POPULATION MODELLING OF TIBETAN ANTELOPE BY ORDINARY DIFFERENTIAL EQUATION

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Modelling Continuous Changes through Ordinary Differential Equations

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Certificate of Originality

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Abstract

POPULATION MODELLING OF TIBETAN ANTELOPE BY ORDINARY DIFFERENTIAL EQUATION

By

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The major focus of this project is on constructing a general model for the population of Tibetan Antelope. The Tibetan Antelope which is also known as Chiru is an endangered species that is distributed in China and India (Aksai Chin and Ladakh). The World Conservation Union (IUCN) lists the Tibetan Antelope as an endangered species due to the sharp decrease in animal numbers and distribution as a result of commercial hunting for the Shahtoosh underfur. In this project, we will construct a mathematical equation model taking into account certain factors around the environment of the Antelope and its commercial hunting to predict the population of the Antelope in the coming years. The model mathematically describes the convergence of population to a certain limit describing the maximum limit or the limitation of the population. Some typical mathematical models such as the exponential model and logistic model are introduced for the same purpose and the solutions of those models are analyzed using graphs and tables. The first step of the project will be to verify the already available data and check whether the model is suitable to be used for the problem or not. After that, we can further move to predict the population change.

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1. INTRODUCTION

Population dynamics, especially the equilibrium states and their stability, have traditionally been analysed using mathematical models. These population models might be the differential models or the different models. But the most interesting part of both these models is the equilibrium states and convergence towards these states. Thus, one fascinating question is how long will it take for the population to approach the equilibrium state or to get extinct. To make a mathematical model useful in practice we need to use quantitative methods that allow us to forecast a population's future and express the numerical results. In order to predict future changes, we need models which can be modified as per the conditions of the concerned species. A model is a mathematical description of changes in population magnitude. These models can be really simple including a few equations with some variables or maybe really complicated and include computer programs with thousands of lines. One of the major difficulties of constructing a mathematical model is that we need to consider a particular situation.

In this project, we will review some simple mathematical models. Particularly we will focus on the logistics growth model and harvesting model. From that, we can see the limitations for the population of Tibetan Antelope. Based on it, a general model is constructed. It may either show the time until extinction or until the population has either decreased or increased to reach an equilibrium level.

1.1 Background and Context

1.1.1 Basic Mathematical Techniques

The population model can always be written in the form of simple equations.
For example,

Rate of Change of Population = Number of births – Number of Deaths (1.1.1)

Now the number of births might also be dependent on the already existing population.
Similarly, the number of deaths might also depend on the already existing population due to the limited amount of food and shelter.
Let P(t) be the population of the species of interest at some time t. Thus the equation (1.1.1) can be written as

$$P'(t) = BP(t) - DP(t) \tag{1.1.2}$$

$$P'(t) = (B-D)P(t) \tag{1.1.3}$$

here B is the birth rate of the species of interest and D is the death rate of the species.
We can see that B and D could be functions of time t or they could be related to the population. It depends on particular species and environmental conditions.
We can make certain assumptions to this model to make it fit for predicting the population of the species we are interested in.

1.1.2 Model 1: When both B and D are constants.

Here if both B and D are constants

- ⇨ Some constant r = B-D

  And equation (1.1.3) becomes

$$P'(t) = rP(t) \tag{1.2.1}$$

This gives us the simplest population model with r being the most significant constant here. r is the constant that tells how fast the population is changing at any given population level. r can be both positive or negative. If r is positive, it tells that the population is increasing and if r is negative, it tells that the population is decreasing. Thus, r can be defined as the rate of growth (when r>0) or the rate of decay (when r<0) of the population. And the model is called the exponential model. Also, it is important to note that r being positive negative depends upon which rate term out of birth rate and the death rate is dominant. In other words, it depends on a number of deaths or births which is greater

Now solving the equation (1.2.1)

$$\frac{\mathbf{dP(t)}}{\mathbf{dt}} = \boldsymbol{rP(t)} \qquad \mathbf{(1.2.2)}$$

$$\int\frac{\boldsymbol{dP(t)}}{\boldsymbol{P(t)}} = \int \boldsymbol{rdt} \qquad \mathbf{(1.2.3)}$$

$$\mathbf{Log\ P(t) = rt + C} \qquad \mathbf{(1.2.4)}$$

$$\mathbf{P(t) = P_o e^{rt}} \qquad \mathbf{(1.2.5)}$$

where P_o represents the initial population size.

1.1.3 Model 2: Either the Birth rate or Death rate is a constant.

1.1.3.a When the Death rate is constant.

The previous model depicts exponential growth without any boundary. But in the real world, it does not happen. The reason for this is that in real-world environment we have to consider certain factors which hamper the death and birth rates. For example, if a species needs grass for food and the area is hit by severe drought decreasing the grassland and shortening the food supplies for the species which in turn will hamper their fertility decreasing the birth rate and increasing death rate.

To solve such a problem we can construct a model in which the birth rate is a linearly decreasing function of population size.

$$\mathbf{B = B_0 - B_1 P(t)} \qquad \mathbf{(1.3.1)}$$

where B_0 and B_1 are constants

Now as we know that the birth rate is the ratio between the number of live born births in the year and the average total population of that year.

- ⇨ Birth rate cannot be greater than 1 and cannot be smaller than 0
- ⇨ $\mathbf{0 \le B_0 - B_1 P \le 1}$ **(1.3.2)**

If we want to control the birth rate then we need to find the range of B_1.

- ⇨ $\mathbf{0 \le B_0 - B_1 P \le 1}$
- ⇨ $\frac{\boldsymbol{Bo-1}}{\boldsymbol{P}} \le \boldsymbol{B1} \le \frac{\boldsymbol{Bo}}{\boldsymbol{P}}$ **(1.3.3)**Thus

  B_1 is between and $\frac{Bo}{P}$.

Let D_0 be the initial death rate at that time, then the population equation becomes

$$\mathbf{P'(t) = (B_0 - B_1 P - D_0)P} \qquad \mathbf{(1.3.4)}$$

$$\mathbf{P'(t) = (B_0 - D_0)P - B_1 P^2} \qquad \mathbf{(1.3.5)}$$

For the purpose of simplification

Let $\mathbf{k = B_1}$ and $\mathbf{M} = \frac{\boldsymbol{Bo-Do}}{\boldsymbol{B1}}$

$$\mathbf{P'(t) = B_1(} \frac{\boldsymbol{Bo-D0}}{\boldsymbol{B1}} \mathbf{P - P^2)}$$

$$\mathbf{P'(t) = k\ (\ MP - P^2\ )}$$

$$\mathbf{P'(t) = kMP - kP^2 = kMP(1 - } \frac{\boldsymbol{P}}{\boldsymbol{M}}\mathbf{)\ \ where\ k \ne 0} \qquad \mathbf{(1.3.6)}$$

If we start with a population of zero, there is no growth, the population stays at zero. If we start with a population greater than zero and less than M then P'(t) is positive thus the population is increasing. If we start at P=M then the population stays at this level. Similarly,

the initial population is greater than M then P'(t) is negative thus the population will decrease.
From the above discussion, it is clear that the equilibrium is stable at P = M while unstable at P = 0. Thus, if the population is above zero, it will go to the carrying capacity M eventually. And if the initial population is greater than k then it will decrease and eventually go to M.
Now solving the differential equation given in equation (1.3.6)

$$\frac{dP}{dt} = kMP(1 - \frac{P}{M}) \tag{1.3.6}$$

$$\frac{dp}{dt} = kP(M - P) \tag{1.3.7}$$

$$\int \frac{dP}{P(M-P)} = \int kdt \tag{1.3.8}$$

$$\frac{1}{M} log\left(\frac{P}{M-P}\right) = kt + c \tag{1.3.9}$$

$$\frac{P}{M-P} = e^{Mkt+C} \tag{1.3.10}$$

$$P(t) = \frac{Me^{Mkt}}{e^{kMt} - C} \tag{1.3.11}$$

where C is the constant of integration.

$$P(t) = \frac{M}{1 - \left(\frac{1}{e^{kMt-C}}\right)} \tag{1.3.12}$$

where M = $\frac{Bo-D0}{D1}$ and k = B_1 and B_0 & B_1 are positive constants and C is also a constant.

⇨ here if t $\rightarrow\infty$ then P(t)$\rightarrow$M.

as we can observe that as time tends to infinity, the population tends to a finite limit.
The value of M is defined in terms of birth rate and death rate expressions and is referred to as a limitation of the population. This result is also at par with the real world expectation.

1.1.3.b When the Birth rate is constant.

Now if the birth rate is constant, let's suppose that the death rate is a linearly increasing function of the population size.

⇨ $$\mathbf{D = D_0 + D_1P} \tag{1.3.13}$$

where D_1 and D_0 are positive constant.
Thus the differential equation becomes

$$\mathbf{P'(t) = (B_0 - D_0 - D_1P)P} \tag{1.3.14}$$

$$\mathbf{P'(t) = (B_0 - D_0)P - D_1P^2} \tag{1.3.15}$$

here k = D_1 and M = $\frac{B0 - D0}{D1}$

$$\mathbf{P'(t) = kMP(1- \frac{P}{M})} \tag{1.3.16}$$

As the equation we get here is of the same form as the previous model thus we can assume they have the same form as a solution or in other words it will also tend to a finite population as time will tend to infinity.

1.1.4 Model 3: Including the harvesting factor

As we know that in many cases the species of interest is also commercially harvested. This implies that besides the environmental factors there is one more factor that affects the population of the species. But this factor will be a constant. So the impact will be a change by a constant.
Thus, if we include the factor of harvesting H too in the previously derived model where the death rate was a constant.
Thus, we get the equation as follows,

$$\frac{dP}{dt} = (B_0 - B_1P - D_0)P - H \qquad (1.4.1)$$
$$\frac{dP}{dt} = (B_0 - D_0)P - B_1P^2 - H \qquad (1.4.2)$$
$$\frac{dP}{dt} = -\{B_1P^2 - (B_0 - D_0)P + H\} \qquad (1.4.3)$$
$$\frac{dP}{dt} = -[\{B_1(P^2 - \frac{Bo-Do}{D1}P)\}+H] \qquad (1.4.4)$$
$$\frac{dP}{dt} = -[k(P^2 - MP)+H] \qquad (1.4.5)$$
$$\frac{dP}{dt} = -(kP^2 - kMP + H) \qquad (1.4.6)$$

Here $k = B_1$ & $M = \frac{Bo-D0}{D1}$ and H is the amount of population being harvested each year.

As we can see from the above equation that P'(t) = 0 when $P(t) = \frac{M+\sqrt{M^2-4\left(\frac{H}{k}\right)}}{2}$ or when $P(t) = \frac{M-\sqrt{M^2-4\left(\frac{H}{k}\right)}}{2}$. So let us say that $\alpha = \frac{M+\sqrt{M^2-4\left(\frac{H}{k}\right)}}{2}$ and $\beta = \frac{M-\sqrt{M^2-4\left(\frac{H}{k}\right)}}{2}$ thus we can say that if we start with a population of zero, there is no growth and the population stays at zero. If we start with a population greater than zero and less than β, P'(t) will become negative and thus the population will decrease. If the starting population is equal to β, P'(t)= 0 thus the population will stay as it is with no change. But if the population is greater β and smaller than α then the population will increase. Similarly, if the population is greater than α then the population will decrease.

Thus, we can conclude that β is an unstable equilibrium point whereas α is a stable equilibrium point.

Now rewriting equation (1.4.6) in terms of α and β, we get

$$\frac{dP}{dt} = -(P-\alpha)(P-\beta) \qquad (1.4.7)$$

$$\frac{1}{\alpha-\beta}\int\left(\frac{1}{P-\alpha} - \frac{1}{P-\beta}\right)dP = -\int dt \qquad (1.4.8)$$

$$-\log\frac{P-\alpha}{P-\beta} = (\alpha-\beta)t + C \qquad (1.4.9)$$

$$\frac{P-\beta}{P-\alpha} = e^{(\alpha-\beta)t+C} \qquad (1.4.10)$$

$$P = \frac{\alpha - \frac{\beta}{e^{(\alpha-\beta)t+C}}}{1 - \frac{1}{e^{(\alpha-\beta)t+C}}} \qquad (1.4.11)$$

Here we can see that as t→∞ and P→α. Thus, this model is also giving results similar to the expected real-life scenarios.

1.2 Scope and Objectives

By this project we aim to construct a model by which we can judge and anticipate the population of the Tibetan Antelope which is native to China and India. At the end of this project we wish to come up with a model that will satisfy not only the existing range of data but will also help us to predict the future curve of population and its limiting value. In the model that we have used, besides including a variable birth rate which is an outcome of the different environmental conditions in the regions of interest which leads to the different reproductive ability of the species with time we have also included the factor of harvesting into our model which will give us a deeper insight into the changes in the population with coming time and how exactly the population of the species is reacting to the harvesting pressure.

1.3 Achievements

Previously we have successfully been able to derive a general model step by step to reach to satisfactory conclusive model that includes both the factors of the variable birth rate as well as the harvesting constant. This model is expected to be closest to the solution of the problem we are dealing with since it includes both the important factors of population change and harvesting.

2. FORMULATION OF THE PROBLEM

2.1 Problem Statement

Due to the geographic distribution of Tibetan Antelope and the circumstance of the Chinese government, the outcome of statistical analysis of Tibetan Antelope remains unsolved before 1950. There are some reasons for that.

First, Tibetan Antelopes are distributed extensively in China. Tibet currently has approximately 149,930 Tibetan Antelopes in a 698,000 square-km area across 103 villages and 18 towns. And also, there are many Tibetan Antelopes distributed in the area of high elevations where the air is thin and no humans inhabit the regions, so performing the census are very difficult, and nothing can be done for the statistical analysis of the wildlife population.

The official data being used in this paper was initially collected after 1950. Because of the special habitat environment of Tibetan Antelopes, we can summarize that from 1900 to 1960's the change in quantities of Tibetan Antelopes depended on the habitat. But soon after that, the antelopes were being hunted for their fur. According to statistics, the population of Tibetan antelope started decreasing rapidly during that period.

Thus, in this project, we have to create a model fit to solve these problems and provide a suitable model.

2.2 Methodology

2.2.1 Data for Tibetan Antelope

According to the IUCN (International Union for Conservation of Nature and Natural Resources), population estimates between 1950 and 1960 ranged from 500,000 to 1,000,000. The following Table 1 shows the available data about Tibetan Antelope between 1950 and 1960.

Date (Years)	Population (Million)
1950	0.500
1951	0.550
1952	0.601
1953	0.645
1954	0.695
1955	0.750
1956	0.816
1957	0.8900.
1958	0.958
1959	1.041
1960	1.130

Table 1

2.2.2 Graph the Data

Considering the population sizes for Tibetan Antelope for the years between 1950 and 1960, we will derive a mathematical model for the Tibetan Antelope.

Using these data, we can plot the graph. It is the following figure 2.

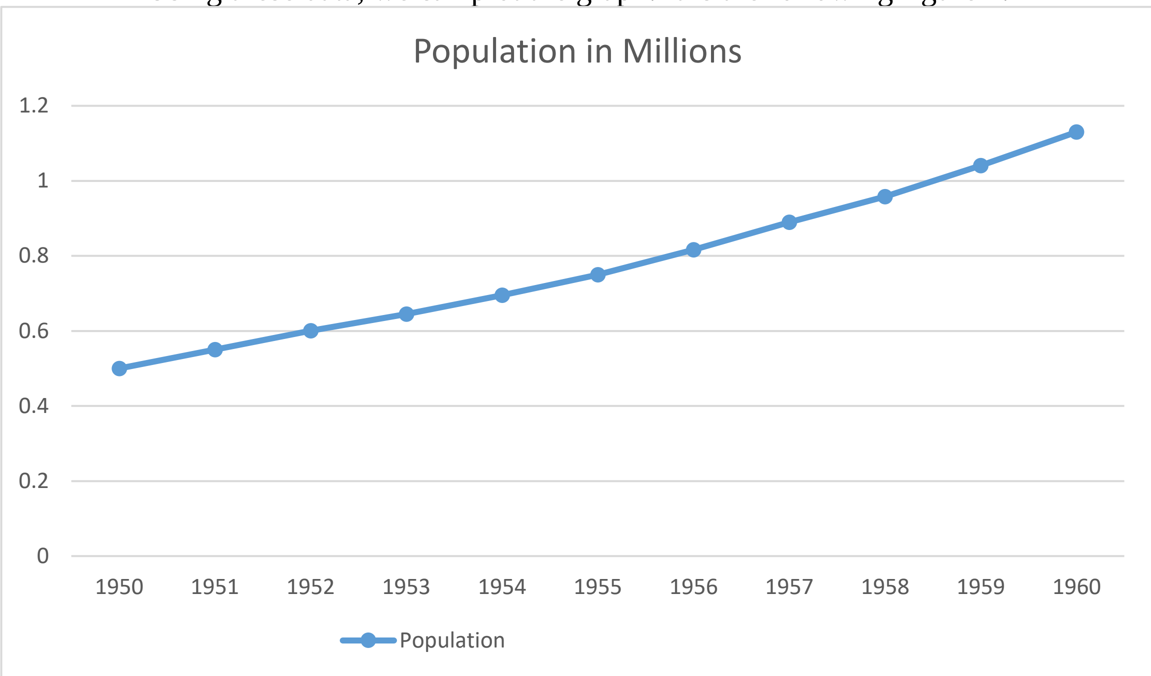


Figure 2

2.2.3 The Mathematical Model

Let’s consider the logistic growth model with form:

$$\mathbf{P' = aP^2 + bP + c} \qquad \mathbf{(2.3.1)}$$

2.2.4 Finding appropriate values of a, b and c.

Since we have discrete data, then we describe the model using a difference equation. We use previous values from the systems to calculate the new ones. The equation (2.3.1) can be expressed by the difference equation version of the following equation:

$$\mathbf{P(t+1) - P(t) = aP^2 + bP + c}$$

It can be rewritten as:

$$\frac{\Lambda \mathbf{P}}{\mathbf{P}} = \mathbf{aP^2 + bP + c}$$

The equation says that the ratio of ΛP to P is a quadratic function of P.

Now we can use the quadratic regression to find the values of a, b and c.

Calculating the ratios on the left-hand side of yields:

1. $a_1 = \frac{P(1951) - P(1950)}{P(1950)} = \frac{0.55-0.5}{0.5} = 0.1$
2. $a_2 = \frac{P(1952) - P(1951)}{P(1951)} = \frac{0.601-0.55}{0.55} = 0.0927$
3. $a_3 = \frac{P(1953) - P(1952)}{P(1952)} = \frac{0.645-0.601}{0.601} = 0.0732$
4. $a_4 = \frac{P(1954) - P(1953)}{P(1953)} = \frac{0.695-0.645}{0.645} = 0.0775$
5. $a_5 = \frac{P(1955) - P(1954)}{P(1954)} = \frac{0.755-0.695}{0.695} = 0.0863$
6. $a_6 = \frac{P(1956) - P(1955)}{P(1955)} = \frac{0.816-0.755}{0.755} = 0.0807$

7. $a_7 = \frac{P(1957) - P(1956)}{P(1956)} = \frac{0.890-0.816}{0.816} = 0.0906$

8. $a_8 = \frac{P(1958) - P(1957)}{P(1957)} = \frac{0.958-0.890}{0.890} = 0.0764$

9. $a_9 = \frac{P(1959) - P(1958)}{P(1958)} = \frac{1.041-0.958}{0.958} = 0.0866$

10. $a_{10} = \frac{P(1960) - P(1959)}{P(1959)} = \frac{1.13-1.041}{1.041} = 0.0854$

Thus, we have the following data:

a	P(t)
0.1	0.500
0.0927	0.550
0.0732	0.601
0.0863	0.695
0.0807	0.755
0.0906	0.816
0.0764	0.890
0.0866	0.958
0.0854	1.041

Table 2

	Y	X	X^2	X^3	X^4	X*Y	$X*Y^2$
	0.05	0.5	0.25	0.125	0.0625	0.025	0.0125
	0.052	0.55	0.3025	0.166375	0.091506	0.0286	0.01573
	0.044	0.601	0.361201	0.217082	0.130466	0.026444	0.015893
	0.05	0.645	0.416025	0.268336	0.173077	0.03225	0.020801
	0.055	0.695	0.483025	0.335702	0.233313	0.038225	0.026566
	0.066	0.75	0.5625	0.421875	0.316406	0.0495	0.037125
	0.074	0.816	0.665856	0.543338	0.443364	0.060384	0.049273
	0.068	0.89	0.7921	0.704969	0.627422	0.06052	0.053863
	0.083	0.958	0.917764	0.879218	0.842291	0.079514	0.076174
	0.089	1.041	1.083681	1.128112	1.174365	0.092649	0.096448
SUM	0.631	7.446	5.834652	4.790008	4.094711	0.493086	0.404374
$(SUM)^2$		55.44292	34.04316	22.94417			

Table to calculate the value of a, b, c using quadratic regression

Formula For quadratic regression model

$$\mathbf{a\,\Sigma x_i^4 + b\,\Sigma x_i^3 + c\,\Sigma x_i^2 = \Sigma x_i^2 y_i}$$
$$\mathbf{a\,\Sigma x_i^3 + b\,\Sigma x_i^2 + c\,\Sigma x_i = \Sigma x_i y_i}$$
$$\mathbf{a\,\Sigma x_i^2 + b\,\Sigma x_i + c\,n = \Sigma y_i}$$

from the given equations the value of a, b and c are

a = 0.0800467

b = -0.0427727

c = 0.0482441

2.2.5The logistic Model

The logistic growth model for the data is as follows

$$P(t+1)-P(t)=rP(1-\frac{P}{K}) \tag{2.5.1}$$

The equation can also be written as,

$$\frac{\Delta P}{P}=r(1-\frac{P}{K}) \tag{2.5.2}$$

The equation says that the ratio of ΛP to P is a quadratic function of P.

Now we can use the quadratic regression to find the values of a and b.

Calculating the ratios on the left-hand side of yields:

1. $a_1 = \frac{P(1951)-P(1950)}{P(1950)} = \frac{0.55-0.5}{0.5} = 0.1$
2. $a_2 = \frac{P(1952)-P(1951)}{P(1951)} = \frac{0.601-0.55}{0.55} = 0.0927$
3. $a_3 = \frac{P(1953)-P(1952)}{P(1952)} = \frac{0.645-0.601}{0.601} = 0.0732$
4. $a_4 = \frac{P(1954)-P(1953)}{P(1953)} = \frac{0.695-0.645}{0.645} = 0.0775$
5. $a_5 = \frac{P(1955)-P(1954)}{P(1954)} = \frac{0.755-0.695}{0.695} = 0.0863$
6. $a_6 = \frac{P(1956)-P(1955)}{P(1955)} = \frac{0.816-0.755}{0.755} = 0.0807$
7. $a_7 = \frac{P(1957)-P(1956)}{P(1956)} = \frac{0.890-0.816}{0.816} = 0.0906$
8. $a_8 = \frac{P(1958)-P(1957)}{P(1957)} = \frac{0.958-0.890}{0.890} = 0.0764$
9. $a_9 = \frac{P(1959)-P(1958)}{P(1958)} = \frac{1.041-0.958}{0.958} = 0.0866$
10. $a_{10} = \frac{P(1960)-P(1959)}{P(1959)} = \frac{1.13-1.041}{1.041} = 0.0854$

Thus, we have the following data:

a	P(t)
0.1	0.500
0.0927	0.550
0.0732	0.601
0.0863	0.695
0.0807	0.755
0.0906	0.816
0.0764	0.890
0.0866	0.958
0.0854	1.041

The linear regression;

	y (a)	x (P(t)	x2	xy
	0.1	0.5	0.25	0.05
	0.0927	0.55	0.3025	0.050985
	0.0732	0.601	0.361201	0.043993
	0.0863	0.695	0.483025	0.059979
	0.0807	0.755	0.570025	0.060929
	0.0906	0.816	0.665856	0.07393
	0.0764	0.89	0.7921	0.067996
	0.0866	0.958	0.917764	0.082963
	0.0854	1.041	1.083681	0.088901
Sum	0.7719	6.806	5.426152	0.579675

Least square fit,

$$m= \frac{N\ \Sigma(xy) - \Sigma x\ \Sigma y}{N\ \Sigma(x2) - (\Sigma x)2}$$

$$m=-0.0145108779$$

$$m=-0.01451$$

$$b = \frac{\Sigma y - m\ \Sigma x}{N}$$

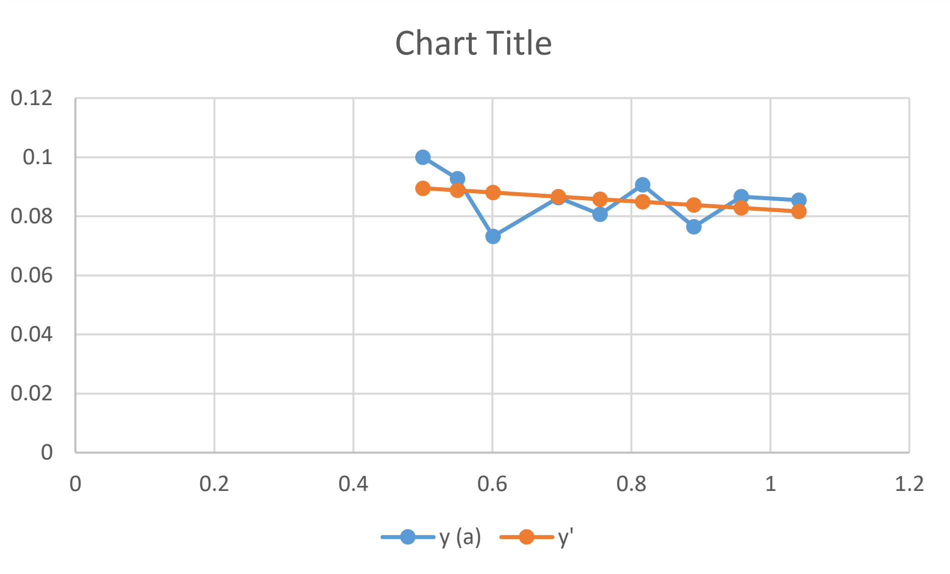
$$b=0.096740115$$

$$b=0.09674$$

The formula after least square fit is

$$y=-0.01451x + 0.09674$$

x (P(t))	y (a)	y'
0.5	0.1	0.089485
0.55	0.0927	0.088759
0.601	0.0732	0.088019
0.695	0.0863	0.086655
0.755	0.0807	0.085784
0.816	0.0906	0.084899
0.89	0.0764	0.083825
0.958	0.0866	0.082839
1.041	0.0854	0.081634



Graph 2.1

2.2.5.a Determining the values of *r* and *K*

We know,

$$y = -0.01451x + 0.09674 \qquad (2.5.3)$$

Substituting the point P(1950) into this equation,

$$y = -0.01451*(0.5) + 0.09674 = 0.089485 \qquad (2.5.4)$$

Similarly, substituting P(1951) into eq 2.5.3,

$$y = -0.01451*(0.55) + 0.09674 = 0.0887595 \qquad (2.5.5)$$

That is to say, we can get values of the ratio, a, where y=a. Then we have,

$$y1 = 0.089485 \text{ and } y2 = 0.0887595$$

Substituting the data of 1950, 1951 and 1952. We have the following two equations:
For 1951,

$$r(1-\frac{0.5}{K}) = 0.089485 \qquad (2.5.6)$$

For 1952,

$$r(1-\frac{0.55}{K}) = 0.0887595 \qquad (2.5.7)$$

Solving equation 2.5.6 and 2.5.7, we get,

$k \approx 6.66713$

And

$r \approx 0.09674$

Therefore we have,

$$P = 0.09674P(1-\frac{P}{6.66713})$$

$$P = 0.09674P - 14509P^2 \qquad (2.5.8)$$

2.2.5.b The Solution for the Logistic Model.

We have equation

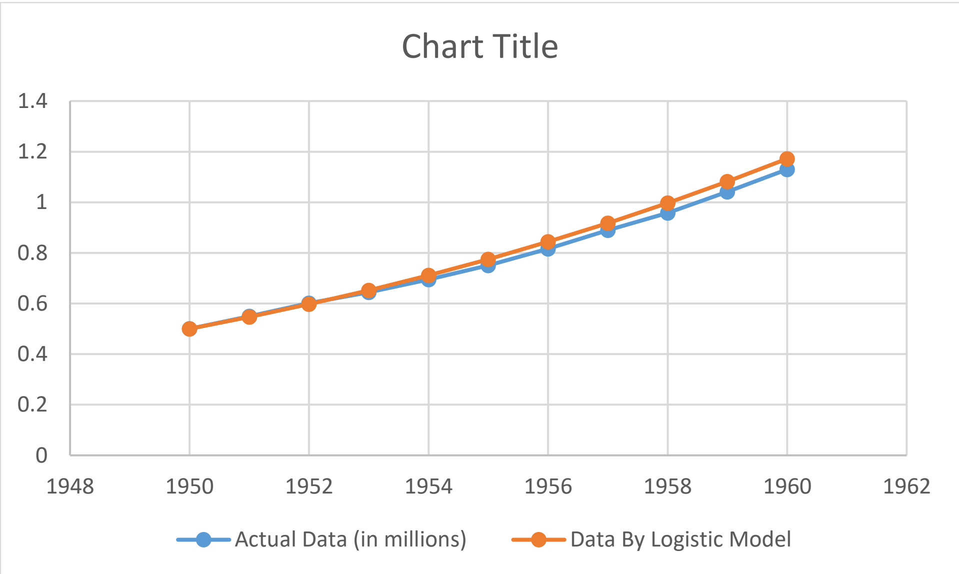
$$\frac{dP}{dt} = 0.09674P - 14509P^2 \qquad (2.5.9)$$

After solving the equation 2.5.9 using the intial condition P(0)= 0.5 we have,

$$y(x) = \frac{6.66759\, e^{0.09674\, x}}{12.3352 + e^{0.09674\, x}}$$

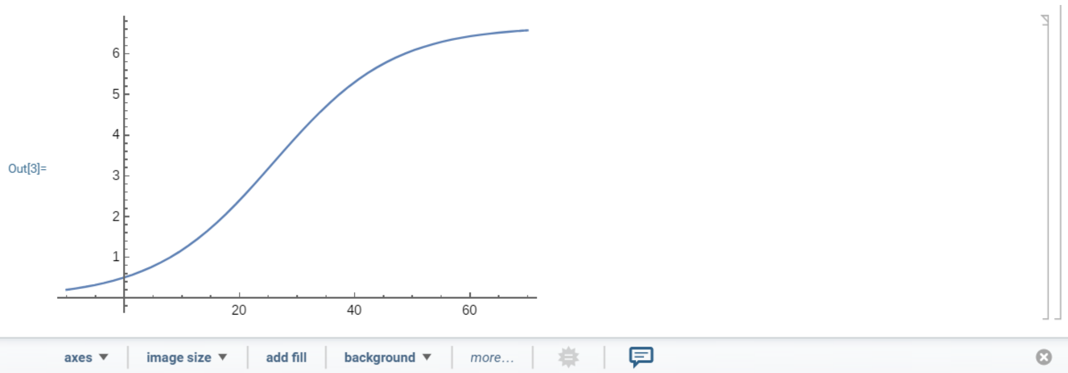
2.2.5.c Values Obtained by Logistic Model

Year	Actual Data (in millions)	Data By Logistic Model
1950	0.5	0.499992501
1951	0.55	0.546622526
1952	0.601	0.597172233
1953	0.645	0.651898939
1954	0.695	0.711053392
1955	0.75	0.774884198
1956	0.816	0.843633499
1957	0.89	0.9175326
1958	0.958	0.996799712
1959	1.041	1.081621102
1960	1.13	1.172171278



Graph 2.2

Solution Curve (graph 2.3)



3. Conclusion

A model for population of the Tibetian Antelope was designed by us. We us the logistic model to find K is which limiting value of the population. According, to our calculations the values of K was 6.66713 million. We then compared the actual population between the 1950 to 1960 to the population data we obtained from our calculation.
In conclusion, our values were close to actual population which is visible in graph 2.2. Though, there are some errors in our data, it can be because of other factors like climate change. Poaching, etc.

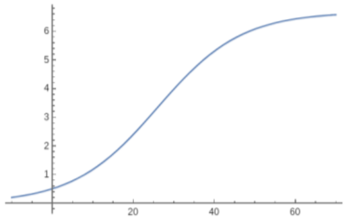
4. References

- Population Modeling by Differential Equations.pdf
- Differential Equations and their Boundary Values
- https://reference.wolfram.com/language/tutorial/DSolveIntroduction.html
- https://www.researchgate.net/publication/229190497_Population_Dynamics_and_Harvest_Potential_of_Mountain_Goat_Herds_in_Alberta

5. Appendix

In[3]:= `Plot[y = (6.66759 E^(0.09674 x)) / (12.3352 + E^(0.09674 x)), {x, -10, 70}]`

Out[3]=



axes ▾ | image size ▾ | add fill | background ▾ | more...

ODE classification

first-order nonlinear ordinary differential equation

Alternate forms

$\{y'(x) = -0.014509\,(y(x) - 6.66759)\,y(x),\ y(0) = 0.5\}$

$\{y'(x) = (0.09674 - 0.014509\,y(x))\,y(x),\ y(0) = 0.5\}$

Alternate form assuming x is real

$\{y'(x) = -0.014509\,y(x)^2 + 0.09674\,y(x) + 0,\ y(0) = 0.5\}$

Differential equation solution — Approximate form — Step-by-step solution

$$y(x) = \frac{6.66759\,e^{0.09674\,x}}{12.3352 + e^{0.09674\,x}}$$

Plots of the solution

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Input

$\{y'(x) = 0.09674\,y(x) - 0.014509\,y(x)^2,\ y(0) = 0.5\}$

ODE names

Separable equation

$$\frac{y'(x)}{0.09674\,y(x) - 0.014509\,y(x)^2} = 1$$

Bernoulli's equation

$$y'(x) = 0.09674\,y(x) - 0.014509\,y(x)^2$$

Bernoulli's equation »

ODE classification

first-order nonlinear ordinary differential equation

Alternate forms

$\{y'(x) = -0.014509\,(y(x) - 6.66759)\,y(x),\ y(0) = 0.5\}$

$\{y'(x) = (0.09674 - 0.014509\,y(x))\,y(x),\ y(0) = 0.5\}$

Alternate form assuming x is real

Input interpretation

$r\left(1-\frac{0.5}{6.66713}\right)=0.089485$

Result

$0.925005\,r=0.089485$

Alternate form

$0.925005\,r-0.089485=0$

Alternate form assuming r is real

$0.925005\,r+0=0.089485$

Number line

Solution Step-by-step solution

$r\approx 0.09674$

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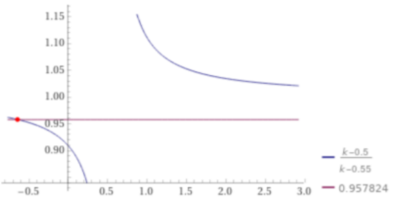

Alternate form

$\frac{k-0.5}{k-0.55}=0.957824$

Expanded form Step-by-step solution

$\frac{k}{k-0.55}-\frac{0.5}{k-0.55}=0.957824$

Solution Step-by-step solution

$k\approx -0.635504$

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$y'(x)=0.0800467\,y(x)^2-0.0427727\,y(x)+0.0482441$

Riccati's equation »

ODE classification

first-order nonlinear ordinary differential equation

Alternate forms

$\{y'(x)=(0.0800467\,y-0.0427727)\,y+0.0482441,\ y(0)=0.5\}$

$\{y'(x)=0.0800467\,(y^2-0.534347\,y+0.602699),\ y(0)=0.5\}$

Differential equation solution Step-by-step solution

$y(x)=0.72892\tan(0.058347\,x+0.309172)+0.26717$

Plots of the solution

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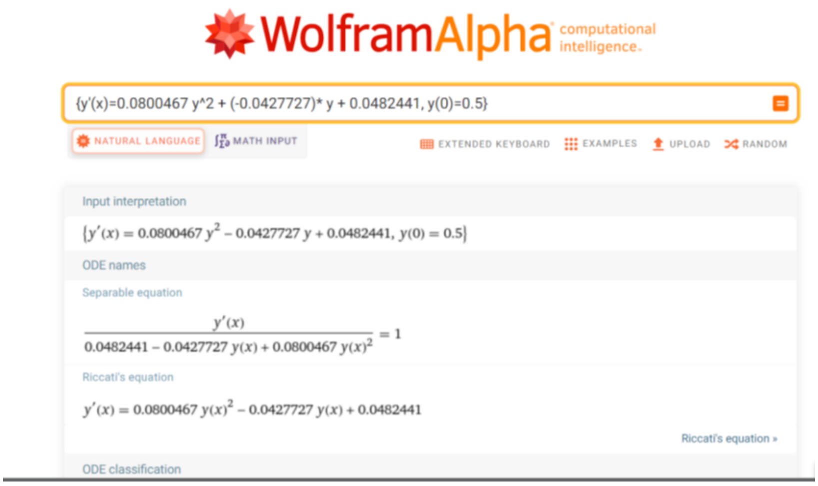
Related Queries:


WolframAlpha computational intelligence.
{y'(x)=0.0800467 y^2 + (-0.0427727)* y + 0.0482441, y(0)=0.5}
NATURAL LANGUAGE
MATH INPUT
EXTENDED KEYBOARD
EXAMPLES
UPLOAD
RANDOM
Input interpretation
$\{y'(x) = 0.0800467\, y^2 - 0.0427727\, y + 0.0482441,\ y(0) = 0.5\}$
ODE names
Separable equation
$\frac{y'(x)}{0.0482441 - 0.0427727\, y(x) + 0.0800467\, y(x)^2} = 1$
Riccati's equation
$y'(x) = 0.0800467\, y(x)^2 - 0.0427727\, y(x) + 0.0482441$
Riccati's equation »
ODE classification