int_{\Omega} gH(-\phi)dxdy$

$$\mathcal{A}_{\mathrm{g}}(\phi + th) = \int_{\Omega} gH(-\phi - th)\mathrm{dxdy},$$

where is t is a small variable and h is an arbitrary function which satisfies $h|_{\partial\Omega} = 0$.

$$\left.\frac{\partial \mathcal{A}_g(\phi + th)}{\partial t}\right|_{t\to 0} = \int_{\Omega} g \left.\frac{H(-\phi - th)}{\partial t}\right|_{t\to 0} dxdy$$

$$= \int_{\Omega} -ghH(-\phi)' dxdy = \int_{\Omega} -gh\delta(\phi)dxdy,$$

i.e.,

$$\phi_t = g\delta(\phi)$$

- $\varepsilon_{g,\lambda,v}(\phi) = \lambda L_g(\phi) + vA_g(\phi)$

$$\frac{\partial \phi}{\partial t} = \lambda\, \delta(\phi) div(g \frac{\nabla \phi}{|\nabla \phi|}) + vg\delta(\phi)$$