Problem1

Since the current date is 03/03/2023, and the options' expiration date is 03/17/2023, so the time to maturity using the calendar days is (17 – 3 +1)/365 = 15/365, which is about 0.041.

For a range of implied volatilities between 10% and 80%, the value of the call and the put could be calculated using the following equations:

$$d_1 = \frac{ln\left(\frac{S}{X}\right)+\left(b+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

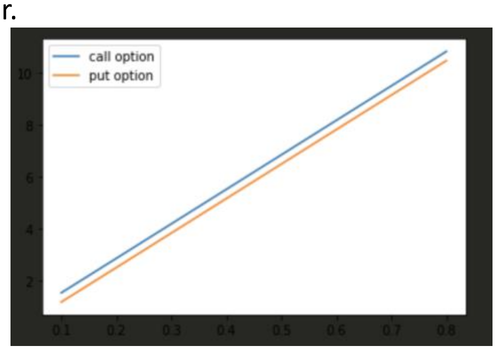
$$Call = Se^{(b-r)T}\Phi\left(d_1\right) - Xe^{-rT}\Phi\left(d_2\right)$$

$$Put = Xe^{-rT}\Phi\left(-\ d_2\right) - Se^{(b-r)T}\Phi\left(-\ d_1\right)$$

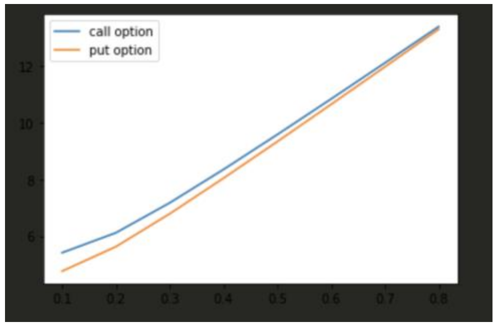
Discussion of these graphs about how the supply and demand affects the implied volatility:

And from the supply and demand perspective: when the strike price for the option is set, the higher the volatility, the more demand for the option to "secure" a price. So the price of the option is higher. That explained why the volatility and value of the option is positively related.

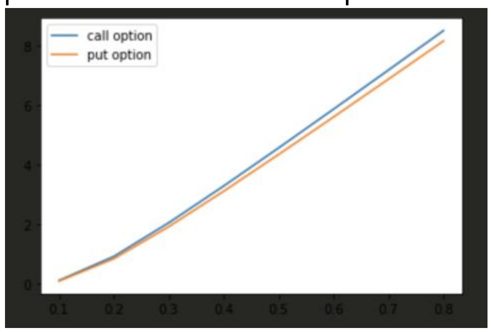
The specific graph of the option value vs volatility depends on the strike price we set. When the strike price for the put and call option is set as 165, the graph is linear. Because the value for b is quite small, and according to the formula for B-S model above, the price of the out and call option are almost linearly influenced by the volatility. More specifically, when the underlying asset's price is exactly equal to the strike price of the option, the option is said to be "at the money" (ATM). At this point, the intrinsic value of both call and put options is zero, because exercising the option would not lead to any profit from the difference between the underlying price and the strike price. The value of an ATM option, therefore, consists solely of its time value, which is influenced by various factors such as time to expiration, risk-free interest rates, dividends, and notably, volatility. Volatility measures the degree to which the underlying asset's price is expected to fluctuate over a period of time. The greater the volatility, the higher the chance that the underlying asset's price will move and thus the higher the potential for the option to gain intrinsic value before expiration. The relationship between volatility and an ATM option's value is almost linear.



When the strike price for the put is set as 170 and call option is set as 160, the graph is:



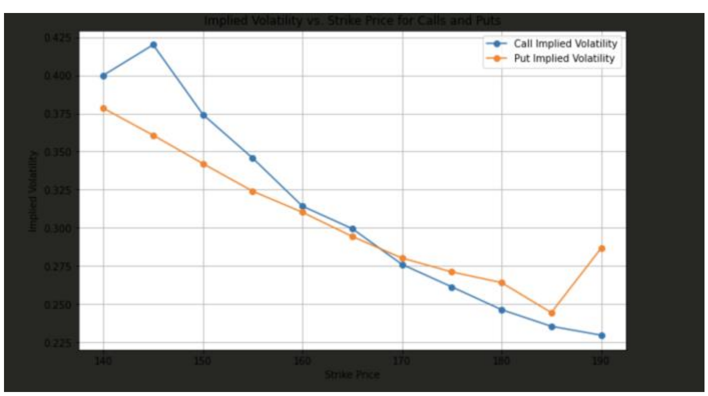
When the strike price for the put is set as 160 and call option is set as 170, the graph is:



They are not strictly linear because: The relationship between volatility and option value is theoretically not perfectly linear; it is more accurately modeled with a complex function that involves a combination of factors. However, near the ATM point, the relationship can be approximated as linear because changes in volatility have a more straightforward and pronounced impact on the time value component of the option's price. But when the option is In-the-money or out-of-money,the relationship may not remain linear due to the complexities of how market participants price options, including adjustments for risk aversion, skew, and other market conditions. Additionally, very high levels of volatility may start to have a less pronounced effect as other factors may dominate option pricing, and the assumption of log-normal price distribution (implied by the Black-Scholes model and others) might break down.

Problem 2

Here is the graph of the implied volatility for the call and put option with different strike price:



Discussions:
The relationship between implied volatility (IV) and the strike price is typically not linear and can manifest in what is known as the "volatility smile" or "volatility skew."

Volatility Smile: In some markets, particularly in currencies and equities, options with strike prices that are far from the current price of the underlying (deep in-the-money and deep out-of-the-money) can have higher implied volatilities than those close to the underlying's price (at-the-money). This phenomenon results in a plot of implied volatility across strike prices that looks like a smile. The volatility smile suggests that traders are expecting extreme movements in the underlying asset, leading to higher demand and higher implied volatility for options with strike prices far from the current price of the underlying asset.

Volatility Skew: In equity markets, particularly for stock options, there is often a skew such that lower strike prices (put options) have a higher implied volatility compared to higher strike prices (call options). This skew reflects the market's greater concern for downside risk, presuming that stock prices are more likely to crash downward than to move upward by an extreme amount. Therefore, the implied volatility increases as the strike price moves lower than the current price of the underlying asset.

Here's why these patterns occur:
Downside Protection: Investors often use options for downside protection. Puts, especially out-of-the-money puts, are commonly used as insurance against a decline in the price of the underlying asset. Higher demand for these puts in anticipation of possible negative events increases their prices and thus their implied volatilities.

Leverage and Speculation: On the call side, especially for out-of-the-money calls, investors might be speculating on an upward move. However, the demand for this sort of speculation is usually less urgent than the need for protection against a downturn, which can result in a less pronounced increase in implied volatility for calls as the strike price increases.

Risk Aversion: The market's natural risk aversion plays a role in skewing implied volatility higher for lower strikes. Traders may be more concerned about significant losses than they are hopeful for equivalent gains, reflecting a risk-averse stance.

Historical Price Movements: Market participants consider historical price movements when determining option prices. Since equities have shown a tendency to fall faster and more dramatically than they rise (due to phenomena like panic selling), this historical perspective can lead to higher implied volatilities for lower strike prices.

Supply and Demand Imbalances: Supply and demand imbalances caused by institutional hedging strategies can also affect the relationship between implied volatility and strike prices.

Understanding this relationship is important for traders when they price options or assess the market's expectations for future volatility. Option pricing models like Black-Scholes assume a log-normal distribution of stock prices, which implies a symmetric volatility structure that does not reflect the actual market, which is why models are adapted or supplemented with empirical data to account for the smile or skew.

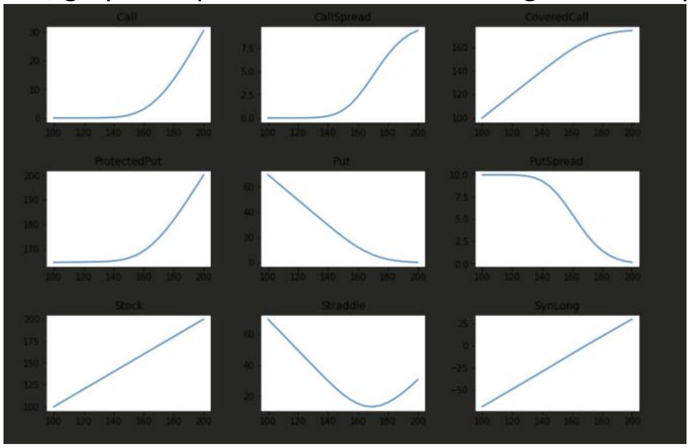
And for the graph I draw using the APLL data, which shows implied volatility plotted against various strike prices for both call and put options, is a typical implied volatility skew graph because:
Put options (yellow line) have higher implied volatilities at lower strike prices. This reflects market concern for downside risk; as the strike price decreases, the implied volatility increases, which is typical in equity markets due to the greater perceived risk of a price fall. This is part of the volatility skew phenomenon.
Call options (blue line) have decreasing implied volatilities as the strike prices increase. The decreasing implied volatility for higher strike prices indicates less concern for large upward price movements. This can also be part of the skew, where there's less demand for out-of-the-money call options compared to put options.
Convergence at At-the-Money: It seems that the implied volatilities for calls and puts converge near the at-the-money strikes, which is typically around the current market price of the underlying asset. This is common as the uncertainty about direction is most balanced at the ATM point, leading to a similar pricing of volatility for calls and puts.
Anomalies: The graph shows an upward spike in implied volatility for put options at the highest strike price shown. This could be an anomaly due to a specific market event, a data error, a sudden change in demand for deep in-the-money puts, or illiquidity at that strike price. Such spikes need to be investigated with market data to understand the reason behind the sudden increase in implied volatility for those options.
This implied volatility structure allows traders to infer market sentiments and expectations. For instance, a higher implied volatility for puts than calls generally suggests that the market is more concerned about potential downside risks than upside potential. Options traders and risk managers analyze these graphs to help in constructing their trading strategies and managing risk appropriately.

Related market dynamics for this graph:
Several market dynamics can create a volatility skew:
Risk Aversion: Investors may be more concerned about significant losses than the potential for gains, reflecting general risk aversion. This typically results in higher implied volatility for puts, especially out-of-the-money (OTM) puts, as investors are willing to pay more for downside protection.
Market Expectations of Downside Risk: If investors expect that the underlying asset is more likely to experience a significant drop than a rise, they may bid up the prices of put options, leading to higher implied volatility on those options.
Leverage Effect: Often, when a stock's price falls, its volatility tends to increase (and vice versa), which can cause a skew. This can occur because a decline in stock price typically signals that the firm's equity is becoming riskier (higher leverage), which can result in higher volatility estimates for OTM puts.

Supply and Demand: If more traders are looking to buy protection against a downturn (by purchasing puts), the increased demand can raise the implied volatility of those options. Conversely, if there is less demand for OTM calls or if sellers are more willing to write these calls, the implied volatility for calls could be lower.
Smile Adjustment for Market Conditions: Sometimes, the implied volatility across strike prices will form what's known as a "volatility smile" in conditions where there's symmetry in the market expectation of an underlying asset moving significantly, either up or down. The skew towards higher implied volatility for puts might be a market adjustment to this phenomenon.
Event Expectations: The anticipation of a specific event (like earnings reports, economic data releases, or geopolitical events) can skew the implied volatility. Traders may expect that the event will have a negative impact, thus they buy more puts to hedge their positions.
Historical Volatility Trends: If the underlying asset has a history of dramatic price falls more than rises, the implied volatility for puts will increase as options traders price in the historical trend into the options.
Illiquidity and Price Gaps: Sometimes, the graph can show a spike in implied volatility if there is a lack of liquidity or if there are large bid-ask spreads. This might especially be true for deep ITM or OTM options, where fewer trades occur, and thus, the market is less efficient.

Problem 3

Part I.
For each of the portfolios, graph the portfolio value over a range of underlying values.



Discuss the shapes:

The graph displays a variety of options trading strategies and the payoff profiles for each, and each of the shapes are related to the characteristics of each strategy:

Graph 1. Call Option: The value of a call option increases as the underlying asset price goes above the strike price, offering potentially unlimited profits as the stock price rises. Below the strike price, the option is out-of-the-money (OTM) and expires worthless, thus the profit remains at zero.

Graph 2. Call Spread: This involves buying a call option at a lower strike price and selling another at a higher strike price. Both options are typically for the same underlying asset and expiration date. The shape reflects a limited profit and loss structure. Profits are capped once the underlying asset price exceeds the higher strike price, as are losses if the price falls below the lower strike.

Graph 3. Covered Call: This strategy combines owning the underlying stock and selling a call option against it. The slope of the stock ownership (linear profit as the stock rises) is flattened due to the call option sold; hence, profits are capped at the strike price of the sold call. If the stock stays below the strike price, the call option expires worthless and the seller keeps the premium.

Graph 4. Protected Put: Also known as a "married put," this involves owning the underlying stock and buying a put option to protect against downside risk. The shape indicates that if the stock price falls below the strike price of the put, the put option will increase in value, offsetting the losses from the stock.

Graph 5. Put Option: The value of a put option increases as the underlying asset price falls below the strike price. The put option gives potentially large profits on the downside, capped to the point where the underlying asset price reaches zero. Above the strike price, the put option is OTM and the profit flatlines at zero, representing the option's premium as the total loss.

Graph 6. Put Spread: Similar to the call spread, but for puts. This strategy involves buying and selling put options at different strike prices. The profit and loss are both limited within a range defined by the two strike prices.

Graph 7. Stock (Long Position): The value is linear, with unlimited potential profit as the stock price increases, and proportional loss as the stock price decreases.

Graph 8. Straddle: A straddle is created by buying a call and a put option with the same strike price and expiration date. The shape shows that profits are potentially unlimited on either side as the stock moves significantly away from the strike price, reflecting the payoff for either the call or the put, depending on the direction of the stock movement. The loss is limited to the premiums paid for both options if the stock price stays at the strike price.

Graph 9. Synthetic Long: This is created by buying a call and selling a put with the same strike and expiration. The shape mirrors that of a long stock position, with unlimited potential profit as the stock price increases and potentially large losses as the stock price decreases, akin to holding the underlying asset itself.

For each strategy, the shape of the graph is crucial for understanding the risk/reward profile. Some strategies like spreads limit both potential gain and loss, providing a more conservative approach, whereas strategies like a naked call or put buying have asymmetric risk/reward profiles with unlimited gain or loss potential on one side of the trade.

The relationship between the portfolio value and the underlying asset value is related to the B-S model we discussed in class:
The Black-Scholes model is a mathematical model used to price European-style options. It uses several factors to calculate the theoretical value of an option: the current underlying price, the option's strike price, the time to expiration, the risk-free interest rate, the dividend yield (if any), and the volatility of the underlying asset.

Here's how the relationships in the graphs you provided are related to option pricing as per the Black-Scholes model: For Call and Put Options, The Black-Scholes model will provide the theoretical price of a call or put option based on its inputs. The graphs of the call and put options depict the payoff at expiration. Black-Scholes gives us the price today, considering the probability of different payoffs, which are reflected in the curvature of the lines in the payoff diagrams as the underlying price changes. And for Spreads (Call Spread, Put Spread), the Spreads involve purchasing one option and selling another. Black-Scholes can be used to price each leg of the spread. The net price of the spread will be the difference between the two. The payoff diagram's shape reflects the limited loss (the net premium paid) and the limited profit (the difference between strike prices minus the net premium). Thirdly, for Covered Call, this strategy involves holding the underlying stock while selling a call option. Black-Scholes would be used to price the call option. The linear increase in the stock portion and the flat payoff above the strike price (due to the call option) are represented in the graph. For the Protected Put: This is a stock plus a put option. The Black-Scholes model would price the put option. The graph reflects the protective nature of the put, which provides a floor to losses if the stock price falls. For Straddle: This involves buying a call and put with the same strike and expiration. Black-Scholes can price each option. The V-shape of the graph is due to the fact that the investor profits from large moves in either direction in the underlying asset. As for Synthetic Long, this position combines a long call and a short put to replicate the payoff of a long stock position. The Black-Scholes model prices each option. The linear payoff matches that of a long stock position.

The Black-Scholes model assumes constant volatility and interest rates, and it doesn't account for early exercise (as it's for European options). Real-world trading conditions can differ, and actual market prices may diverge from the Black-Scholes theoretical values due to these and other factors, like supply and demand dynamics.

Part II.
Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss:

	Mean	VaR	ES
Portfolio			
Call	0.604972	15.797181	18.914327
CallSpread	-0.160003	1.986177	2.990954
CoveredCall	-0.130899	2.454929	4.122473
ProtectedPut	0.584594	16.685168	20.277700
Put	0.098163	0.273888	0.276006
PutSpread	0.077448	0.157878	0.159198
Stock	0.535874	17.441673	21.771266
Straddle	0.703135	14.538330	16.547867
SynLong	0.506809	17.056031	21.280787

The table in the graph shows several portfolios (options strategies and a stock position) and their associated risk metrics. Here's an analysis of each:

Mean

The mean represents the average expected return of each portfolio. It is important to note that these means do not necessarily represent the actual future returns, but they give an indication of how each strategy has performed on average over the period analyzed.

Call, ProtectedPut, Stock, Straddle, SynLong: These portfolios show a positive mean return, suggesting that they could be expected to gain value over time.

CallSpread, CoveredCall: These show a negative mean return, indicating that, on average, they would be expected to lose value. This could be due to the cost of the hedge within the spread or the covered call strategy underperforming in a rising market.

Value at Risk (VaR)

VaR is a risk measure that estimates the maximum loss expected (with a certain level of confidence) over a given period. It is a way to quantify the amount of financial risk within a firm or portfolio over a specific time frame.

Put, PutSpread: These strategies have the lowest VaR, suggesting that they have the lowest risk in terms of potential loss. This makes sense for the PutSpread, as it is a hedged strategy limiting downside risk.

Stock, ProtectedPut, SynLong: These exhibit the highest VaR, indicating they are potentially the riskiest strategies in terms of the maximum amount they could lose. The stock and Synthetic Long (SynLong) have unlimited upside but also substantial downside risk, which contributes to a higher VaR.

Expected Shortfall (ES)

ES is a risk measure that provides an average loss according to the distribution tail when the loss exceeds the VaR threshold at a certain confidence level. It is considered a more accurate measure of risk than VaR because it takes into account the severity of the losses.

Put, PutSpread: As with VaR, these strategies show the lowest ES, aligning with their defensive nature.
Stock, ProtectedPut, SynLong: These strategies, which had the highest VaR, also have the highest ES. This suggests that when losses occur, they can be severe.

The portfolios show a wide range of risk profiles, from conservative (low VaR and ES) to aggressive (high VaR and ES). Investors can choose a strategy that aligns with their risk tolerance. In addition, the graph shows the trade-offs Between Return and Risk: Portfolios with higher mean returns also come with higher risk (VaR and ES), which is typical in financial markets. Plus, the negative mean of the CallSpread and CoveredCall strategies implies that hedging may not always lead to positive expected returns. This could be during market conditions where the underlying asset's growth outpaces the premium earned from writing options.
The AR(1) model assumes returns are normally distributed, which may not capture extreme market events ("black swan" events). If the actual price distribution is having heavy tails, the risk could be underestimated.
In conclusion, the choice of portfolio strategy should align with an investor’s objectives and risk appetite. While strategies with higher mean returns may be attractive, the associated risk levels must also be acceptable to the investor. It is also important to remember that past performance is not indicative of future results, and risk metrics like VaR and ES are based on historical data and assumptions that may not hold true in the future.