INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 70 No. of students: 12

Spring End Semester Exams, 2012 Dept: *Computer Science & Engineering* Sub No: CS60060

M.Tech (Elective) Sub Name: ***Formal Systems***

Instructions: *Answer Question 1, Question 2, and any one from the rest.*

Answer all parts of a question in the same place.

---

1. [Timed Systems] [Compulsory Question] [10 + 3 + 3 + 5 + 6 + 3 + 5 + 5 = 40 marks]

When the door of a refrigerator is opened, the light inside the refrigerator turns ON. If the door is kept open for more than one minute, then a beeper is activated. The beeper, when activated produces a beep, and then repeats the beep at least once in every 30 seconds. When the door is closed, the light goes OFF and the beeper (if active) is deactivated.

(a) Draw a timed automaton for the door controller which controls the light and the beeper.

(b) Give the formal definition of a timed trace. Give an example of a timed trace in your timed automaton.

(c) Give the formal definition of a time convergent path. Does your timed automaton have a time convergent path?

(d) What is a xeno behavior? Does your timed automaton have zeno behaviors? If so, describe a timed trace exhibiting xeno behavior and then modify the timed automaton to eliminate xeno behaviors.

(e) Write the following properties in TCTL

- The beeper never sounds when the light is OFF
- Whenever the beeper is active, a beep is heard within 30 seconds.
- When the light turns ON, it remains ON for at least 30 seconds

(f) Which of the properties of part (e) are true in your timed automaton?

(g) Formally define the meaning of TS |= φ, where TS is a timed transition system and φ is a TCTL property. [In other words, present the formal semantics of TCTL]

(h) Explain the steps in constructing a region automaton from your timed automaton. Note that you do not have to construct the full region automaton – you have to state and explain the steps of the procedure, preferably showing a few regions only.

2. [Hybrid System Modeling] [Compulsory Question] [10 marks]

A system to cool a nuclear reactor is composed by two independently moving rods. Initially the coolant temperature, *x*, is 510 degrees and both rods are outside the reactor core. The temperature inside the reactor increases according to the following system:

$$\dot{x} = 0.1x - 50$$

When the temperature reaches 550 degrees, the reactor must be cooled down using the rods. There are three possibilities:

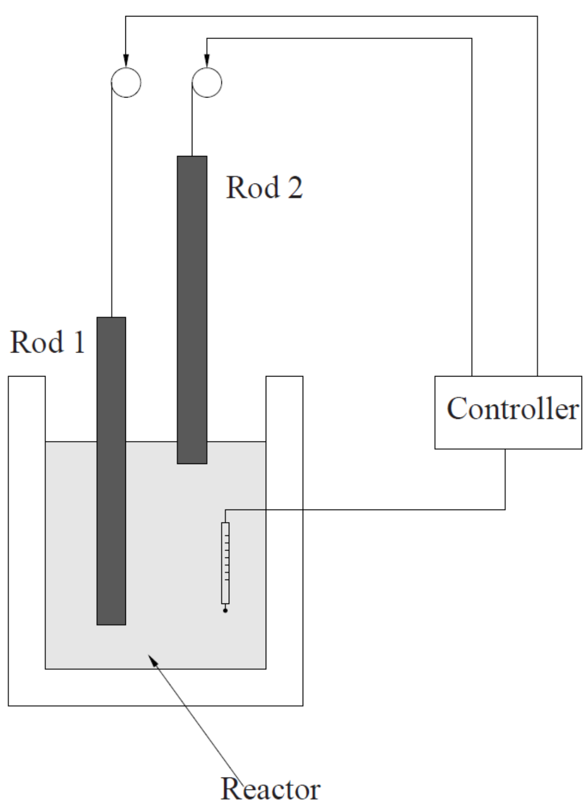
- The first rod is lowered into the reactor core
- The second rod is lowered into the reactor core
- None of the rods can be lowered into the reactor core

For mechanical reasons, the rods can be lowered into the core only if it has not been there for at least 20 seconds. Both rods cannot be inside the reactor at the same time. The two rods can refrigerate the coolant according to the following equations:

Rod-1: $\dot{x} = 0.1x - 56$

Rod-2: $\dot{x} = 0.1x - 60$

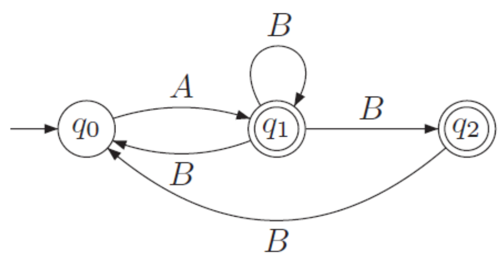
When the temperature decreases to 510 degrees, the rod is removed from the reactor core. Model the system, including the controller, as a hybrid automaton.



3. [LTL, Buchi Automata, and ω-Regular Languages] [2 + 5 + 5 + 5 + 3 = 20 marks]

(a) Give the formal definition of a Generalized Non-deterministic Buchi Automaton (GNBA).

(b) Consider the GNBA shown below with acceptance sets $F_1 = \{ q_1 \}$ and $F_2 = \{ q_2 \}$. Construct an equivalent NBA following the standard conversion technique from GNBA to NBA.



(c) Describe the language accepted by the above GNBA using a ω-regular expression. Can it be expressed using a LTL formula?

(d) Outline the construction of a GNBA for the LTL property, p U q.

(e) Outline the steps for model checking a LTL property φ on a transition system, TS.

4. [BDDs and CTL model checking] [4 + 5 + 2 + 6 + 3 = 20 marks]

(a) Draw the ROBDD for the function $F = (x_1 + x_6)(x_2 + x_5)(x_3 + x_4)$.

(b) Consider a 2-bit gray counter whose counting sequence is shown below. You wish to represent the state transition relation of the counter using a ROBDD. Determine the function that the state transition BDD will represent.

00 → 01 → 11 → 00 → ….

[Hint: Develop the state transition table. Develop the functions for each of the two next state bits. Thereafter define a *characteristic function* capturing both next state functions.]

(c) State the syntax (grammar) for *Computation Tree Logic* (CTL). What is meant by a computation tree?

(d) For each of the following pairs of formulas, determine whether the two formulas are equivalent. For the ones which are non-equivalent give a sample transition system where one is true and the other is false.

- $\forall\Diamond p \wedge \forall\Box q$ and $\forall\Diamond(p \wedge \forall\Box q)$
- $\forall\Diamond p \wedge \forall\Box q$ and $\forall\Box(\forall\Diamond(p \wedge q))$
- $\forall[p \text{ U } q] \wedge \forall[q \text{ U } r]$ and $\forall[p \text{ U } r]$

(e) Outline the steps for computing the set of states satisfying $\exists(p \text{ U } q)$ using the set of states satisfying p, the set of states satisfying q, and the transition relation.