Verification and Control of CPS: Assignment 6
Due Date: Thursday, Dec 4, 2014 (4:30 PM)

P1 A reset oscillator has a model given by state variables x_1, x_2 with dynamics given by:

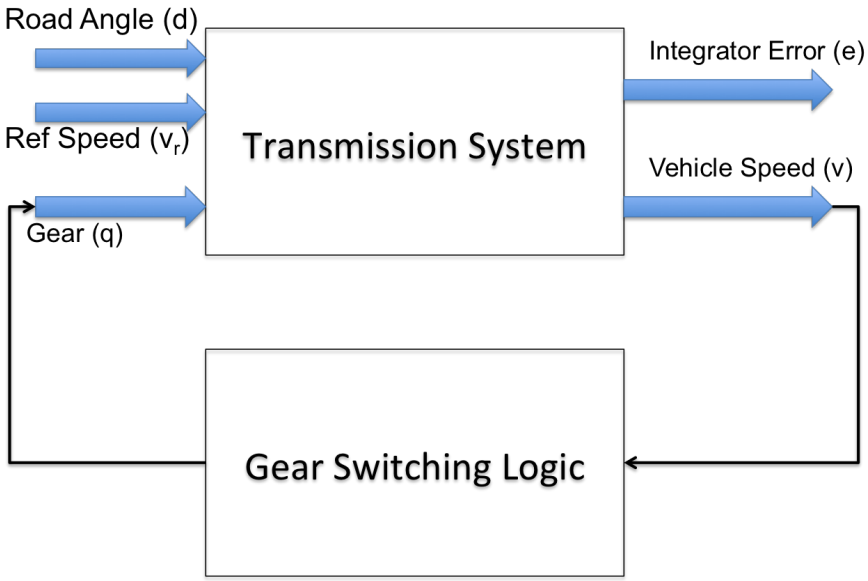
$$\begin{array}{rcl} \frac{dx_1}{dt} & = & x_2 \\ \frac{dx_2}{dt} & = & -x_1 + 0.2x_2 \end{array}$$

Every time the system hits the condition $x_1 < 0 \ \wedge \ x_2 = 0$, it executes a discrete transition that executes the jump

$$x_1 := -x_1$$

Write down a hybrid automaton that models the reset oscillator. Implement the oscillator in simulink. Use a "hit-crossing" block in simulink to detect the condition $x_2 = 0$ as a zero crossing of the signal x_2 from below. Plot the signal $x_1(t)$ as a function of time.

P2 We will consider an automatic gearbox system with four possible gears $q = \{1, 2, 3, 4\}$. The block diagram is shown below:



The system is subject to a disturbance input given by the road angle disturbance $d(t)$ that changes randomly fluctutes between $[-\frac{\pi}{4}, \frac{\pi}{4}]rad$. To model realistic road conditions, pass a random number signal source through an integrator. Furthermore, the reference speed $v_r(t)$ is input signal, kept constant at $30m/s$.

The transmission system is modeled by state variables v and e with dynamics given by the ODE

$$\begin{array}{rcl} \frac{dv}{dt} & = & \frac{P(q)K(q)}{m}(v_r(t) - v) + \frac{P(q)}{m}e - g\sin(d(t)) \\ \frac{de}{dt} & = & \frac{K(q)}{T_r}(v_r(t) - v) \end{array}$$

Note that $P(q), K(q)$ are constants that vary according to the chosen gear q, while T_r, m, g are constants. Finally, we recall that $v_r(t)$ and $d(t)$ are external input signals. The meanings and

values of various parameters are given below:

$P(q)$	Transmission ratio for gear q	$P(1) = 50, P(2) = 32, P(3) = 20, P(4) = 14$
$K(q)$	Integral gain for gear q	$K(1) = 3.75,\ K(2) = 5.86,\ K(3) = 9.375,\ K(4) = 13.39$
m	Mass of Vehicle	$1500kg$
g	acceleration due to gravity	$10m/s^2$
T_r	Integrator time constant	$40s$

The mode transmission logic switches between gear q to gear $q+1$ according to maintain maximum and minimum engine torques. The maximum desired engine torque is $w_{hi} = 500rad/s$ while minimum is $w_{lo} = 230rad/s$

For each mode $q \in \{1, 2, 3\}$ we switch to a higher gear $q \to q+1$ if

$$v \geq \frac{w_{hi}}{P(q)}$$

Likewise, for each mode $q \in \{2, 3, 4\}$, we switch to a lower gear $q \to q-1$ if

$$v \leq \frac{w_{lo}}{P(q)}$$

The initial gear is $q = 2$ with initial conditions $v(0) = 30, e(0) = 0$.

P1 Write down a hybrid automaton model with four modes (one for each gear) and 6 transitions. For each mode, write down the ODE clearly.

P2 Implement the automaton inside Simulink/Stateflow.

P3 Simulate the system as specified here.