

Q.1) Consider the following functions. Does there exist $k \in \mathbb{R}$ such that the function becomes a probability mass function ??

$$a) f(x) = \begin{cases} \frac{k}{x^2} & ; x \in \{1, 2, 3, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

$$b) f(x) = \begin{cases} \frac{k}{|x|} & ; x \in \mathbb{Z} \setminus \{0\} \\ 0 & \text{otherwise} \end{cases}$$

\mathbb{Z} : denotes the set of all integers.

Q.2) Define a random variable on a probability space (Ω, \mathcal{F}, P) . Further, state Lebesgue decomposition theorem and discuss different types of random variables based on the obtained Lebesgue decomposition.

Q.3) Define the geometric distribution and exponential distribution. State and prove memoryless property in case of each of these distributions.

Q.4) Consider a quadratic

equation $ax^2 + bx + c = 0$

where the coefficients

a, b, c are chosen randomly

from the following pmf

independent of each other.

x	0	-1	1	-2	2	-3	3
f(x)	1/7	1/7	1/7	1/7	1/7	1/7	1/7

What is the probability that
the quadratic equation has
repeated roots?

Q.5) In a class of 50 students
15 are from AGFE department,
20 are from CS department and
15 are from EE department.
10% of the AGFE department
students are likely to drop the
course without completing it.
The same probabilities for the
CS & EE students are 0.07
and 0.05 respectively. What is
the probability that a student
who dropped the course without
completing is from the AGFE
department??

Q.6) 2 fair dice are thrown.
what is the probability to
obtain two sixes given that
i) one six has already
appeared.
ii) sum of the two faces is
greater than 6.

Q.7) Consider the following function.

$$f(x) = \frac{1}{2} e^{-|x-2|} \quad \text{for } x \in \mathbb{R}$$

- i) Is $f(x)$ a probability density function?
- ii) If yes, compute mean, mode and median.

Q.8) Consider 4 independent Bernoulli trials with probabilities of success as P_1, P_2, P_3, P_4 respectively.

Here, $P_{ith} = \frac{P_i}{2}$ for $i=1,2,3$

and $P_i = \frac{2}{3}$

let X count the number of successes in these 4 independent trials. What is the probability that X is greater than or equal to 2 ??

Q.9) Consider a random variable X with the following pdf

$$f(x) = \begin{cases} (1-p)p^i & \text{for } x \in [i, i+1) \\ 0 & \text{otherwise} \end{cases}$$

where $i = 0, 1, 2, \dots$ and $p \in (0, 1)$.

Compute the value of p such that median of X is 2.

Q.10) Consider the function

$$f(x) = \begin{cases} \frac{k}{2^x} & x = 0, 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

for some $k \in \mathbb{R}$. Determine the value of k so that $f(x)$ is a probability mass function. Then compute the probability that a number chosen randomly is either a multiple of 2 or a multiple of 3.