CHALMERS
UNIVERSITY OF TECHNOLOGY

MMF062 – Vehicle Dynamics

Department of Mechanics and Maritime Sciences
Vehicle Dynamics Group, Division Vehicle and Autonomous Systems

Assignment 3
Vertical Dynamics

Submitted by:

Group-9

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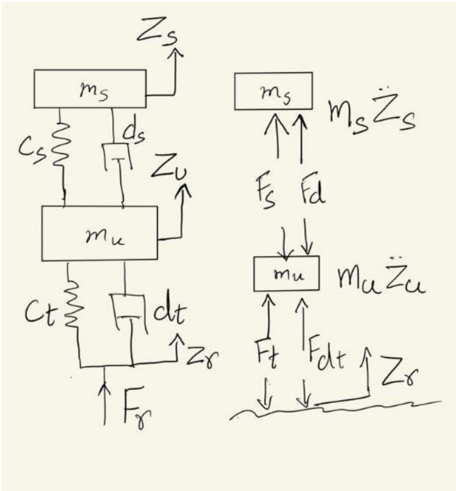
Free body diagram of spring mass system for front wheel of quarter car model.



Equilibrium:

$$m_s * \ddot{z}_s = F_{sz} - m_s * g$$

$$m_u * \ddot{z}_u = F_{rz} - F_{sz} - m_u * g$$

Constitution:

$$F_{sz} = c_s(z_u - z_s) + d_s(\dot{z}_u - \dot{z}_s) + m_s * g$$

$$F_{rz} = c_t(z_r - z_u) + d_t(\dot{z}_r - \dot{z}_u) + (m_s + m_u) * g$$

Excitation:

$$z_r = z_r(t)$$

Combining the above equations,

$$m_s * \ddot{z}_s - c_s * z_u + c_s * z_s - d_s * \dot{z}_u + d_s * \dot{z}_s = 0 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (1)$$

$$m_u * \ddot{z}_u + (c_t + c_s)z_u - c_s * z_s + (d_t + d_s)\dot{z}_u - d_s * \dot{z}_s = d_t * \dot{z}_r + c_t * z_r \dots \dots (2)$$

Expressing the above equation in state space form,

$$\dot{x} = A * x + B * u$$

$$\begin{bmatrix} \dot{z}_s \\ \dot{z}_u \\ \ddot{z}_s \\ \ddot{z}_u \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c_s}{m_s} & -\frac{c_s}{m_s} & \frac{d_s}{m_s} & -\frac{d_s}{m_s} \\ \frac{c_s}{m_u} & -\frac{(c_s+c_t)}{m_u} & \frac{d_s}{m_u} & \frac{(d_t-d_s)}{m_u} \end{bmatrix} * \begin{bmatrix} z_s \\ z_u \\ \dot{z}_s \\ \dot{z}_u \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{c_t}{m} & \frac{d_t}{m} \end{bmatrix} * \begin{bmatrix} z_r \\ \dot{z}_r \end{bmatrix}$$

Task 1.2: Transfer function matrix

The output of the system, y, can be written in state space form as,

$$y = C * x + D * u$$

From the handout,

The system is time invariant since the matrices A, B, C and D are constant coefficient matrices. Further, assume that there is a harmonic input to the system given by

$$\boldsymbol{u}=\boldsymbol{U}.e^{jwt} \quad (3)$$

Outputs and states can also be assumed harmonic:

$$\boldsymbol{y}=\boldsymbol{Y}\cdot e^{jwt},\ \boldsymbol{x}=\boldsymbol{X}\cdot e^{jwt} \quad (4)$$

where in general the elements of $\boldsymbol{X}$, $\boldsymbol{U}$and $\boldsymbol{Y}$are complex. Insert the assumptions into the state-space equations (1) and (2). Show that the transfer function matrix H(ω) defined by

$$\boldsymbol{Y}=\boldsymbol{H}(\omega)\cdot\boldsymbol{U}$$

Hence, the transfer function can be calculated as,

$$H(w) = c * (jw * I_n - A)^{-1} * B + D$$

Task 1.3: Transfer function plot

Case 1:

Input- Road displacement, z_r

Output- Ride comfort, $\ddot{z}_s$

$$H(\omega)_{zr\to\ddot{z}s} = -\omega^2 * C_1 f * \left(i * \omega * I - A_f\right)^{-1} * B_f + D_1 f$$

Where,

$C_1 f\ is\ [1 \quad 0 \quad 0 \quad 0]$

$D_1 f\ is\ 0$

Case 2:

Input- Road displacement, z_r

Output- Suspension travel, $(z_u - z_s)$

$$H(\omega)_{zr\to(z_u - z_s)} = C_2 f * \left(i * \omega * I - A_f\right)^{-1} * B_f + D_2 f$$

Where,

$C_2 f\ is\ [1 \quad -1 \quad 0 \quad 0]$

$D_2 f\ is\ 0$

Case 3:

Input- Road displacement, z_r

Output- Ride grip, ΔF_{rz}

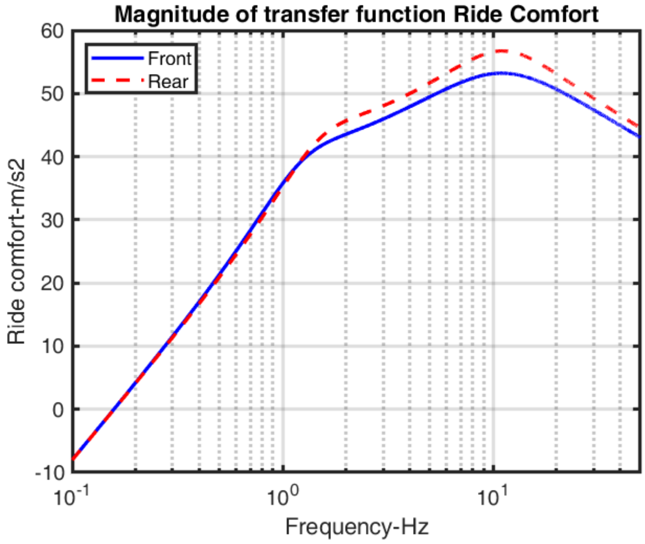
$$H(\omega)_{zr\to\Delta F_{rz}} = -\omega^2 * C_1 f * \left(i * \omega * I - A_f\right)^{-1} * B_f + D_1 f$$

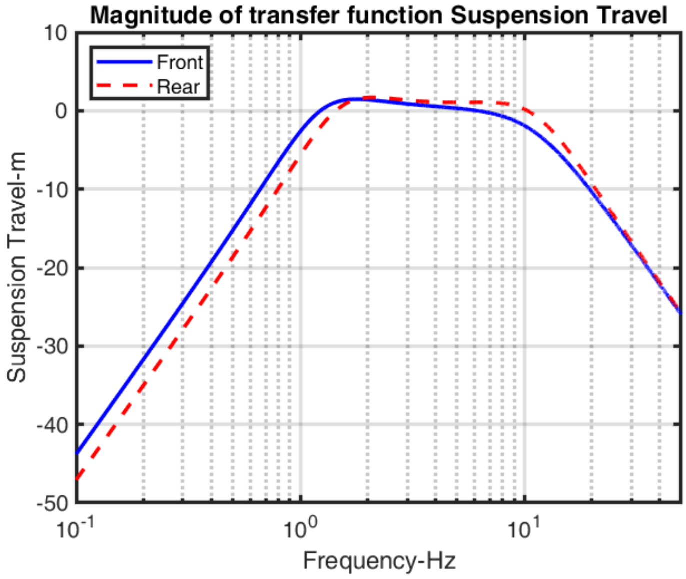
Where,

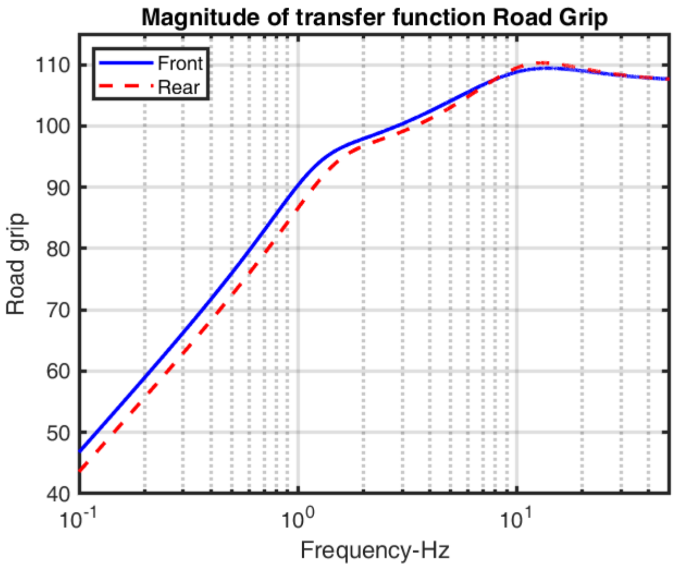
$C_3 f\ is\ [0 \quad 1 \quad 0 \quad 0]$

$D_3 f\ is\ 0$

The magnitude of the three transfer functions for one of the fronts and the rear wheels are plotted against frequencies from 0.1 to 50 Hz.







Task 1.4: Natural frequencies

Natural frequencies for the front and rear wheel for bounce and hop can be found by the expression,

$$\omega_{bounce} = \sqrt{\frac{1/(\frac{1}{C_S} + \frac{1}{C_t})}{m_s}}$$

$$\omega_{wheelhop} = \sqrt{\frac{C_s + C_t}{m_u}}$$

The values are tabulated.

Tyre	Bounce natural frequency	Hop natural frequency
Front	1.2448	11.9397
Rear	1.4131	11.9191

TASK 2: Study of the suspension stiffness and damping

Consider one front wheel of a vehicle moving at constant speed. If the longitudinal velocity vx is constant, the road spectrum can be calculated according to equation in the compendium stating

$$G_{z_r}(\omega) = V_x^{w-1} * \omega^{-w} * \Phi$$

Where,

w=waviness

Φ=road severity[m^2/(rad/m)].

The given values are w =2.5, $\Phi=10*10^{-6}$ and V_x=80 km/h.

Task 2.1: Response spectrum plot and RMS values calculation

The Power Spectral Density(PSD) is plotted and the RMS values are calculated for,

- Sprung mass acceleration (Ride comfort)
- Tyre force (Road Grip)

Given bandwidth, $\Delta\omega = 2 * \pi * 0.01$

The formulae used to calculate the response spectra and rms values are,

$$G(\omega) = \mid H(\omega) \mid^2 * G_{Z_r}(\omega)$$

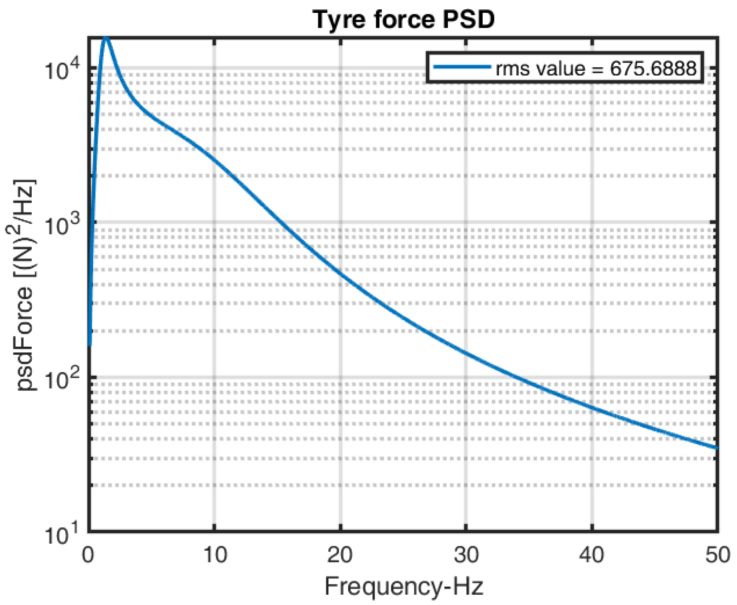
$$rms = \sqrt{\sum_{i}^{N} G(\omega_i) * \Delta\omega}$$

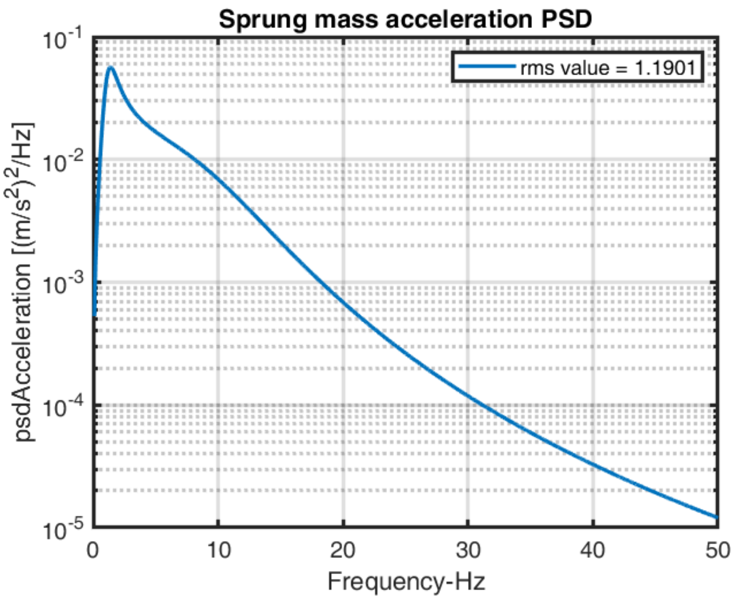
The PSD acceleration can be calculated using the formula,

$$PSD_{acc} = \mid H(\omega)_{zr \rightarrow \ddot{z}s} \mid^2 G_{zr}(\omega)$$

$$rms_{acc} = \sqrt{\sum_{i}^{N} PSD_{acc} * \Delta\omega}$$

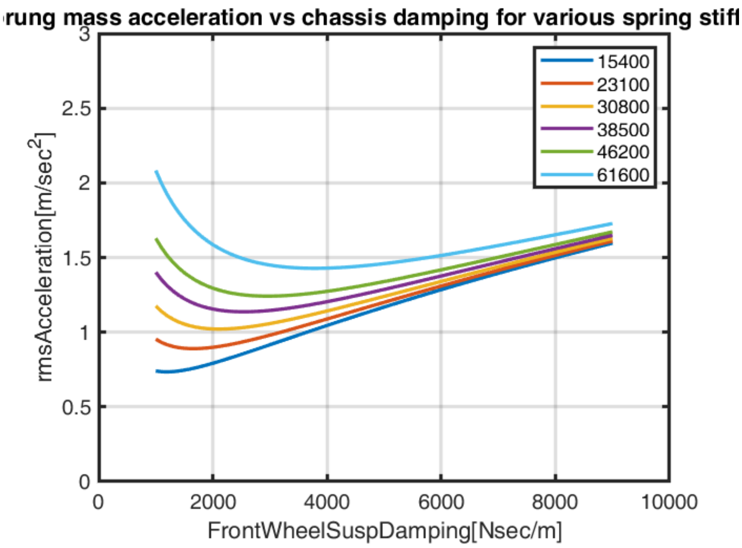
The plots are attached below.

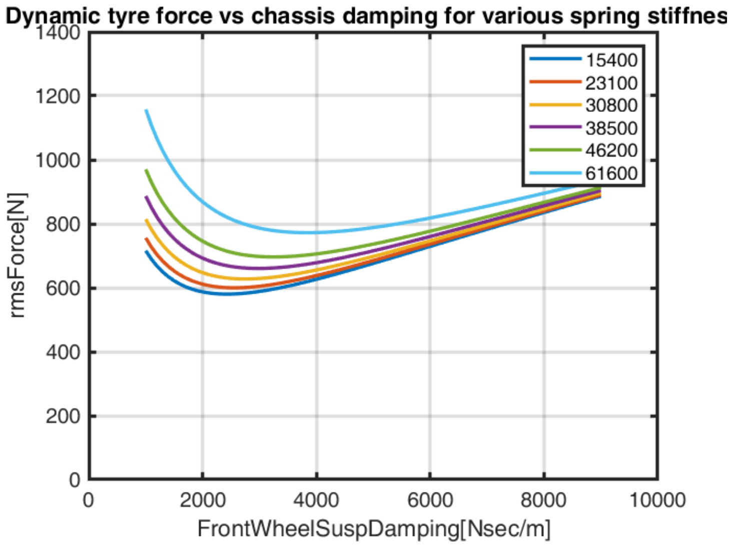




Task 2.2: Balance ride comfort and Tyre force/road grip

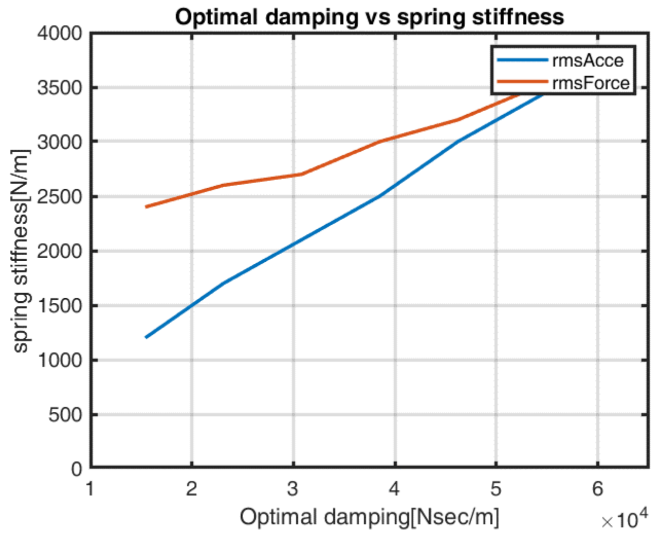
The spring and damper stiffness are related to road comfort and road grip in the plots below. For each of the given spring stiffness values, the damping values are varied. The rms values for the sprung mass acceleration and the tyre force for each setting of stiffness and damping are calculated.





Spring Stiffness	RMS acceleration	Optimal damping	RMS tyre force	Optimal damping
0.5	0.73	1500	580.07	2200
0.75	0.89	1800	599.76	2400
1	1.02	2000	627.43	2700
1.25	1.13	2200	660.30	3000
1.5	1.24	2700	696.34	3300
2	1.42	3800	772.08	3900

The effect of spring stiffness with respect to the acceleration rms and the tyre force rms can be understood from the curve below.



High levels of sprung mass accelerations and road grip will result in poor ride comfort and bad road grip respectively.

TASK 3: Ride comfort and the use of ISO-2631

Management of different road types is considered and studied in this task. ISO standard (ISO2631) is used to calculate ISO-filtered vertical whole-body vibration levels and the linked time averaged vibration exposure value. The EC Directive 2002/44 is also made use of to compare your calculated value with the existing vibration exposure limits.

Considering the given scenario between city A and B which is 80 km long, the vertical profile of the path is categorised in three different types,

- Good road (ϕ=1E-6, w=3)
- Bad road (ϕ=10E-6, w=2.5)
- Very bad road (ϕ=100E-6, w=2)

The distribution of these road types is as follows: good road-70% (56km), bad road-17.5% (14km) and very bad road-12.5% (10km).

The driver works 8 hours per day and the maximum speed driven is 110 km/hr. In the beginning, the driver rides at the maximum speed regardless of the road and by that he will be able to drive to the customer back and forth 5 times per day. After the first working day, the driver expresses that the ride comfort has been very poor: the vibration levels were too high. He proposes to his manager that to improve the ride comfort he needs to lower the vehicle speed on some parts of the road. This will not only affect the ride comfort level but also the number of times he can drive to the customer and back per day. Some suggestions are made to make the decision.

Task 3.1: Calculation of daily whole-body exposure values

The whole-body vibration exposure value when driving on the specified road in 110 km/h during an eight-hour period is first calculated. The value of time weighted rms acceleration is found to be **0.9556 m/s²**.

Task 3.2: Modification of vehicle velocity

The vehicle speed is individually modified based on the road condition in such a way that the vibration exposure limit(1.15 m/s^2) is not exceeded.

RMS acceleration on smooth road	0.3940 m/s^2
RMS acceleration on rough road	1.13 m/s^2
RMS acceleration on very rough road	1.15 m/s^2
Velocity on smooth road	110 km/hr
Velocity on rough road	99 km/hr
Velocity on very rough road	9 km/hr

According to the above values the number of trips which can be driven per day is 2.1047 which is **approximately 2 trips**.

REFERENCES:

Bengt J H Jacobsson, Vehicle Dynamics Compendium, Vehicle Dynamics Group, Division Vehicle and Autonomous Systems, Department of Mechanics and Maritime Sciences, Chalmers University of Technology.