INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Computer Science and Engineering

Foundations of Computing Science (CS60005)

Class Test – I (Autumn)

Place: CSE-107 *Date:* Wed, Sep 05, 2018 *Time:* 8:15am-9:15am (FN)

Students: 77 *Marks:* 30

Answer ALL the questions (following question specific instructions)

Q1: Choose the last non-zero digit from your roll number and any other non-zero digit, excluding the first digit to ensure that the resulting number is not a prime number. Construct the lattice of divisors for the number. 5

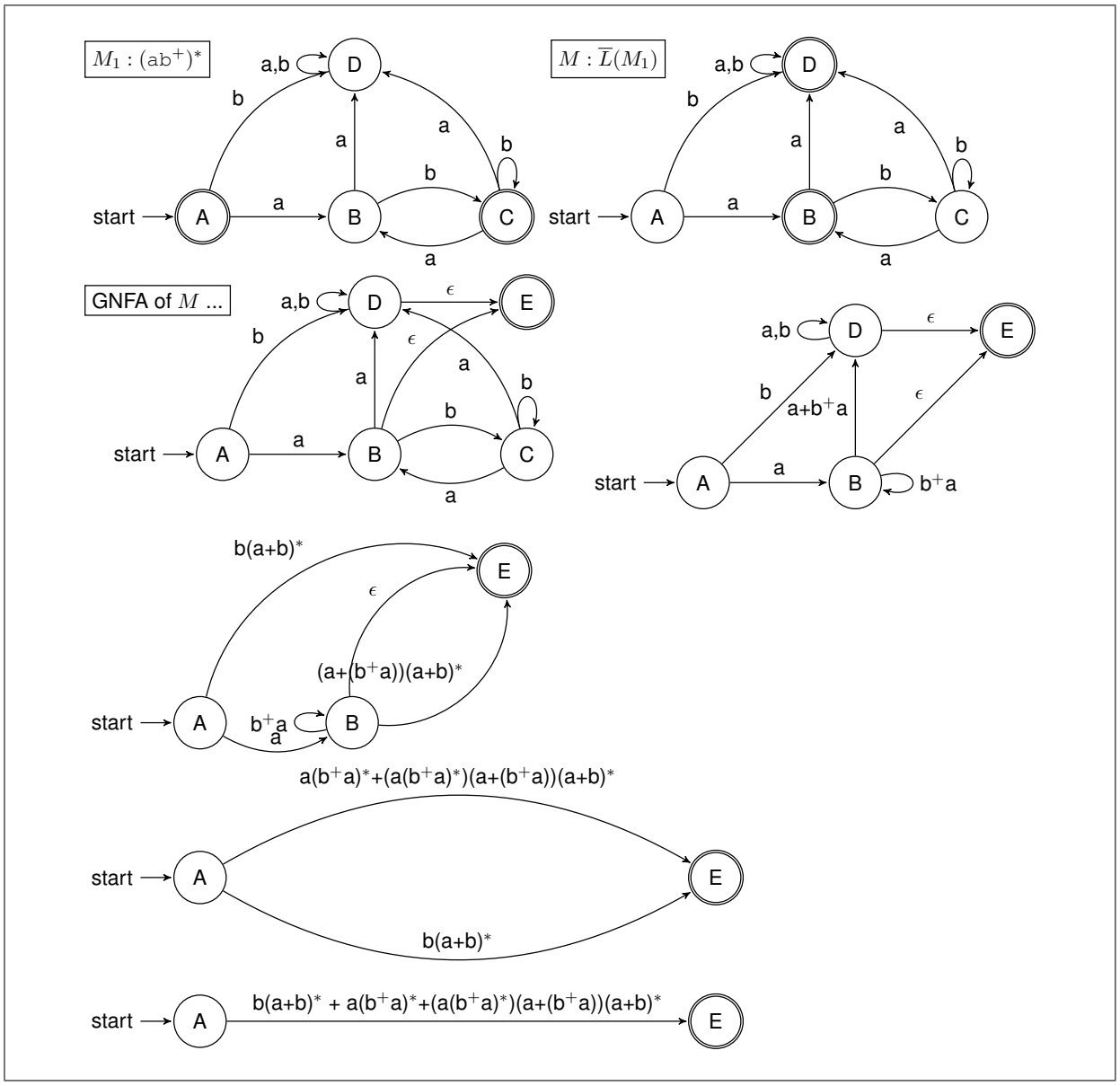
Q2: Prove that $x^{2n+1} + y^{2n+1}$ is divisible by $x + y$ for all $n \geq 0$. 5

- Base case for $n = 0$ is satisfied
- Assuming hypothesis for k ($x^{2k+1} + y^{2k+1}$ is divisible by $x + y$ for $k \geq 0$), need to prove for $k + 1$
- $x^2x^{2k+1} + y^2y^{2k+1} = x^2x^{2k+1} + \left(y^2 + x^2 - x^2\right) y^{2k+1} = x^2\left(x^{2k+1} + y^{2k+1}\right) + \left(y^2 - x^2\right) y^{2k+1}$ which is divisible by $x + y$

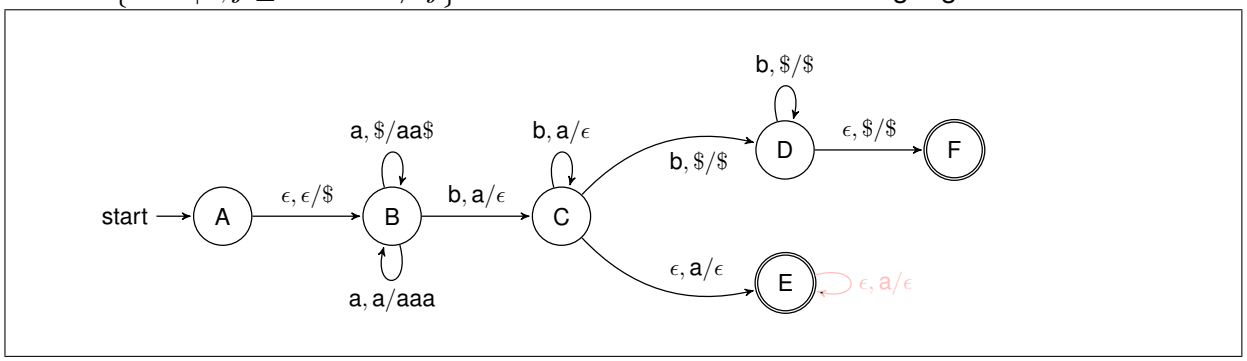
Q3: Consider the language $\{w \mid w$ is any string not in $(\mathtt{ab}^+)^*\}$

(a) Construct the DFA to recognise this language 5

(b) Construct the GNFA corresponding to the constructed DFA to yield the RE for this language 5



Q4: Let $L = \{\mathrm{a}^i\mathrm{b}^j \mid i, j \geq 0 \text{ and } 2i \neq j\}$. Show that L is a context free language. 5



- Let $L_1 = \left\{ \mathtt{a}^i \mathtt{b}^j \mid i, j \geq 0 \text{ and } 2i < j \right\}$ and $L_2 = \left\{ \mathtt{a}^i \mathtt{b}^j \mid i, j \geq 0 \text{ and } 2i > j \right\}$, $L = L_1 \cup L_2$
- For L_1, $S \rightarrow EB$, $E \rightarrow \mathtt{a}E\mathtt{bb} \mid \epsilon$, $B \rightarrow \mathtt{b}B \mid \mathtt{b}$
- For L_2, $S \rightarrow AE \mid AE\mathtt{b}$, $E \rightarrow \mathtt{a}E\mathtt{bb} \mid \epsilon$, $A \rightarrow \mathtt{a}A \mid \mathtt{a}$

Q5: Prove that the subset of a context free language with strings starting and ending in the same symbol is context free. *Hint: Utilize closure properties of CFLs and RLs.* 5

- Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$
- Let L_1 be defined by $\mathtt{a} \mid \mathtt{a}\,(\mathtt{a} \mid \mathtt{b})^*\,\mathtt{a}$ and L_2 by $\mathtt{b} \mid \mathtt{b}\,(\mathtt{a} \mid \mathtt{b})^*\,\mathtt{b}$
- $L_1 \cup L_2$ is a RL
- Required language is $L \cap (L_1 \cup L_2)$ which is a CFL, being the intersection of a CFL and a RL and L being the the CFL under consideration