Foundations of Computing Science (CS60005)

TUTORIAL 2

1. Given an NFA that recognizes L, to build an NFA to recognize the reverse of L, that is L^r containing every string of L in reverse, it suffices to swap the initial and final states and reverse edges.
   Do you agree with the above statement?

2. Construct a DFA that will accept the following languages over the alphabet $\{0, 1\}$

   (a) All strings that start with 0 and have odd length OR start with 1 and have even length.

   (b) The set of all strings with number of 0's divisible by 3 and number of 1's divisible by 2.

3. Write regular expressions for the following languages

   (a) The set of all strings of 0's and 1's whose fourth symbol from the right end is 1.

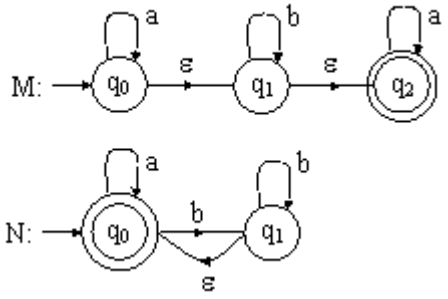
   (b) The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's tan 1's, nor two more 1's than 0's

   (c) The set of all strings of 0's and 1's whose number of 0s divisible by three.

   (d) The set of all strings of 0's and 1's with at most one pair of consecutive 1's.

   (e) The set of all strings of 0's and 1's with at least one 0 and one 1.

4. Consider the finite machines M and N



   Find the regular expression for $L(M) \cap L(N)$.

5. Draw a DFA for the language of strings representing binary sums. A string of the language is represented as a series of triplets $a_i b_i s_i$, where s_i is the sum bit, and a_i and b_i are the bits being added, delivered to the DFA from LSB (bit 0) to MSB (bit N). An example of a valid string representing 0100 + 0101 = 1001 would be 011 000 110 001 (spaces inserted only for readability), to be interpreted as 0+1=1, 0+0=0, 1+1=0 (1), 0+0+(1)=1. An invalid string would be one representing an incomplete sum (like 011000110, 01100) or an incorrect sum (like 011111001001). Have as few states as you can, or the DFA will look messy.
   [Hint: A bit sum produces a carry or does not – yielding two clusters of states in the DFA].

6. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes? [inclusion exclusion principle]

7. From the following implications which one is not a tautology?

   (a) $(\neg p \wedge (p \vee q)) \rightarrow q$

   (b) $(p \wedge (p \rightarrow q)) \rightarrow q$

   (c) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

   (d) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$