MMF062 – Vehicle Dynamics

Department of Mechanics and Maritime Sciences

Vehicle Dynamics Group, Division Vehicle and Autonomous Systems

Assignment 2
Lateral Dynamics

Submitted by:

Sundar Murugan Ramaswamy- 199809188239

(murugan@student.chalmers.se)

Surya Prakash Avaragere Lokesh- 199509044815

(prsurya@student.chalmers.se)

Nitesh Anil Kumar Nikkam- 199411259238

(nitesha@student.chalmers.se)

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Task 1: Steady state cornering characteristics

The given load cases are:

Load case 1: l_f=0.37*L

Load case 2: l_f=0.63*L

Load case 3: l_f=0.47*L

Task 1.1: Steady state cornering equation for a single-track vehicle model at high speeds

The assumptions made are'

- Small steering angle
- Small slip angle
- Linear tyres

As the rate of change do not apply for steady state, the **lateral force equilibrium equation** can be written as,

$$m * \omega_z * V_x = F_{fy} + F_{ry}$$

Also, we know that,

$$V_x = R * \omega_z$$

Where,

m= mass of the vehicle

$\omega_z = yaw\ rate$

$V_x = Longitudinal\ velocity$

$F_{fy} = Lateral\ force\ on\ front\ wheel$

$F_{ry} = Lateral\ force\ on\ rear\ wheel$

$R = Radius\ of\ the\ path$

The yaw equilibrium equation can be written as,

$$J * \dot{\omega}_z = F_{fy} * l_f - F_{ry} * l_r$$

For steady state $\dot{\omega}_z = 0$, hence the equation becomes

$$F_{fy} * l_f - F_{ry} * l_r = 0$$

Where,

$l_f = Distance\ of\ COG\ from\ front\ axle$

$l_r = Distance\ of\ COG\ from\ rear\ axle$

$\dot{\omega}_z = Rate\ of\ change\ of\ yaw\ velocity$

$J = Moment\ of\ inertia$

Constitution:

The lateral force is a function of lateral slip and the cornering stiffness along the y-direction. The unknowns in the above equation $F_{fy}\ and\ F_{ry}$ can be found by,

$$F_{fy} = -C_f * S_{fy}$$

$$F_{ry} = -C_r * S_{ry}$$

Where,

$C_f = cornering\ stiffness\ of\ front\ axle$

$C_r = cornering\ stiffness\ of\ rear\ axle$

$S_{fy} = lateral\ slip\ of\ front\ axle$

$S_{ry} = lateral\ slip\ of\ rear\ axle$

The values of $C_f\ and\ C_r$ can be found by,

$$C_f = C_o * F_{fz} + C_1 * F_{fz}^2$$

$$C_r = C_o * F_{rz} + C_1 * F_{rz}^2$$

Where,

$C_o = 30.7\ rad^{-1}$ and
$C_1 = -0.00235\ (Nrad)^{-1}$

$F_{fz} = Normal\ force\ on\ front\ axle$

$F_{rz} = Normal\ force\ on\ rear\ axle$

Compatibility:

As our vehicle is a front axle steered vehicle, the steering angle applies only for the front axle. The compatibility equations can be written as,

Front axle: $$\delta_f + S_{fy} = \frac{V_{fy}}{V_x} = \frac{V_y + l_f * \omega_z}{V_x}$$

Rear axle: $$S_{ry} = \frac{V_{ry}}{V_x} = \frac{V_y - l_r * \omega_z}{V_x}$$

Where,

$V_{fy} = Front\ axle\ lateral\ velocity$

$V_{ry} = Rear\ axle\ lateral\ velocity$

$\delta_f = Front\ wheel\ steering\ angle$

$V_y = Lateral\ velocity$

Combining all the constitution and compatibility equations with the equilibrium equation, the steering angle can be derived as follows,

$$\omega_z = \frac{C_f * C_r * L + (C_r * l_f + C_r * l_r) * F_{fxw}}{C_f * C_r * L^2 + (C_r * l_r - C_f * l_f) * m * V_x^2} * V_x * \delta_f$$

Where,

$F_{fxw} = longitudinal\ force\ on\ front\ axle$

$L = Wheelbase$

Rearranging we get,

$$\delta_f = \frac{C_f * C_r * L^2 + (C_r * l_r - C_f * l_f) * m * V_x^2}{C_f * C_r * L + (C_r * l_f + C_r * l_r) * F_{fxw}} * \frac{\omega_z}{V_x}$$

Elongating the terms inside the parentheses by multiplying and Substituting $l_f + l_r = L$ and $\frac{V_x}{\omega_z} = R_p$ in the above rearranged equation,

$$\delta_f = \frac{L}{1 + \frac{F_{fxw}}{C_f}} * \frac{1}{R_p} + \frac{(C_r * l_r - C_f * l_f)}{C_f + F_{fxw}} * \frac{m * V_x^2}{R_p}$$

Assuming $\frac{F_{fxw}}{C_f} = 0$ (minimizing the longitudinal force) the equation becomes,

$$\delta_f = \frac{L}{R_p} + \frac{(C_r * l_r - C_f * l_f)}{C_f * C_r * L} * \frac{m * V_x^2}{R_p}$$

In the above equation $\frac{(C_r * l_r - C_f * l_f)}{C_f * C_r * L} = K_u$, where K_u is the Understeer gradient.

$$\boldsymbol{\delta_f = \frac{L}{R_p} + K_u * \frac{m * V_x^2}{R_p}}$$

Task 1.2 Understeer Gradient

The understeer gradient for the three load cases can be found by the equation,

$$Understeer\ gradient, K_u = \frac{(C_r * l_r - C_f * l_f)}{C_f * C_r * L}$$

By substituting the values in the above equation we get understeer gradient in $rad/_N$ or $rad/_{kgm/s^2}$. So, in order to get the values in $rad/_{m/s^2}$ we need to multiply the values by the mass in kg. Therefore, the equation becomes,

$$Understeer\ gradient, K_u = m * \frac{(C_r * l_r - C_f * l_f)}{C_f * C_r * L}$$

The corresponding values of understeer gradient for the three load cases are found to be,

Load case 1: K_u= 0.0012

Load case 2: K_u= -0.0012

Load case 3: K_u= 2.668*10^{-4}

From the above values we can infer that,

- If the Center Of Gravity is closer to the front axle, the understeer gradient is positive.
- If the Center Of Gravity is closer to the rear axle, the understeer gradient is negative.
- If the Center Of Gravity is in between the front and rear axle, the understeer gradient will be zero.

Task 1.3 Critical and characteristic speeds

The critical speed is the speed above which the vehicle becomes unstable. It can be found using the formula,

$$V_{critical} = \sqrt{\frac{L}{-K_u * m}}$$

The values of critical velocities for the three load cases are found to be,

Load case 1: $V_{critical}$= 47.7766 *i* m/s

Load case 2: $V_{critical}$= 47.7766 m/s

Load case 3: $V_{critical}$= 100.13 *i* m/s

The characteristic speed occurs when the required steering increases to about twice needed for low speed for the same path radius. It can be found by,

$$V_{characteristic} = \sqrt{\frac{L}{K_u * m}}$$

The values of characteristic velocities for the three load cases are found to be,

Load case 1: $V_{characteristic}$= 47.7766 m/s

Load case 2: $V_{characteristic}$= 47.7766 *i* m/s

Load case 3: $V_{characteristic}$= 100.1318 m/s

Task 1.4 Steering wheel angle for one certain operating point

The values of lateral acceleration and longitudinal velocity are given.

$a_y = 4\, m/s^2$

$V_x = 100\ km/hr$

The path radius can be found by,

$$R_p = \frac{V_x^2}{a_y}$$

The value of path radius is found to be **192.9m**.

The steering angle can be found using the previously derived equation,

$$\boldsymbol{\delta_f = \frac{L}{R_p} + K_u * \frac{m * V_x^2}{R_p}}$$

The values of steering angles are found to be,

Load case 1: $\boldsymbol{\delta_f}$= 16.9035 degrees

Load case 2: $\boldsymbol{\delta_f}$= 8.3626 degrees

Load case 3: $\boldsymbol{\delta_f}$= 13.6053 degrees

Task 1.5 Steering wheel angle for varying operating point

The value of path radius is given as 200m.

The steering angle is found for a range of velocity(vector) from 0 to 50 m/s. The steering angles can be found by,

$$\boldsymbol{\delta_f = \frac{L}{R_p} + K_u * \frac{m * V_x^2}{R_p}}$$

The steering angle values are plotted against the velocities.

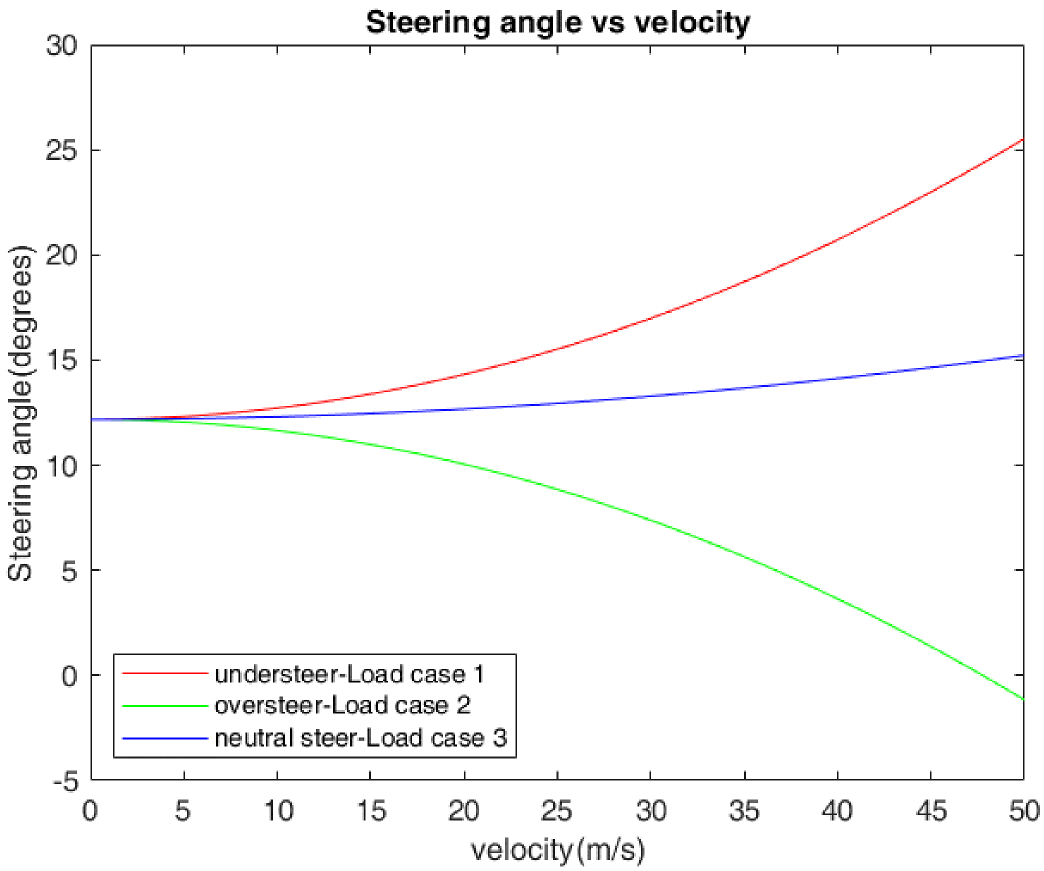


Figure 1 steering angle vs velocity

From the above plot we can infer that,

- Load case 1 is understeer (steering angle increases with velocity)
- Load case 2 is oversteer (steering angle decreases with velocity)
- Load case 3 is relatively neutral steer (with a very low understeer gradient)

Task 2: Simulation with single track model (without load transfer and without combined tire slip)

Task 2.1: Model Implementation

The following are the equations of motions in the XY plane for the single-track vehicle model at high speed

$$\dot{v_x} = (m * \omega_z * v_y + F_x(1) * cos(\delta) - F_y(1) * sin(\delta) + F_x(2))/m,$$

$$\dot{v_y} = (-m * \omega_z * v_x + F_x(1) * sin(\delta) + F_y(1) * cos(\delta) + F_y(2))/m,$$

$$\dot{\omega_z} = ((F_y(1) * sin(\delta) + F_y(1) * cos(\delta)) * l_f - F_y(2) * l_r)/J,$$

where,
$\dot{v_x}$ = derivative of the velocity in longitudinal direction
$\dot{v_y}$ = derivative of the velocity in lateral direction
$\dot{\omega_z}$ = derivative of yaw rotational velocity
m = mass of the vehicle
ω_z = Rotational velocity
$v_x = longitudinal\ velocity$
v_y = lateral velocity
F_x = force acting on the axle in longitudinal direction
F_y = force acting on the axle in lateral direction
δ = steering wheel angle
l_f = distance of COG from front axle
l_r = distance of COG from the rear axle
J = vehicle inertia about the Z axis

The following are the equations for lateral slip

$$\alpha_f \ = \ atan(v_{fyw}/v_{fxw})$$

$$\alpha_r \ = \ atan(v_{ryw}/v_{rxw})$$

The unknowns in the above equation can be found by,

$$v_{fxw} = \left(v_y + l_f * \omega_z\right) * \sin(\delta) + (v_x * \cos(\delta))$$

$$v_{fyw} = \left(v_y + l_f * \omega_z\right) * \cos(\delta) - (v_x * \sin(\delta))$$

$$v_{rxw} = \ v_x$$

$$v_{ryw} \ = \ v_y \ - \ l_r * \omega_z$$

Where,
α_f = front lateral tire slip
α_r = rear lateral tire slip
v_{fxw}, v_{fyw} = Components of the velocity vector in front wheel
v_{rxw}, v_{ryw} = Components of the velocity vector in rear wheel

The above equations were incorporated into the respective blocks the Simulink model and with varying load case changes in the Initmodel.m file and the simulations were carried out. The vehicle data and the initial conditions are declared in the InitModel.m MATLAB script.

Task 2.2: Model verification for moderate a_y

With the steering angle values from task 1 for respective load cases, the Simulation was carried out and the vehicle reaches 4 m/s^2 lateral acceleration. The plots are provided below.

	Load case (m)	Steering wheel angle (deg)	Lateral acceleration (m/s^2)
1	l_f = 0.37*L	16.9035	4.0043
2	l_f = 0.63*L	8.3626	3.9535
3	l_f = 0.47*L	13.6053	3.9864

Lateral acceleration plots

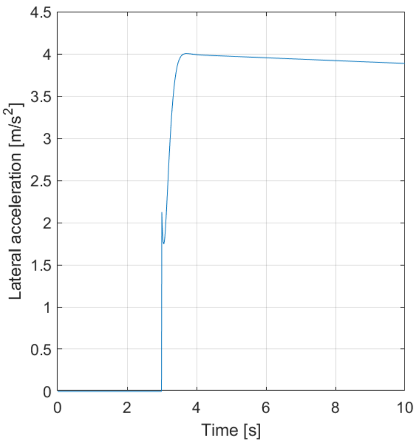


Figure 1 Case 1

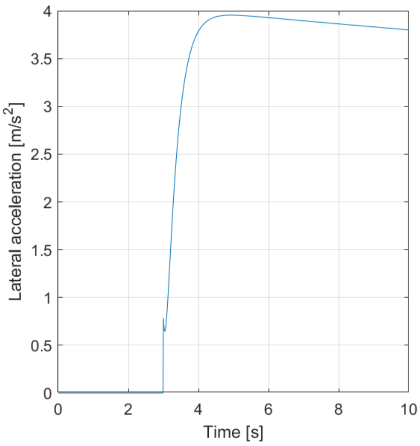


Figure 2 Case 2

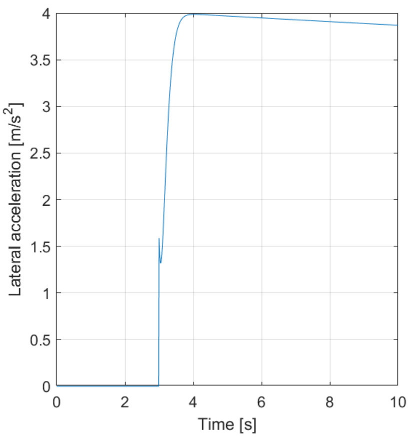


Figure 3 Case 3

Task 2.3: Transient response

To find the quickest response to the vehicle setup is by comparing the maximum lateral accelerations of each vehicle setup and the time taken to reach the same.

Following table gives the values of the maximum lateral acceleration and the time taken to reach the same.

Load Case	Lateral acceleration (m/s^2)	Time
1	4.0043	3.710
2	3.9535	4.93
3	3.9864	4.02

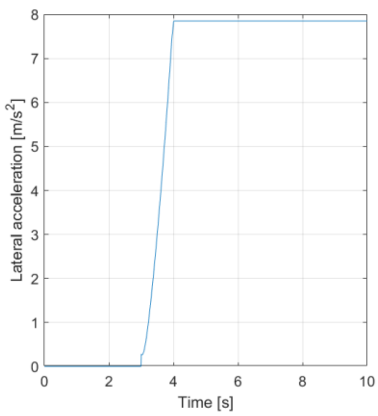
Table 1 Lateral acceleration verification

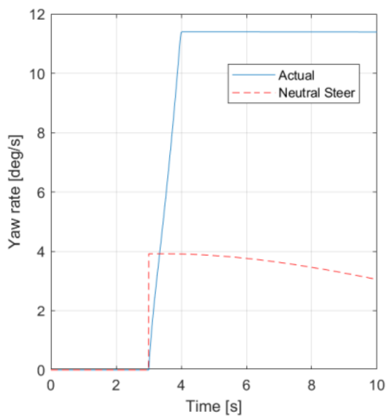
From the above table, it can be observed that the quickest response to the vehicle setup is during Load case 1 i.e., understeer vehicle.

Task 2.4: Model behaviour at over-critical speed

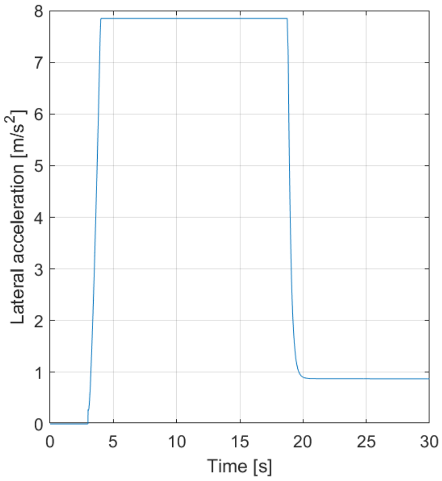
With the vehicle setup in oversteer case and a speed beyond critical speed (200kmph), steering wheel angle set at 3 degrees, following observations are made for simulation time t=10s and t=30s.

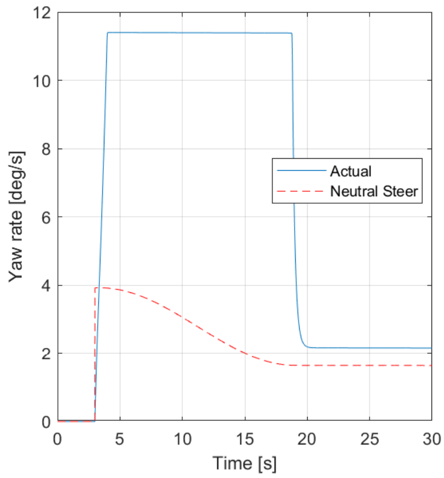
Scenario 1: t=10s





Scenario 2: t=30s





Comparing the above results with that of task 1, the results obtained do not match with that of the above scenarios. The steering angle tends to increase with time and becomes constant when the simulation is run for t=10s. But, when the simulation is run for t=30s, a decrease in the lateral acceleration and yaw rate can be observed but the value does not reach zero as it is observed in task 1, instead it is constant with time.

Task 3: Load transfer

Load transfer will be calculated for the single-track model by considering longitudinal and lateral load transfer on both wheels of an axle which can be used to calculate the individual tyre stiffness and in turn can be combined to form axle stiffness. Neglecting the aerodynamic drag and the gravity components on the road plane, the equations are derived.

Task 3.1: Add load transfer to the model

The Longitudinal load transfer equations for the individual wheels are as follows,

Front left:

$$\delta_{Fflz} = -\frac{(m * a_x * h)}{2 * L}$$

Front right:

$$\delta_{Frlz} = -\frac{(m * a_x * h)}{2 * L}$$

Rear left:

$$\delta_{Frlz} = \frac{(m * a_x * h)}{2 * L}$$

Rear right:

$$\delta_{Frrz} = \frac{(m * a_x * h)}{2 * L}$$

Where,

$m = mass$

$a_x = longitudinal\ acceleration$

$L = wheel\ base$

$h = Center\ of\ Gravity\ height$

The Lateral load transfer equations for the individual wheels are as follows,

Front left:

$$\delta_{Fflz} = -a_y * m * \frac{h_1 * l_r}{L * w} + (\frac{dh}{w} * \frac{C_{Froll}}{C_{roll}})$$

Front right:

$$\delta_{Ffrz} = a_y * m * \frac{h_1 * l_r}{L * w} + \left(\frac{dh}{w} * \frac{C_{Froll}}{C_{roll}}\right)$$

Rear left:

$$\delta_{Frlz} = -a_y * m * \frac{h_2 * l_f}{L * w} + \left(\frac{dh}{w} * \frac{C_{Rroll}}{C_{roll}}\right)$$

Rear right:

$$\delta_{Frlz} = a_y * m * \frac{h_2 * l_f}{L * w} + (\frac{dh}{w} * \frac{C_{Rroll}}{C_{roll}})$$

Where,

$m = mass$

$a_y = lateral\ acceleration$

$h_1 = front\ roll\ center\ height$

$h_2 = rear\ roll\ center\ height$

$L = wheel\ base$

$l_f = distance\ of\ COG\ from\ front\ axle$

$l_r = distance\ of\ COG\ from\ rear\ axle$

$dh = height\ of\ COG\ from\ roll\ axis$

$w = wheel\ track$

$C_{Froll} = front\ roll\ stiffness$

$C_{Rroll} = rear\ roll\ stiffness$

The above equations are updated to the Load transfer block. Its verified that the model runs and behaves reasonable.

Task 3.2: Influence from load transfer on steering response

The roll stiffness distribution is kept as 0.65 (65% roll stiffness front) and the three load cases are simulated. The values are found to be,

Load case 1: a_y =3.868 $^m/_{s^2}$

Load case 2: a_y =3.734 $^m/_{s^2}$

Load case 3: a_y =3.834 $^m/_{s^2}$

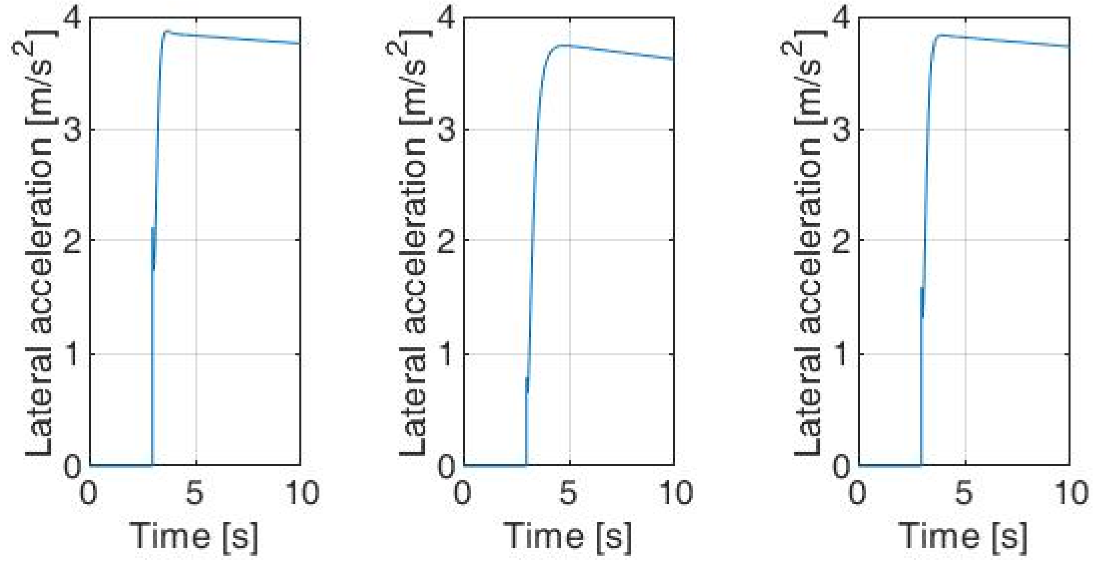


Figure 4 Lateral acceleration with load transfer

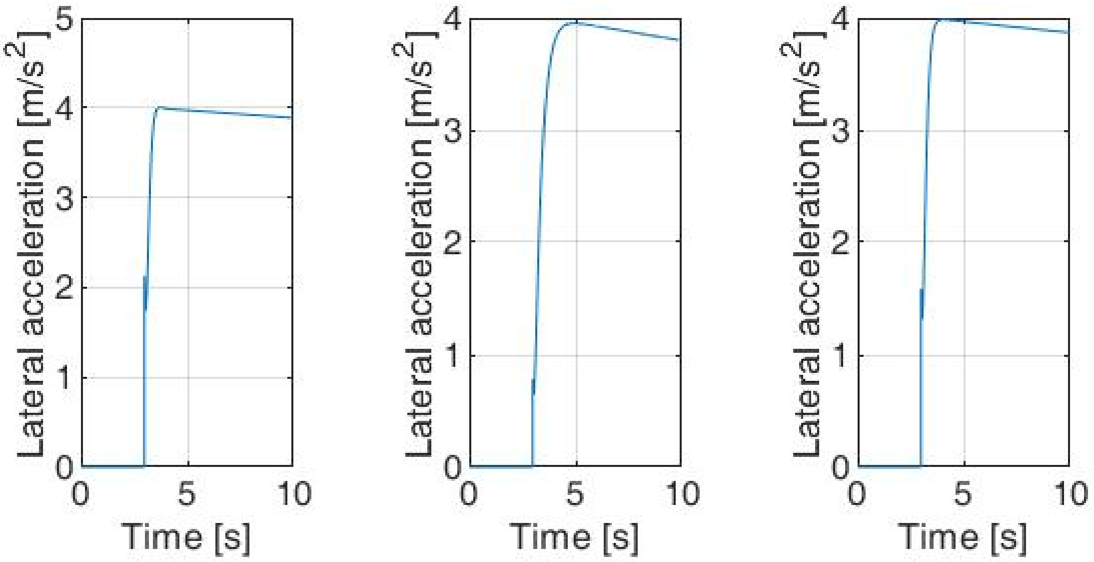


Figure 5 Lateral acceleration without load transfer

From the above values it is inferred that the values of lateral acceleration differ from task 2 where the value of lateral acceleration was 4 $^m/_{s^2}$ for all cases. The change in peak lateral acceleration is found to be,

Load case 1: 3.41%

Load case 2: 7.12%

Load case 3: 4.33%

From the above values of percentage change, we can infer that the linear 2 DOF single track model can be used to analyse the handling characteristics of the vehicle in its linear range because a 7% change in lateral acceleration values is negligible.

Task 3.3: Influence of roll stiffness distribution on steering response

The roll stiffness distribution is studied by simulating load case 3 with a steering angle of 30 degrees and roll stiffness distribution values of 0.45 and 0.65 to compare the trajectory of the vehicle and the yaw rate.

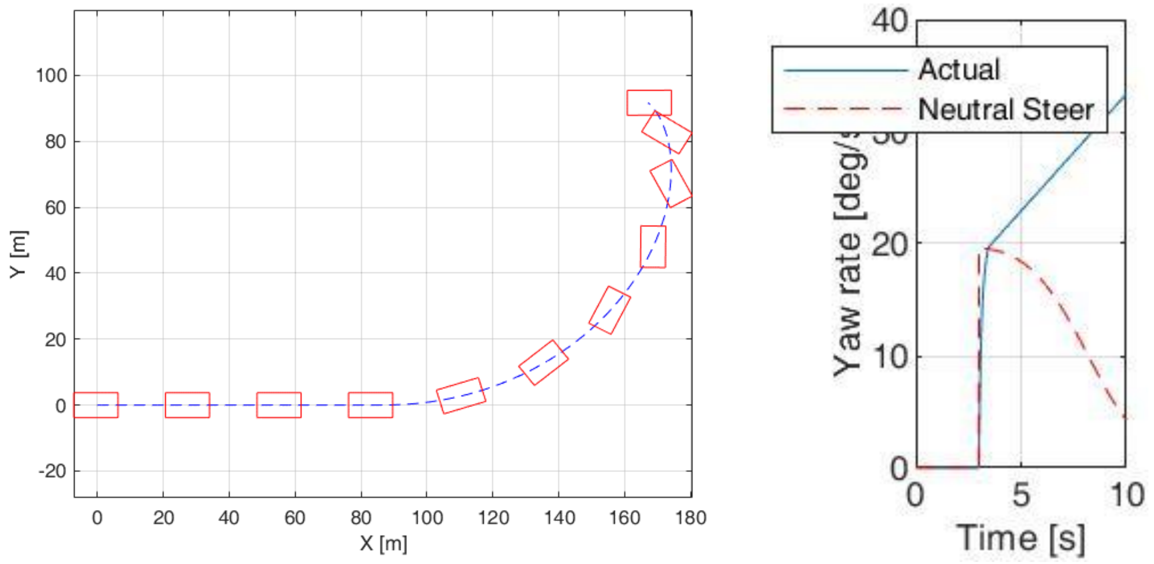


Figure 6 Influence of roll stiffness distribution (0.45 roll stiffness)

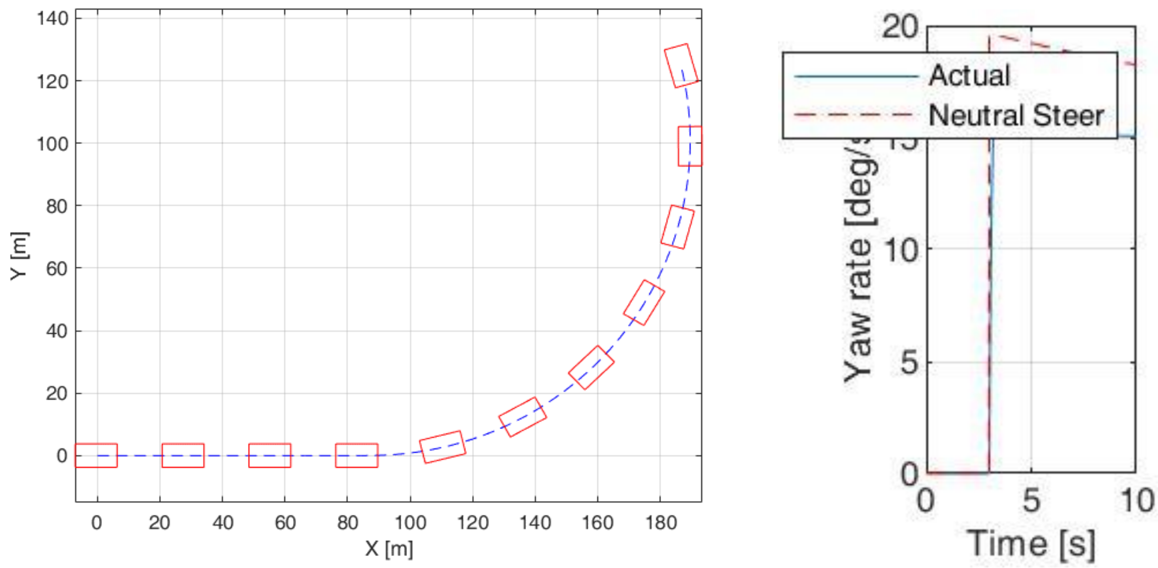


Figure 7 Influence of roll stiffness distribution (0.65 roll stiffness)

Increasing the roll stiffness on the rear axle leads to less rear cornering stiffness making the vehicle more oversteered. Accordingly, in a 0.45 roll stiffness distribution (45% roll stiffness front), the roll stiffness of the rear axle is high, hence the vehicle oversteers leading to a drastic increase in the yaw rate. And in a 0.65 roll stiffness distribution, more roll stiffness acts on the front axle and hence the vehicle understeers more with increasing lateral acceleration (less front cornering stiffness). Also, the yaw rate is less. Hence the difference.

Task 3.4: Propose roll stiffness distribution

For load case 3, for a steering wheel angle of 10 degrees and an initial velocity of 100 km/hr, the roll stiffness distribution is tuned to 0.23 and verified such that the actual line coincides with the neutral steer line in the yaw rate versus time plot.

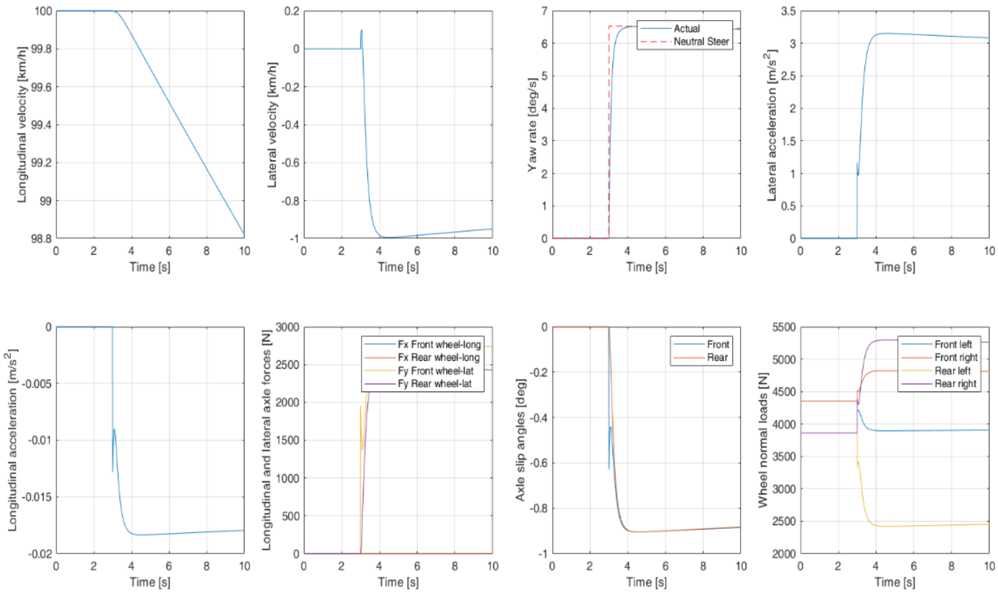


Figure 8 Tuned roll stiffness distribution plot

I would not recommend the same roll stiffness distribution for any production car with the same load case because the steering angle considered here is just 10 degrees. If the value of steering angle or the longitudinal velocity is increased further, for the roll stiffness of 0.23 (77% roll stiffness in the rear) the cornering stiffness will be more on the rear axle leading to very high increase in the yaw rate and more oversteering. Hence, I will not propose this roll stiffness value for a production vehicle with the same load case. The simulated results for a case with velocity of 120 km/hr and steering angle of 30 degrees are simulated for 1 minute, compared and attached below.

Also, when the steering angle and longitudinal velocity inputs are kept constant and the rolling stiffness alone is varied, it is inferred that the vehicle understeers to a very minimal extent and the change is not much predominant.

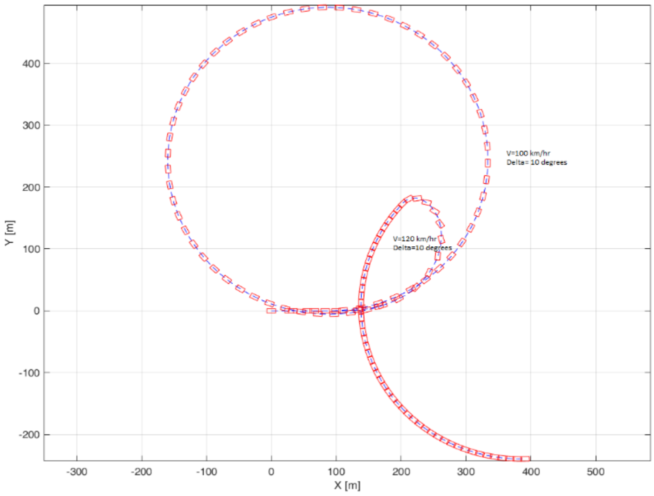


Figure 9 Simulation with varied velocities

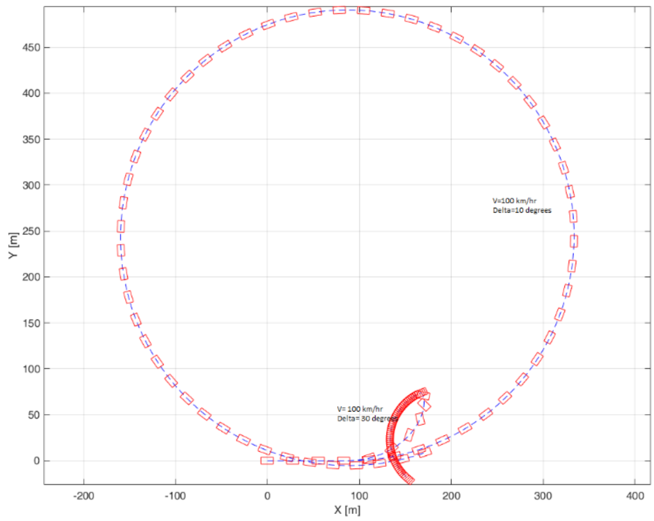


Figure 10 Simulation with varied steering angles

Task 4: Combined tyre slip

Task 4.1: Add combined slip to model

Combined slip equation for front and rear are as below, implemented in the 'LatForce'-block inside the 'Tyre model'-block with the correction factor for front and rear denoted as Cf_corr and Cr_corr.

$$Cf = (c0 * FzWhl(1) + (c1 * FzWhl(1)\,\hat{}\,2)) + (c0 * FzWhl(2) + (c1 * FzWhl(2)\,\hat{}\,2))$$

$$Cr = (c0 * FzWhl(3) + (c1 * FzWhl(3)\,\hat{}\,2)) + (c0 * FzWhl(4) + (c1 * FzWhl(4)\,\hat{}\,2))$$

$$Cf\_corr = (sqrt((mu * FzAxle(1))^{\wedge}2 - Fx(1)^{\wedge}2) / (mu * FzAxle(1))) * Cf$$

$$Cr\_corr = (sqrt((mu * FzAxle(2))^{\wedge}2 - Fx(2)^{\wedge}2) / (mu * FzAxle(2))) * Cr$$

Adding a brake demand of 4000 N and keeping the default brake distribution ratio of 0.5 (i.e., 50%brake distribution in the front and rear), with the steering wheel angle of 10degrees at 100km/hr (Load case 3: l_f=0.47*L).

Observation form the slip angle plot are the rear wheels negative slip angle is greater than the front which depicts oversteer of the vehicle. Wheel normal loads on the front wheels are higher in the transition compared to the rear wheels, hence there is a oversteer of the vehicle.

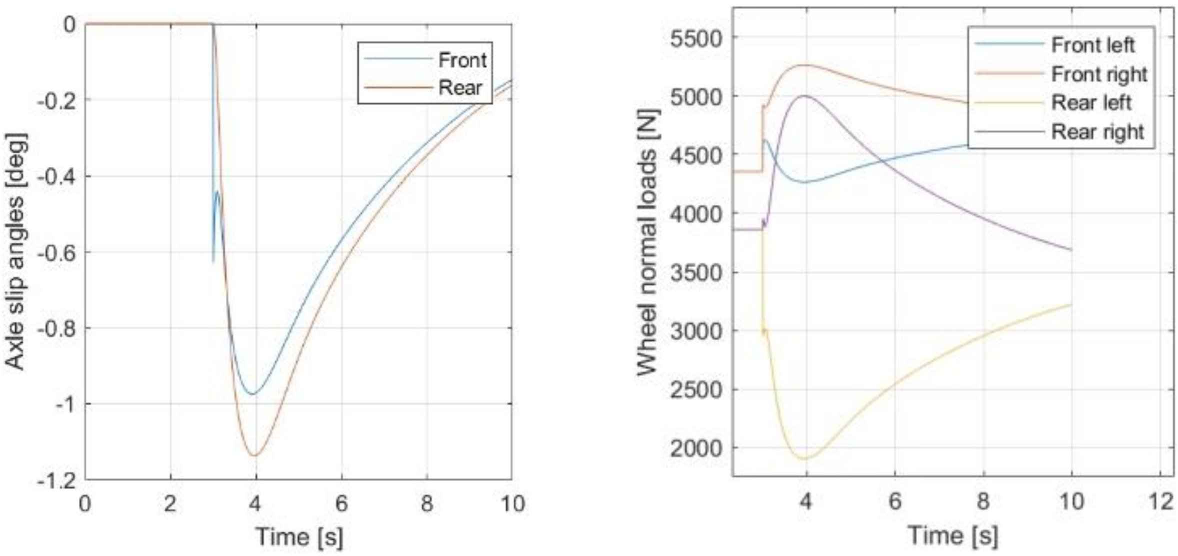


Figure 11 Axle slip angle and Wheel normal loads for 50% brake force distribution on front wheel

Task 4.2: Influence of brake force distribution

Brake force distribution ratio of 0.2 (i.e., 20%brake distribution in the front), for a brake demand of 4000 N with the steering wheel angle of 10degrees at 100km/hr (Load case 3: l_f=0.47*L) was considered and the plots are below.

Observations from the plots depicts oversteer of the vehicle. The rear wheel slip angle is greater than the front wheel as observed with the brake force distribution ratio of 0.5. Similar trend is observed in the wheel normal loads plot where the front wheel loads are higher than the rear wheel loads.

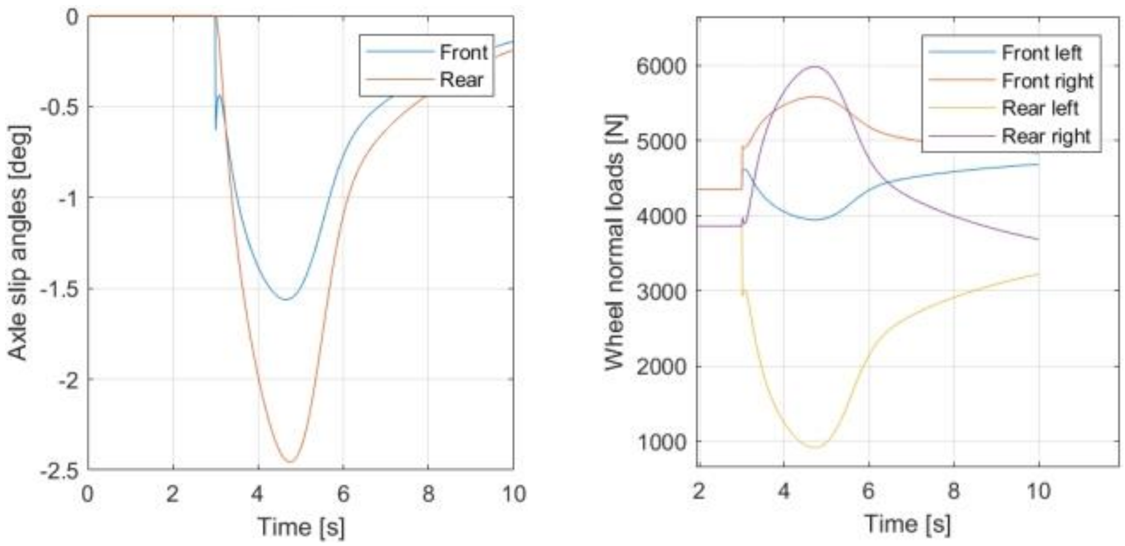


Figure 12 Axle slip angle and Wheel normal loads for 20% brake force distribution on front wheel

The track travelled by the vehicle for 0.2 and 0.5 brake force distribution is plotted in the graph below. Observations depict decrease in brake force distribution in the front of the vehicle tends to oversteer predominantly i.e., 0.2. Deviation can be observed in the yaw rate plots fitted with the neutral steer plots.

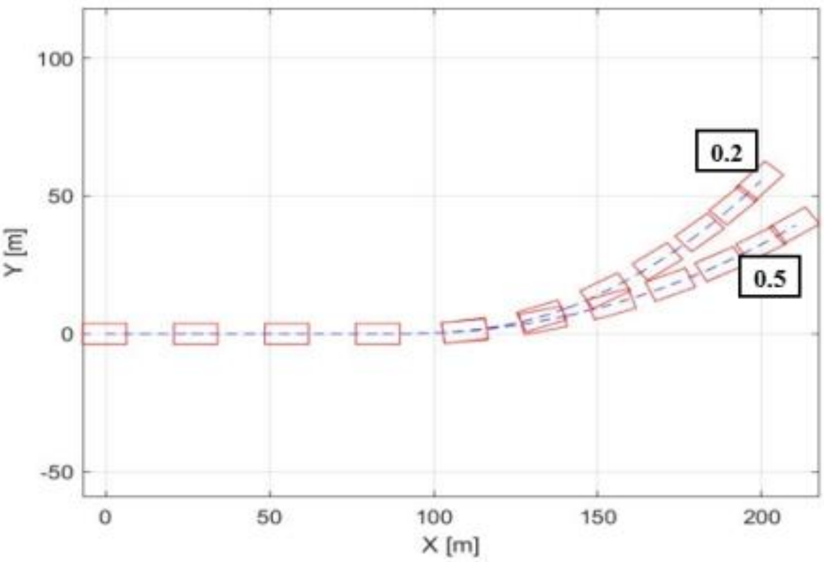


Figure 13 Trajectory travelled by vehicle for 20% & 50% brake force distribution on front wheel

Task 4.3: Propose brake force distribution

To attain a neutral steer the braking distribution ratio is 80% in the front wheels. Figure:4.4 depicts that the yaw rate of neutral steer and actual vehicle follow the same path. The value of brake distribution force is varied to obtain a neutral steer befitting the yaw rate and slip angle curve.

There is a slight offset in the slip angle plot depicting same angle of oversteer of the vehicle.

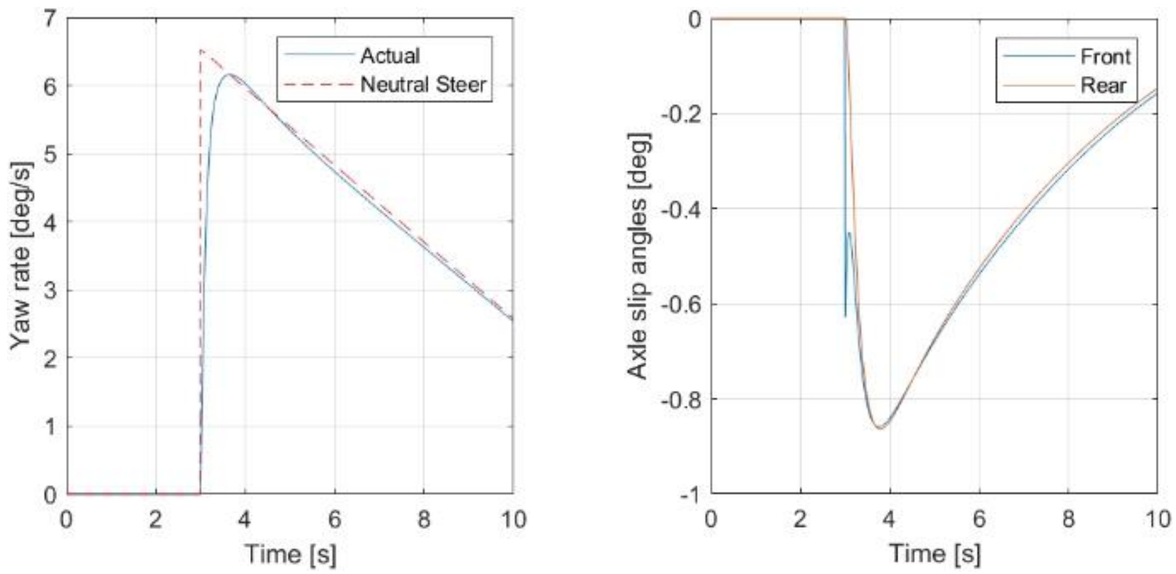


Figure 14 Yaw rate and Axle slip angle for 80% brake force distribution on front wheel

Task 5: Simulator driving experience

Task 5.1: Model validation

	Longitudinal acceleration (m/s^2)		Lateral acceleration (m/s^2)		Yaw rate (deg/s)		Yaw acceleration (deg/sec^2)	
	min	max	min	max	min	max	min	max
Case 1	-0.01282	0	0	1.917	4.953	4.953	-0.264	53.8532
Case 2	-1.194	0	0	0	0	0	0	0
Case 3	-0.5982	0	0	0.5708	1.505	1.505	-0.0401	16.0699

Table 2 Model validation table

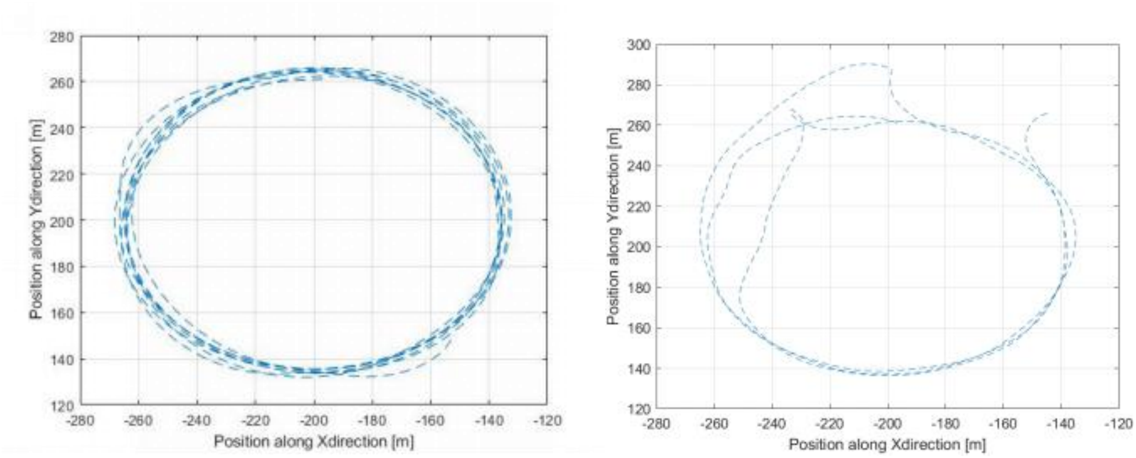
The maximum and minimum values for longitudinal, lateral and yaw accelerations in each of the cases are recorded in the above table.

Task 5.2: CASTER driving session

Task 5.3: Documentation

Driving simulators are used for real time model verification and development, evaluating key vehicle attributes and efficiently used for safety testing. Real time seating position with high responsive steering feedback and consisted of automatic gears, which was neglected in our model building. A scalable road track with skid pad and flat track gave the visualisation of the vehicle response and 6DOF motion system response gave the braking and acceleration, shift of mass experience in the vehicle. Motion and shift of mass on the vehicle (i.e., mass distribution) and steering feedback justified the steering conditions of the vehicle.

The position plot (figure 5.1) of the vehicle in driving session on a skid pad for brake force distribution ratio for 80% on the front gave the vehicle understeer and this assisted steering response for the vehicle to travel with steady state cornering with lesser feedback input on the steering experienced by the drivers. Mass distribution of 20% on the vehicle front wheels tends it to oversteer. The comparison between the mass distribution for 80% on the front and 20% on the front wheel can be observed from the plots below.



Understeer **Oversteer**

Figure 15 Position of vehicle for 80% &20%brake force distribution on front wheel-Driving session results

REFERENCES:

Bengt J H Jacobsson, Vehicle Dynamics Compendium, Vehicle Dynamics Group, Division Vehicle and Autonomous Systems, Department of Mechanics and Maritime Sciences, Chalmers University of Technology.