Foundations of Computing Science (CS60005)

TUTORIAL 3

1. Prove with the help of pumping lemma that the following languages are not regular.
   (a) L1 = {w | w has an equal number of 0s and 1s}
   (b) $L2 = \{ww \mid w \in \{a, b\}^\star\}$
   (c) $L3 = \{a^i b^j | i > j\}$
2. One of the following languages is regular, and the other one is not regular. Identify which is which with respective proofs:
   $L_a = \{a^i b^j \mid i, j >= 0 \text{ and } i + j >= 10\}$
   $L_b = \{a^i b^j \mid i, j >= 0 \text{ and } i - j >= 10\}$
3. Prove that the following language L over the alphabet $\{a, b, c\}$ is not regular.
   $L = \{wcx : w, x \in \{a, b\}^\star$ and the number of a's in w is equal to the number of b's in x.$\}$
4. For each of the following languages, give a context-free grammar.
   (a) $L_1 = \{xy \mid |x| = |y| \text{ and } x \neq y\}.\Sigma = \{a, b\}$.
   (b) $L_2 = \{w \mid w$ has the same number of a's as b's and c's together $\}.\Sigma = \{a, b, c\}$.
   (c) $L_3 = \{a^i b^j c^k d^l \mid i + k = j + l, i, k, j, l \geq 0\}.\Sigma = \{a, b, c, d\}$.
   (d) $L_4 = \{w\#x \mid w^R$ is a substring of x for $w, x \in \{0, 1\}^\star\}$.
   (e) $L_5 = \{w \mid w$ has twice as many a's as b's$\}$.
5. What is the language defined by the following grammar
   (a) $S -> AS|\epsilon$
   $A -> 0A1|A1|0$
   (b) $S -> A1B$
   $A -> 0A|\epsilon$
   $B -> 0B|1B|\epsilon$
6. Consider the following statements about the context free grammar.
   $G = (\{S\}, \{a, b\}, \{ S -> SS, S -> ab, S -> ba, S -> \epsilon\}$, S)
   I. G is ambiguous
   II. G produces all strings with equal number of a's and b's
   III. G can be accepted by a deterministic PDA

Which combination below expresses all the true statements about G (Explain briefly)?
A. I only
B. I and III only
C. II and III only
D. I, II and III

7. A cassette tape reader/recorder head has two moves, namely going forward one tape cell (R) and going back by one tape cell (L). A string RLRRR represents a sequence of moves of the tape head. The tape head is initially at the beginning of the tape. A string of moves which requires the tape to move left of the beginning of the tape is an invalid string. For example, RLRLL is invalid.

   (a) Consider the set of strings in which the tape has a finite number of cells, N. Is the language defined by legal moves over such a tape regular? If Yes, draw a DFA for N=5, else give a proof using the pumping lemma / closure properties for Regular Languages.

   (b) Consider the set of all strings in which the tape in not finite and the head returns to the beginning of the tape. Show that this language is not regular.

   (c) Give a context free grammar for the language in (b).