Tutorial – Complexity Theory

Foundations of Computing Science

Pallab Dasgupta
Professor,
Dept. of Computer Sc & Engg

1. Which of the following are True/False. Provide a clear reason for your response.

   a. If languages $L_1 \in NP$ and $L_2 \in co\text{-}NP$, then $L_1 \cap L_2 \in P$
   b. The complement of a satisfiable Boolean formula is unsatisfiable.
   c. The class NP is closed under complementation
   d. If language $L \in NP$ and $L1 \subseteq L$, then $L_1 \in P$
   e. All problems in NP can be solved using a *deterministic* Turing Machine in *polynomial space.*

2. A coloring of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Finding whether a graph has a coloring with k colors is NP-complete for $k \geq 3$. Now consider the following problem.

   You are given a list of final examinations $F_1, \ldots, F_m$ to be scheduled, and a list of students $S_1, \ldots, S_n$. Each student is taking some specified subset of these exams. You must schedule these exams into slots so that no student is required to take two exams in the same slot. The problem is to determine if such a schedule exists that uses only h slots.

   Prove that the above problem is NP-complete by using the fact that graph coloring is NP-complete. Give an example which clearly demonstrates the reduction.

3. The SUBSET-SUM problem is defined as follows:

SUBSET-SUM = $\{ \langle S, t \rangle \mid S = \{x_1, \ldots, x_k\}$ and for some $Q \subseteq S$, we have $t = \Sigma_{x \in Q}\, x\}$

In other words, given a set S of integers and a number t, the SUBSET-SUM problem asks whether the sum of the integers in any subset of S equals t. This problem is known to be NP-complete. Use this fact to prove that the EQIPARTITION problem is NP-complete using a clean mapping reduction (no hand-waving arguments).

EQUIPARTITION = $\{ S \mid S = \{x_1, \ldots, x_k\}$ and for some $Q \subseteq S$, we have $\Sigma_{x \in Q}\, x = \Sigma_{x \notin Q}\, x\}$

In other words, given a set S of integers, the EQUIPARTITION problems asks whether the set S can be partitioned into two parts such that the sum of the integers in both parts are equal.

4. In mobile telephony, the frequency allocation problem is stated as follows. There are a number of transmitters deployed and each of them can transmit on any of a given set of frequencies. Different transmitters have different frequency sets. Some transmitters are so close that they cannot transmit at the same frequency, because then they would interfere with each other.

You are given the frequency range of each transmitter and the pairs of transmitters that can interfere if they use the same frequency. The problem is to determine if there is any possible choice of frequencies so that no transmitter interferes with any other.

(a) Show that the frequency allocation problem is in NP

(b) Given that the Graph Coloring problem is known to be NP-complete, show that the frequency allocation problem is NP-complete. Indicate which problem is being reduced to which problem and clearly show the steps in reducing one problem to the other

(c) Consider the following variant of the frequency allocation problem: Is the minimum number of frequencies required to get a proper allocation exactly equal to k? What can you say about the complexity of this problem?