Problem1

In this problem, we are given the following assumptions:

- Current Stock Price $151.03
- Strike Price $165
- Current Date 03/13/2022
- Options Expiration Date 04/15/2022
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Firstly, we implement the closed form Greeks for GBSM:

```python
# Greeks calculations
def black_scholes_greeks(S, K, T, r, q, sigma):
    D1 = d1(S, K, T, r, q, sigma)
    D2 = d2(S, K, T, r, q, sigma)
    call_delta = norm.cdf(D1)
    put_delta = -norm.cdf(-D1)
    gamma = norm.pdf(D1) / (S * sigma * np.sqrt(T))
    vega = S * norm.pdf(D1) * np.sqrt(T) * 0.01  # The vega is usually represented in percentage terms
    call_theta = (-S * sigma * norm.pdf(D1) / (2 * np.sqrt(T)) - r * K * np.exp(-r * T) * norm.cdf(D2)) / 365
    put_theta = (-S * sigma * norm.pdf(D1) / (2 * np.sqrt(T)) + r * K * np.exp(-r * T) * norm.cdf(-D2)) / 365
    call_rho = K * T * np.exp(-r * T) * norm.cdf(D2) * 0.01  # The rho is usually represented in percentage terms
    put_rho = -K * T * np.exp(-r * T) * norm.cdf(-D2) * 0.01
```

The Greeks for this are:

call_gamma: 0.016830979206204362
put_gamma: 0.016830979206204362
call_delta: 0.08301107089626869
put_delta: -0.9169889291037313
call_theta: -0.022456481874505486
put_theta: -0.0033178341735710703
call_rho: 0.011025939156368188
put_rho: -0.13758003122735787
call _vega: 0.06942036604441162
put _vega: 0.06942036604441162

Call Greeks:
  Delta: 0.08301107089626869
  Gamma: 0.016830979206204362
  Vega: 6.942036604441163%
  Theta: -8.126522359668838%
  Rho: 1.1025939156368187%
  Carry Rho: 1.132953825011723%

Put Greeks:
  Delta: -0.9169889291037313
  Gamma: 0.016830979206204362
  Vega: 6.942036604441163%

Theta: -1.9409914783019566%
Rho: -13.758003122735788%
Carry Rho: -12.515271800549371%

Secondly, we implement a finite difference derivative calculation:

```python
# Calculate first order derivative
def first_order_der(func, x, delta):
    return (func(x + delta) - func(x - delta)) / (2 * delta)

# Calculate second order derivative
def second_order_der(func, x, delta):
    return (func(x + delta) + func(x - delta) - 2 * func(x)) / delta ** 2

def cal_partial_derivative(func, order, arg_name, delta=1e-3):
    arg_names = list(inspect.signature(func).parameters.keys())
    derivative_fs = {1: first_order_der, 2: second_order_der}

    def partial_derivative(*args, **kwargs):
        args_dict = dict(list(zip(arg_names, args)) + list(kwargs.items()))
        arg_val = args_dict.pop(arg_name)

        def partial_f(x):
            p_kwargs = {arg_name: x, **args_dict}
            return func(**p_kwargs)

        return derivative_fs[order](partial_f, arg_val, delta)

    return partial_derivative
```

And the Greeks for this are:

Call:
({'delta': 0.08297130307255429,
'gamma': 0.016817658377021868,
'theta': 8.201791134286553%,
'vega': 22.543363068695754%,
'rho': 22.5552079749729%},

Put:
{'delta': -0.9165496337004696,
'gamma': 0.01691091711109038,
'theta': 2.016661124660999%,
'vega': 5.597801398522506%,
'rho': 5.456555021225995%})

The results show consistency in Delta, Vega and Gamma across both methods, indicating similar modeling of basic price movements and curvature. Some slight differences in Theta and Rho suggest divergent approaches to time decay, and interest rate sensitivity. These differences could stem from different assumptions of the methods, computational techniques in the two methods.

Next, we calculate the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:

- Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put:

Binomial tree value without dividend for call: 9.349599724132323
Binomial tree value without dividend for put: 22.756696757667296
Binomial tree value with dividend for call: 5.943325900144015
Binomial tree value with dividend for put: 30.226821632852445

In addition, we also calculate the Greeks of each:

The following are greeks_call_with_dividend, greeks_put_with_dividend, greeks_call_no_dividend, greeks_put_no_dividend

```
({'price': 5.943325900144015,
  'delta': 0.31491723133259686,
  'gamma': 0.008035264186402614,
  'theta': -40.664020637951275%,
  'vega': 15.300335670813347%,
  'rho': 2.6123541553424445%},
 {'price': 30.226821632852445,
  'delta': -0.6439305735493484,
  'gamma': 0.009297238052048256,
  'theta': -147.55478851498364%,
  'vega': 14.673856245393413%,
  'rho': -11.72046488687073%},
 {'price': 9.349599724132323,
  'delta': 0.42641967654359736,
  'gamma': 0.011603243083945261,
  'theta': -82.86069401391454%,
  'vega': 18.08344348373936,%
  'rho': 4.728038087513653%},
{'price': 22.756696757667296,
  'delta': -0.577471616786322,
  'gamma': 0.013017980071740421,
  'theta': -76.51746265375296%,
  'vega': 18.024328802098033%,
  'rho': -8.188856681350387%})
```

The sensitivity of the put and call to a change in the dividend amount:

call_dividend_sensitivity, put_dividend_sensitivity

-2.9414166825542765, 8.99173044723618

The result (-2.9414166825542765, 8.99173044723618) represents the sensitivity of the call and put option prices, respectively, to a change in the dividend amount. Here's what each number signifies: Call Option Sensitivity is -2.9414.This means that for a one-unit increase in the dividend amount, the price of the call option is expected to decrease by approximately 2.9414 units. Call options are generally negatively affected by an increase in dividends because as dividends increase, the expected future stock price decreases (since cash is being taken out of the company and paid to shareholders), making the call option less valuable.
Put Option Sensitivity is 8.9917. This indicates that for a one-unit increase in the dividend amount, the price of the put option is expected to increase by approximately 8.9917 units. Put options tend to become more valuable with an increase in dividends for the opposite reason that call options decrease in value. As the expected future stock price drops due to higher dividends, the put option, which profits from a decrease in the stock price, becomes more valuable. The sensitivities are quite significant, which may be due to several factors such as the proximity of the option's expiration date to the dividend date, the size of the dividend relative to the stock price, or the volatility of the stock. If the options are close to the money (where the stock price is close to the strike price), even small changes in expected future stock prices due to dividends can have a large impact on the value of the options.

Problem2

In this problem, we usethe options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming :

- American Options
- Current Date 03/03/2023
- Current AAPL price is 165
- Risk Free Rate of 4.25%
- Dividend Payment of $1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES:

	Mean	VaR($)	ES($)
Portfolio		6.193469	6.562662
Call	0.952987	4.023378	4.362484
CallSpread	0.000725	13.850938	18.803451
CoveredCall	-0.479103	7.643353	8.041389
ProtectedPut	1.334180	4.352356	4.649988
Put	0.948173	2.625317	2.854175
PutSpread	0.373035	17.739029	22.800091
Stock	0.548978	1.346503	1.385781
Straddle	1.901160	19.005682	24.336698
SynLong	0.004814		

Calculate VaR and ES using Delta-Normal:

Portfolio	Mean	VaR	ES
Call	0	9.51986	11.938288
CallSpread	0	5.26384	6.601068
CoveredCall	0	10.901461	13.670872
ProtectedPut	0	12.141348	15.225741
Put	0	8.187534	10.267498
PutSpread	0	4.91698	6.166092
Stock	0	17.627131	22.105134
Straddle	0	1.332326	1.67079
SynLong	0	17.707393	22.205786

Compare to last week's result:

Portfolio	Mean	VaR	ES
Call	0.761264	6.148784	6.427707
CallSpread	-0.155469	3.987674	4.240254
CoveredCall	-0.963381	13.026246	16.570004
ProtectedPut	0.801492	8.270248	8.806453
Put	1.001422	4.440605	4.621886
PutSpread	0.390362	2.686044	2.827056
Stock	-0.034535	16.884799	20.522397
Straddle	1.762686	1.376917	1.386781
SynLong	-0.240158	17.133547	20.791651

Portfolio	currentValue	VaR95	ES95	VaR99	ES99	Standard_De	min	max	mean	VaR95_Pct	VaR99_Pct	ES95_Pct	ES99_Pct
Straddle	13.37	1.5928718	1.59974193	1.60279469	1.60309908	3.07899465	-1.6032637	28.6683037	0.71532127	0.11913776	0.11987993	0.1196516	0.1199027
SynLong	1.05	15.0239765	18.7003346	21.2041791	23.9803542	9.57415296	-32.37642	40.9258842	0.08568502	14.3085491	20.1944563	17.8098425	22.8384325
CallSpread	4.54	3.51345183	3.88524632	4.13762681	4.27921772	2.23396121	-4.5009665	5.34573839	-0.1115173	0.77388807	0.91137154	0.85578113	0.94255897
PutSpread	3.17	2.4893742	2.73656284	2.89493546	2.97701885	1.91193672	-3.1484861	6.381965	0.19944637	0.78529154	0.91322885	0.86326904	0.93912267
Stock	170.15	14.8630142	18.5548848	21.0697166	23.8552879	9.57806934	-32.271881	41.0704903	0.27715548	0.08735242	0.12383025	0.10905016	0.14020152
Call	7.21	6.02144764	6.47039467	6.77088383	6.92938514	5.51029494	-7.1698627	34.7970939	0.40050314	0.83515224	0.93909623	0.89741951	0.96107977
Put	6.16	5.1134155	5.50291695	5.75038099	5.87433388	4.49543727	-6.1287903	25.2065575	0.31481812	0.83009992	0.93350341	0.89333067	0.95362563
CoveredCall	165.52	10.726336	14.2082184	16.5864179	19.3119942	5.57598324	-27.650033	8.64519219	-0.2026311	0.06480387	0.10020794	0.08583989	0.11667469
ProtectedPut	174.47	7.77495042	8.58727852	9.14111904	9.48572037	6.45864524	-10.088472	36.7674229	0.49297632	0.04456325	0.05239364	0.04921923	0.05436878
Total	545.64	42.2434899	47.938536	51.7891622	55.206999	33.841773	-64.432418	186.942849	2.17175729	0.07742008	0.09491453	0.08785744	0.10117843

We could find that the calculation results are quite close. The mean are derived close to 0, and the VaR and ES are compared as follows:
For the Delta-Normal:
There's a general trend of decreasing risk (as measured by VaR and ES) from the Delta-Normal of results to the last week's for most of the portofolio. This suggests that the last week's method is reflecting a less volatile or less risky market environment for most portfolios. The mean returns have generally decreased as well, which might indicate a less profitable or a more conservative market scenario assumptions in last week's model. This comparison indicates a general trend towards lower risk and lower returns in last week's model.

The Normal method, Most of the calculation results are quite close. For Synlong, PutSpread, CoveredCall, the Var and ES are lower for last week's results, which indicates the different distribution may influence the portofolio's risk to some degree.

Problem3

Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the given stocks.

Based on the past 10 years of factor returns, find the expected annual return of each stock:

AAPL	0.171146
META	0.762903
UNH	-0.037641
MA	0.271602
MSFT	0.192290
NVDA	0.950026
HD	0.282706
PFE	0.005805
AMZN	0.316488
BRK-B	0.146426
PG	0.145970
XOM	0.110124
TSLA	0.139654
JPM	0.407970
V	0.260227
DIS	0.462612
GOOGL	0.262884
JNJ	-0.038059
BAC	0.371124
CSCO	0.263864

Construct an annual covariance matrix for the 10 stocks:

	AAPL	META	UNH	MA	MSFT	NVDA	HD	PFE	AMZN	BRK-B	PG	XOM	TSLA	JPM
AAPL	0.187678	0.312484	0.043679	0.127405	0.152881	0.430491	0.104033	0.039731	0.203032	0.077363	0.051068	0.050553	0.228246	0.100238
META	0.312484	1.579908	0.031448	0.243338	0.322789	0.931842	0.233174	0.083442	0.503301	0.129622	0.069139	0.042602	0.382796	0.192019
UNH	0.043679	0.031448	0.058855	0.039281	0.043127	0.091133	0.033209	0.031809	0.046285	0.031884	0.031573	0.029309	0.044878	0.046263
MA	0.127405	0.243338	0.039281	0.164553	0.127062	0.368959	0.096262	0.044228	0.172149	0.071805	0.046433	0.045088	0.151263	0.108628
MSFT	0.152881	0.322789	0.043127	0.127062	0.196004	0.451145	0.113558	0.043527	0.228225	0.074625	0.047474	0.042509	0.193638	0.098659
NVDA	0.430491	0.931842	0.091133	0.368959	0.451145	1.910206	0.298844	0.094890	0.645605	0.198885	0.096143	0.124574	0.765912	0.286210
HD	0.104033	0.233174	0.033209	0.096262	0.113558	0.298844	0.171198	0.044521	0.174285	0.064209	0.052242	0.023037	0.120132	0.081309
PFE	0.039731	0.083442	0.031809	0.044228	0.043527	0.094890	0.044521	0.074653	0.051819	0.036963	0.032475	0.022817	0.026637	0.046317
AMZN	0.203032	0.503301	0.046285	0.172149	0.228225	0.645605	0.174285	0.051819	0.487487	0.104121	0.047306	0.056700	0.313940	0.137938
BRK-B	0.077363	0.129622	0.031884	0.071805	0.074625	0.198885	0.064209	0.036963	0.104121	0.068186	0.033367	0.044392	0.084380	0.078316
PG	0.051068	0.069139	0.031573	0.046433	0.047474	0.096143	0.052242	0.032475	0.047306	0.033367	0.066419	0.005961	0.031868	0.048631
XOM	0.050553	0.042602	0.029309	0.045088	0.042509	0.124574	0.023037	0.022817	0.056700	0.044392	0.005961	0.160104	0.057039	0.046797
TSLA	0.228246	0.382796	0.044878	0.151263	0.193638	0.765912	0.120132	0.026637	0.313940	0.084380	0.031868	0.057039	0.778792	0.113882
JPM	0.100238	0.192019	0.046263	0.108628	0.098659	0.286210	0.081309	0.046317	0.137938	0.078316	0.048631	0.046797	0.113882	0.172478
V	0.110482	0.198846	0.037018	0.139296	0.106985	0.311906	0.084076	0.040044	0.147164	0.062826	0.043515	0.034882	0.139432	0.097726
DIS	0.159249	0.351335	0.032889	0.151763	0.162759	0.495162	0.125715	0.037424	0.259931	0.089518	0.047661	0.059793	0.234115	0.132593
GOOGL	0.176847	0.446682	0.037246	0.133338	0.194404	0.512562	0.117765	0.038315	0.272511	0.084616	0.041641	0.045074	0.224714	0.109814

Assume the risk-free rate is 0.0425. Find the super-efficient portfolio:

We need to calculate the most efficient portfolio which has the highest Sharpe ratio:

```
The most efficient portfolio consists of:
       weights(%)
AAPL         0.00
META         4.27
UNH          0.00
MA           0.00
MSFT         0.00
NVDA         0.63
HD           7.76
PFE          0.00
AMZN         0.00
BRK-B        0.00
PG           8.35
XOM          0.00
TSLA         0.00
JPM         58.01
V            0.00
DIS         15.91
GOOGL        0.00
JNJ          0.00
BAC          0.00
CSCO         5.07
The Portfolio's Sharpe Ratio is: 1.0462140296202906
```