

Foundations of Computing Science (CS60005)

TUTORIAL 4

1. Prove with the help of pumping lemma that the following languages are not Context Free.
 - (a) $L1 = \{ww \mid w \in \{0,1\}^*\}$
 - (b) $L2 = \{0^n 1^n 0^n \mid n \geq 0\}$
 - (c) $L3 = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$
2. Design TM for the following languages:
 - $\{a^n b^n c^n \mid n \geq 0\}$
 - $\{w \mid w \text{ contains equal number of 0's as 1's}\}$
 - $\{w \mid w \text{ contains twice as many 0's as 1's}\}$
3. Design a TM that transforms a string containing only a 's, b 's, and c 's by replacing each letter preceding an a to a b . (Do not worry about the case when the string begins with an a .) Thus, $bccb$ would remain unchanged while $caccaa$ would change to $bacbb a$. The Turing machine should always eventually enter an accepting state to terminate.
4. Design a TM that acts like a comparator when input string is in the form of $a0b$, where a and b are sequence of 1's. For example, 11101111 , here $a = 111$ and $b = 1111$ and that corresponds to $a < b$. Machine should have states corresponding to $a = b$, $a > b$ and $a < b$.
5. Construct a TM that takes its input on the tape, shifts it to the right by one position, and put a $\$$ on the leftmost position on the tape when $\Sigma = \{a, b, \dots, z\}$
6. Prove with the help of pumping lemma that the following languages are not regular
$$L_a = \{1^{n^2} \mid n \geq 0\}$$
$$L_a = \{1^n \mid n \text{ is prime}\}$$
7. Draw the PDA and then write the CFG for the following language.
$$L = \{a^m b a^m \mid m \geq 0\}$$