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import numpy as np
from nnrl.layers import *
import pdb

def conv_forward_naive(x, w, b, conv_param):
    """
    A naive implementation of the forward pass for a convolutional layer.

    The input consists of N data points, each with C channels, height H and width W. We convolve each input with F different filters, where each filter spans all C channels and has height HH and width WW.

    Input:
    - x: Input data of shape (N, C, H, W)
    - w: Filter weights of shape (F, C, HH, WW)
    - b: Biases, of shape (F,)
    - conv_param: A dictionary with the following keys:
      - 'stride': The number of pixels between adjacent receptive fields in the horizontal and vertical directions.
      - 'pad': The number of pixels that will be used to zero-pad the input.

    Returns a tuple of:
    - out: Output data, of shape (N, F, H', W') where H' and W' are given by
      H' = 1 + (H + 2 * pad - HH) / stride
      W' = 1 + (W + 2 * pad - WW) / stride
    - cache: (x, w, b, conv_param)
    """
    out = None
    pad = conv_param['pad']
    stride = conv_param['stride']

    # ===== #
    # YOUR CODE HERE:
    # Implement the forward pass of a convolutional neural network.
    # Store the output as 'out'.
    # Hint: to pad the array, you can use the function np.pad.
    # ===== #

    N, C, H, W = x.shape
    F, C, HH, WW = w.shape
    h_out = int((H + 2 * pad - HH) / stride + 1)
    w_out = int((W + 2 * pad - WW) / stride + 1)
    out = np.zeros([N, F, h_out, w_out]) # [N, 32, 32, 32]

    for n, x_n in enumerate(x):
        x_pad = (np.pad(x_n, ((0, 0), (pad, pad), (pad, pad)), 'constant'))
        # for f, filter in enumerate(w):
        for f in range(h_out):
            for j in range(w_out):
                out[n, :, f, j] = np.sum(x_pad[:, :, f*stride:f*stride+HH, j*stride:j*stride+WW] * w, axis=tuple(range(1, w.ndim))) + b

    # ===== #
    # END YOUR CODE HERE
    # ===== #

    cache = (x, w, b, conv_param)
    return out, cache

def conv_backward_naive(dout, cache):
    """
    A naive implementation of the backward pass for a convolutional layer.

    Inputs:
    - dout: Upstream derivatives.
    - cache: A tuple of (x, w, b, conv_param) as in conv_forward_naive

    Returns a tuple of:
    - dx: Gradient with respect to x
    - dw: Gradient with respect to w
    - db: Gradient with respect to b
    """
    dx, dw, db = None, None, None

    N, F, out_height, out_width = dout.shape
    x, w, b, conv_param = cache

    stride, pad = [conv_param['stride'], conv_param['pad']]
    xpad = np.pad(x, ((0, 0), (0, 0), (pad, pad), (pad, pad)), mode='constant')
    num_filts, _, f_height, f_width = w.shape

    # ===== #
    # YOUR CODE HERE:
    # Implement the backward pass of a convolutional neural network.
    # Calculate the gradients: dx, dw, and db.
    # ===== #

    F, C, HH, WW = w.shape
    dx = np.zeros(x.shape)
    dx_pad = np.pad(dx, ((0, 0), (0, 0), (pad, pad), (pad, pad)), mode='constant')
    dw = np.zeros(w.shape)
    db = np.zeros(b.shape)

    db = np.sum(np.sum(np.sum(dout, axis=3), axis=2), axis=0)

    for n, x_pad_n in enumerate(xpad):
        for f, filter in enumerate(w):
            for i in range(out_height):
                for j in range(out_width):
                    dw[f] += dout[n, f, i, j] * x_pad_n[:, :, i * stride:i * stride + HH, j * stride:j * stride + WW]
                    dx_pad[n, :, i * stride:i * stride + HH, j * stride:j * stride + WW] += dout[n, f, i, j] * filter

    dx = dx_pad[:, :, pad:-pad, pad:-pad]

    # ===== #
    # END YOUR CODE HERE
    # ===== #

    return dx, dw, db

def max_pool_forward_naive(x, pool_param):
    """
    A naive implementation of the forward pass for a max pooling layer.

    Inputs:
    - x: Input data, of shape (N, C, H, W)
    - pool_param: dictionary with the following keys:
      - 'pool_height': The height of each pooling region
      - 'pool_width': The width of each pooling region
      - 'stride': The distance between adjacent pooling regions

    Returns a tuple of:
    - out: Output data
    - cache: (x, pool_param)
    """
    out = None

    # ===== #
    # YOUR CODE HERE:
    # Implement the max pooling forward pass.
    # ===== #

    N, C, H, W = x.shape

    h_out = int((H - pool_param['pool_height']) / pool_param['stride'] + 1)
    w_out = int((W - pool_param['pool_width']) / pool_param['stride'] + 1)
    out = np.zeros((N, C, h_out, w_out))
    for n, x_n in enumerate(x):
        for c, channel in enumerate(x_n):
            for i in range(h_out):
                x_i = i * pool_param['stride']
                for j in range(w_out):
                    y_j = j * pool_param['stride']
                    out[n, c, i, j] = np.amax(channel[x_i:x_i + pool_param['pool_height'], y_j:y_j + pool_param['pool_width']])

    # ===== #
    # END YOUR CODE HERE
    # ===== #

    cache = (x, pool_param)
    return out, cache

def max_pool_backward_naive(dout, cache):
    """
    A naive implementation of the backward pass for a max pooling layer.

    Inputs:
    - dout: Upstream derivatives
    - cache: A tuple of (x, pool_param) as in the forward pass.

    Returns:
    - dx: Gradient with respect to x
    """
    dx = None
    x, pool_param = cache
    pool_height, pool_width, stride = pool_param['pool_height'], pool_param['pool_width'], pool_param['stride']

    # ===== #
    # YOUR CODE HERE:
    # Implement the max pooling backward pass.
    # ===== #

    dx = np.zeros(x.shape)

    F, C, h_out, w_out = dout.shape

    for n, x_n in enumerate(x):
        for c, channel in enumerate(x_n):
            for i in range(h_out):
                for j in range(w_out):
                    x_i = i * pool_param['stride']
                    y_j = j * pool_param['stride']
                    target_area = channel[x_i:x_i + pool_param['pool_height'], y_j:y_j + pool_param['pool_width']]
                    max_x, max_y = np.unravel_index(np.argmax(target_area, axis=None), target_area.shape)
                    dx[n, c, max_x + x_i, max_y + y_j] = dout[n, c, i, j]

    # ===== #
    # END YOUR CODE HERE
    # ===== #

    return dx

def spatial_batchnorm_forward(x, gamma, beta, bn_param):
    """
    Computes the forward pass for spatial batch normalization.

    Inputs:
    - x: Input data of shape (N, C, H, W)
    - gamma: Scale parameter, of shape (C,)
    - beta: Shift parameter, of shape (C,)
    - bn_param: Dictionary with the following keys:
      - mode: 'train' or 'test'; required
      - eps: Constant for numeric stability
      - momentum: Constant for running mean / variance. momentum=0 means that old information is discarded completely at every time step, while momentum=1 means that new information is never incorporated. The default of momentum=0.9 should work well in most situations.
      - running_mean: Array of shape (D,) giving running mean of features
      - running_var: Array of shape (D,) giving running variance of features

    Returns a tuple of:
    - out: Output data, of shape (N, C, H, W)
    - cache: Values needed for the backward pass
    """
    out, cache = None, None

    # ===== #
    # YOUR CODE HERE:
    # Implement the spatial batchnorm forward pass.
    # You may find it useful to use the batchnorm forward pass you implemented in HW #4.
    # ===== #

    N, C, H, W = x.shape
    x_2d = np.reshape(x, (N * H * W, C)) # 2-d reshape of permuted matrix
    out_2d, cache = batchnorm_forward(x_2d, gamma, beta, bn_param)
    out = out_2d.reshape((N, H, W, C)).transpose(0, 3, 1, 2) # reshape and permute back

    # ===== #
    # END YOUR CODE HERE
    # ===== #

    return out, cache

def spatial_batchnorm_backward(dout, cache):
    """
    Computes the backward pass for spatial batch normalization.

    Inputs:
    - dout: Upstream derivatives, of shape (N, C, H, W)
    - cache: Values from the forward pass

    Returns a tuple of:
    - dx: Gradient with respect to inputs, of shape (N, C, H, W)
    - dgamma: Gradient with respect to scale parameter, of shape (C,)
    - dbeta: Gradient with respect to shift parameter, of shape (C,)
    """
    dx, dgamma, dbeta = None, None, None

    # ===== #
    # YOUR CODE HERE:
    # Implement the spatial batchnorm backward pass.
    # You may find it useful to use the batchnorm forward pass you implemented in HW #4.
    # ===== #

    N, C, H, W = dout.shape

    dx = np.zeros(dout.shape)
    dout_2d = np.reshape(dout, (N * H * W, C))
    dx_2d, dgamma, dbeta = batchnorm_backward(dout_2d, cache)
    dx = dx_2d.reshape((N, H, W, C)).transpose(0, 3, 1, 2)

    # ===== #
    # END YOUR CODE HERE
    # ===== #

    return dx, dgamma, dbeta

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