MMF062 – Vehicle Dynamics

Department of Mechanics and Maritime Sciences
Vehicle Dynamics Group, Division Vehicle and Autonomous Systems

Assignment 1
Longitudinal Dynamics

Submitted by:

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Abstract

The learning objective of this assignment is to observe and understand the various factors affecting the longitudinal slip of the vehicle.

To draw free body diagrams (FBD) and form mathematical equations to solve for the various forces and reactions acting on the body.

To propose an optimum driving condition of a car for the drag race under dry and wet surface road condition.

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Task 1 : Tire characteristics on different road conditions

Task 1A: Plot of tire model

In this task, using the Magic Tyre Formula, a plot of normalised tyre traction force against longitudinal slip is plotted for various road conditions.

Magic Tyre Formula

$$F\_x/F\_Z = D \times \sin\left(C \times \arctan\left(B \times s\_x\right) - E \times \left(B \times s\_x - \arctan\left(B \times s\_x\right)\right)\right)$$

Where

F_x/F_z = Normalised longitudinal tyre force

$K = (3 \times \pi)/180$

$B = 100 \times (\arctan(K))/(C \times D)$

$0 < s\_x < 1$

The parameters for different road conditions

	D	C	E
Dry Asphalt	1.0	1.45	-4.0
Wet Asphalt	0.6	1.35	-0.20
Ice	0.10	1.50	0.80

Using the above equations and values, the following plot is obtained.

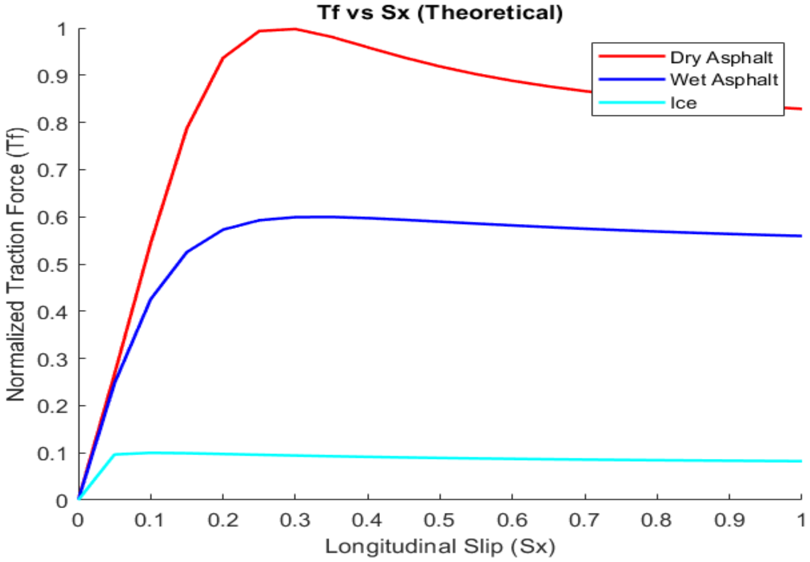

Figure 1 Normalized traction force vs longitudinal slip

Task 1B: Plot of experimental tire behaviour

The experimental data is given below

Slip, sx	0.0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Normalized longitudinal tyre force, $FxFz/$	0.0	0.25	0.53	0.77	0.89	0.95	0.94	0.92	0.90	0.86

Slip, sx	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Normalized longitudinal tyre force, $FxFz$	0.85	0.83	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74

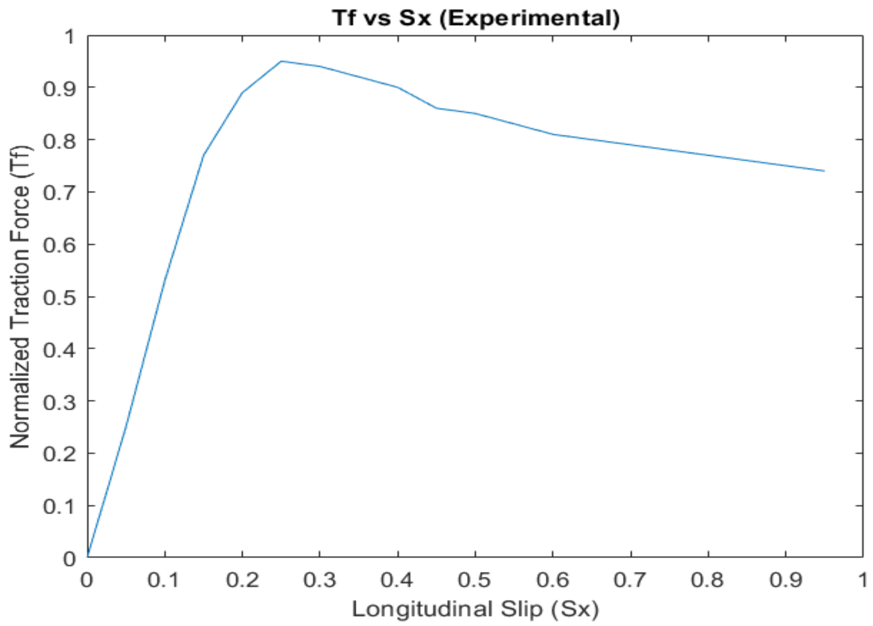

Figure 2 Experimental normalized traction force vs slip

Now, altering the values of C, D and E, the plot obtained by the Magic tyre formula is curve fitted with the experimental values.

Substituting D=.95, C=1.5 and E=-4.0 in the Magic Tyre Formula and plotting the same along with the experimental values, the following plot is obtained

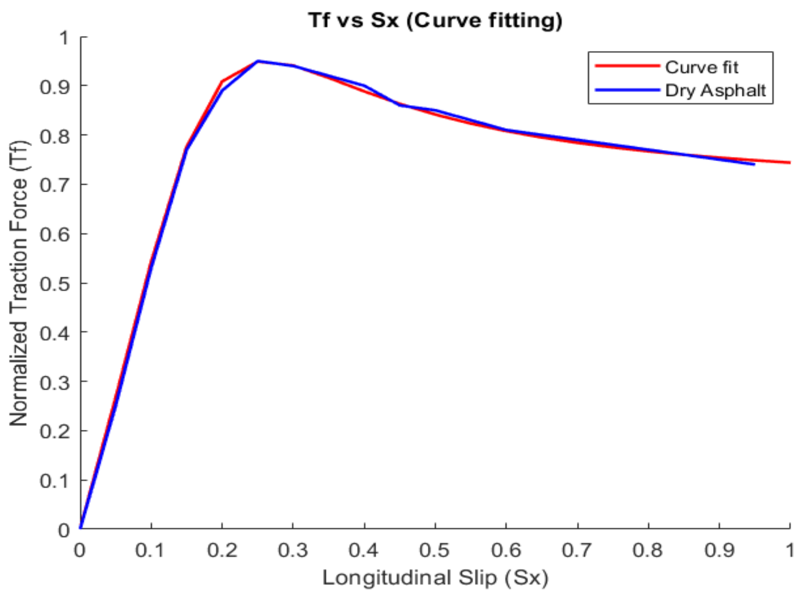

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Figure 3 Curve fitting

When substituting E=0 in the Magic Tyre Formula,

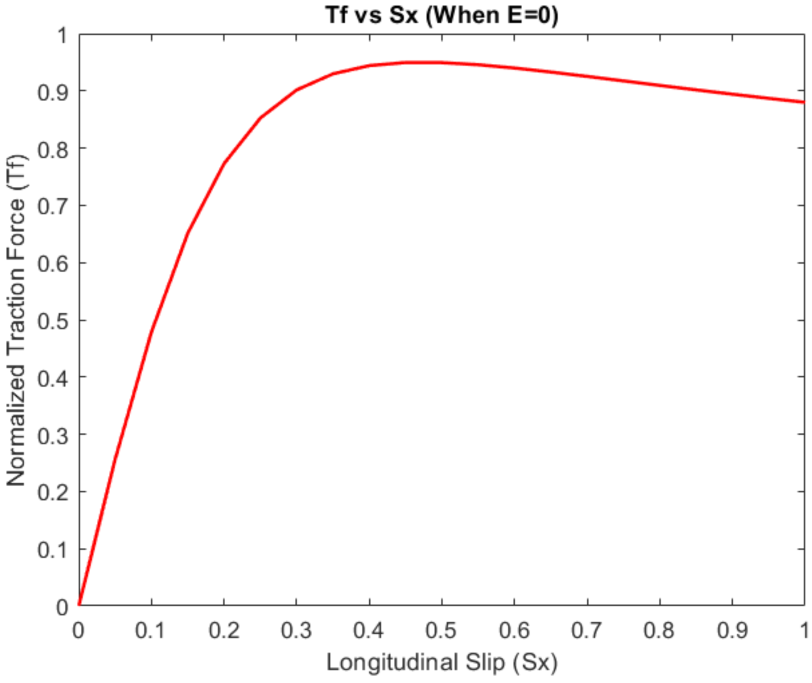

Figure 4 Normalized traction force vs slip when E=0

In the magic tire formula by substituting E=-4 is illustrated in figure 3. When E=-4, the slope after the peak value is higher as the value of normalized traction force reduces as the slip increases. But when E=0, The slope is less after the peak value and the normalized traction force do not reduce much.

Task 2: Vehicle model

Task 2A: Physical Model

The free body diagram of the vehicle with mass m separately for the chassis, wheels and the ground. The suspension of the vehicle is assumed to be rigid so that there is neither heave nor pitch. The relevant variable and parameters for forming the equations of motion are,

- F_fx- Traction force on front axis
- F_rx- Traction force on rear axle
- F_fz- Normal force on front axle
- F_rz- Normal force on rear axle
- $⟦\dot{\omega}⟧\_f$- - Front wheel rotational acceleration
- $⟦\dot{\omega}⟧\_r$- Rear wheel rotational acceleration
- a- Vehicle acceleration

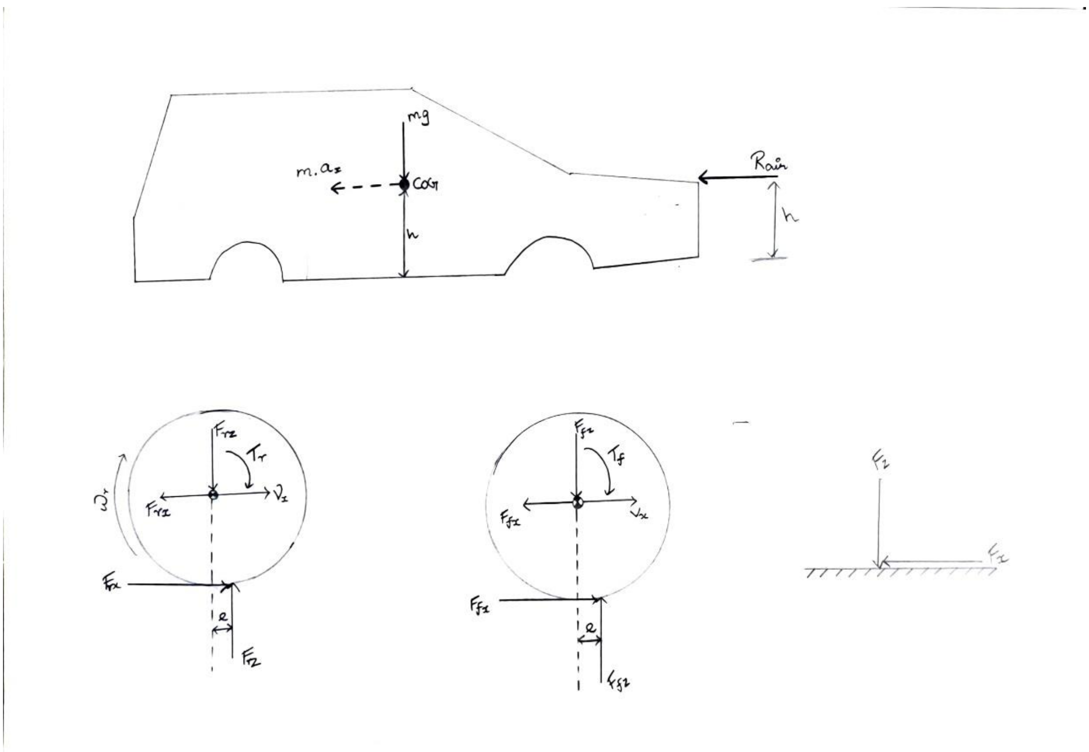

Figure 5 Free body diagram of a moving vehicle

SYMBOL	NOMENCLATURE
m	mass
g	Acceleration due to gravity
a_x	Acceleration of the vehicle
F_fz	Normal force on front axle
F_rz	Normal force on rear axle
F_fx	Traction force on front axis
F_rx	Traction force on rear axis
T_f	Front wheel torque
T_r	Rear wheel torque
e	Horizontal offset of vertical force to hub
COG	Center Of Gravity
h	Height of COG
R_{air}	Drag resistance
h_{air}	Height of drag resistance
ω_r	Rear wheel acceleration

Task 2B: Mathematical model

The equations of motion are derived for plain ground from the free body diagram.

The equations are,

$$F\_fx + F\_rx - (m * a) - F\_air = 0$$

$$F\_fz * (L\_f + L\_r) + (m * a * h) + (F\_air * h) - (m * g * L\_r) == 0$$

$$F\_rz * (L\_f + L\_r) - (m * a * h) - (F\_air * h) - (m * g * L\_f) = 0$$

$$T\_f - (J * \omega\_f) - (F\_fx * R) - (F\_fz * e) = 0 \ldots \ldots \ldots \ldots (4)$$

$$T\_r - (J * \omega\_r) - (F\_rx * R) - (F\_rz * e) = 0 \ldots \ldots \ldots \ldots (5)$$

$$F\_fx - (\mu\_f * F\_fz) = 0 \ldots \ldots \ldots \ldots (6)$$

$$F\_rx - (\mu\_r * F\_rz) = 0 \ldots \ldots \ldots \ldots (7)$$

The above equations are independent of each other and they can be solved.

Task 2C: Update models with road gradient

The vehicle is driving up a slope of road inclination ϴ. The updated free body diagram is drawn.

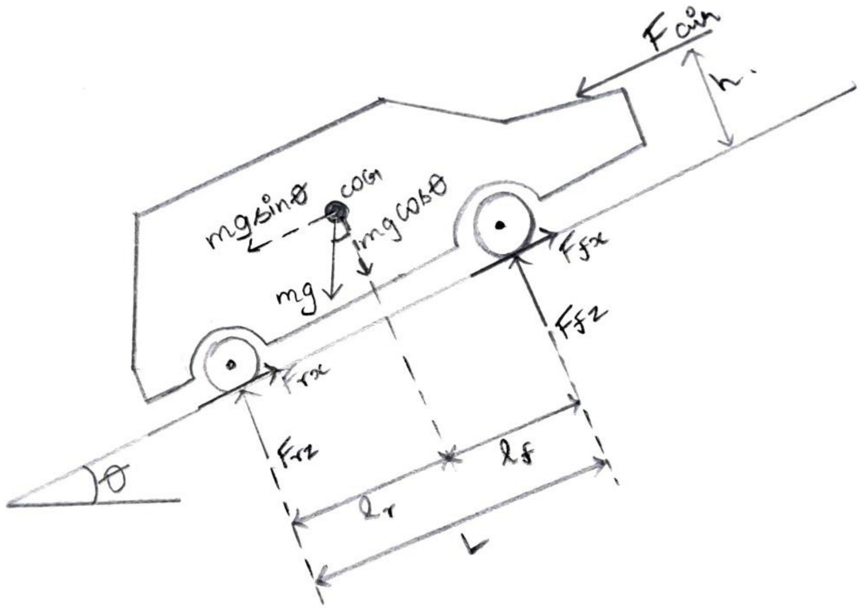

Figure 6 Free body diagram of a grading vehicle

SYMBOL	NOMENCLATURE
ϴ	Angle of inclination
mgsinϴ	Horizontal component of mg
mgcosϴ	Vertical component of mg
l_r	Distance of COG from rear axle
l_f	Distance of COG from front axle
L	Wheel base

The updated equations are as follows,

$$\mathrm{F\_fx + F\_rx - (m * a) - Fair - (m * g * \sin(slope)) = 0} \ldots\ldots\ldots\ldots (1)$$

$$\mathrm{F\_fz * (L\_f + L\_r\,) + (m * a * h) + (F\_air * h) - (m * g * L\_r * \cos\,(slope)\,)}$$
$$\mathrm{+(m * g * h * \sin(slope)) = 0} \ldots\ldots\ldots\ldots (2)$$

$$\mathrm{F\_rz * (L\_f + L\_r\,) - (m * a * h) - (F\_air * h) - (m * g * L\_f * \cos\,(slope)\,)}$$
$$\mathrm{-(m * g * h * \sin(slope)) = 0} \ldots\ldots\ldots\ldots (3)$$

$$\mathrm{T\_f} - (\mathrm{J} * \omega_{\mathrm{f}}) - (\mathrm{F_{fx}} * \mathrm{R}) - (\mathrm{F_{fz}} * \mathrm{e}) = 0 \ldots\ldots\ldots\ldots (4)$$

$$\mathrm{T\_r} - (\mathrm{J} * \omega_{\mathrm{r}}) - (\mathrm{F_{rx}} * \mathrm{R}) - (\mathrm{F_{rz}} * \mathrm{e}) = 0 \ldots\ldots\ldots\ldots (5)$$

$$F_{fx} - (\mu_f * F_{fz}) = 0 \dots\dots\dots\dots (6)$$

$$F_{rx} - (\mu_r * F_{rz}) = 0 \dots\dots\dots\dots (7)$$

Task 2D: Solving the equations

Acceleration,

$$\mathbf{a} = -(F_{air} * L_f + F_{air} * L_r + F_{air} * h.* \mu_f - F_{air} * h.* \mu_r + L_f * g * m * \sin(slope) + L_r * g * m * \sin(slope) - L_f * g * m.* \mu_r.* \cos(slope) - L_r * g * m.* \mu_f.* \cos(slope) + g * h * m.* \mu_f.* \sin(slope) - g * h * m.* \mu_r.* \sin(slope))/(L_f * m + L_r * m + h * m.* \mu_f - h * m.* \mu_r)$$

Rotational acceleration of front wheel,

$$\dot{\omega}_f = (L_f * T_f + L_r * T_f + T_f * h.* \mu_f - T_f * h.* \mu_r - L_r * e * g * m * \cos(slope) - L_r * R * g * m.* \mu_f.* \cos(slope) + e * g * h * m.* \mu_r.* \cos(slope) + R * g * h * m.* \mu_f.* \mu_r.* \cos(slope))/(J * (L_f + L_r + h.* \mu_f - h. * \mu_r))$$

Rotational acceleration of rear wheel,

$$\dot{\omega}_r = (L_r * T_r + L_r * T_r + T_r * h.* \mu_r - T_r * h.* \mu_f - L_r * e * g * m * \cos(slope) - L_r * R * g * m.* \mu_r.* \cos(slope) + e * g * h * m.* \mu_f.* \cos(slope) + R * g * h * m.* \mu_r.* \mu_f.* \cos(slope))/(J * (L_f + L_r + h.* \mu_r - h.* \mu_f))$$

Normal force on front axle,

$$F_{fz} = (L_r * g * m * \cos(slope) - g * h * m.* mu_r.* \cos(slope))/(L_f + L_r + h. * \mu_f - h.* \mu_r)$$

Normal force on rear axle,

$$F_{rz} = (L_f * g * m * \cos(slope) + g * h * m.* \mu_f.* \cos(slope))/(L_f + L_r + h.* \mu_f - h.* \mu_r);$$

Contribution of F_{air} to the vertical load transfer between axles:

Whole vehicle moment equation from the rear patch in plain ground is given by the equation,

$$-mg * l\_f + F\_rz * L - m * a\_x * h - R\_air * h\_air = 0$$

$$Frz = (mg * l\_f)/L + (m * a\_x * h)/L + (R\_air * h\_air)/L$$

Assuming that COG is in between front and rear axle,

$$Frz = mg/2 + (m * a\_x * h)/L + (R\_air * h\_air)/L$$

By partially integrating the above equation with R_{air}, we get the influence of R_{air} on F_{rz},

$$(\partial F\_rz)/(\partial R\_air) = \partial/(\partial R\_air)(mg/2 + (m * a\_x * h)/L + (R\_air * h\_air)/L)$$

$$(\partial F\_rz)/(\partial R\_air) = \partial/(\partial R\_air)((m * a\_x * h)/L + (R\_air * h\_air)/L) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The acceleration a_x is related to the force F_x by,

$$a\_x = (F\_X - R\_air)/m$$

Substituting the above equation in equation 1,

$$\boldsymbol{(\partial F\_rz)/(\partial R\_air) = (h\_air - h)/L}$$

Hence by the above equation R_{air} influences F_{rz}. This condition satisfies only if $h_{air} \neq 0$ or only $h_{air} \neq h$.

Similarly the influence of drag resistance on vertical load transfer of the front axle can be found.

Task 2E: Euler Integration

The task is to integrate the system of equations using Euler's method. The input values are extracted from the code Sub_Read_data.m and Saab_93_datasheet.m. The input values for the road condition (dry for our case), the values of slip, constant slope and constant propulsion torques were selected. The value of μ, μ_f and μ_r is obtained from the code Sub_magic_tireformula.m. Values at time zero are initialized for distance travelled, vehicle speed, rotational speed for front and rear tyres and slip of front and rear tyres.

Initial Distance travelled, $s = 0$

Initial velocity, $v = 0.1$

Initial rotational velocity of the front wheel, $w\_f = ((1.05 * \mathrm{v}))/(\mathrm{R\_wheel})$

Initial rotational velocity of the rear wheel, $w\_r = ((1.05 * \mathrm{v}))/(\mathrm{R\_wheel})$

The time interval is defined from 0 to 20 seconds in steps of 0.0005s. Vectors with zeros of size t are created. The values of slip was founded by,

$$slip\_f = (w\_f * R\_wheel - v)/(abs(w\_f * R\_wheel))$$

$$slip\_r = (w\_r * R\_wheel - v)/(abs(w\_r * R\_wheel))$$

In order to make sure that the slip is not greater than the optimum ones, the optimal value of slip is calculated which is found to be 0.281. This optimum value of slip is updated for all the values which are greater than the optimum. The equations of motion of the vehicle which was earlier derived is brought in again and the new values are updated in the vectors which was created earlier with zeros in it. All the vectors are plotted against t and the plots are obtained.

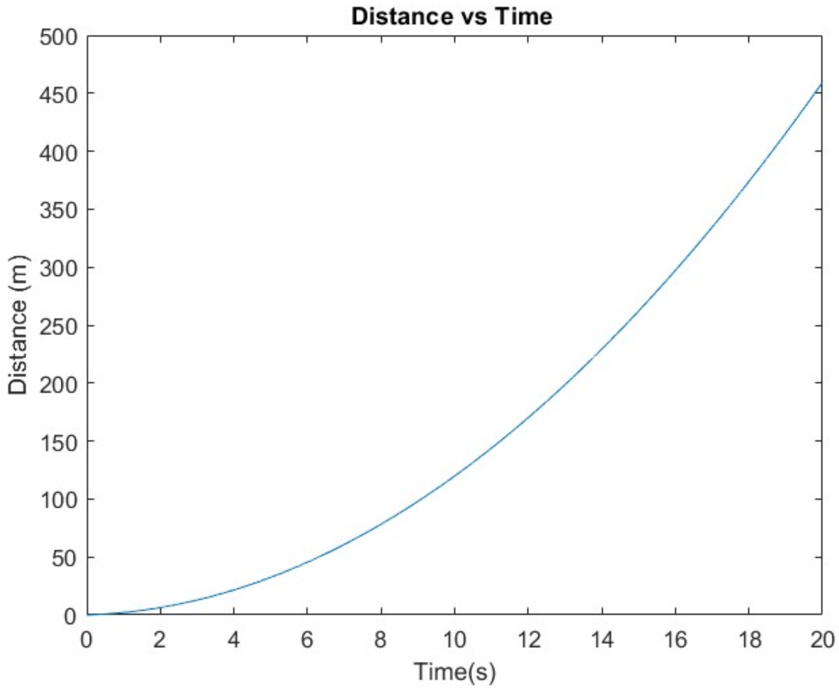

Figure 7 Distance travelled vs Time

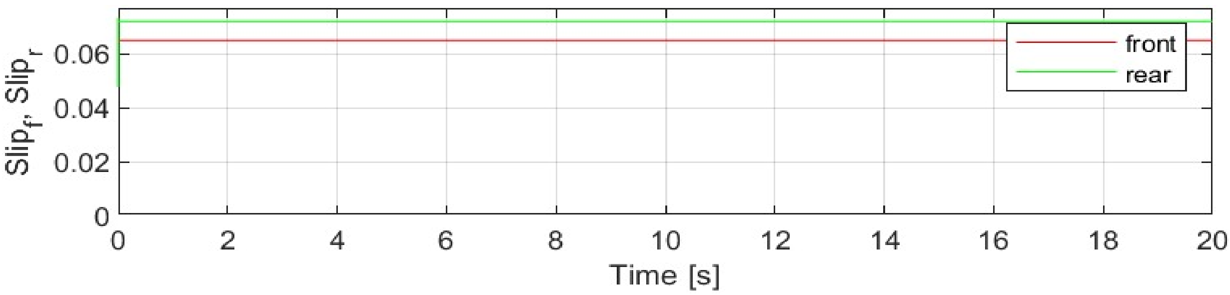

Figure 8 Front and rear wheel slip

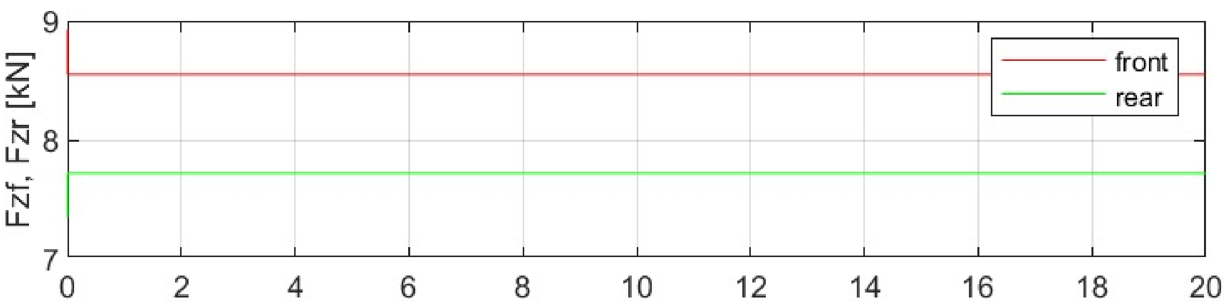

Figure 9 Normal forces on front and rear axle

Task 3: Simulation of Longitudinal dynamics

Task 3A: Implement and validate model

The Simulation MATLAB files have been updated and the solutions are plausible.

Task 3B: Select between FWD and RWD

In this task, the time taken to cover a distance of 100m drag race is required under various driving conditions and surface conditions.

The following plots are obtained in the simulation MATLAB code with varying condition of driving axle and surface condition.

The time taken to cover the 100m distance can be obtained from the plot of Dist.(m) vs Time.

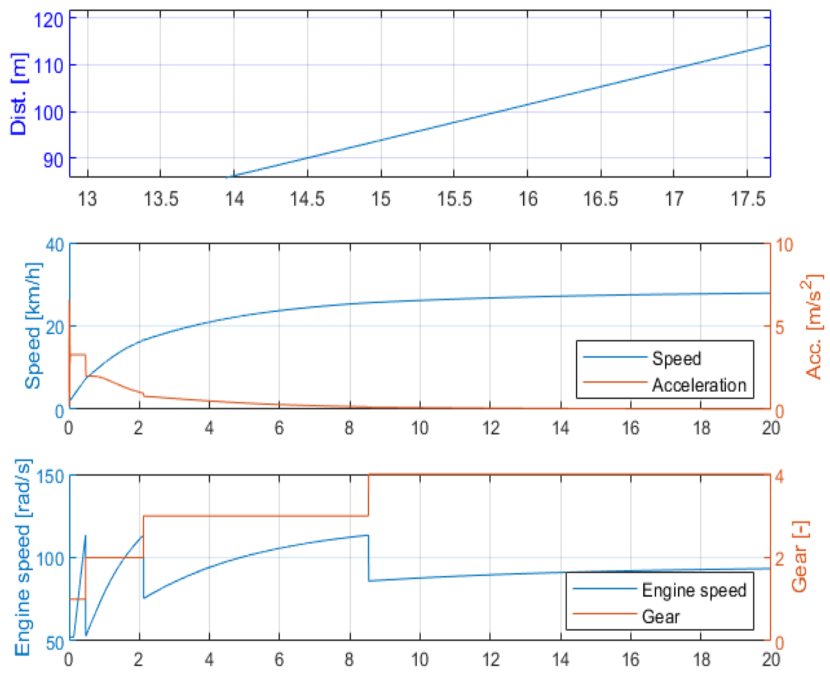

Figure 10 Front wheel drive vehicle under dry road condition

The time taken to cover 100 m distance is 15.75 seconds.

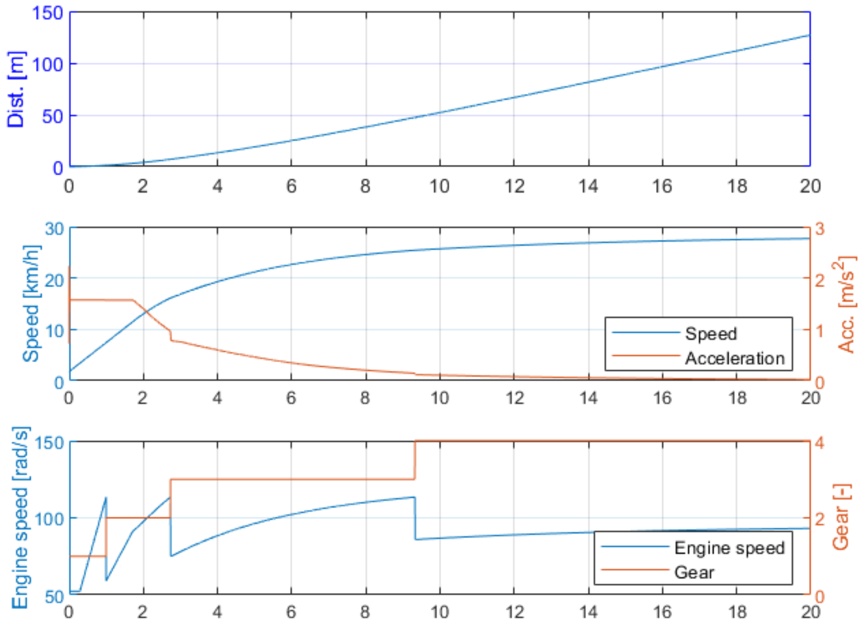

Figure 11 Front wheel drive vehicle under wet road condition

The time taken to cover 100m distance is 16.4 seconds.

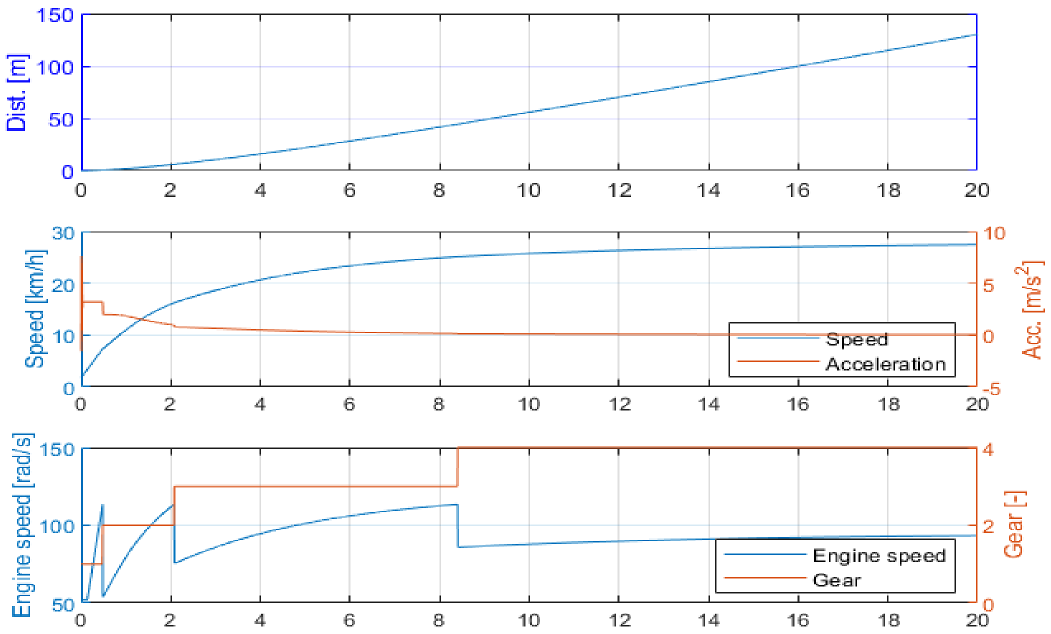

Figure 12 Rear wheel drive vehicle under dry road condition

The time taken to cover 100m distance is 15.99 seconds.

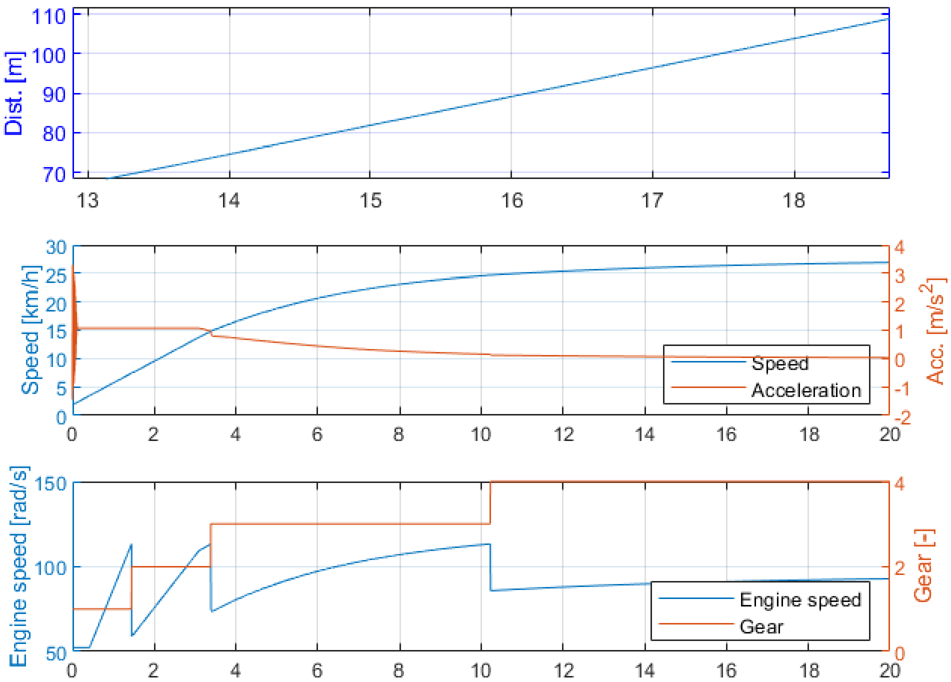

Figure 13 Rear wheel drive vehicle under wet road condition

The time taken to cover 100m distance is 17.56 seconds.

	Front wheel drive	Rear wheel drive
Dry surface condition	15.75 seconds	15.99 seconds
Wet surface condition	16.4 seconds	17.56 seconds

Table: Time taken to cover 100m distance in various road surface condition and driving axle configuration.

The setting for the dry and wet conditions differ in the choice of tyre and the traction properties of each of the wheels and the propulsion torque to the wheels to overcome the friction changes to move forward without any slip.

From the above observations, it is best suited to have a front wheel drive for both wet and dry surface conditions. Additionally, as it is a drag race, the front wheel drive vehicle is chosen as the time taken to reach 100 meters is lesser compared to the rear wheel drive configuration.

REFERENCES:

Bengt J H Jacobsson, Vehicle Dynamics Compendium, Vehicle Dynamics Group, Division Vehicle and Autonomous Systems, Department of Mechanics and Maritime Sciences, Chalmers University of Technology.