3D Geometry

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Problem. Circle ω touches side BC of triangle $\triangle\ ABC$ at point D. AD meets ω again at E. The tangent to ω at E meets AB and AC at J and I respectively. Both BI and CI intersect ω at points K, L, M and N as shown in the figure below. Show that $A = KL \cap MN$

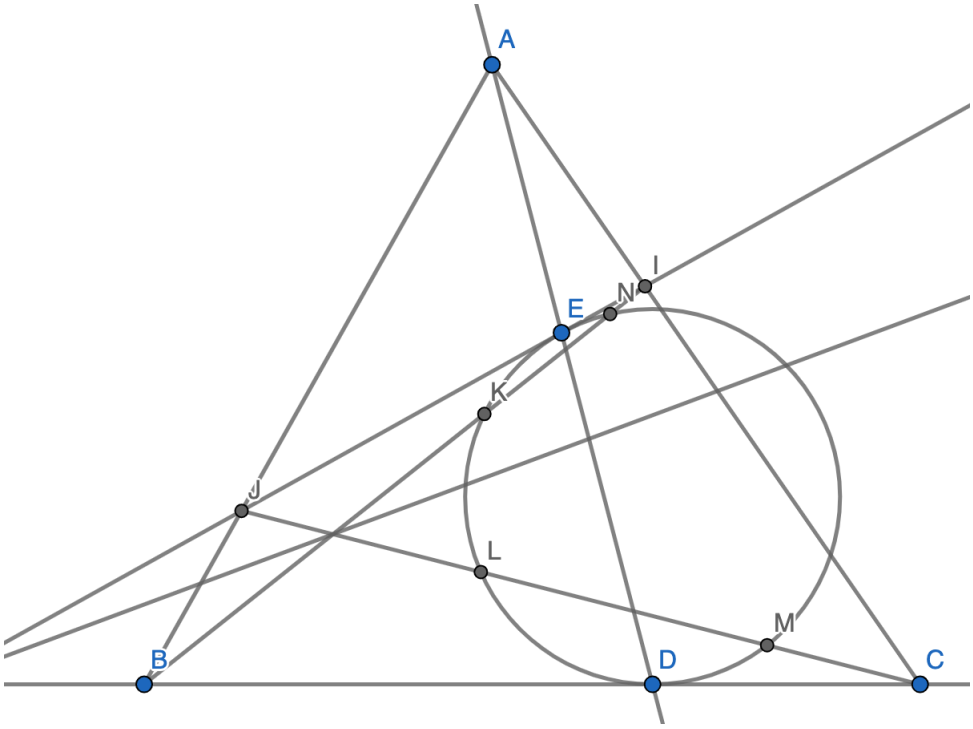

Figure 1: A is moving on the fixed median WOW!

My Solution. We will project the figure from this plane to another plane via the Perspective point which we can choose somewhere in the third dimension as so:

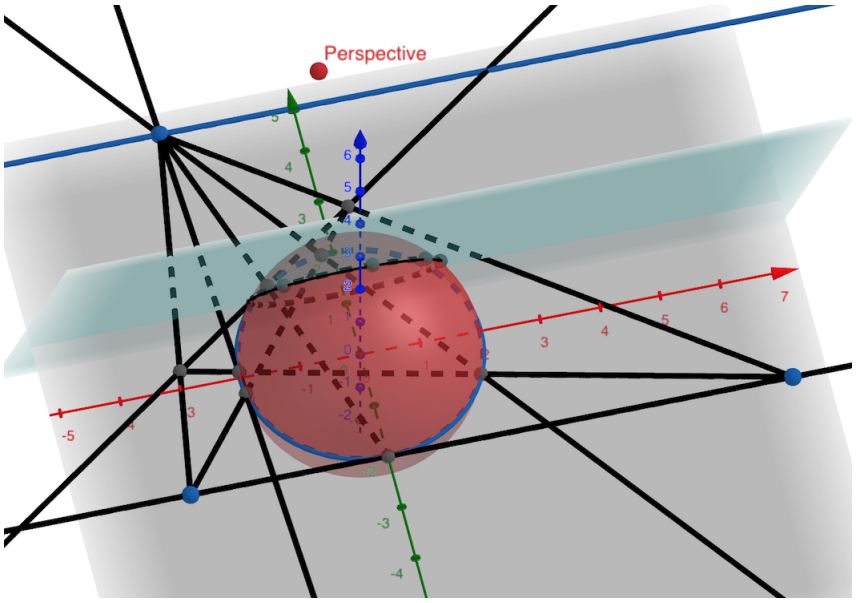

Figure 2: This figure shows initial figure and the perspective point

And since we can preserve a circle and send any point to infinity line (the rules of projective transformation which can be proved and are allowed at the IMO):

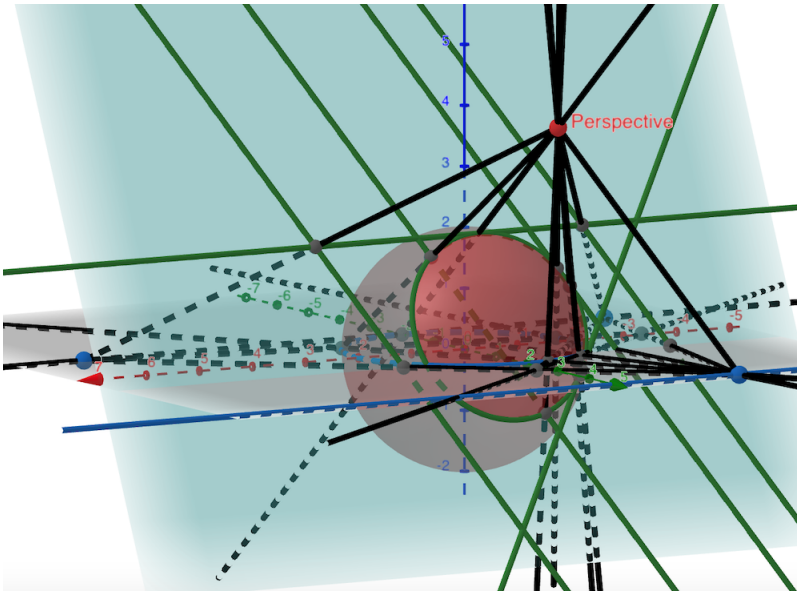

Figure 3: Here we show the figure after projection with the initial figure visible so you can visualize what's going on

And now we clear things up a bit:

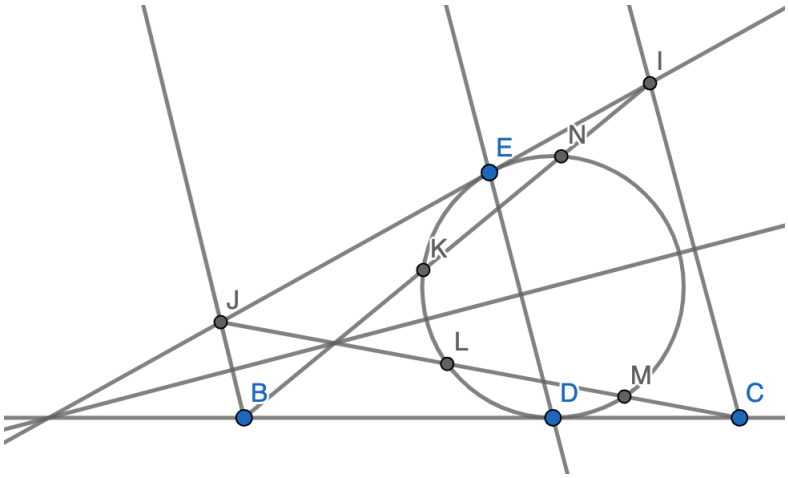

Figure 4: This is the figure after projection

and how elegant does that look, now we just need to prove BJ, CI, KL and MN are parallel which is true by symmetry. □