Q.1) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Derive the distributions of the following random variables.

a) $T_1 = \dfrac{X_1 - X_2 + X_3 - X_4 \cdots (-1)^n X_n}{n}$

b) $T_2 = \dfrac{X_1 + X_2 + \cdots + X_m}{m}$ for $1 < m < n$

c) $T_3 = \dfrac{1}{\sigma^2} \sum_{i=1}^{m} (X_i - T_2)^2$ where T_2 is defined in part (b) above.

Q-2) Let $Y_1, Y_2, \ldots, Y_{10}$ be independent Chi-squared random variables with $n_1, n_2, \ldots, n_k$ degrees of freedom.

Derive the distribution of the following random variables.

i) $Y_1 + Y_3 + \cdots + Y_9$

ii) $\dfrac{Y_2 + Y_4 + Y_6}{Y_5}$

Q.3) Consider an n-dimensional random vector $Z = (Z_1, \dots, Z_n)^T$ and $Z_i \in \{0,1\}$ and $\sum_{i=1}^{n} Z_i = 2$.

i) If Z is continuous of discrete random vector.

ii) How many elements are there in the range of Z?

iii) Is it possible to set up a uniform probability space for this random vector Z?? If yes, write down the pdf/pmf of Z.

Q.4) Let $X \sim N(3,2)$. The compute $a > 0$ such that

$$P\left(\frac{(X-3)^2}{2} \leq a\right) = 0.7$$

Then compute $P(-a \leq X \leq a)$.

Q.5) Let $X_1, X_2, \cdots, X_n \overset{iid}{\sim} U(0,1)$.

Then compute densities of the following random variables??

i) $\min\{X_1, \cdots, X_n\}$

ii) $\max\{X_1, \cdots, X_n\}$

Q.6) Let $X \sim U[0, \pi]$ and define

$Y = \sin(X)$.

Compute the median of Y.

Q.7) Let X be a gamma random variable with mean 2 and varianc 4. Compute $P(X \geq 8 \mid X \geq 5)$.

Q.8) Let $X_1, X_2, \cdots, X_n$ be a random sample from $f_\theta(x)$ where θ is the unknown parameter vector. Explain in details the method of maximum likelihood estimator to estimate the parameter vector θ. What might be the challenges one might face in implmenting this method??

Q.9) Let $X_1, X_2, \dots, X_n$ be the independent and identically distributed random variables with mean μ and variance σ^2.

Let $n = 2k$ be a positive integer such that $k \geq 30$.

Define $Y_1 = X_1 + X_3 + \cdots + X_{n-1}$

$$Y_2 = X_2 + X_4 + \cdots + X_n$$

Using CLT, discuss the approximate distribution of $Y_1 - Y_2$.

Q.10)

Discuss how binomial probabilities can be approximated by

a) Normal density??

b) Poission pmf??