Foundations of Computing Science (CS60005)

TUTORIAL 7

1. Prove that k-clique is NP Complete.

2. A coloring of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Finding whether a graph has a coloring with k colors is NP-complete for $k3$. Now consider the following problem.
   You are given a list of final examinations $\{F_1, \ldots, F_m\}$ to be scheduled, and a list of students $\{S_1, \ldots, S_n\}$. Each student is taking some specified subset of these exams. You must schedule these exams into slots so that no student is required to take two exams in the same slot. The problem is to determine if such a schedule exists that uses only h slots.
   Prove that the above problem is NP-complete by using the fact that graph coloring is NP-complete. Give an example which clearly demonstrates the reduction.

3. Consider the SET-COVER problem defined as follows:
   SET-COVER $= \{U, S, k|$U is a finite set of numbers, S is a collection of sub-sets of U, there is a k-sized cover of U from the collection $S\}$
   A cover $C \subseteq S$ is a collection of sub-sets whose union is U. Prove that the SET-COVER problem is NP-Complete. Clearly indicate which problem is being reduced to which problem and clearly show the steps of the reduction, proving the reduction is in P.
   [Hint: Use the fact that VERTEX-COVER is NP-Complete]

4. The SUBSET-SUM problem is defined as follows:
   SUBSET-SUM $= \{< S, t > |S = \{x_1, \ldots, x_k\}$ and for some $Q \subseteq S$, we have $t = \Sigma_{x \in Q} x\}$
   In other words, given a set S of integers and a number t, the SUBSET-SUM problem asks whether the sum of the integers in any subset of S equals t. This problem is known to be NP-complete. Use this fact to prove that the EQIPARTITION problem is NP-complete using a clean mapping reduction (no hand-waving arguments).

   EQUIPARTITION $=\{S|S = \{x_1, \ldots, x_k\}$ and for some $Q \subseteq$S, we have $\Sigma_{x \in Q} x = \Sigma_{x \notin Q} x\}$
   In other words, given a set S of integers, the EQUIPARTITION problems asks whether the set S can be partitioned into two parts such that the sum of the integers in both parts are equal.