FOCS TUTORIAL 3 Solution

1. (a) Let's assume L1 is regular.

pumping length : p

If we will take string s as :

$s = 0^p 1^p$ — (i)

we will not find any assignments for x, y, z such that $s = xyz$

1. $\forall i \geq 0,\ xy^iz \in L1,$
2. $|y| > 0,$
3. $|xy| \leq p$

$$s = \overbrace{000 - - - 0}^{p}\ \overbrace{111 - - - 1}^{p}$$

(a.) If we place y into 0's part only, by applying condition (1.), we will get more 0's tha 1's,

~~hence~~ $s \notin$

(b.) If y contains only 1's : Not possible because of condn. (3.)

(c.) If y contains both 0's & 1's : Not possible because of condn. (3.)

There is no possible assignment for x, y, z

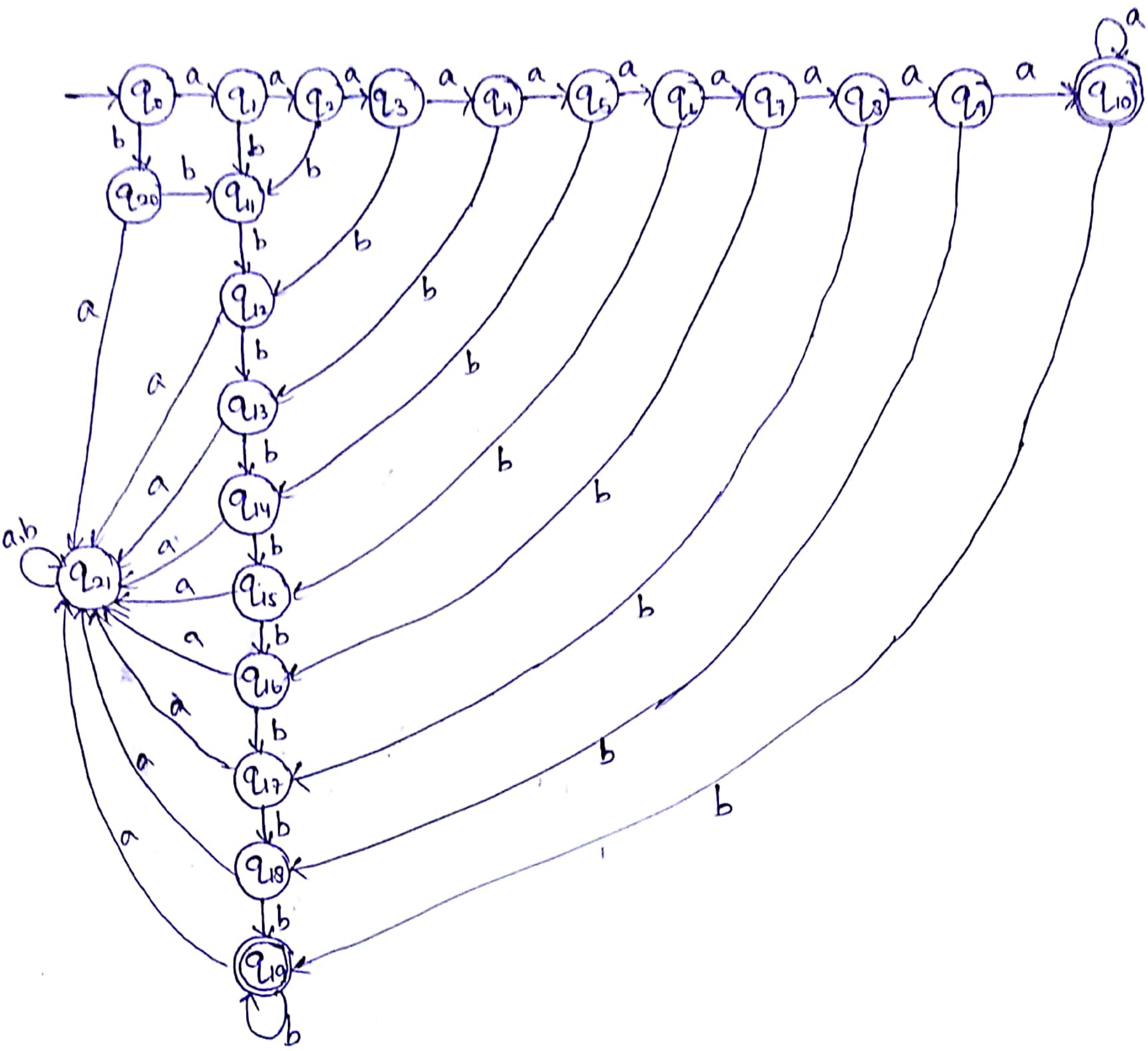
Hence $L1$ is not regular [contradiction].

1.(b) Apply same process as 1.(a)

take $s = a^p b a^p b$

1.(c) take $s = a^{p+1} b^p$

2. $L_a = \{ a^i b^j \mid i,j \geq 0 \text{ and } i+j \geq 10 \}$



Since, we can draw the DFA for LA

∴ LA is regular.

$L_b = \{ a^i b^j \mid i,j \geq 0 \text{ and } i-j \geq 10 \}$

Assume L_b is regular. Let p be the pumping length and let $s = a^{p+11} b^p$. ~~Let~~

Since, $s \in L_b$ $\therefore$ pumping lemma guarantees that above string can be broken into 3 pieces xyz such that

(i) $\forall i \geq 0$ $xy^iz \in L_b$

(ii) $|y| > 0$

(iii) $|xy| \leq p$

Now, we consider the following cases to prove that the above conditions are not met.

CASE 1: When y ~~belongs~~ contains only a's

Let $x = \epsilon$

and $y = a^p$

Then, according to pumping lemma

$xy^iz \in L_b$

i.e., $(a^p)^i a^{11} b^p \in L_b$

If $i=0$, then resulting string is $a^{11} b^p \notin L_b$

$\therefore$ contradiction arises

CASE 2: When y contains only b's. This contradicts the third condition of pumping lemma which says that $|xy| \leq p$

$\therefore$ Again not possible

CASE 3: When y contains some a's and some b's.

Again not possible because $|xy| \leq p$

Thus, we see in all the possible cases that no such division of L_b in 3 pieces exists which satisfies pumping lemma.

∴ Our assumption was wrong

The given language is NOT REGULAR.

3. $L = \{ w c x : w, x \in \{a, b\}^* \}$

Take $s = a^p c b^p$

Tutorial 3

4: (a) $S \to AB$

$A \to a \mid aAa \mid aAb \mid bAa \mid bAb$

$B \to b \mid aBa \mid aBb \mid bBa \mid bBb$

(b) $S \to SaSbS \mid SbSaS \mid$

$SaScS \mid ScSaS \mid \epsilon$

(c) $L_3 = \{a^i b^j c^k d^l \mid i+k = j+l,\ i,j,k,l \geq 0\}$

without loss of generality $j = i+k-l$

$L_3 = \{a^{l+(i-l)} b^{k+(i-l)} c^k d^l\}$

$S \to S_1$

$S_1 \to aS_1d \mid S_2$

$S_2 \to S_3 S_4$

$S_3 \to aS_3b \mid \epsilon$

$S_4 \to bS_4c \mid \epsilon$

(d) $L_4 = \{w\#x \mid w^R$ is a substring of x for $w, x \in \{0,1\}^*\}$

$S \to TR$

$T \to 0T0 \mid 1T1 \mid \#R$

$R \to RR \mid 0 \mid 1 \mid \epsilon$

(e) $L_5 = \{w \mid w$ has twice as a's as b's$\}$

$S \to SaSaSbS \mid SaSbSaS \mid SbSaSaS \mid \epsilon$

5(a). $S \to AS \mid \epsilon$

$A \to 0A1 \mid A1 \mid 0$

language $L = \{0^m 1^n \; , \; m > 0 \text{ and } n \geq 0\}$

(b) $S \to A1B$

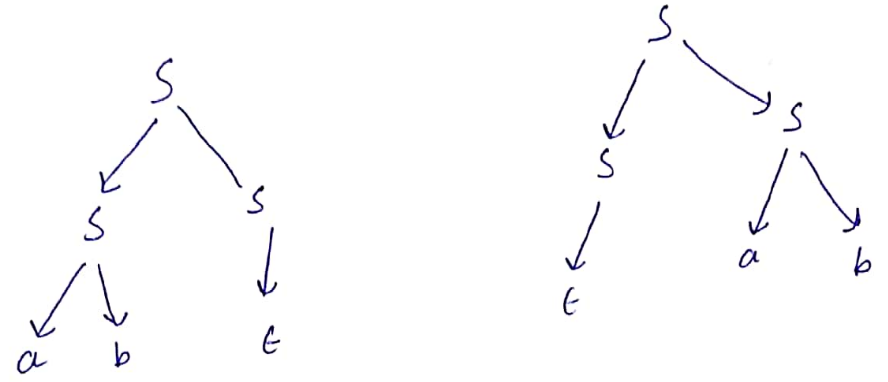
$A \to 0A \mid \epsilon$

$B \to 0B \mid 1B \mid \epsilon$

language $L = \{0^n 1 x \mid x \in \{0,1\}^* \text{ and } n \geq 0\}$

6. $G = (\{S\}, \{a, b\}, \{S \to SS, S \to ab, S \to ba, S \to \epsilon\}, S)$

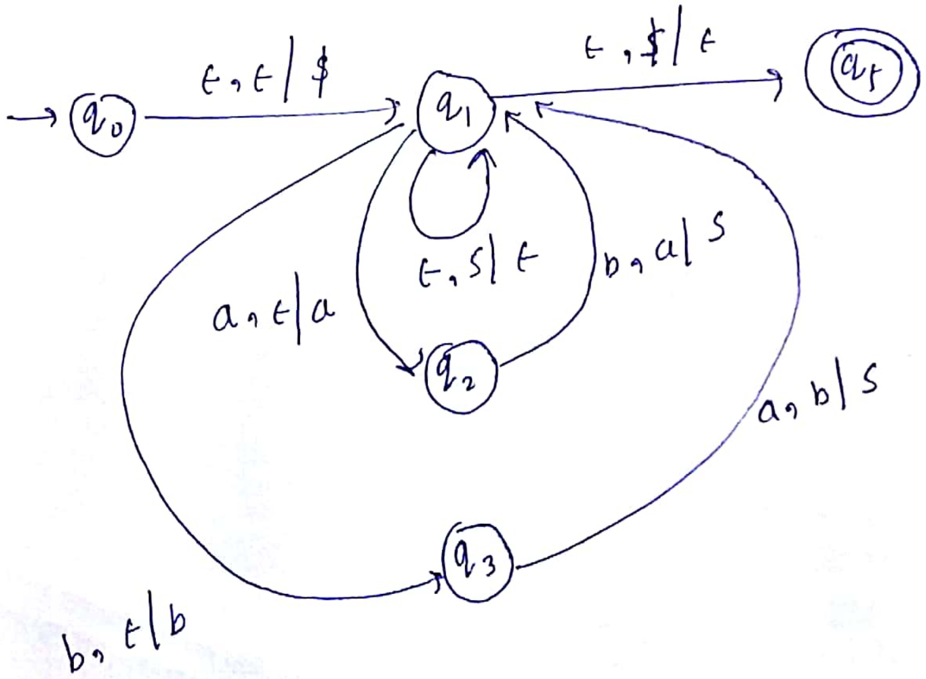
(I.)



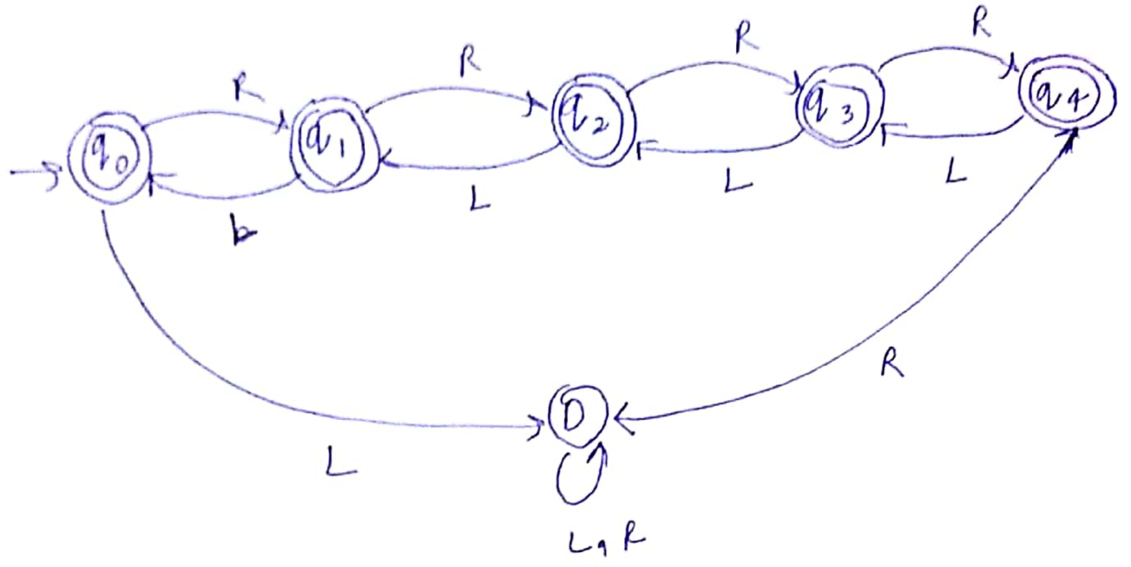
G is ~~ab~~ ambiguous as it generates 2 parse trees for string ab. (True)

(II.) aa bb can not be generated. (False)

(III.)



7. (a)



(b) $L_T = \{ w \mid w \in \{L, R\}^*$, ~~~~ Each prefix in w has no. of R's greater than or equal to no. of L's and w starts with R $\}$

Take $s = R^P L^P$ and apply pumping lemma

(c) $G = (\{S\}, \{L, R\}, \{S \to RSLS \mid \epsilon\}, S)$