INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 80 No. of students: 45

Autumn End Semester Exams, 2011 Dept: *Comp. Sc & Engg.* Sub No: CS60005

M.Tech (Core) Sub Name: ***Foundations of Computing Science***

Instructions: *Part-A must be answered on the question paper itself.*

Part-B must be answered on the answer book.

Answer all parts of a question in the same place

PART- A (20 marks)

Answer all questions

Roll No: ______ Name: ______

1. Indicate whether the following statements are True/False in the box provided with an accurate reason in the line provided. [5 x 2 = 10 marks]

(a) If languages $L_1 \in$ NP and $L_2 \in$ coNP, then $L_1 \cap L_2 \in$ NP $\cap$ coNP.

(b) TIME(2^n) = TIME(2^{2n})

(c) NP is closed under complementation

(d) The following statement is valid:

$$((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$$

(e) A *f(n)*-time non-deterministic Turing machine can always be simulated by a deterministic Turing machine in $(f(n))^2$ space.

2. Answer the following : [5 X 2 = 10 marks]

(a) Write the most general unifier for ***P(f(g(w), y), w)*** and ***P(f(x, Einstein)), g(y))***

(b) Write a regular expression for the following language with alphabet {0,1}:

L = {w | w contains an even number of 0's and any number of 1's}

(c) State Godel's Incompleteness Theorem

(d) What is an *enumerator* for a language?

(e) Define the complexity class Σ_2P

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 80 No. of students: 45

Autumn End Semester Exams, 2011 Dept: *Comp. Sc & Engg.* Sub No: CS60005

M.Tech (Core) Sub Name: ***Foundations of Computing Science***

Instructions: *Part-A must be answered on the question paper itself.*

Part-B must be answered on the answer book.

Answer all parts of a question in the same place

PART- B (60 marks)

Answer any three.

---

3. [Computational Complexity] [10 + 7 + 3 = 20 marks]

(a) Consider the following languages:

EQ_{CFG} = { ⟨G, H⟩ | G and H are CFGs and L(G) = L(H) }

MAX-CLIQUE = { ⟨G, k⟩ | the largest clique in the graph G is of size exactly k }

k-PATH = { ⟨G, s, t, k⟩ | graph G contains a path of length exactly k from s to t }

Now answer the following questions carefully with brief justification (keep in mind the hierarchy of complexity classes):

i. Which of the above problems are Turing recognizable?
ii. Which of the above problems are in P?
iii. Which of the above problems are in NP?
iv. Which of the above problems are NP-complete?
v. Which of the above problems are in coNP?

(b) The SUBSET-SUM problem is defined as follows:

SUBSET-SUM = { ⟨S, t⟩ | S = {x_1, …, x_k} and for some $Q \subseteq S$, we have $t = \sum_{x \in Q} x$ }

In other words, given a set S of integers and a number *t*, the SUBSET-SUM problem asks whether the sum of the integers in any subset of S equals *t*. This problem is known to be NP-complete. Use this fact to prove that the EQIPARTITION problem is NP-complete using a clean mapping reduction (no hand-waving arguments).

EQUIPARTITION = { S | S = {x_1, …, x_k} and for some $Q \subseteq S$, we have $\sum_{x \in Q} x = \sum_{x \notin Q} x$ }

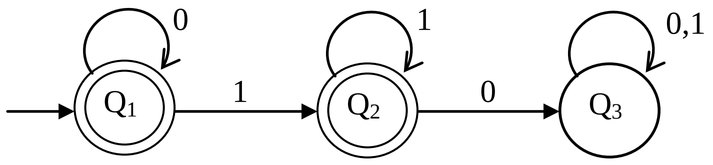
In other words, given a set S of integers, the EQUIPARTITION problems asks whether the set S can be partitioned into two parts such that the sum of the integers in both parts are equal.

(c) Two problems are said to be polynomially equivalent if each is reducible to the other in polynomial time. Are SUBSET-SUM and EQUIPARTITION polynomially equivalent?

4. [Miscellaneous] [5 + 5 + 5 + 5 = 20 marks]

(a) Prove that there is no string z in $(a \cup b)^*$ such that az = zb.

(b) Write the regular expression corresponding to the finite automaton given below.



(c) When do we call a grammar *ambiguous*? Show that the following grammar is ambiguous:

$S \rightarrow a \mid abSb \mid aAb$

$A \rightarrow bS \mid aAAb$

(d) Draw the polynomial hierarchy

5. [Miscellaneous] [3 + 3 + 7 + 7 = 20 marks]

(a) Give the definition of a countable set.

(b) Prove that the set of all finite length strings over a finite alphabet is countable.

(c) Prove that the set of all languages over a finite alphabet is uncountable.

(d) Let J = { w | either w = 0x for some $x \in A_{TM}$, or w = 1y for some $y \in \overline{A_{TM}}$ }. Show that neither J nor $\overline{J}$ is Turing recognizable.

6. [Complexity] [5 + 9 + 6 = 20 marks]

(a) Suppose P=NP, and you have an algorithm, A, for solving the decision version of the Traveling Salesperson Problem (TSP) (that is, whether a tour of cost at most *k* exists) in polynomial time. How would you use this algorithm to find the minimum cost tour in polynomial time? [Recall that a tour is a Hamiltonian cycle, and its cost is the sum of its edge weights. Assume that all weights are integers.]

(b) State and prove Savitch's theorem

(c) Present brief arguments supporting that PSPACE is closed under union and complementation.

7. [Logic] [12 + 2 + 6 = 20 marks]

(a) Consider the following statements: **A topper is a student with highest CGPA. For each topper, either everyone likes the topper or the topper dislikes everyone. Raj dislikes everyone.**

   i. Express these statements in first-order logic. Use meaningful predicate names, such as, Topper(x), Likes(x, y) (for x likes y).

   ii. Convert the statements into normal form.

   iii. For each of the following statements, determine whether it is entailed from the above statements. For the ones that are entailed, provide a resolution-refutation proof. For the ones that are not entailed (if any), provide a justification.

   - **G1: Raj is the topper**
   - **G2: Every topper dislikes everyone**

(b) What can you say about the complexity of determining the truth of a quantified Boolean formula with *k* alternations?

(c) What is meant by the *completeness* of a proof procedure for first-order logic? Why doesn't the completeness of a proof procedure guarantee termination? Would anyone use a proof procedure which is not complete?