

## Tutorial 1 :

1.) Solution : The proof does not assure that all the identity elements pairs of type  $(a,a)$  are present in  $R$ .

3.) Solution :- Considering all the diagonal relations we will have  $2^{n(n-1)}$  reflexive relations.

eg:-  $A = \{a, b, c\}$

$$\begin{bmatrix} (a,a) & (a,b) & (a,c) \\ (b,a) & (b,b) & (b,c) \\ (c,a) & (c,b) & (c,c) \end{bmatrix}$$

Now, by pairing symmetrical sets i.e.  $(a,b)$  &  $(b,a)$   
we will have  $2^{\frac{n(n-1)}{2}}$  symmetrical and reflexive relations.

6.) For a Partial order set, It should satisfy the conditions of reflexivity, ~~sym~~ anti-symmetric and transitive.

The set  $A = [0, 2)$  of real numbers on operation  $\leq$  is.

① Reflexive :- Since every element is equal to itself i.e.  $0 \leq 0, 0.1 \leq 0.1$ .

② Anti-Symmetric :- Symmetric elements are not present. i.e.  $(0.1 \leq 0.2)$ , then  $(0.2 \leq 0.1)$  will not be in the set.

3) transitive  $\therefore$  if  $a \leq b$  &  $b \leq c$  then  $a \leq c$

Hence, the set  $A = [0, 2)$  of real numbers with the operator  $\leq$  is a poset or partial order.

Also, every pair of elements will have a meet and join hence the poset is a lattice.

( $\because$  A poset is a lattice if every pair of elements have a meet and a join.)

Q. 2  $\therefore$  Proof by Induction:

Basic Step  $\therefore R^1 = R$  is symmetric which is true.

Inductive Step  $\therefore$  Assume that  $R^n$  is symmetric.

To Prove  $\therefore R^{(n+1)}$  is symmetric.

$R^{(n+1)}$  is symmetric if for all  $(x, y) \in R^{(n+1)}$ ,

we have  $(y, x) \in R^{(n+1)}$  as well.

= Assume that  $(x, y) \in R^{(n+1)}$ .

Now,  $R^{n+1} = R^n \circ R = R \circ R^n$ .

We know that if  $(x, y) \in R \circ R^n$ , then by the definition of composition there exists a  $z \in A$  such that  $x R z$  and  $z R^n y$  i.e.  $(x, z) \in R$  and  $(z, y) \in R^n$ .

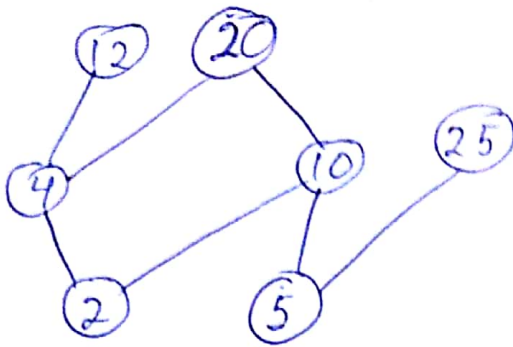
And we also know that  $R$  and  $R^n$  are symmetric, which implies that  $(z, x) \in R$  and also  $(y, z) \in R^n$ .

Therefore, by definition of composition  $(y, x) \in R \circ R^n$  i.e.  $(y, x) \in R^{n+1}$

Hence Proved.

Sol<sup>n</sup>

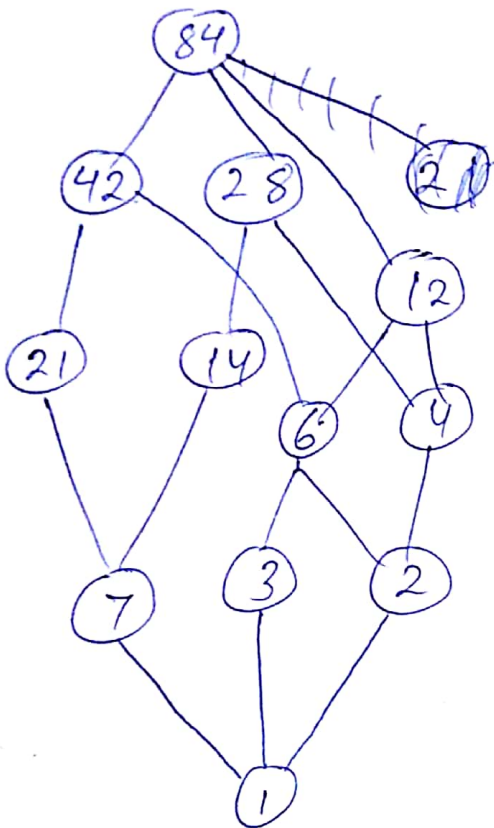
Q.4:



maximal :- 25, 20, 12

minimal :- 2, 5

Q.5:



(7.)  $(p, q) R (r, s) \text{ iff } p - s = q - r$

Test for Reflexive:

$$(p, q) R (p, q)$$

$$\Rightarrow p - q = q - p \quad (\text{not possible})$$

Hence it is not reflexive.

Test for Symmetric:

$$(p, q) R (r, s) \rightarrow (r, s) R (p, q)$$

$$p - s = q - r$$

$$r - q = s - p$$

$$\underline{q - p = r - s}$$

$$\underline{r - q = s - p} \quad \checkmark$$

It is symmetric.

Option (c)