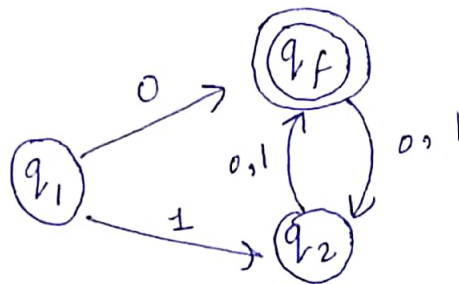


## FOCS TUTORIAL 2 Solutions

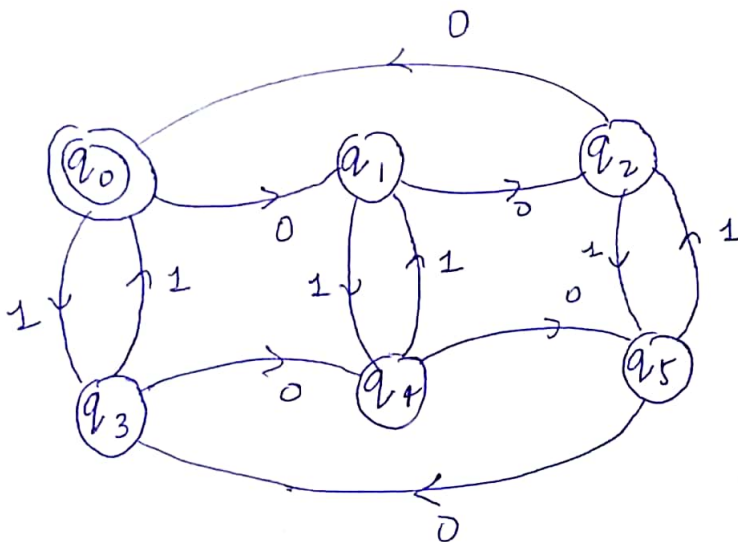
1. NO.

We need to put a dummy final state first.  
(Before swapping)

2. (a)



(b)



3. (a)  $(0+1)^* 1 (0+1) (0+1) (0+1)$

(b)  $(01+10)^*$

(c)  $(1^* 0 1^* 0 1^* 0 1^*)^*$

(d) There can be either zero or one pair of consecutive 1's

$p$ : No consecutive 1's and last symbol 0.

$q$ : No consecutive 1's and last symbol 1.

$r$ : One consecutive 1's and last symbol 1.

$s$ : One consecutive 1's and last symbol 0.

$p$ :  $(0+10)^*$        $q$ :  $(0+10)^* 1$

$r$ :  $(0+10)^* 11 (00^* 1)^*$

$s$ :  $(0+10)^* 11 (00^* 1)^* 00^*$

solution:  $p + q + r + s$ .

(e)  $(0+1)^* (01+10) (0+1)^*$

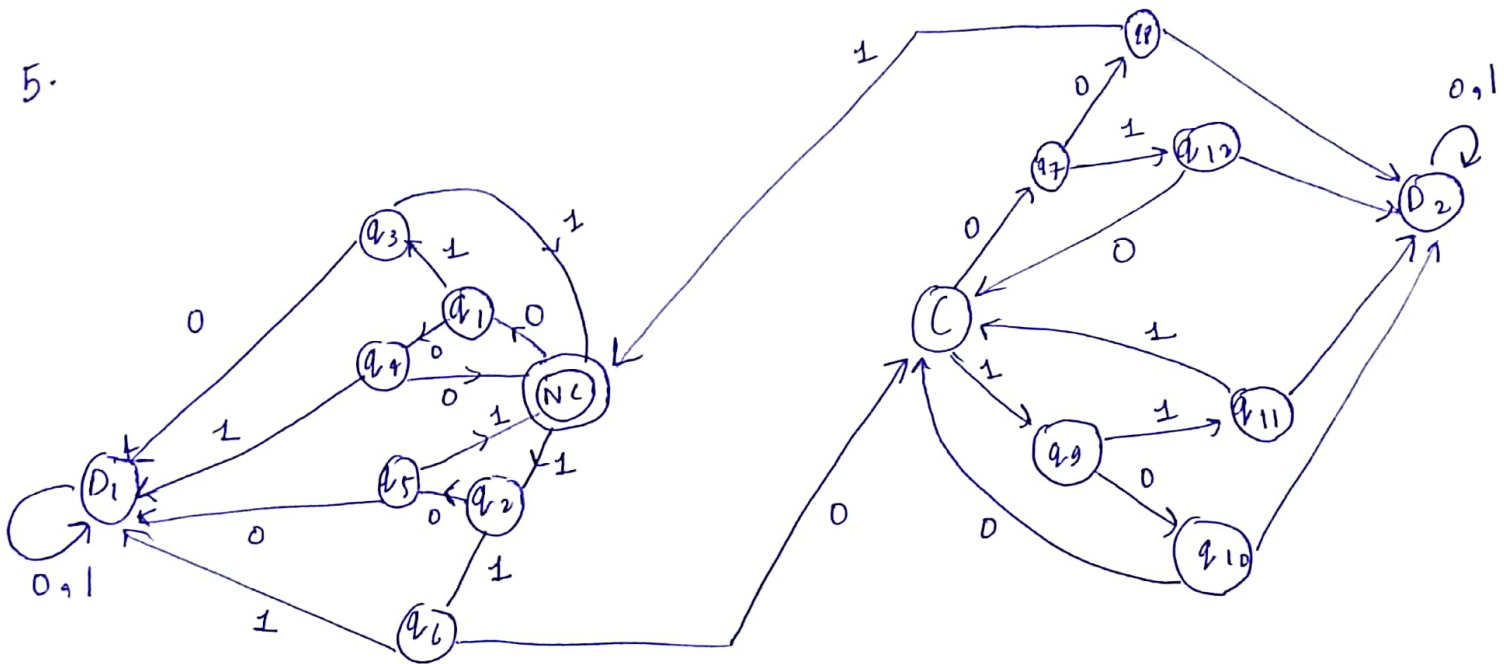
4.  $L(M) = a^* b^* a^*$

$L(N) = (a+b)^*$

$$L(M) \cap L(N) = (a^* b^* a^*) \cap (a+b)^*$$

$$= a^* b^* a^*$$

5.



$$6. \quad 10 + 13 + 9 - \frac{10+13+9}{3} - 20 = 3$$

(10+13+9) + 10+13+9 + 10+13+9

$$\begin{aligned}
 7. (a) \quad & (\bar{p} \wedge (p \vee q)) \rightarrow q \\
 & \equiv \overline{\bar{p} \wedge (p \vee q)} \vee q \\
 & \equiv p \vee (\overline{p \vee q}) \vee q \\
 & \equiv p \vee (\bar{p} \wedge \bar{q}) \vee q \\
 & \equiv (p \vee \bar{p}) (p \vee \bar{q}) \vee q \\
 & \equiv p \vee \bar{q} \vee q \\
 & \equiv p \vee T \\
 & \equiv T \Rightarrow \text{Tautology}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & (p \wedge (p \rightarrow q)) \rightarrow q \\
 & \equiv \overline{p \wedge (p \rightarrow q)} \vee q \\
 & \equiv \bar{p} \vee (\overline{p \rightarrow q}) \vee q \\
 & \equiv \bar{p} \vee (\bar{p} \vee q) \vee q \\
 & \equiv \bar{p} \vee (p \wedge \bar{q}) \vee q \\
 & \equiv \bar{p} \vee [(p \vee q) \wedge (\bar{q} \vee q)] \\
 & \equiv \bar{p} \vee p \vee q \\
 & \equiv T \vee q \\
 & \equiv T \Rightarrow \text{Tautology}
 \end{aligned}$$

$$(c) ((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

$$\equiv ((\bar{p} \vee q) \wedge (\bar{q} \vee r)) \rightarrow (\bar{p} \vee r)$$

$$\equiv \overline{((\bar{p} \vee q) \wedge (\bar{q} \vee r))} \vee \bar{p} \vee r$$

$$\equiv (\overline{\bar{p} \vee q}) \vee (\overline{\bar{q} \vee r}) \vee \bar{p} \vee r$$

$$\equiv (p \wedge \bar{q}) \vee (q \wedge \bar{r}) \vee \bar{p} \vee r$$

$$\equiv [(p \vee \bar{p}) \wedge (\bar{q} \vee \bar{p})] \vee [(q \vee r) \wedge (\bar{r} \vee r)]$$

$$\equiv \bar{q} \vee \bar{p} \vee q \vee r$$

$$\equiv T \vee \bar{p} \vee r$$

$$\equiv T \Rightarrow \text{Tautology}$$

$$(d) (\bar{p} \wedge (p \rightarrow q)) \rightarrow \bar{q}$$

$$\equiv [\bar{p} \wedge (\bar{p} \vee q)] \rightarrow \bar{q}$$

$$\equiv (\bar{p} \vee \bar{p}q) \rightarrow \bar{q}$$

$$\equiv \overline{\bar{p} \vee (\bar{p} \wedge q)} \vee \bar{q}$$

$$\equiv [p \wedge (\overline{\bar{p} \wedge q})] \vee \bar{q}$$

$$\equiv [p \wedge (p \vee \bar{q})] \vee \bar{q}$$

$$\equiv p \vee p\bar{q} \vee \bar{q}$$

$$= p \vee \bar{q} \Rightarrow \text{Not Tautology.}$$