
Tutorial 4

— LQR Based Control Design —
Technique with Matlab

Adaptive Cruise Control: State Space Model

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6.0476 & -5.2856 & -0.238 \end{bmatrix};$

$B = \begin{bmatrix} 0 \\ 0 \\ 2.4767 \end{bmatrix};$

$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix};$

$D = 0;$

Source-Code can be found at: <https://goo.gl/vQuwvM>

Adaptive Cruise Control: LQR Based Approach

- Derive open loop poles and check for stability of the system [use command `pole()`].
- If unstable then design discrete controller to stabilize the system.
- We use LQR based control design technique for designing the controller.

Stepwise Design of LQR Based Controller

- First discretize the plant and get the discrete state space model [use command : `c2d()`]
- Choose suitable Q and R matrix for designing the controller.
- We select $Q=10^3*(C'*C)$ and $R=10$.
- Design controller using the command `dlqr()` which takes the Q,R matrix together with the dynamic matrix of the plant and returns the gain **K** which is needed to place the **optimal pole**.
- Next design discrete closed loop system (A-BK).
- Now check the closed loop system poles for stability and step response.

Response vs Varying Sampling Period

- Change sampling period and check the system response (keeping Q, R unchanged).
- Vary sampling period as follows: 1 ms -> 10ms -> 100ms -> 1sec.
- Run simulation for 5 sec for a step response and observe the output.
- For this system with increasing sampling period settling time gets increased.
- Also notice the changes in peak overshoot.

Response vs Varying R (Keeping Q fixed)

- Change the weight matrix R (keeping sampling period fixed).
- Reduce R as: 10 \rightarrow 1 \rightarrow 0.1
- Run simulation for 10 sec for a step response and observe the output.
- For this system with settling time and peak overshoot gets decreased.
- Increase R as: 10 \rightarrow 100 \rightarrow 1000 (keeping Q fixed)
- For this system with settling time and peak overshoot gets increased.

Conclusion:

- Increasing R means more weight on control inputs and system response becomes slow and vice versa (provided Q is constant).

Response vs Varying Q (Keeping R fixed)

- Change the weight matrix Q (keeping sampling period fixed).
- Increase Q as follows: $10*(C'*C) \rightarrow 100*(C'*C) \rightarrow 1000*(C'*C)$ (keeping R fixed)
- Run simulation for 10 sec for a step response and observe the output.
- For this system with increasing sampling period **settling time** and **peak overshoot** gets decreased.

Conclusion:

- Increasing Q means more weight on state variables and system response becomes fast and vice versa (provided R is constant).

Simulation under Control Execution Drop

- Discretize the plant and design the LQR controller using the step discussed earlier.
- Set an execution sequence. We choose 111110 (one drop followed by 5 consecutive samples).
- Set initial state valuation as $[0;0;0]$ and reference as $[30;0;0]$ (i.e, 30km/hr speed).
- Set initial LQR cost and control input as 0.
- Run simulation for 5 sec.
- During simulation update control actuation according to the execution sequence and also the update the LQR cost depending on current value of \mathbf{x} and \mathbf{u} .

Contd...

- Change execution sequence by incorporating **more drops** and observe the output response.
- For this case, you will see an increasing settling time and more oscillation in the response of the plant due to more drop.
- Now consider a periodical spike with **period of 2 sec and amplitude of 60**. This essentially induces a state deviation by changing the speed limit to 60 km/hr (where our reference was 30km/hr) with an interval of 2 sec.
- Again run the simulation under this noisy situation and observe the output response.
- Under this noisy situation you can further change the execution sequence and observe output.