Solution to analysis in Home Assignment 3

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Analysis

In this report, i have analysed the questions related to home assignment 3. I consent that the results produced are my own results.

1 Approximations of mean and co-variance

a,b,c,d

The plots below represent the approximated mean and covariance for the Extended Kalman Filter, Unscented Kalman Filter and Cubature Kalman Filter.

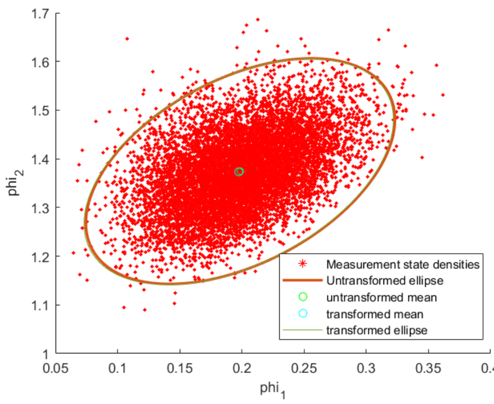

Figure 1: Mean and Covariance Approximation Using EKF:case 1

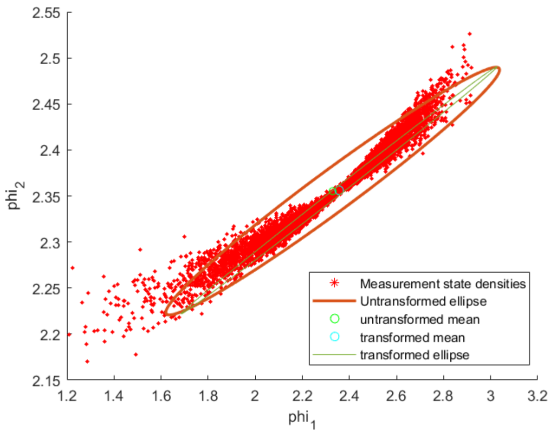

Figure 2: Mean and Covariance Approximation Using EKF:case 2

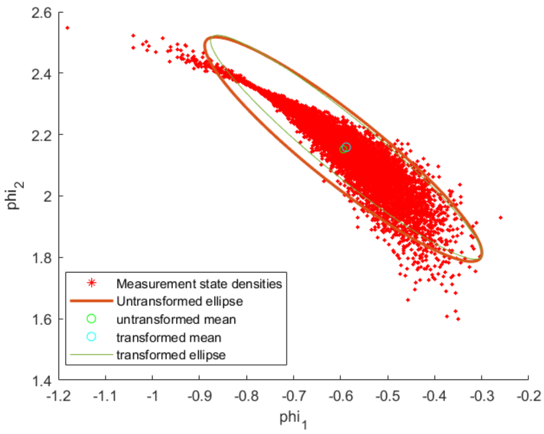

Figure 3: Mean and Covariance Approximation Using EKF:case 3

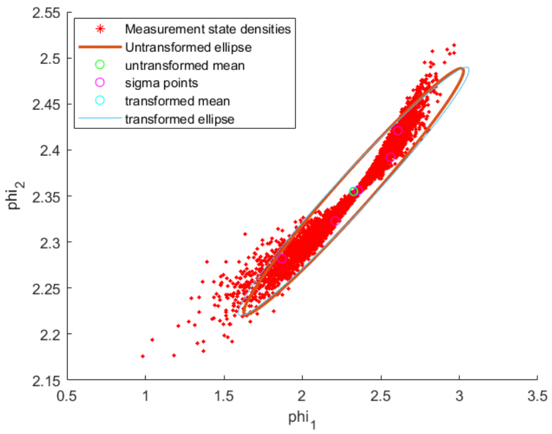

Figure 5: Mean and Covariance Approximation Using UKF:case 2

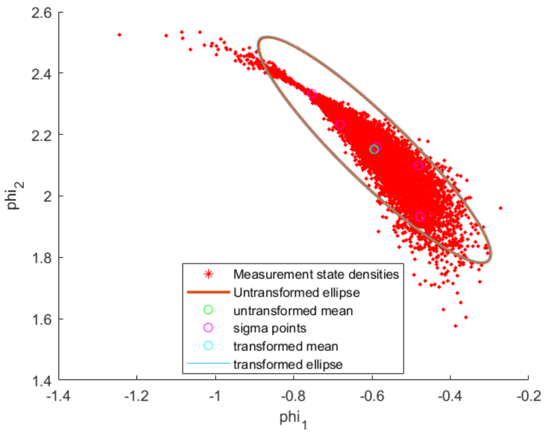

Figure 6: Mean and Covariance Approximation Using UKF:case 3

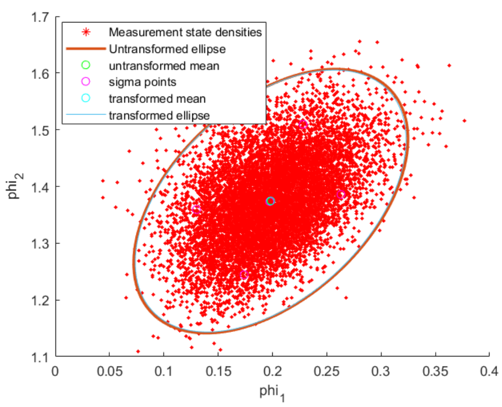

Figure 4: Mean and Covariance Approximation Using UKF:case 1

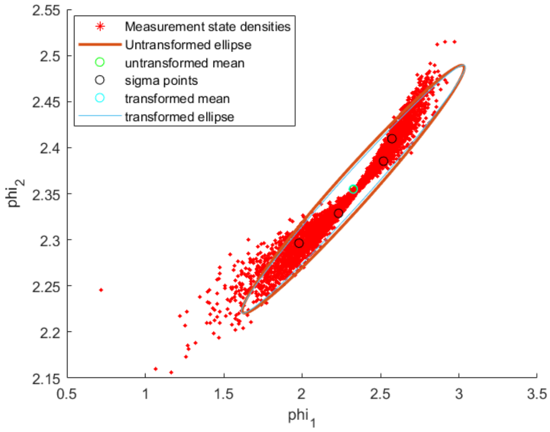

Figure 8: Mean and Covariance Approximation Using CKF:case 2

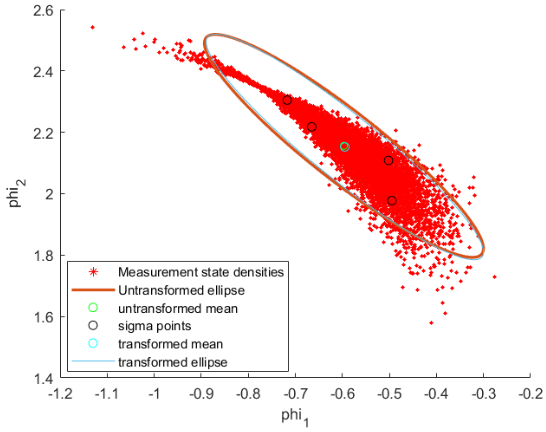

Figure 9: Mean and Covariance Approximation Using CKF:case 3

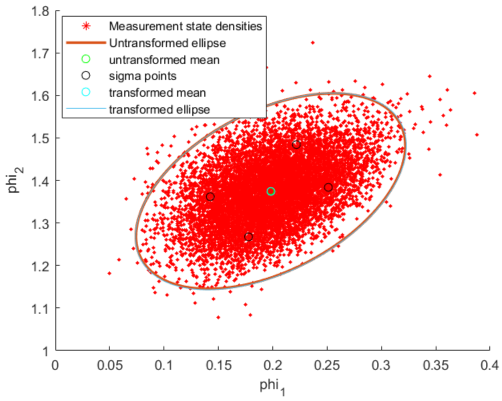

Figure 7: Mean and Covariance Approximation Using CKF:case 1

From the above figures it can be observed that the sample mean and covariance approximates very well with the actual values. i.e., accomodates withing the ellipse. Considering case 1, it can be inferred that all the three kalman filters provide approximations of the mean and covariance well. Considering case 2, there is a drastic difference in the performance of the three kalman filters. Comparing the peerformance of the EKF's of case 1 and 2, as the former is fairly gaussian, the EKF works better and as the latter is non linear, the EKF performs worse. As the UKF and CKF are sigma point methods, the difference between their performances are negigible. Case 3 is also similar to case 2.

2 Non-linear Kalman filtering

a,b The below figures represent sensor positions, measurements, true position sequence, estimated position sequence and 3 contours at every 5th estimate for all the three Kalman filters.

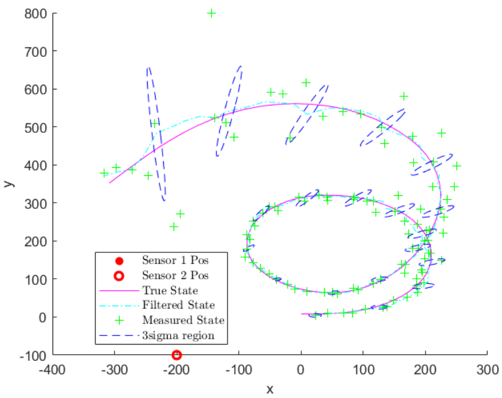

Figure 10: Sensor Position, Measurements and True Position for Noise Model 1: EKF

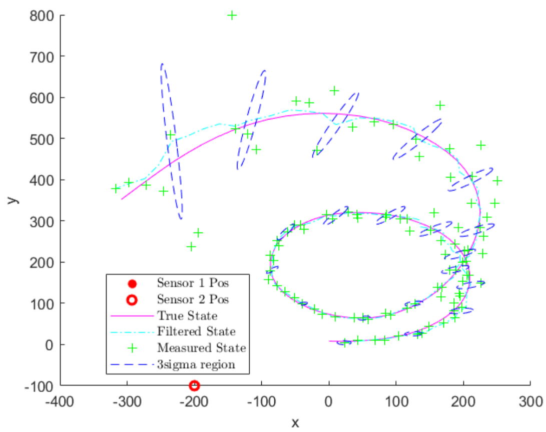

Figure 11: Sensor Position, Measurements and True Position for Noise Model 1: UKF

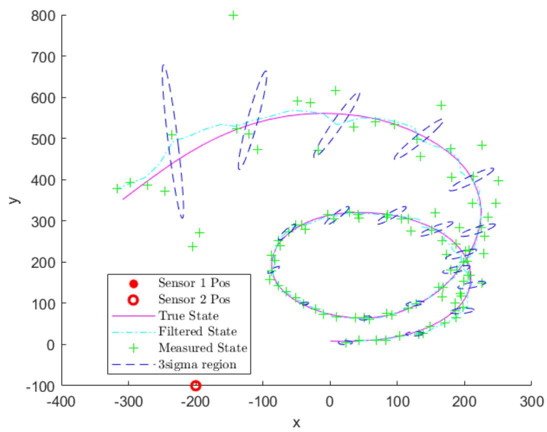

Figure 12: Sensor Position, Measurements and True Position for Noise Model 1: CKF

[h]
The EKF is found to perform worse when compared to UKF and CKF by observing the above figures.On evaluating the performance of different Kalman filters, it can be clearly observed that the UKF and CKF performs better than the EKF. This is because of the way the Extended Kalman Filter performs the prediction and update steps. Linearization of the non-linearities of the models around the prior mean takes place in case of EKF.EKF performs poor with non linear models whereas the UKF and CKF being sigma point methods perform well in that case estimating posterior densities better.
The above figures represent the histogram of the estimation errors for the position states. Comparing the three filters in all the three cases, The histogram of the error produced by EKF(the two graphs on the very left) shows that the EKF performs very poo when compared to the other two Kalman Filters. Comparing UKF(middle graph) and CKF(The

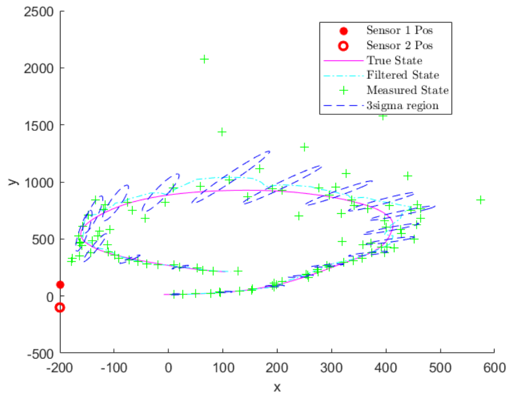

Figure 13: Sensor Position, Measurements and True Position for Noise Model 2: EKF

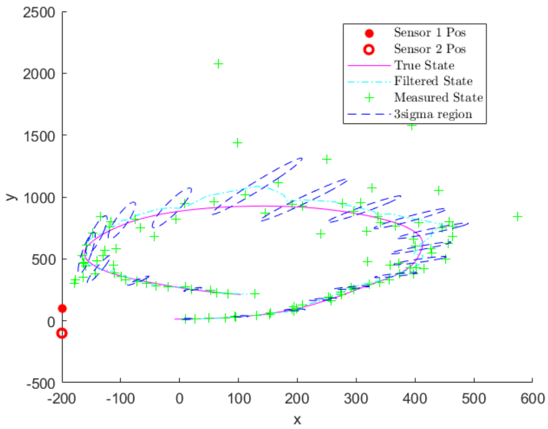

Figure 14: Sensor Position, Measurements and True Position for Noise Model 2: UKF

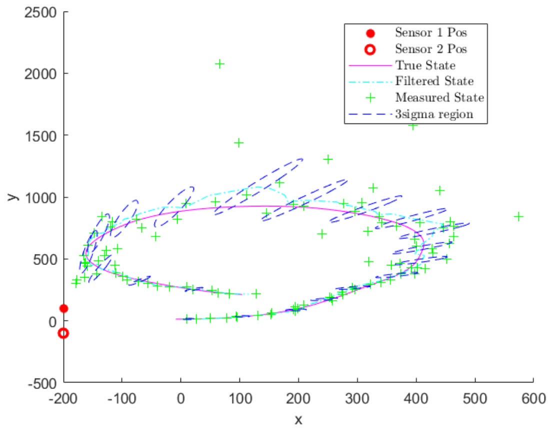

Figure 15: Sensor Posit2on, Measurements and True Position for Noise Model 2: CKF

grpahs to the very right), there is no much difference between the two. However, The CKF is found to perform better in terms of mean and covariance approximation. As a high number of samples are placed outside the 3 sigma region, the histogram is not gaussian.

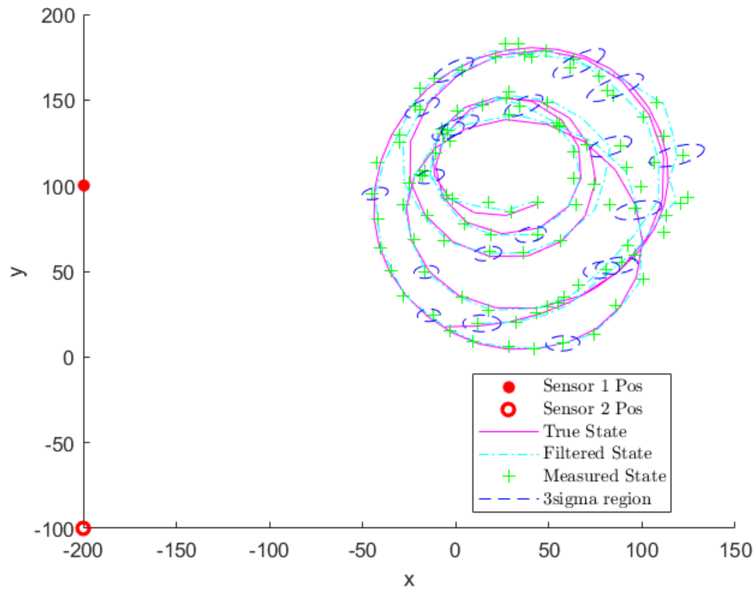

Figure 16: Sensor Position, Measurements and True Position for Noise Model 3: EKF

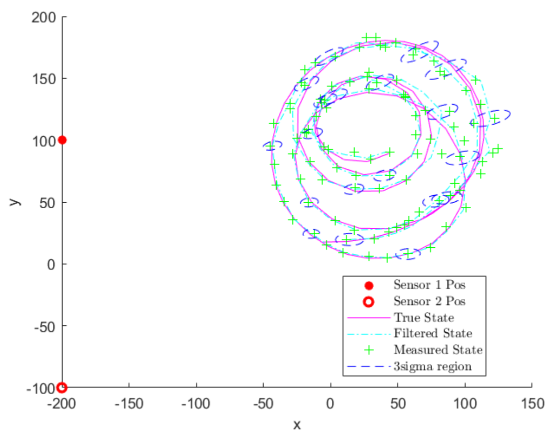

Figure 17: Sensor Position, Measurements and True Position for Noise Model 3: UKF

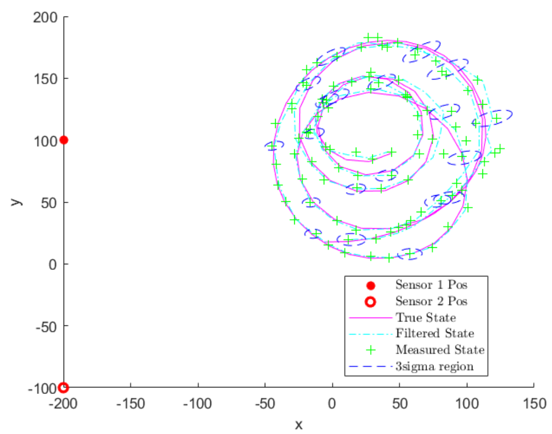

Figure 18: Sensor Position, Measurements and True Position for Noise Model 3: CKF

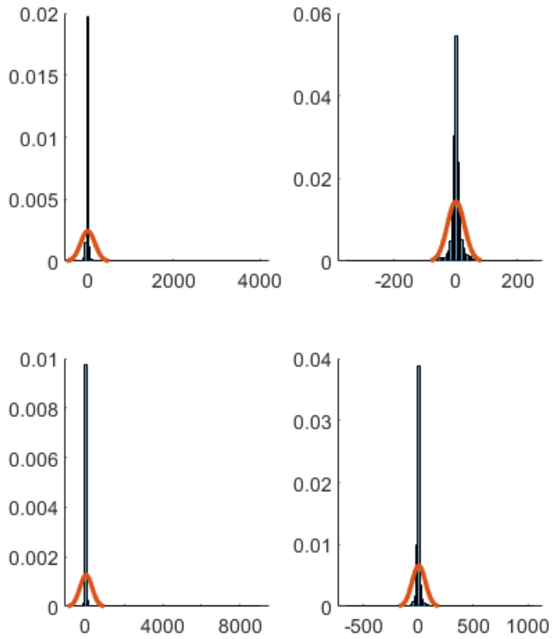

Figure 19: Histogram of the estimation errors for the position states in case 2

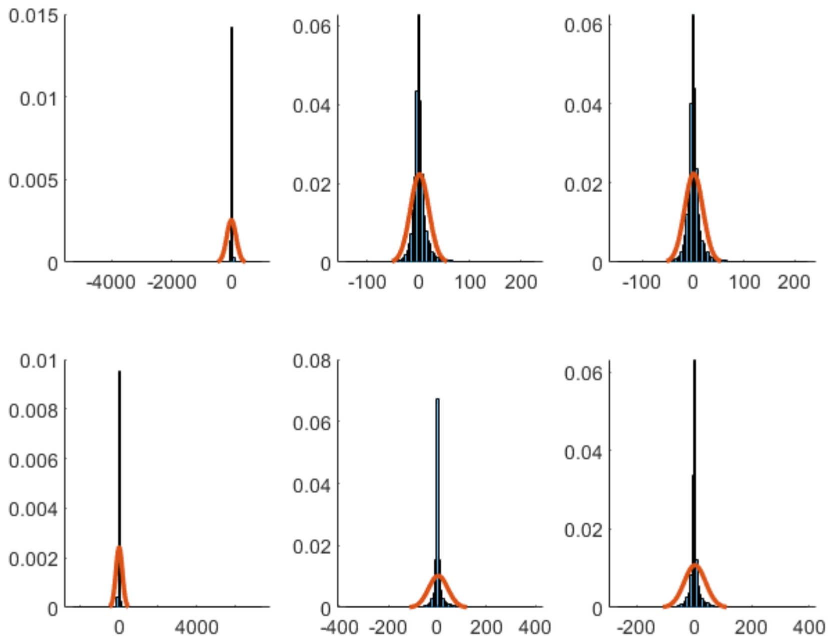

Figure 20: Histogram of the estimation errors for the position states in case 3

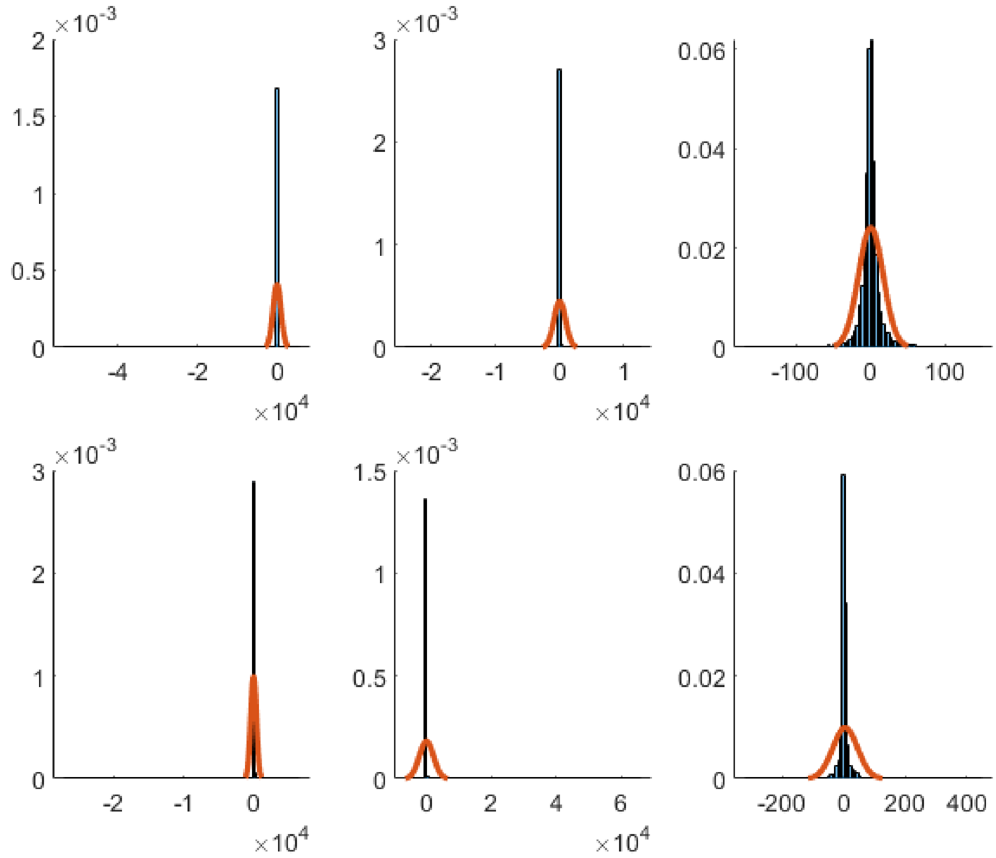

Figure 21: Histogram of the estimation errors for the position states in case 1

3 Tuning non-linear filters

b The speed is assumed to be constant throughout. A low velocity noise would be better in case of a kalman filter as the prior information is quite precise and also since the high velocity noise is needed only during the acceleration periods where the prior info is not reliable. The below figures represent sensor positions, the positions corresponding to the measurements, the true position sequence, and, the estimated position sequence with corresponding covariance contours for high process noise, low process noise and well tuned process noise.

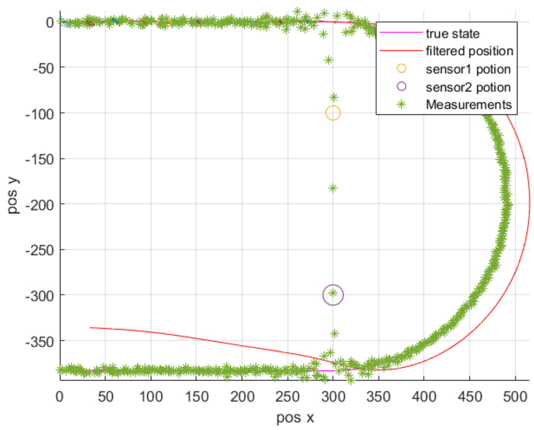

Figure 22: Low Process Noise

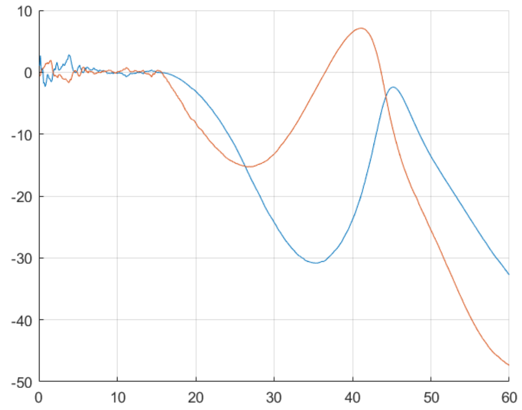

Figure 23: Low Process Noise

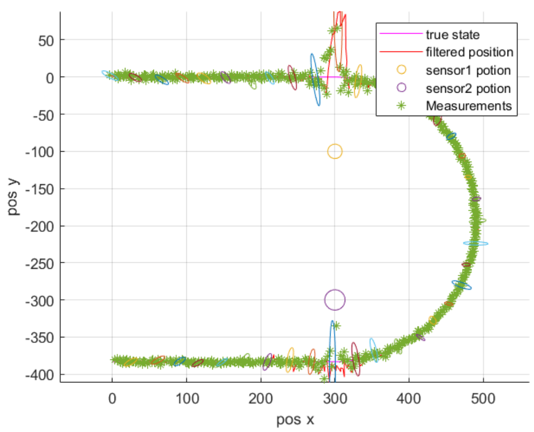

Figure 24: High Process Noise

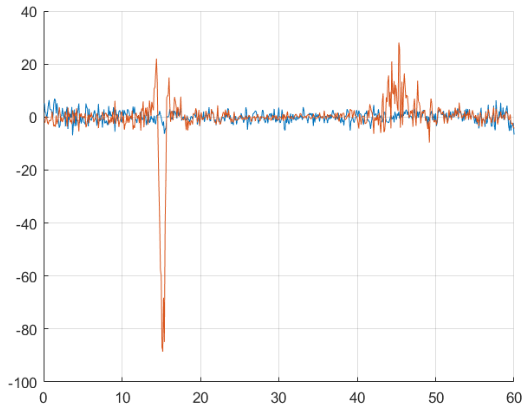

Figure 25: High Process Noise

c From the figures representing high process noise, it can be inferred that when the noise covariance is higher, the filter trusts the measurements more than the model which leads to unrealistic outputs. The trajectory is observed to be straight and hence there exists a high positional error.
From the figures representing high process noise, it can be inferred that the covariance of the noise is low. The trajectory is also observed to be curved and the error increases rapidly.
Once he model is tuned which can be obbserved from the tuned process noise plots, the error is very low throughout the entire path. In this case, the filter performs better in both the straight and curved paths.

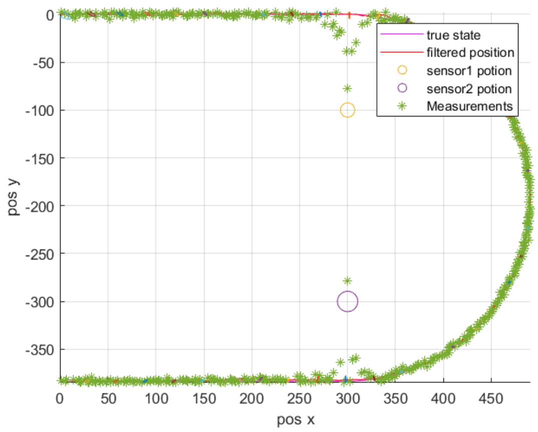

Figure 26: Tuned Process Noise

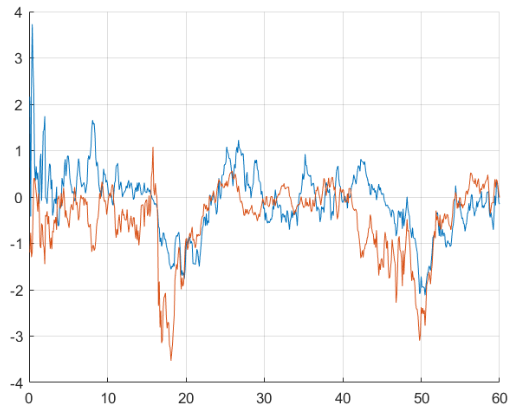

Figure 27: Tuned Process Noise

d As the straight and curved trajectories have different values for noise covariance, it is not possible to eliminate error completely and have very precise and accurate estimates. During the straight trajectory, it is preferable to keep the velocity and the yaw covariance to be as low as possible since a low value for covariance allows only a low output error. Also, acceleration is made difficult when changing from a straight to curved trajectory when the process noise is low which inturn demands high value of covariance. The above issues should be taken into mind while adjusting the filter parameters to provide better estimates.