INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Computer Science and Engineering

Foundations of Computing Science (CS60005))

Mid-semester (Autumn)

Place: NC341/G, NC342/E, NC343/E, NC344/E, NC333/H, NC334/G *Time:* 2m-4pm (AN)

Date: Wed, Sep 19, 2018 *Students:* 80 *Marks:* 45

Answer ALL the questions (following question specific instructions)

Q1: If $\langle A_1, \leq_1 \rangle$ and $\langle A_2, \leq_2 \rangle$ are two posets, then the dictionary order $\leq$ on $A = A_1 \times A_2$ is given by $(x_1, x_2) \leq (y_1, y_2)$ iff $x_1 <_1 y_1 \vee (x_1 = y_1 \wedge x_2 \leq_2 y_2)$.

Prove that the Cartesian product $A_1 \times A_2$ of two posets A_1 and A_2 forms a poset under the dictionary order $\leq$. 5

Q2: If $\langle A, \leq \rangle$ is a bounded distributive lattice with minimum 0 and maximum 1, then prove that complements of elements are unique, provided they exist (let x and y be complements of a, show that $x = y$). 5

Q3: Let $D = \{0, 1\}$ and let $T = D \times D$ be the alphabet $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

A correct multiplication of a number in binary notation by 2 can be represented as a string in T^*. For example, the multiplication of $10 \times 2 = 20$ ($001010 \times 10 = 010100$) could be represented as follows:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Show that the set of all strings in T^* that represent correct multiplications is a regular language (provide FA). 5

Q4: Consider the language L for $\mathtt{a}^n\mathtt{b}^m\mathtt{c}^{m+n}$ over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, $m, n \geq 0$.

(a) Show that it is not regular (apply PL-RL). 5

(b) Show that it is context free (provide PDA or CFG for L). 5

(c) Show that its complement is also context free (provide PDA for $\overline{L}$). 5

Q5: Top-down parsing involves reconstructing the leftmost derivation. How does bottom-up parsing work? Give the schematic of a NPDA for a bottom-up parser for a given CFG. 5

Q6: Consider the language ww over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

(a) Show that it is not context free (apply PL-CFL). 5

(b) Show that its Turing decidable (provide decider TM). 5