INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 70 No. of students: 11

Spring End Semester Exams, 2018 Dept: *Computer Science & Engineering* Sub No: CS60030

M.Tech (Elective) Sub Name: ***Formal Systems***

Instructions: Answer Q1, Q2, Q3, and any one from the rest.

Answer all parts of a question in the same place.

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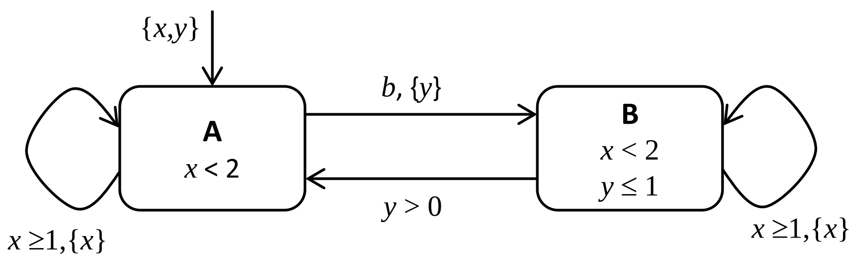
1. [Timed Automata] Answer the following questions. [4+4+5+5 = 18 marks]

   (a) Give the formal definition of a timed automaton as a tuple. Then use the timed automaton shown below to explain each of the items of the tuple.

   (b) For the timed automaton shown below, draw the *standard time regions*

   (c) List all possible regions that are successors of the region $\langle$ A, x=0, $y > 1$ $\rangle$ in this timed automaton

   (d) List all possible regions that are successors of the region $\langle$ B, $0 < x = y < 1$ $\rangle$ in this timed automaton



2. [Hybrid Automata] Answer the following questions. [3+6+9 = 18 marks]

   (a) Give the formal definition of a hybrid automaton as a tuple

   (b) Explain the following terms:
      - Time convergent and time divergent state traces (state sequences)
      - Zeno behaviors
      - Time locked states

   (c) Consider the following scenario. Two trains are heading toward each other on a single track at constant speeds: the train *E* is travelling east at a fixed speed v_e, and the train *W* is travelling west at a fixed speed v_w. A bee *B* is initially travelling west at a fixed speed v_b along the line joining the two trains. When the bee reaches the train *E*, it reverses its direction, heads east at the same speed v_b, and reverses its direction again when it reaches the train *W*. This cycle repeats. Model the scenario as a hybrid automaton. The hybrid automaton can have two locations, one each corresponding to the direction in which the bee is travelling, and three state variables that capture the positions of the train *E*, the train *W*, and the bee *B*.

- Draw the hybrid automaton showing the transition guards, location invariants, and the location dynamics.
- Write down a time convergent state sequence, if one exists in the automaton.
- Is a time locked state reachable in your automaton? Is so, which state is that?
- Is a state reachable from where zeno behavior is possible? If so, which state is that?

3. [Program Verification] Answer the following questions. [6+4+8 = 18 marks]

(a) Recall the rules for computing weakest pre-conditions and fill up the following:

- *wp*(`x:=e, Q(x)`) = ___________
- *wp*(S_1;S_2, Q) = ____________
- *wp*(`if b then` S_1 `else` S_2, `Q`) = ______________

(b) Now use these rules to compute *wp*(*A*, `x<y`) in as simple form as you can, where A is the following code fragment:

```
if (x>y) then
     tmp :=x;
     x:= y;
     y := tmp;
endif
```

(c) We wish to use abstract interpretation on the following program to determine whether the assertion is true. Use the sign domain as your abstract domain, which has only three elements, [+, –, 0], representing +ve, –ve, and zero respectively. Show the values collected in each location. Also indicate the outcome:

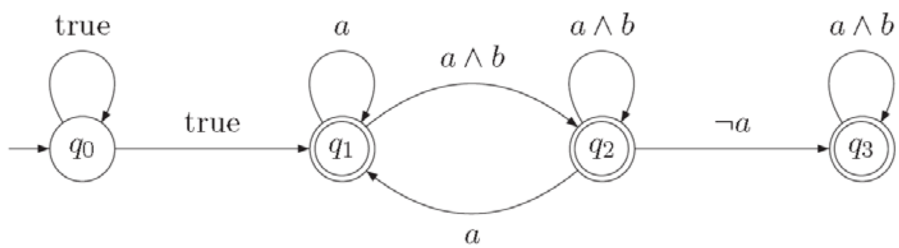
```
x = 1;
if (y <= 10){
     y = 10;
}
else {
     while (x<y) {
          x = 2*x;
          y = y-1;
     }
}
x = y+1;
assert (x>0);
```

4. [Linear Temporal Logic] Answer the following questions. [8+8 = 16 marks]

(a) Express the following LTL properties over a trace, $\pi = s_0, s_1, \ldots$

- If q occurs twice then q occurs infinitely often
- q holds at all states s_{3k} and does not hold at states s_{3k+1} and s_{3k+2} where $k = 0, 1, 2, \ldots$
- If q holds at a state s_k, then r must hold in at least one of the two states just before s_k that is, s_{k-1} and s_{k-2}
- q never holds at less than two consecutive states, that is, if q holds at s_k then it also holds at s_{k-1} or at the state s_{k+1}

(b) Consider the GNBA G in the figure below with alphabet $\Sigma = 2^{\{a,b\}}$ and the accepting sets of states as F = {{q1,q3}, {q2}}.



- What is the acceptance criterion for this GNBA?
- Provide an LTL formula φ with Words(φ) = L_ω(G).
- Depict the NBA *A* with L_ω(A) = L_ω(G)

5. [Miscellaneous] Answer the following questions. [6+6+4 + 16 marks]

(a) For each of the following, indicate whether the statement is True / False with a brief justification:

- DBAs and NBAs have the same expressive power
- Omega regular languages are closed under intersection
- GNBAs are more powerful than NBAs

(b) Explain the steps of CTL model checking. Do we start with the negation of the property as in LTL model checking?

(c) Explain the use of the widening and narrowing operations in the context of abstract interpretation of programs.