Solving Linear Matrix Inequalities using the Multi Parametric Toolbox

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Where innovation starts

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From LMI to Matlab code

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Motivating Examples

- Quadratic Lyapunov Function

$$\text{Find } P \text{ s.t.} \begin{cases} A^T P + PA < 0 \\ P > 0 \end{cases}$$

- Common Quadratic Lyapunov Function

$$\text{Find } P \text{ s.t.} \begin{cases} A_1^T P + PA_1 < 0 \\ A_2^T P + PA_2 < 0 \\ P > 0 \end{cases}$$

- PWL system, continuity conditions and S-procedure

$$\text{Find } P_i, U_i \text{ and } W_i \text{ s.t.} \begin{cases} A_i^T P_i + P_i A_i + E_i^T U_i E_i < 0 \\ P_i - E_i^T W_i E_i > 0 \\ Z_{ij}^T [P_i - P_i] Z_{ij} = 0 \qquad (i,j) \in \mathcal{J} \end{cases}$$

Example: From LMI to Matlab Code

- Quadratic Lyapunov Function: LMI's

$$\begin{cases} A^T P + PA < Q \\ P > 0 \end{cases}$$

- Matlab code

```
A = [-1 2 0;-3 -4 1;0 0 -2];
P = sdpvar(3,3); % Unknown 3x3 Hermitian matrix
Q = 1/100 * eye(3,3); % Decrease rate

L1 = set(A'*P + P*A + Q < 0); % Constr.  1
L2 = set(P > 0); % Constr.  2
L = L1 + L1; % Combine all constraints

solve(L); % Solving for P (matlab workspace)
P = double(P); % Converts to standard format
```

General Procedure

- Define unknowns: `sdpvar(..,..)` (matrices are symmetric)
- Define constraints L_i
- Combine constraints $L = \sum L_i$
- Solve for unknowns: `solvesdp(L)` or `solvesdp(L,c)`
- Convert solutions: `double(..)`
- Check solutions:
  - Substitute in constraints and check manually (recommended)
  - Check yalmip output (sometimes puzzling, overkill for this course)

Commands and Technicalities

- Equality operators: ==
- Inequality operators: <=, >=, >, <
- Condition on matrix (eigenvalues): `set(M < 0)`
- Condition on matrix elements: `set(M(:) < 0)`
- Solve while minimizing a parameter, matrix elements or eigenvalues: `solvesdp(L,MinCond)`
- Avoiding the zero solution:

```
set(M + epsilon * eye(size(M)) < 0)
```

Matlab Examples

1. Quadratic Lyapunov Function
2. + multiple Lyapunov Functions: stability of PWL system
3. + relaxing the problem: S-procedure

4. Synthesis of a stabilizing controller

Final Notes

For more info type: `yalmipdemo`

On studyweb

- Examples
- Presentation
- Toolbox and installation manual