Sensor Fusion and Non Linear Filtering - Project Report

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TASK 1

The gyroscopic measurements are considered to be the inputs to the system in this project as it is considered to be accurate since it measure the true angular velocities. It is better to include the angular velocity in state vector when the values are when the measurements are not accurate. Also considering linear acceleration as input makes it much harder to compute.

TASK 2

1 Histograms of Measurements

The measurement histograms of the three sensors fitted inside the gaussian distribution curve is shown.The histograms appears to fit inside the Gaussian distribution curve and hence the measurements are normally distributed.

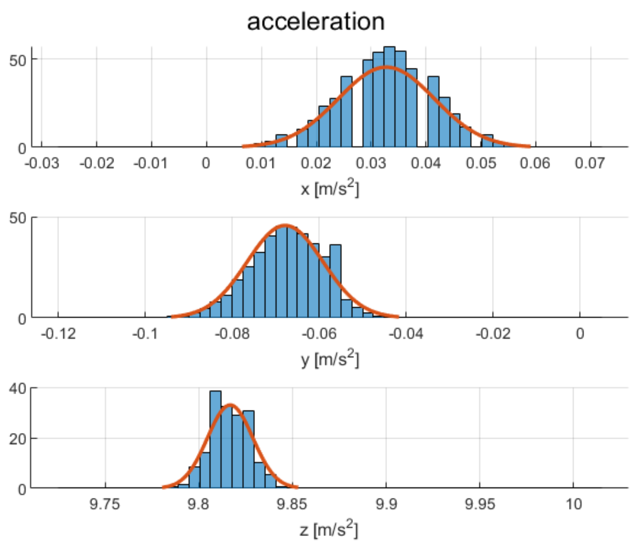

Figure 1: accelerometer histogram

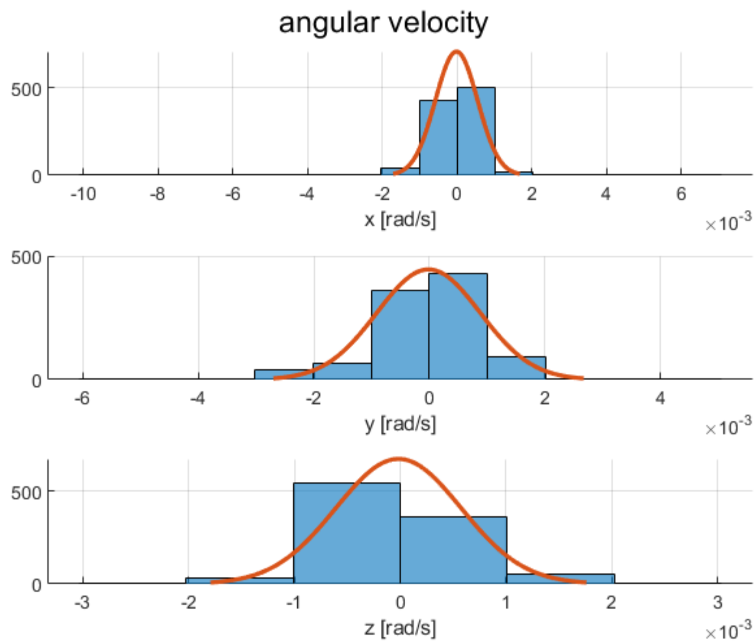

Figure 2: Gyroscope histogram

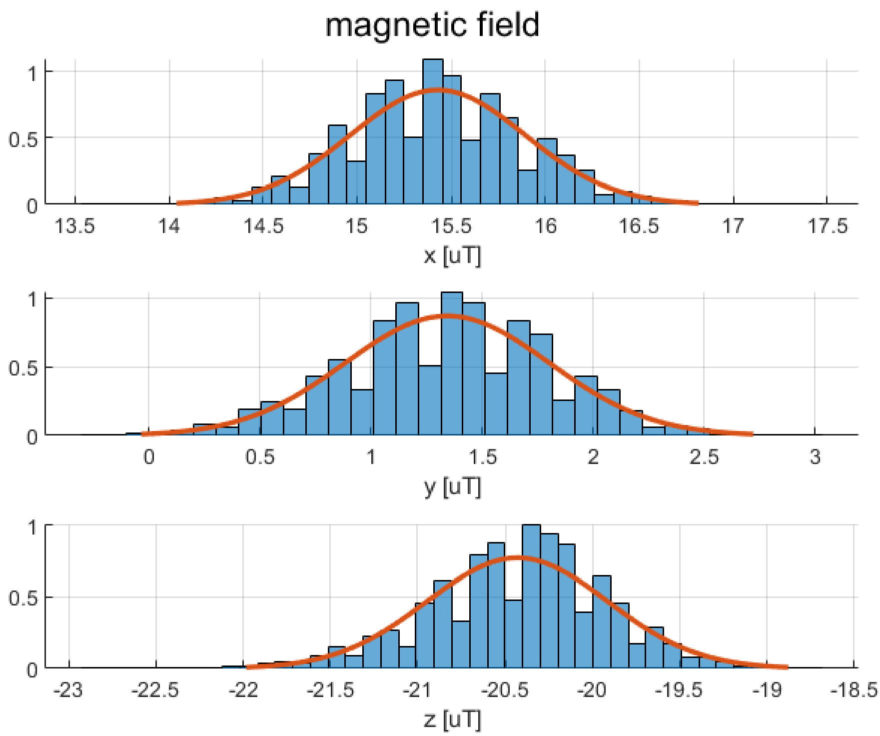

Figure 3: magnetometer histogram

2 Signals over time plot

The smartphone is placed flat on the table and the readings from the sensors are shown below. By observing the plots we can infer that there are no drifts and the sensors are quite accurate.

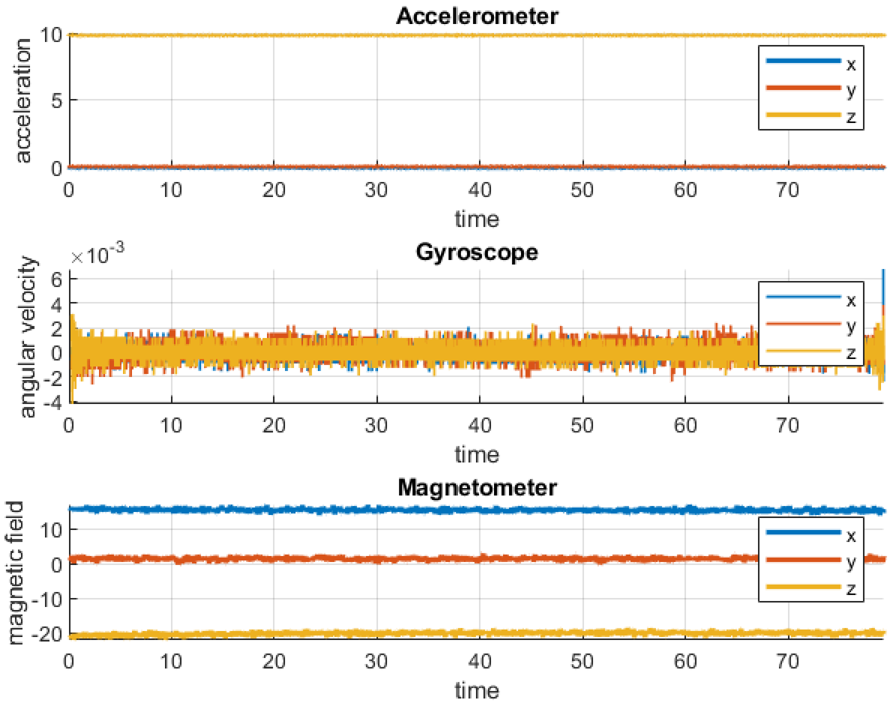

Figure 4: Measurement signals when smartphone is on flat table

3 Mean and covariances

The mean and covariances of the acceleration vector, angular velocity vector, and magnetic field when the smartphone is placed flat on table are,
g0 = [0.0696 -0.0410 9.8216]'
mag mean = [-87.1983 48.6800 -73.3492]
m0= [0 99.86 -73.35]'
gyr mean =1.0e-03 * [0.0389 0.2201 -0.0710]'

$$\text{Ra}= 1.0\text{e-}03 * \begin{bmatrix} 0.0723 & 0.0014 & -0.009 \\ 0.0014 & 0.0624 & 0.007 \\ -0.009 & 0.007 & 0.1149 \end{bmatrix}$$

$$\text{Rm}= \begin{bmatrix} 0.1939 & 0.0101 & -0.0088 \\ 0.0101 & 0.1984 & 0.0242 \\ -0.0088 & 0.0242 & 0.1963 \end{bmatrix}$$

$$\text{Rw}= 1.0\text{e-}06 * \begin{bmatrix} 0.2858 & 0.0594 & -0.0128 \\ 0.0594 & 0.5395 & 0.0362 \\ -0.0128 & 0.0362 & 0.3160 \end{bmatrix}$$

The covariance values are found to be around zero and the accelerometer value on the z axis is 9.82 which is reasonable and hence the sensor can be considered precise.

TASK 3

The motion model is given by the equation,

$$\dot{q}(t) = \frac{1}{2}S(\omega_{k-1} + v_{k-1})q(t), for \;\; t \in [t_{k-1}, t_k) \tag{1}$$

where,
$\sim \mathcal{N}$(0,R_v).
The matrix S is also considered to be constant as both v_{k-1} and ω_{k-1} are piecewise constants during the sampling intervals. The analytical solution for (t) is given by,

$$q(t+T) = exp(\dot{q}(t)) \cdot T \tag{2}$$

By substituting we get,

$$q(t+T) = exp\Big(\frac{1}{2}S(\omega_{k-1} + v_{k-1})T\Big)q(t) \tag{3}$$

Adding an identity matrix does not induce any change in the equation but it will be helpful in simplification. Hence,

$$q(t+T) = \Big(I + \frac{1}{2}S(\omega_{k-1} + v_{k-1})T\Big)q(t) \tag{4}$$

$q(t_k)$ is written as q_k and $q(t_{k-1})$ is written as q_{k-1}. The model can be derived as,

$$q_k = \Big(I + \frac{1}{2}S(\omega_{k-1})T\Big)q_{k-1} + \frac{1}{2}\bar{S}(q_{k-1}) \cdot T * v_{k-1} \tag{5}$$

In the above equation q_k will not be gaussian but an EKF can be used to approximate it to a gaussian random variable.

$$g(q_{k-1}, v_{k-1}) \approx g(\hat{q}k-1, \hat{v}k-1) + \frac{\delta g(q_{k-1}, v_{k-1})}{\delta v_{k-1}}(v_{k-1} - \hat{v}k-1) + \frac{\delta g(qk-1, v_{k-1})}{\delta q_{k-1}}(q_{k-1} - \hat{q}_{k-1}) \tag{6}$$

The mean is given by,

$$E[g(q_{k-1}, v_{k-1})] \approx g(\hat{q}k-1, \hat{v}k-1) = 0 \tag{7}$$

(since $\hat{v}k-1$ tends to zero)
The covariance is given by,

$$\begin{aligned} Cov[g_{k-1}] &= \frac{\delta g(q_{k-1}, v_{k-1})}{\delta v_{k-1}} \cdot Cov[v_{k-1}] \frac{\delta g(q_{k-1}, v_{k-1})}{\delta v_{k-1}}^T + \frac{\delta g(q_{k-1}, v_{k-1})}{\delta q_{k-1}} \cdot Cov[q_{k-1}] \frac{\delta g(q_{k-1}, v_{k-1})}{\delta q_{k-1}}^T \\ &= \frac{T}{2} \cdot R_\omega \cdot \bar{S}(\hat{q}_{k-1})^T \cdot \frac{T}{2} \\ &= G(\hat{q}k-1) \cdot R\omega \cdot G(\hat{q}_{k-1})^T \end{aligned} \tag{8}$$

The final expressions for mean and covariance are given by

$$\begin{aligned} E = \hat{q}k = F(\omega k-1) \cdot \hat{q}_{k-1} \\ Cov = P_k = G(\hat{q}k-1)R\omega G(\hat{q}k-1)^T + F(\omega k-1)P_{k-1}F(\omega_{k-1})^T \end{aligned} \tag{9}$$

The motion model can also be derived as follows.

$$q_k = G(\hat{q}k-1)vk-1 + F(\omega_{k-1})\hat{q}_{k-1} \tag{10}$$

TASK 4

The motion model can be described using the equations below which can be used for predicting. If the angular velocity is not available, a random walk motion model is used which comes into existence when the sensor fails.

TASK 5

The EKF estimates the orientation using the motion model thus providing a relationship between the state space variables and gyroscopic readings and the gyroscope measures the angular velocities but lacks absolute orientation. Making use of the gyroscopic readings alone does not give us information about the initial orientation and the error cannot be compensated and also as we measure only the angular velocity, the numerical integration has approximations which causes orientation errors.
There will be an offset in the orientation when we start the filter with the phone on the side instead of laying face up on the desk. While shaking the phone, the angular accelerations increase rapidly causing estimation errors which gets added to the offset causing drift.

TASK 6

The accelerometer measurement can be expressed as,

$$y^a k = Q^T(qk)(g^0 + f_k^a) + e_k^a \tag{11}$$

where,
g^0=nominal gravity vector
f_k^a= external forces
e_k^a= measurement noise
In EKF filter, in order to linearize the model,

$$h'(x) = \begin{bmatrix} \delta Q^T/\delta q_1 & \delta Q^T/\delta q_2 & \delta Q^T/\delta q_3 & \delta Q^T/\delta q_4 \end{bmatrix} * g^0 \tag{12}$$

The innovation covariance is calculated using,

$$S_k = h'(x_{k|k-1})P_{k|k-1}h'(x_{k|k-1})^T + R_a \tag{13}$$

The Kalman gain is given by,

$$K_k = P_{k|k-1}h'(x_{k|k-1})^T S_k^{-1} \tag{14}$$

The update step is given by the equation,

$$\hat{x}k|k = \hat{x}k|k-1 + K_k(y^a k - h(xk|k-1)) \tag{15}$$

$$P_{k|k-1} - K_k S_k K_k^{-1} \tag{16}$$

TASK 7

The update step is added to the EKF.The smartphone is placed on a flat table which allows only the gravitational force to act on the body. As we have earlier assumed that the smartphone is not accelerating on the world frame (i.e., $f_k = 0$) and if the smartphone is slided back and forth quickly,f_k becomes large and it does not perform well.

TASK 8

A simple outlier rejection algorithm is implemented. This eliminates the effect of the unwanted forces which are vulnerable to cause errors in the estimation.

TASK 9

The magnetometer measurement can be expressed similar to the accelerometer equations in task 6 as,

$$y^m k = Q^T(qk)(m^0 + f_k^m) + e_k^m \tag{17}$$

where,
m^0=Earth's magnetic field
f_k^m= external magnetic field
e_k^m= measurement error

TASK 10

When a smartphone is introduced near a magnetic disturbance like a computer, due to the effect of the magnetic field, the filter will try to cause changes in the estimates. And when the outlier rejection is not implemented, estimation error occurs.

TASK 11

The assumptions made are that the measured reference m_0 is the true value of the earth's magnetic field and also it is assumed that the measurements are taken away from the other magnetic fields (so that the measurements are estimated without any errors). These assumptions are made to detect the outliers and to implement the outlier rejection. Once the outlier is implemented, introducing a magnetic field will not have remarkable effects as the filter does not consider the affected measurements.

TASK 12

Accelerometer and Gyroscope

The filter does not produce up to the mark results when it is used without the magnetometer. The reference point is lost and uncertainty is introduced in the orientation.

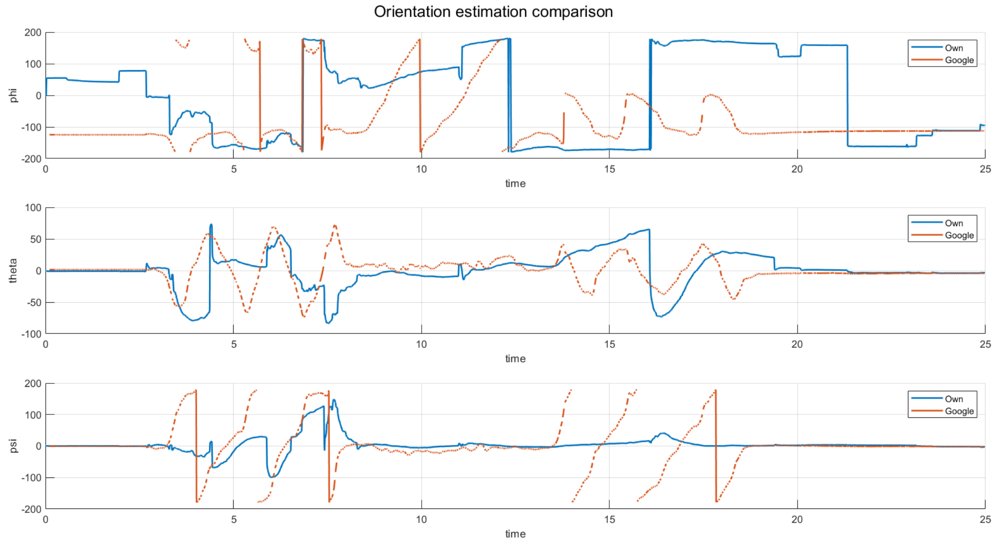

Figure 5: Orientation without magnetometer

Accelerometer and magnetometer

The filter produces partially good results when used without a gyroscope. The model makes use of a process noise of $0.1I_4$. The deviations in the plots are due to the uncertainties.

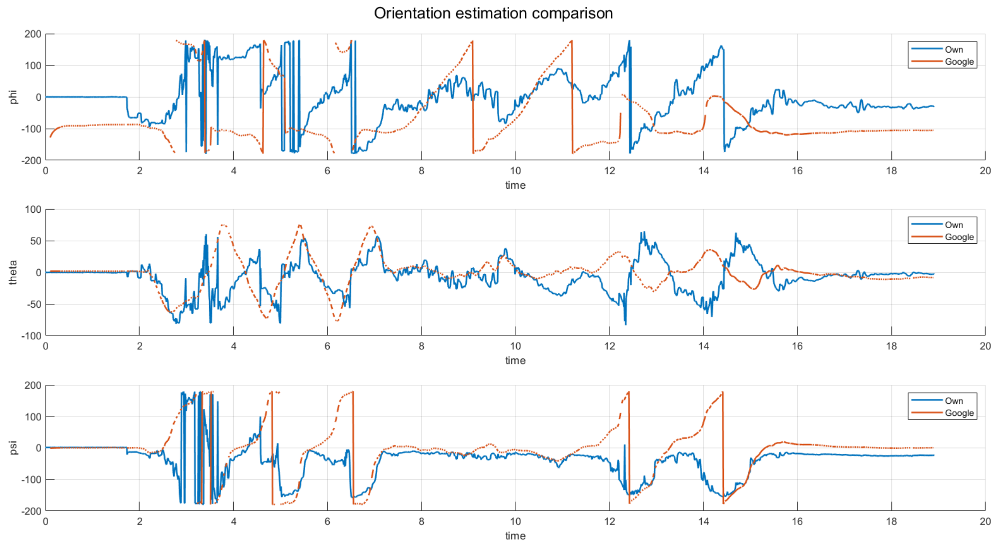

Figure 6: Orientation without Gyroscope

Magnetometer and gyroscope

The filter does not produce remarkable results when used without an accelerometer since it loses one degree of freedom.

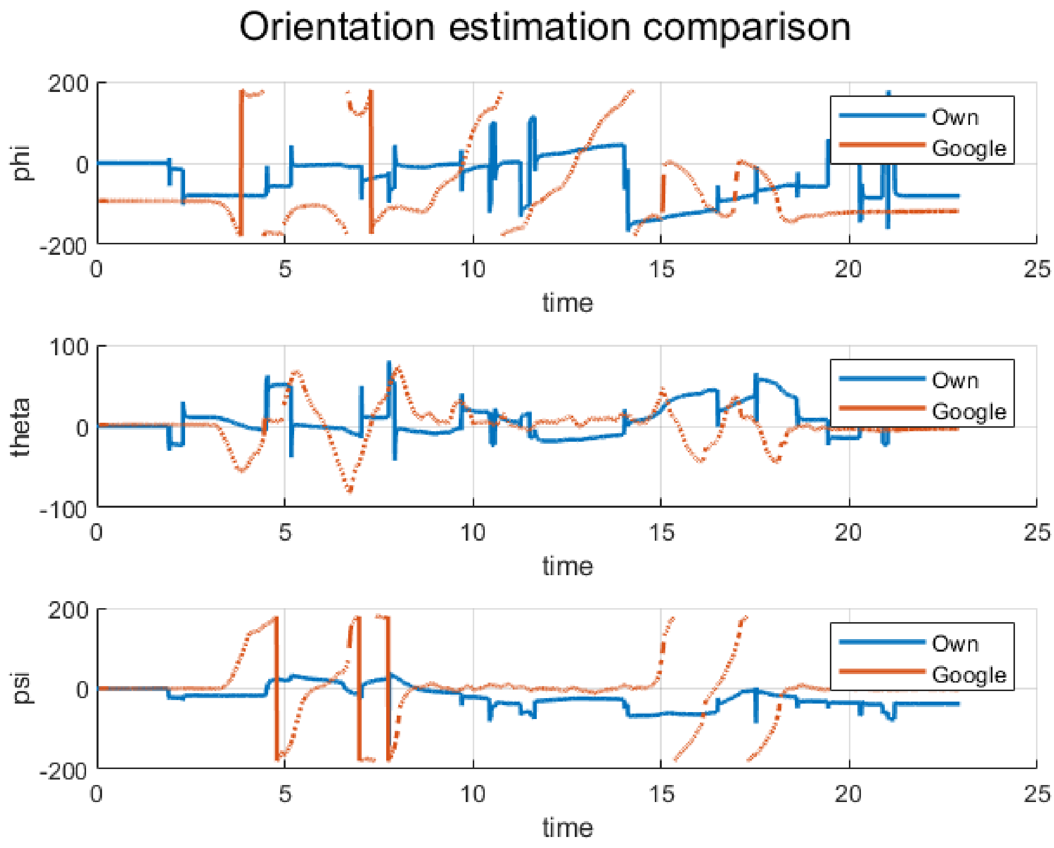

Figure 7: Orientation without Accelerometer

Accelerometer, Gyroscope and Magnetometer

The results when all three sensors are used in the filter is shown below.

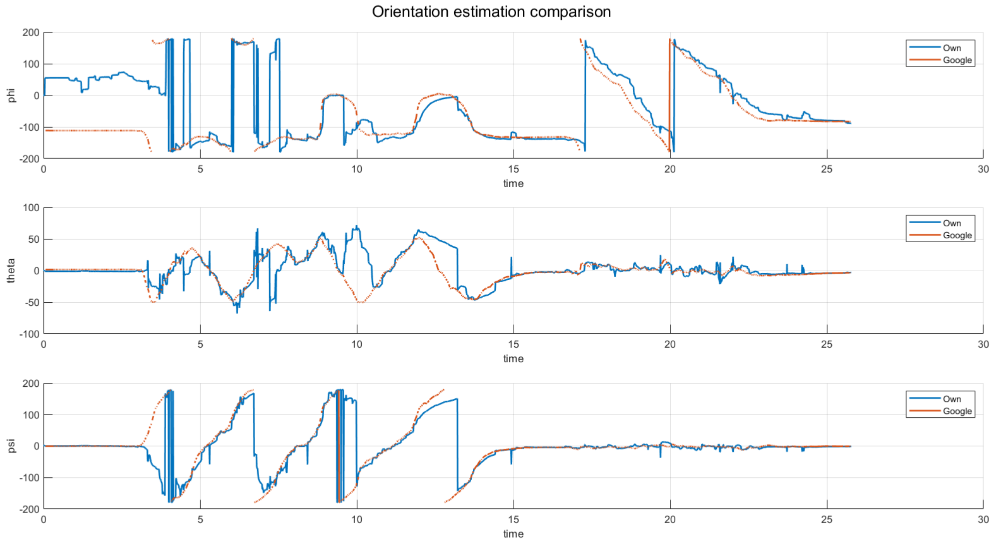

Figure 8: Orientation with all three sensors