INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 85 No. of students: 19

Spring End Semester Exams, 2013 Dept: *Computer Science & Engineering* Sub No: CS60060

M.Tech (Elective) Sub Name: ***Formal Systems***

Instructions: Answer Question 1, Question 2, and any one from the rest.

Answer all parts of a question in the same place.

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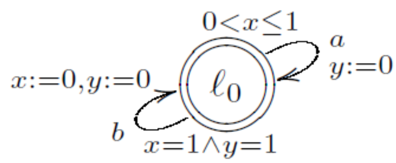
1. [Timed Automata] (Compulsory Question) [12 + 6 + 12 = 30 marks]

(a) Draw a region graph of the following timed automaton.



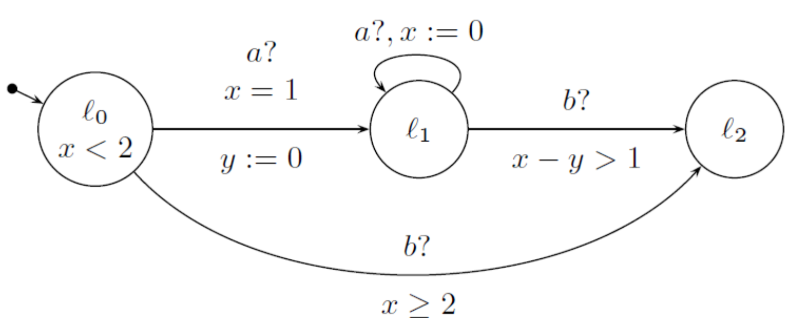
Using the region graph decide whether the following configurations are reachable from the initial configuration.

- (l_0, v) where $v(x) = 0.7$ and $v(y) = 0.61$
- (l_0, v) where $v(x) = 0.2$ and $v(y) = 0.41$

(b) Define the following notions for timed systems

(i) Time divergent paths (ii) Zeno behaviors (iii) Timelocks

(c) Consider the following timed automaton, $\mathcal{A}$



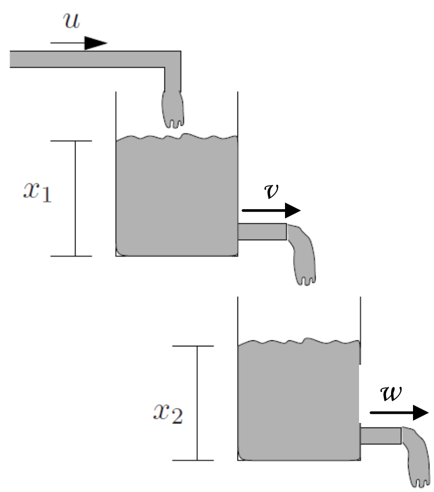
- Does $\mathcal{A}$ have a computation path with Zeno behavior? If so, which one?
- Does $\mathcal{A}$ have a computation path with timelock? If so, which one?
- Does $\mathcal{A}$ have a run? Explain.
- Is the location l_2 reachable? Explain.

2. [Hybrid Automata] (Compulsory Question) [6 + 4 + 10 + 10 = 30 marks]

(a) A hybrid automaton is a 6-tuple, ***H* = (*Loc, Var, Lab, Edg, Act, Inv*).** Define each of these six components. Use a suitable example to demonstrate.

(b) What is the forward time closure of a state of the hybrid automaton.

(c) Consider the following hybrid system:



There are three taps in the system, namely Tap-1 having a flow rate of u = 5, Tap-2 having a flow capacity of v = 2, and Tap-3 having a flow capacity of w = 4. Tap-2 and Tap-3 are always on. Tap-1 is switched on when $x_1 + x_2$ falls below 10 and is switched off when x_1 exceeds 80. Initially, we have x_1 = 50 and x_2 = 50. Draw a hybrid automaton for the system. Explain the dynamics of the system.

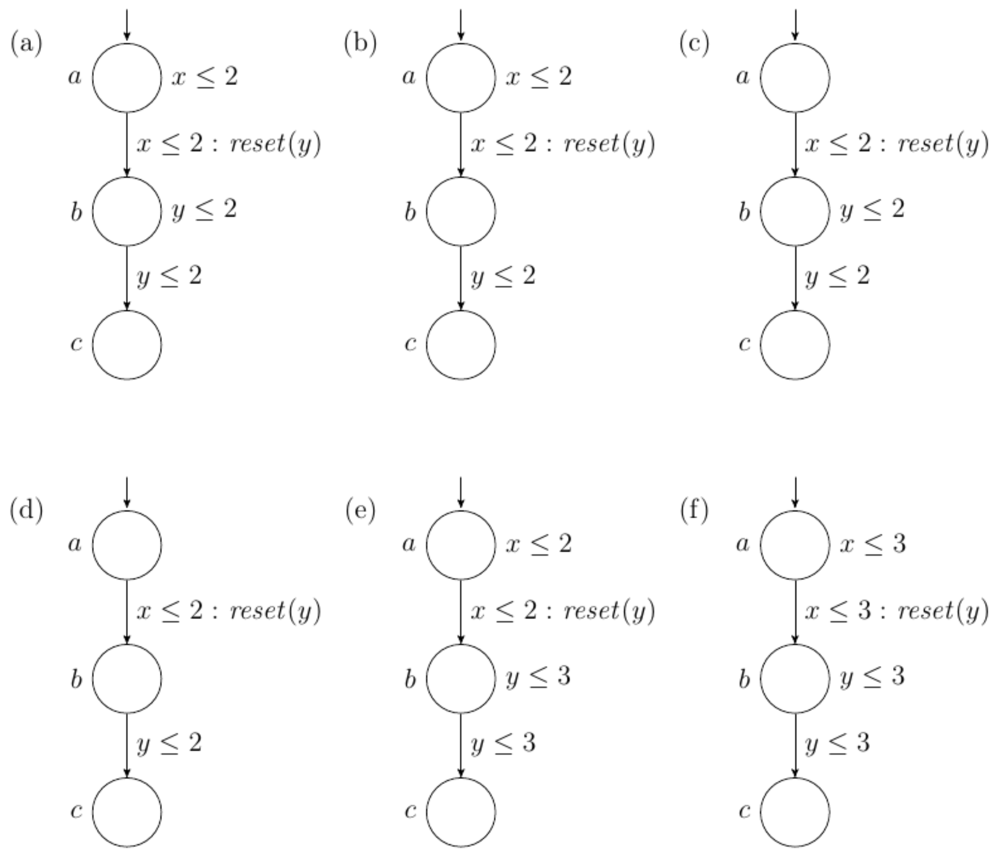
(d) Which of the following states are reached by the hybrid automaton of part (b)? If reachable, indicate the time at which it is reached for the first time.

(i) A state with x_2 = 0 (ii) A state with x_1 = 0 (iii) A state with x_2 = 80

(iv) A state with x_1 = 80 (v) A state with x_1 = 10

3. [Temporal Logic] [12 + 8 + 5 = 25 marks]

(a) Consider the following timed automata.



For each automaton, one of the TCTL formulae below distinguishes it from all other ones. Map every automaton to a corresponding formula.

(i) $\forall \Diamond^{\leq 4} c$

(ii) $\forall \Diamond \exists \Box b$

(iii) $(\forall \Diamond^{\leq 5} c) \wedge (\exists \Box^{\leq 5} \neg c)$

(iv) $(\exists \Box a) \wedge (\exists \Diamond \exists \Box b)$

(v) $(\exists \Box a) \wedge (\neg \exists \Diamond \exists \Box b)$

(vi) $(\forall \Diamond^{\leq 6} c) \wedge (\exists \Box^{\leq 6} \neg c)$

(b) Consider an elevator system that services N>0 floors numbered 0 through N–1. There is an elevator door at each floor with a call-button and an indicator light that signals whether or not the elevator has been called. For simplicity consider N=4. Present a set of atomic propositions – try to minimize the number of propositions – that are needed to describe the following properties of the elevator system as LTL formulae and give the corresponding LTL formulae:

- The doors are *safe*, that is, a floor door is never open if the elevator is not present at the given floor
- A requested floor will be served sometime
- Again and again the elevator returns to floor 0
- When the top floor is requested, the elevator serves it immediately and does not stop on the way there.

(c) Write down the steps for checking whether a given LTL formula is satisfiable. For example, the LTL property, $\Diamond c \wedge \Box \neg c$, is not satisfiable.

4. [Miscellaneous] [6 + 6 + 6 + 7 = 25 marks]

(a) Let P be a linear time property. Define *closure*(P). Prove that *closure*(P) is a safety property.

(b) Draw a Buchi automaton that accepts the following ω-regular language:

$$L = \{ \sigma \in \{A, B\}^\omega \mid \sigma \text{ contains ABA infinitely often, but AA only finitely often} \}$$

(c) Explain the working of Bounded Model Checking. Write down the clauses generated by unfolding the LTL property p U (q U r) over three cycles.

(d) A Muller automaton $\mathcal{A}$ is a tuple $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$, where $F \subseteq 2^Q$. It is therefore similar in structure as a generalized Buchi automaton where $F = \langle F_1, .., F_k \rangle$ and each F_i is a subset of Q. An accepting run, π, is one in which the set of states of $\mathcal{A}$ that are visited infinitely often is exactly equal to some $F_j \in F$. Show that any generalized non-deterministic Buchi automaton (GNBA) can be converted into a non-deterministic Muller automaton accepting the same language. Starting with the tuple for the given GNBA construct the tuple for the equivalent non-deterministic Muller automaton.