1 Problem 1 a

If $r_i(t)$ and $r_j(t)$ are correlated, then:

$$r_i(t) = \alpha\, r_j(t)$$

Thus:

$$< r_i(t)r_j(t) >= \alpha \ < r_j(t)^2 >$$
$$< r_i(t) >< r_j(t) >= \alpha \ < r_j(t) >^2$$

In Addition:

$$(< r_i(t)^2 > - < r_i(t) >^2) = (\alpha^2 < r_j(t)^2 > -\alpha^2 < r_j(t) >^2) = \alpha^2(< r_j(t)^2 > - < r_j(t) >^2)$$

Thus the denominator of ρ_{ij} is:

$$\sqrt{(< r_i(t)^2 > - < r_i(t) >^2)(< r_j(t)^2 > - < r_j(t) >^2)} = \sqrt{\alpha^2(< r_j(t)^2 > - < r_j(t) >^2)^2}$$

And the numerator of ρ_{ij} is:

$$< r_i(t)r_j(t) > - < r_i(t) >< r_j(t) >= \alpha < r_j(t)^2 > -\alpha < r_j(t) >^2 = \alpha(< r_j(t)^2 > - < r_j(t) >^2)$$

And finally:

$$\rho_{ij} = \frac{\alpha(< r_j(t)^2 > - < r_j(t) >^2)}{\sqrt{\alpha^2(< r_j(t)^2 > - < r_j(t) >^2)^2}} = \frac{\alpha}{\sqrt{\alpha^2}}$$

Meaning that if $\alpha > 0$, $\rho_{ij} = 1$ and if $\alpha < 0$, $\rho_{ij} = -1$

2 Problem 1 b

$$log[1 + q_i(t)] =$$
$$log[\frac{p_1(t-1)}{p_1(t-1)} + \frac{(p_1(t) - p_1(t-1))}{p_1(t-1)}] =$$
$$log[\frac{p_1(t)}{p_1(t-1)}] =$$
$$log[p_1(t)] - log[p_1(t-1)]$$

3 Problem 2 a

$$w_{ij} = \sqrt{2(1 - \rho_{ij})}$$

4 Problem 4

$$\alpha = \frac{1}{|V|} \sum_{v_i \in V} P(v_i \in S_i)$$

The first method utilizes the minimum spanning tree (MST) generated in the previous question. Q_i is the set of neighbors from the MST that share the same sector, while N_i is the set of all neighbors in the MST:

$$P(v_i \in S_i) = \frac{|Q_i|}{|V_i|}$$

The second method effectively considers the entire graph. S_i is the total number of stocks in the same sector, while V is the set of all stocks (vertices):

$$P(v_i \in S_i) = \frac{|S_i|}{|V|}$$

Note that if we take the **first method of the full graph, these two values are equivalent.** Since the full graph is fully connnected:

$$Q_i = S_i, N_i = V$$

Thus,

$$\alpha_1 = \alpha_2$$