Nice Geometry Problems

Dedicated to Mathscopers

Problem 1: $(O_1),(O_2)$ are the B,C mixtilinear excircles of ΔABC. (I) is the incenter of ΔABC, it touches AB,AC,BC at K,L,F resp. CK cuts (O_2) at M, BL cuts (O_1) at N, MB cuts CN at P. Prove that $\widehat{BAF}=\widehat{CAP}$.

Problem 2: Let ABC be a triangle inscribed in circumcircle (O). Denote $A_1,B_1.C_1$ respectively to be the projections of A,B,C on to BC,CA,AB. Let A_2,B_2,C_2 respectively be the intersections of AO,BO,CO with BC,CA,AB. A circle Ω_a passes through A_1,A_2 and lies tangent to the arc of BC that does not contain A of (O) at T_a. The same denition holds for T_b,T_c. Prove that AT_a,BT_b and CT_c are concurrent.

Problem 3: Let ABC be a triangle. Point M and N lie on sides AC and BC respectively such that $MN||AB$. Points P and Q lie on sides AB and CB respectively such that $PQ||AC$. The incircle of triangle CMN touches segment AC at E. The incircle of triangle BPQ touches segment AB at F. Line EN and AB meet at R, and lines FQ and AC meet at S. Given that $AE=AF$, prove that the incenter of triangle AEF lies on the incircle of triangle ARS.

Problem 4: Given three parallel lines l_1, l_2, l_3, points A, B, C are on l_1, l_2, l_3, respectively AC intersect l_2 at point N, angle $ABC=90$ point D is on line l_2 such that N is midpoint of BD, let circles a, b wich goes through point D such that a is tangent to l_1, b is tangent to l_3, a, b intersects at point E, J is midpoint of DE. Prove that line BJ goes through midpoint of AC.

Problem 5: BD,CE are the altitudes of ΔABC. M is the midpoint of BC. BD,CE cut (EDM) at S,R resp. SC cuts BR at Q. ES cuts DR at G. Prove that A,Q,G are collinear.

Problem 6: Given triangle ABC with its circumcircle (O) and its orthocenter H. Let $H_aH_bH_c$ be the orthic triangle of triangle $ABC.P$ is an arbitrary point on Euler line. AP,BP,CP intersect (O) again at A_1,B_1,C_1. Let A_2,B_2,C_2 be the reflections of A_1,B_1,C_1 wrt H_a,H_b,H_c, respectively. Prove that H,A_2,B_2,C_2 are concyclic.

Problem 7: A triangle $\triangle ABC$ is given with orthocenter H and circumcenter O and let P be, an abitrary point on OH. We denote the points $K\equiv(O)\cap BP,\ L\equiv(O)\cap CP$, where (O) is the circumcircle of $\triangle ABC$ and let be the points $X,\ Z$, as the symmetric points of $K,\ L$ respectively, with respect to the points $E\equiv AC\cap BH$ and $F\equiv AB\cap CH$. The lines through the points $X,\ Z$ and perpendicular to $HX,\ HZ$, intersect the line segments $KH,\ LH$ respectively, at points $M,\ N$. Prove that $MN\parallel KL$.

Problem 8: $\odot O_1$ and $\odot O_2$ are mixtillnear excircles, they cut lines $\overline{AB},\overline{BC}$ and $\overline{CA}$ at points D,E,F,G respectively. Points M,N are the midpoints of $\overline{DE}$ and $\overline{GF}$, connect M to F and connect $\overline{NF}$, $\overline{MF}\cap\overline{O_1O_2}=W,\overline{NE}\cap\overline{O_1O_2}=V$. Prove that $\dfrac{WB}{VC}=\dfrac{AB}{CA}$

Problem 9: Let ABC be a triangle. Let Γ be the inscribed circle. A', B', C' are the points of tangency on the side BC, AC, AB. The point T is the intersection between the extension of BC and $C'B'$. Let Q be the intersection of AA' and the inscribed circle Γ. Show that TQ is tangent to Γ .

Problem 10: The A mixtilinear incircle of ΔABC cuts (ABC) at P. D is on BC such that $AC + BD = AB + CD$. Prove that $\widehat{DPC} = \widehat{ACB}$.

Problem 11: ABC a triangle. P: a point inside ΔABC.1, 2, 3: the resp. circumcircles of the triangles PBC, PCA, PAB. U the midpoint of the arc BPC. V, W the resp. midpoints of the arcs CA, AB which doesn't contain P. Prove that: P, U, V, W are concyclic.

Problem 12: Let ABC be an acute-angled triangle, and let P and Q be two points on its side BC. Construct a point C_1 in such a way that the convex quadrilateral $APBC_1$ is cyclic, $QC_1 \parallel CA$, and the points C_1 and Q lie on opposite sides of the line AB. Construct a point B_1 in such a way that the convex quadrilateral $APCB_1$ is cyclic, $QB_1 \parallel BA$, and the points B_1 and Q lie on opposite sides of the line AC. Prove that the points B_1, C_1, P, and Q lie on a circle.

Problem 13: Let (P) and (Q) be two circles on a plane such that (Q) lies inside (P). Three circles $\omega_A, \omega_B, \omega_C$ are internally tangent to $(P), (Q)$ at $P_A, Q_A, P_B, Q_B, P_C, Q_C$, respectively. A_1, A_2 are the intersections of ω_B and ω_C, and similarly define B_1, B_2, C_1, C_2. Denote by Γ_A the circumcircle of $P_A Q_A A_1 A_2$ (it is easy to see it's cyclic), and similarly Γ_B, Γ_C.
a) Show that $\Gamma_A, \Gamma_B, \Gamma_C$ are coaxial, their centers lying on a line ℓ.
b) Show that the envelope of ℓ as ω_A varies is a conic.

Problem 14: Given B, C mixtilinear excircles of ΔABC touch the lines BC, AC, AB at D, G; E; F resp. Points M, N are midpoints of GF, DE. ME cuts NF at P. The line GF cuts the line DE at T, the lines TA, PA cuts the line BC at H, Q resp. Prove that $BQ = CH$.

Problem 15: Given M, N are the C, B excenters of ΔABC, the A mixtilinear excircles of ΔABC touch the lines AB, AC at H, I resp. Prove that MI, HN, BC are concurrent.

Problem 16: Given ΔDEF is the orthic triangle of ΔABC. I_a, I_b, I_c are the A, B, C excenters of $\Delta AEF, \Delta BDF, \Delta CDE$. $I_a I_b$ cuts DE at M. $I_a I_c$ cuts DF at K. $I_c I_b$ cuts FE at L. Prove that L, F, K are collinear.

Problem 17: $(O_1), (O_2), (O_3)$ are the A, B, C mixtilinear incircles of ΔABC. They touch (ABC) at D, E, F reps. FO_1 cuts DO_3 at Z. FO_2 cuts EO_3 at X. DO_2 cuts EO_1 at Y. Prove that YO_3, XO_1, ZO_2 are concurrent.

Problem 18: The B, C mixtilinear excircles of ΔABC touch CB at D, W resp. They meet the extentions of AC, AB at G, H resp. The incircles of ΔABC touches AB, AC at Q, N resp. WH cuts DG at M. QD cuts WS at P. Prove that $\widehat{BAM} = \widehat{PAC}$.

Problem 19: P is arbitrary point on the side AB of a $\triangle ABC$ with circumcircle (O) and CP cuts (O) again at D..$(I_1),(I_2)$ are ordinary incircles of $\triangle APC, \triangle BPC$ with incenters I_1, I_2.. $(J_1),(J_2)$ are Thebault circles with centers J_1, J_2, tangent to the rays PD, PA resp PD, PB and internally tangent to the circumcircle (O). Prove that the center lines I_1I_2, J_1J_2 intersect on the line AB.

Problem 20: Let ABC be a triangle with circumcircle (O). P, Q are two point on BC. AP, AQ cut (O) again at M, N, respectively. $(I_1),(I_2),(I_3),(I_4)$ are incircles of triangle PAB, QAC, QAB, PAC, respectively. $(J_1),(J_2),(J_3),(J_4)$ are Thebault circles tangent to the rays PB, PM; QC, QN; QB, QN; PC,PM, respectively and internally tangent to the circumcircle (O). Prove that I_1I_2 ,I_3I_4 ,J_1J_2, J_3J_4 and BC are concurrent.

Problem 21: Let ABC be a triangle and a point P. PA, PB, PC cut BC, CA, AB at A_1, B_1, C_1, respectively. The intersections of B_1C_1 and BC, C_1A_1 and CA, A_1B_1 and AB are collinear on a line d. PA, PB, PC cut d at A_2, B_2, C_2, respectively. ℓ is a line cutting BC, CA, AB at A_3, B_3, C_3, respectively. $B_3B_2 \cap C_3C_2 \equiv A_4, C_3C_2 \cap A_3A_2 \equiv B_4, A_3A_2 \cap B_3B_2 \equiv C_4$. B_4C_4, C_4A_4, A_4B_4 cut d at A_2, B_2, C_2 ,respectively. Let A_5, B_5, C_5 be the points such that cross ratio $(B_4C_4A_2A_5) = (C_4A_4B_2B_5) = (A_4B_4C_2C_5) = -1$.

a) Prove that A_4A_5, B_4B_5, C_4C_5 are concurrent at point Q.

b) PQ cut ℓ, d at R, S, respectively. Prove that $(PQRS) = -1$.

Problem 22: Let ABC be a triangle with orthocenter H. M, N are points on CA, AB, respectively. Constructed triangle $\triangle PBC \sim \triangle HNM$ and they are in opposite directions such that P and A are in the same side with BC. Prove that $PH \perp MN$.

Problem 23: Let ABC be a triangle and a point P. Q is isogonal conjugate of P with respect to triangle ABC. K is midpoint of PQ. H is projection of P on BC. Ray HP cuts circle $(Q, 2KH)$ at A'. Circumcircle $(HQA') \cap (K, KH) = \{H, M\}$. Prove that HK passes through M.

Problem 24: Let ABC be a triangle and A-excircle touches BC at D. d is a line passing through D. d cut CA, AB at E, F, respectively. M is E-excenter of triangle DCE, N is F-excenter of triangle DBF, P is incenter of triangle AEF. Prove that A, M, N, P are concyclic.

Problem 25: A circle (O) through B,C is tangent to the incircle (I) of $\triangle ABC$ at X. (O) cut CA,AB at P,Q and BP meet CQ at N. AH is the altitude of $\triangle ABC$. M is the midpoint of AH. I_1, I_2 are the incenters of $\triangle PNQ$, $\triangle CNB$. AX meet (O) at Y (different from X).Prove that:

(1) D,I_2,M,X are collinear, where it's well known that D,M,X are collinear.

(2) I,I_2,Y are collinear.

(3) X,I_1,I_2,Y are cyclic.

(4) when a tangent to (I) at X cut BC at K, a circle (K, KX) pass I_1.

Problem 26: Let ABC be a triangle with circumcenter O. P, Q are two isogonal conjugate with respect to triangle such that P, Q, O are collinear. Prove that four nine-point circles of triangles ABC, APQ, BPQ, CPQ have a same point.

Problem 27: Let ABC be a triangle and O is circumcenter I is incenter. OI cuts BC, CA, AB at D, E, F. The lines passing through D, E, F and perpendicular to BC, CA, AB bound a triangle MNP. Let Fe, G be Feuerbach points of triangle ABC and MNP. Prove that OI passes through midpoint of FeG.

Problem 28: Let (O_1) and (O_2) be two circles and d is their radical axis. I is a point on d. IA, IB tangent to $(O_1), (O_2)$ $(A \in (O_1), B \in (O_2))$ and A, B have same side with O_1O_2, respectively. IA, IB cut O_1O_2 at C, D. P is a point on d. PC cut (O_1) at M, N such that N is between M and C. PD cut (O_2) at K, L such that L is between K and D. MO_1 cuts KO_2 at T. Prove that $TM = TK$.

Problem 29: Let ABC be a triangle with circumcircle (O) and P, Q are two isogonal conjugate points. A_1, B_1, C_1 are midponts of BC, CA, AB, resp. PA, PB, PC cut (O) again at A_2, B_2, C_2, resp. A_2A_1, B_2B_1, C_2C_2 cut (O) again at A_3, B_3, C_3. A_3Q, B_3Q, C_3Q cut BC, CA, AB at A_4, B_4, C_4. Prove that AA_4, BB_4, CC_4 are concurrent.

Problem 30: Let ABC and $A'B'C'$ be two triangles inscribed circle (O). Prove that orthopoles of $B'C', C'A', A'B'$ with respect to triangle ABC and orthopoles of BC, CA, AB with respect to triangle $A'B'C'$ lie on a circle.

Problem 31: Let ABC be a triangle inscribed circle (O) and orthocenter H. P is a point on (O). d is Steiner line of P. M is Miquel point of d with respect to triangle ABC. Prove that M, P, H are collinear iff $OP \perp d$.

Problem 32: Let ABC be a triangle with orthocenter H and circumcenter O. (ω) is a circle center O and radius k. $(K), (L)$ are two circle passing through O, H and touch (ω) at M, N.
a) Prove that $(K), (L)$ have the same radius.
b) Prove that MN passes through H.
c) Prove that circle (Ω) center H radius k also touches (K) and (L) at P, Q and PQ passes through O.

Problem 33: Let ABC be triangle inscribed (O). (O_a) is A-mixtilinear incircle of ABC. Circle (ω_a) other than (O) passing through B, C and touches (O_a) at A'. Similarly we have B', C'. Prove that AA', BB', CC' are concurrent.

Problem 34: Let O be circumcenter of triangle ABC. D is a point on BC. (K) is circumcircle of triangle ABD. (K) cuts OA again at E.
a) Prove that B, K, O, E are concyclic on circle (L).
b) (L) cuts AB again at F. G is on (K) such that $EG \parallel OF$. GK cuts AD at S. SO cuts BC at T. Prove that O, E, T, C are concyclic.

Problem 35: $(O_1), (O_2), (O_3)$ are the A, B, C mixtilinear incircles of ΔABC. They touch (ABC) at D, E, F resp. $(O_2), (O_3)$ touch BC at W, V resp. I is the incenter of ΔABC. DV cuts FI at Y, EF cuts BC at Z, DW cuts EI at X. Prove that X, Y, Z are collinear.

Problem 36: The A mixtilinear incircles of ΔABC touches $AB, AC, (ABC)$ at G, H, D resp. BI cuts DG at S. DH cuts IC at T. Prove that $ST \parallel GH$.

Problem 37: The A, B, C mixtilinear incircles of ΔABC touch the ABC at D, E, F resp and touch BC, CA, AB at W, V, H, T, U, G resp. FW cuts DG at N, VE cuts DH at Y, CY cuts FT at K, BN cuts EU at P. Prove that $PK \parallel BC$.

Problem 38: ΔABC, $AB > BC > CA$. ΔDEF is the orthic triangle of ΔABC. I_a, I_b, I_c are the A, B, C excenters of triangles AEF, BDF, CDE resp. Prove that:
a) $\widehat{BI_aC} + \widehat{AI_bC} - \widehat{BI_cA} = 180$.
b) $\dfrac{BI_a}{CI_a}.\dfrac{CI_b}{AI_b}.\dfrac{AI_c}{BI_c}$.

Problem 39: AB, BE, CF are concurrent. The angle bisector of $\widehat{AEB}$ cuts the angle bisector of $\widehat{ADB}$ at Z. The angle bisector of $\widehat{BFC}$ cuts the angle bisector of $\widehat{BEC}$ at X.The angle bisector of $\widehat{ADC}$ cuts the angle bisector of $\widehat{AFC}$ at Y. Prove that AX, BY, CZ are concurrent.

Problem 40: AB, BE, CF are concurrent. The angle bisector of $\widehat{AEB}$ cuts the angle bisector of $\widehat{AFC}$ at T. The angle bisector of $\widehat{BFC}$ cuts the angle bisector of $\widehat{BDA}$ at K.The angle bisector of $\widehat{CEB}$ cuts the angle bisector of $\widehat{ADC}$ at J. Prove that KE, FJ, DT are concurrent.

Problem 41: $(O_1), (O_2), (O_3)$ are the three mixtilinear incircles of ΔABC. They cut (ABC) at D, E, F resp. Their points tangency on BC, CA, AB are W, V, H, T, Y, G resp. UT cuts EF at X, DF cuts GW at Y. Prove that $AX \parallel BY$.

Problem 42: ΔDEF is the orthic triangle of ΔABC. I_a, I_b, I_c are the A, B, C excenters of ΔAEF, ΔBDF, ΔCDE. Prove that DI_a, EI_b, FI_c are concurrent.

Problem 43: $(O_1), (O_2)$ are the B, C excircles of ΔABC resp, the points of tangency are D, K, E, F, L, G. GK, DL cuts $(O_1), (O_2)$ at M, N resp. ML cuts NK at P. Prove that $AP \perp BC$.

Problem 44: Circle (O) and circle (P) are externally tangent at T. A, B, M, N are on (O) such that AN and BM are tangent to (P) at F, E resp. NT, MT meet (P) at R, Q resp. FR cuts EQ at U. Prove that $\widehat{UTR} = \widehat{UTQ}$

Problem 45: $(O_1), (O_2)$ are the B, C mixtilinear excircles of ΔABC resp. The points of tangency are G, F, E, D. M is the midpoint of DE. MF cuts O_1O_2 at W. Prove that $BW \perp O_1O_2$.

Problem 46: The Mixtilinear excircles of ΔABC cuts each side at H, I, D, E, F, G. IE cuts DF at K. HF cuts GE at J. JC cuts BK at S. Prove that IJ, HK, AS are concurrent.

Problem 47: $(S), (P), (Q)$ are the A, B, C mixtilinear incircles of ΔABC resp. $A-$ mixtillinear touches the circumcircle of ΔABC at D. DQ cuts (Q) at M. DP cuts (P) at N. Prove that

$MN \parallel PQ$.

Problem 48: $(O_1), (O_2), (O_3)$ are the mixtilinear excircle of ΔABC, H, I, D, E, F, G are the points of tangency on BC, CA, AB resp. GO_2 cuts DO_3 at X, define Y, Z similarly. Prove that AX, BC, CZ are concurrent.

Problem 49: The three excircles of ΔABC touch three sides at I, J, D, G, H, E resp. M, N, F, T, P, K are the midpoints of IJ, HG, DJ, GD, HE, EL. Prove that PF, KT, MN are concurrent.

Problem 50: ΔDEF is the orthic triangle of ΔABC. The incenters of $\Delta ADF, \Delta BDE, \Delta CEF$ are K, I, X resp. The points tangency are G, M, W, Q, N, H. IM cuts KN at Y, KH cuts IG at Z. Prove that X, Y, Z are collinear.

Problem 51: The B, C excircles of ΔABC touch AC, AB at F, G resp. They touch BC at D and E resp. EF, DG cut the two circles at L, M. CG, FB cut the two excircles at T, S. TB cuts SC at H. MB cuts LC at P. Prove that H, P, A are collinear.

Problem 52: The A mixtilinear incircle of ΔABC touches AB, AC at G, H resp. B, C mixtilinear incircles touch BC at V, W resp. M, N are the midpoints of GW, HV resp. GN cuts HM at P. Prove that MV, WN, AP are concurrent.

Problem 53: The mixtilinear incircles wrt $\widehat{B}, \widehat{C}$ for ΔABC touches the (ABC) at E, F resp. The mixtilinear incircle wrt $\widehat{A}$ touches AB, AC at G, H resp. FH cuts GE at P. T is on BC such that $AB + BT = p$. Prove that $\widehat{BAT} = \widehat{PAC}$.

Problem 54: The mixtilinear excircles of ΔABC touch three sides at H, I, D, E, F, G resp. J, L, N, K, M, P are midpoints of BC, CA, AB, EF, GH, ID resp. Prove that ML, PN, JK are concurrent.

Problem 55: The mitilinear incircle and mixtilinear excircle wrt $\widehat{A}$ of ΔABC touches the ABC at T, X resp. AX cuts the mixtilinear incircle wrt $\widehat{A}$ at S. The incircle of ΔABC touches AB, AC at E, F resp. Prove that $\dfrac{ES}{SF} = \dfrac{TF}{TE}$.

Problem 56: ΔDEF is the orthic triangle of ΔABC. The incircles of triangles BDE, CEF, ADF are $(I), (J), (K)$ resp. $(I), (K)$ cut AB, BC, CA at H, G, L, S resp. KS cuts JH at M. JG cuts IL at N. Prove that MN, AB, LK are concurrent.

Problem 57: A, B, C, D are on (O). (P) externally touches (O) at G, it also touches AB, BC at F, E resp. The angle bisectors of $\widehat{FDC}, \widehat{DCE}$ cut EF at L, K resp. Prove that $\widehat{ABG} = \widehat{LKG}$

Problem 58: B, C are on (O). The A mixtilinear incircle of ΔABC touches (O) and AB at P, D resp. (BDP) cuts BC, CP at H, G resp. Prove that $BG \parallel DH$.

Problem 59: The A mixtilinear incircle of ΔABC touches AB, AC at G, H resp. It also touches the (ABC) at D. (BGD) and (HCD) cut BC at J, I resp. Prove that $\widehat{IGJ} = \widehat{IHJ}$.

Problem 60: Let O be the circumcenter of the acute triangle ABC. Suppose points A', B' and C' are on sides BC, CA and AB such that circumcircles of triangles $AB'C', BC'A'$ and $CA'B'$ pass through O. Let l_a be the radical axis of the circle with center B' and radius $B'C$ and circle with center C' and radius $C'B$. Define l_b and l_c similarly. Prove that lines l_a, l_b and l_c form a triangle such that it's orthocenter coincides with orthocenter of triangle ABC.

Problem 61: Let ABC be a triangle. P is a point inside ABC. AP, BP, CP cut BC, AC, AB at E, F, D respectively. The angle bisector of $\angle ADC$ cuts the angle bisector of $\angle AFB$ at K. The angle bisector of $\angle BDC$ cuts the angle bisector of $\angle AEB$ at I. The angle bisector of $\angle AEC$ cuts the angle bisector of $\angle CFB$ at J.Prove that CJ, BI, AK are concurrent.

Problem 62: Let AA', BB', CC' be three diameters of the circumcircle of an acute triangle ABC. Let P be an arbitrary point in the interior of $\triangle ABC$, and let D, E, F be the orthogonal projection of P on BC, CA, AB, respectively. Let X be the point such that D is the midpoint of $A'X$, let Y be the point such that E is the midpoint of $B'Y$, and similarly let Z be the point such that F is the midpoint of $C'Z$. Prove that triangle XYZ is similar to triangle ABC.

Problem 63: Let $\omega, \omega_1, \omega_2$ be three mutually tangent circles such that ω_1, ω_2 are externally tangent at P, ω_1, ω are internally tangent at A, and ω, ω_2 are internally tangent at B. Let O, O_1, O_2 be the centers of $\omega, \omega_1, \omega_2$, respectively. Given that X is the foot of the perpendicular from P to AB, prove that $\angle O_1XP = \angle O_2XP$.

Problem 64: Let ABC be a triangle with altitudes AD, BE, CF, medians AM, BN, CP and three Feuerbach points F_a, F_b, F_c, (E) is nine points circles. Tangents at F_a, F_b, F_c of (E) intersect base a triangle XYZ.
a) Prove that two triangle XYZ and DEF are perspective.
b) Prove that two triangle XYZ and MNP are perspective.

Problem 65: Suppose that the B, Cmixtilinear incircles (with centers atO_b, O_c ,respectively) of the triangle touch the Circumcircle of triangle ABC at D, E . respectively. The lines EO_b, DO_cmeet at F. Show that the line AF passes through the incenter of the triangle.

Problem 66: Let ABC be a triangle with circumcircle (O) and a point P. PA, PB, PC cut (O) again at D, E, F. Let d_a, d_b, d_c be Simson line of D, E, F with respect to triangle ABC, respectively. Prove that d_a, d_b, d_c intersect base a triangle that is orthologic with triangle ABC.

Problem 67: Let ABC be a triangle with mixtilinear incircles $(O_a), (O_b), (O_c)$. (O_a) cuts BC at A_1, A_2 such that A_1 is between B, A_2. (O_b) cuts CA at B_1, B_2 such that B_1 is between C, B_2. (O_c) cuts AB at C_1, C_2 such that C_1 is between A, C_2. Prove that A_2B_1, B_2C_1, C_2A_1 intersect base a triangle that is perspective with triangle ABC.

Problem 68: Let ABC be a triangle and a point P. (P) is a circle center (P). Let D, E, F be inverse points of A, B, C with respect to (P). X, Y, Z lie on BC, CA, AB such that $DX \perp PA, EY \perp PB, FZ \perp PC$. Prove that X, Y, Z are collinear.

Problem 69: Let DEF be pedal triangle of a point P with respect to triangle ABC. X, Y, Z lie on EF, FD, DE, respectively such that $PX \perp PA, PY \perp PB, PZ \perp PC$. Prove that X, Y, Z are collinear.

Problem 70: Let K be orthopole of a line d with respect to triangle ABC. x, y are two perpendicular lines passing through K. x, y cut BC at A_1, A_2, x, y cut CA at B_1, B_2, x, y cut AB at C_1, C_2, x, y cut d at X, Y. Assume that midpoints of A_1A_2, B_1B_2, C_1C_2 are collinear on a line then that line passes though midpoint of XY.

Problem 71: In the acute-angled triangle ABC with orthocenter H and circumcircle k is drawn a line l through H (non-intersecting AB). K and L are the intersection points of k and l (K is from the smaller arc AC, L is from the smaller arc BC). M, N and P are the feets of the perpendiculars from the vertices A, B and C to l, respectively (M, N, P are internal to k). Prove that $PH = |KM - LN|$.

Problem 72: Let (I) be a circle inside circle (O). P, Q are two points on (I). There are two circles (K) and (L) which is passing through P, Q and intouch (O) at M, N, respectively. Prove that MN always passes through a fixed point when P, Q move on (I).

Problem 73: Let ABC be a triangle with altitudes AD, BE, CF. (N) is nine point circle. X, Y, Z are in inverse points of A, B, C with respect to (N). Prove that DX, EY, FZ are concurrent.

Problem 74: Let ABC be triangle with incircle (I). (I) touches BC, CA, AB at D, E, F, respectively. (ω_1) and (ω_2) are two circles center I. Let M, N, P are in inverse points of D, E, F with respecto circle (ω_1). Let X, Y, Z are in inverse points of A, B, C with respecto circle (ω_2). Prove that MX, NY, PZ are concurrent.

Problem 75: Let ABC be a triangle and a point P. DEF is pedal triangle of P with respect to ABC. (P) is circle center P. Let X, Y, Z be polars of lines EF, FD, DE with respect to circle (P), respectively. Prove that AX, BY, CZ are concurrent.

Problem 76: Given triangle ABC and its circumcircle (O). (O_1) is a circle that tangents AB, AC at N, K and circle (O). Circles (O_2) and (O_3) are defined similarly. According to the given diagram, prove that $\triangle BNM \sim \triangle LKC$.

Problem 77: From the three sides of ΔABC draw rectangles $ACDE, BCIH, ABGF$. V, W, Y are the midpoints of HE, FI, DG. Prove that the lines perpendicular ro EF, GH, DI that go through Y, W, V resp. are concurrent.

Problem78: Let $ABCDEF$ be bicentric hexagon with incircle (I) and circumcircle (O).
a) Prove that AD, BE, CF are concurrent at point K on OI.
b) Constructed circles $(K_1), (K_2), (K_3), (K_4), (K_5), (K_6)$ as in figure. Let G, H, J, L, M, N be incenters of triangles $KAB, KBC, KCD, KDE, KEF, KFA$, respectively. Prove that X, G, L, T; Y, H, M, U; Z, J, N, V are concyclic on circles $(O_1), (O_2), (O_3)$, respectively.
c) Prove that three circles $(O_1), (O_2), (O_3)$ are coaxal with radical axis is OI.

Problem 79: Let $ABCD$ be circumscribed quadrilateral and M is its Miquel point. (M) is a circle center M. Let A', B', C', D' invert A, B, C, D through (M), respectively. Prove that $A'B'C'D'$ is circumscribed quadrilateral.

Problem 80: Let ABC be a triangle with circumcenter O. d is a line passing though O. d cuts circumcircle of triangle OBC, OCA, OAB again at X, Y, Z, respectively. A circle center O cuts rays OA, OB, OC at A', B', C', respectively. Prove that $A'X, B'Y, C'Z$ are concurrent.

Problem 81: ΔABC,AD the angle bisector.M the midpoint of AD.$\odot O_1$, whose diameter is AB, intersects CM at F.$\odot O_2$, whose diameter is AC , intersects BM at E. Prove that: $MEFD$ concyclic and $BCEF$ concyclic.

Problem 82: Let B and D be points on segments $[AE]$ and $[AF]$ respectively. Excircles of triangles ABF and ADE touching sides BF and DE is the same, and its center is I. BF and DE intersects at C. Let $P_1, P_2, P_3, P_4, Q_1, Q_2, Q_3, Q_4$ be the circumcenters of triangles IAB, IBC, ICD, IDA, IAE, IEC, ICF, IFA respectively.
a) Show that points P_1, P_2, P_3, P_4 concylic and points Q_1, Q_2, Q_3, Q_4 concylic.
b)Denote centers of theese circles as O_1 and O_2. Prove that O_1, O_2 and I are collinear.

Problem 83: ABC is a non-isosceles triangle. T_A is the tangency point of incircle of ABC in the side BC (define T_B,T_C analogously). I_A is the ex-center relative to the side BC (define I_B,I_C analogously). X_A is the mid-point of I_BI_C (define X_B,X_C analogously). Prove that X_AT_A,X_BT_B,X_CT_C meet in a common point, colinear with the incenter and circumcenter of ABC.

Problem 84: Line intersects sides (or extension) of triangle ABC (or extension) with A_1 ,B_1,C_1. Let O,H,O_a,H_a,O_b,H_b,O_c,H_c be the circumcenters and orthocenters of triangles ABC , AC_1B_1 , BA_1C_1 , CA_1B_1 respectively. Prove that midperpendicular of OH,O_aH_a,O_bH_b, O_cH_c intersects one point.

Problem 85: Given $(O_1), (O_2)$ are B excircle and C excircle of ΔABC, they touch AB, AC at R, T, E, F, the line CR and BT intersect $(O_1), (O_2)$ at H, P resp, the line CR intersects BT at J, the line HR intersects EC at L, the line PT intersects BF at K, the line PL intersects KH at I. Prove that BF, IJ, CE are concurrent.

Problem 87: Let $ABCDE$ be bicentric pentagon with incircle (I) and circumcircle (O). M, N, P, Q, R are intersections of diagonals a figure. Constructed circles $(K_1), (K_2), (K_3), (K_4), (K_5)$ as in figure. Prove that XM, YN, ZP, TQ, UR are concurrent at a point S on OI.

Problem 88: Let ABC be a triangle and P, Q are two arbitrary point. $A_1B_1C_1$ is pedal triangle of P with respect to triangle ABC. A_2, B_2, C_2 are symmetric of Q through A_1, B_1, C_1, respectively. A_3, B_3, C_3 are reflection of A_2, B_2, C_2 through BC, CA, AB ,respectively. Prove that Q, A_3, B_3, C_3 are concyclic.

Problem 89: Let $ABCD$ be a parallelogram. (O) is circumcircle of triangle ABC. P is a point on BC. K is circumcenter of triangle PAB. L is in AB such that $KL \perp BC$. CL cuts (O) again at M. Prove that M, P, C, D are concyclic.

Problem 90: Let ABC be triangle and P, Q are two isogonal conjugate points with respect to triangle ABC. Prove that circumcenter of the triangles PAB, PAC, QAB, QAC are concyclic.

Problem 91: Let O be circumcenter of triangle ABC. D is a point on BC. (K) is circumcircle of triangle ABD. (K) cuts OA again at E.
a) Prove that B, K, O, E are concyclic on circle (L).
b) (L) cuts AB again at F. G is on (K) such that $EG \parallel OF$. GK cuts AD at S. SO cuts BC at T. Prove that O, E, T, C are concyclic.

Problem 92: Let $ABCD$ be a cyclic quadrilateral. Circle pass through A, D cuts AC, DB at E, F. G lies on AC such that $BG \parallel DE$, H lies on BD such that $CH \parallel AF$. $AF \cap DE \equiv X, DE \cap CH \equiv Y, CH \cap GB \equiv Z, GB \cap AF \equiv T$. M lies on AC. N lies on BD such that $MN \parallel AB$, P lies on AC such that $NP \parallel BC$, Q lies on BD such that $PQ \parallel CD$. d_M passes though M, d_P passes though P such that $d_M \parallel d_P \parallel DE$. d_Q passes though Q, d_N passes though N such that $d_Q \parallel d_N \parallel AF$. $d_Q \cap d_M \equiv U, d_M \cap d_N \equiv V, d_N \cap d_P \equiv W, d_P \cap d_Q \equiv S$. Prove that XU, ZW, SY, TV are concurrent.

Problem 93: Let ABC be a triangle with circumcircle (O). D, E are on (O). DE cuts BC at T. Line passes though T and parallel to AD cuts AB, AC at M, N. Line passes though T and parallel to AE cuts AB, AC at P, Q. Perpendicular bisector of MN, PQ cut perpendicular bisector of BC at X, Y, resp. Prove that O is midpoint of XY.

Problem 94: Let ABC be triangle with circumcircle (O, R). P, P^* are two isogonal conjugate points with respect to triangle ABC. Q is reflection of P through BC. AP, AP^* cut (O) again at D, D'. DQ cuts (O) again at E. EP^* cuts (O) again at E'. Prove that $AE \parallel D'E'$.

Problem 95: Let ABC be a triangle with circumcircle (O). P is a point on line BC outside (O). T is a point on AP such that BT, CT cuts (O) again at M, N, resp, then $MN \parallel PA$. Q is reflection of P through MB, R is reflection of P through NC. Prove that $QR \perp BC$.

Problem 96: Let ABC be a triangle and a circle (ω) passes through B, C. The circle (K) touches to segment AC, AB at E, F and externally tangent to (ω) at T. Prove that intersection other than T of circumcircle (TEC) and (TFB) is incenter of triangle ABC.

Problem 97: Let ABC be triangle with incenter I. A circle (ω) passes through B, C. T is a point on (ω). Circumcircle (BIT) cuts AB again at F. Circumcircle (CIT) cuts AC again at E. (K) is cirumcircle of triangle TEF.
a) Prove that K lies on AI.
b) (K) cuts AB, AC again at P, Q, resp. Prove that PQ, EF, AI are concurrent.

Problem 98: Let ABC be triangle and a point P. $A_1B_1C_1$ is pedal triangle of P with respect to triangle ABC. $A_2B_2C_2$ is circumcevian triangle of P. $A_3B_3C_3$ is pedal triangle of P with respective to triangle $A_2B_2C_2$. Prove that $A_1B_1C_1$ and $A_3B_3C_3$ are persective if only if $A_1B_1C_1$ and $A_2B_2C_2$ are perspective.

Problem 99: Let ABC be triangle with circumcircle (O) and a point D. $(O_1), (O_2)$ are circumcircles of triangles ABD, ACD, resp. DO_1 cuts (O_2) again at E. DO_2 cuts (O_1) again at F.
a) Prove that A, E, F, O_1, O_2 lie on a circle (K).
b) DB cuts (O_2) again at M, DC cuts (O_1) again at N, EM cuts (K) again at P, FN cuts (K) again at Q. Prove that B, P, F; C, Q, E; P, O, O_1; Q, O, O_2 are collinear, resp.

Problem 100: Let ABC be triangle with circumcircle (O). P is a point and PA, PB, PC cuts (O) again at $A'.B', C'$. Tangent at A of (O) cuts BC at T. TP cuts (O) at M, N. Prove that triangle $A'B'C'$ and $A'MN$ have the same A'-symmedian.

Problem 101: Let ABC be triangle. A circle (K) passing through B, C cuts CA, AB at E, F. BE cuts CF at G. AG cuts BC at H. L is projection of H on EF. M is midpoint of BC. MK cuts circumcircle (KEF) again at N. Prove that $\angle LAB = \angle NAC$.

Problem 102: Given $\Delta AMB, \Delta AKC, \Delta BNC$ are isosceles right triangles. Their incenters are S, T, R resp. $\widehat{M} = \widehat{N} = \widehat{K} = 90$. U, X, J are the midpoints of SR, ST, TR resp. Prove the lines through X, U, J that are perpendicular to BC, CA, AB resp are concurrent.

Problem 103: Let M, N be two points interior to the circle (O) such that O is the midpoint of MN. Let S be an arbitrary points lies on (O) and E, F are the second intersections of the lines SM, SN with (O), resp. The tangents in E, F with respect to the circle (O) intersect each other at I. Prove that the perpendicular bisector of segment MN passes through the midpoint of SI.

Problem 104: The A mixtilinear incircle and the A mixtilinear excircle of ΔABC touch the (ABC) at D, Q resp. QD cuts BC at P. Prove that $AP \perp AO$.

Problem 105: In an acute triangle ABC, M is the midpoint of BC, and P is the point on BC such that $\angle BAP = \angle CAM$. The tangent of A to the circumcircle of ABC meets BC at N. The circumcircles of NAB and CAM meet again at Q. The line through Q perpendicular to PQ meets AB and AC at D and E, respectively, and BE meets CD at R. QR meets AN and AM at F and G, respectively, and DG meets FE at S. Prove that $AS \parallel BC$.

<u>Problem 106:</u> I' is the reflection of incenter I over BC, $EH \perp BC, DS \perp BC, DG \perp AB$, $EF \perp AB$. HF cuts GS at M. Prove that A, M, I' are collinear.

<u>Problem 107:</u> The three mixtilinear excircles of ΔABC touch the three sides BC, CA, AB at H, I, D, E, F, G resp. IE cuts DF at K, HF cuts GE at J, JB cuts KC at Q. Prove that HF, EI, AQ are concurrent.

<u>Problem 108:</u> $(O_a), (O_b), (O_c)$ are the A, B, C mixtilinear incircles of ΔABC resp. They touch (O) at D, E, F resp. Let X be the intersection of FO_b, EO_c. Similarly we have Y, Z. Prove that ΔDEF and ΔXYZ are perspective.

<u>Problem 109:</u> $(O_1), (O_2$ are the B, C mixtilinear incircles of ΔABC. They touch BC at G, D resp. (O_1) touches AB at F. (O_2) touches AC at E. DE, GF cut O_1O_2 at K, J resp. CK cuts BJ at N. Prove that AN bisects $\widehat{BAC}$.

<u>Problem 110:</u> The incircle of ΔABC touches BC, CA, AB at D, E, F resp. AD cuts EF at G. BE cuts DF at L. GL cuts AC at M. DE cuts CF at K. AD cuts ZF at J. KM cuts EF at H. Prove that GL, HJ, BC are concurrent.

<u>Problem 111:</u> $(O_1), (O_2)$ are the B, C mixtilinear excircles of ΔABC. They touch AB, AC at F, E resp. S, H are resp on AB, AC such that $O_1H \perp AC, O_2S \perp AB$. O_1H cuts O_2S at P. Prove that:
a) EF, O_1O_2, SH are concurrent.
b) $\widehat{AEF} = \widehat{ASH}$.