Foundations of Computing Science (CS60005)

TUTORIAL 1

1. Find the error in the 'proof' of the following 'theorem'.

   Theorem: Let R be a relation on a set A that is symmetric and transitive. Then R is reflexive.

   Proof: Let $a \in A$. Take an element $b \in A$ such that $(a, b) \in R$. Because R is symmetric, we also have $(b, a) \in R$. Now using the transitive property, we can conclude that $(a, a) \in R$ because $(a, b) \in R$ and $(b, a) \in R$.

2. Let R be a symmetric relation. Show that R^n is symmetric for all positive integers n.

3. How many reflexive, symmetric relations can be there in a set of n elements?

4. Find the maximal and minimal elements of the poset $(2, 4, 5, 10, 12, 20, 25, |)$. Draw the Hasse diagram.

5. Construct the Hasse diagram for the divisors of 84.

6. Is the set $A = [0, 2)$ of real numbers with the operation $\leq$ a partial order? Is it a lattice?

7. Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in \mathcal{R}$ if and only if $p - s = q - r$. Which one of the following is true about R? (Give reason for your answer)

   (a) Symmetric and Reflexive

   (b) Reflexive but not symmetric

   (c) Not Reflexive but Symmetric

   (d) Neither Reflexive nor Symmetric