

Long Division

When integrating a fraction where the degree of the numerator \geq the degree of the denominator, you will have to use long division (or creative thinking) to “split the fraction.”

Example: Integrate $\int \frac{x^2 - 4x + 2}{x^2 + 2} dx$

Assigned Problems:

1. $\int \frac{x+1}{x-1} dx$

2. $\int \frac{x^3}{x^2+1} dx$

3. $\int_0^1 \frac{x^2}{x+1} dx$

4. $\int \frac{5x^2}{x^2+1} dx$

5. $\int \frac{2x^2-4}{x+1} dx$

6. $\int_2^3 \frac{y+1}{y-1} dy$

Completing the Square

Example: Complete the square to find $\int \frac{1}{x^2+4x+8} dx$.

Assigned Problems:

1. $\int \frac{1}{t^2-10t+32} dt$

2. $\int_{-3}^{-1} \frac{1}{\sqrt{7-x^2-6x}} dx$

3. $\int \frac{4}{t^2-4t+20} dt$

4. $\int_{-1}^0 \frac{1}{\sqrt{3-x^2-2x}} dx$

5. $\int \frac{3}{x^2-6x+18} dx$

Solutions:

Long Division:

Example $x - 2\ln(x^2 + 2) + C$

1. $x + 2\ln|x - 1| + C$

2. $\frac{1}{2}x^2 - \frac{1}{2}\ln(x^2 + 1) + C$

3. $-\frac{1}{2} + \ln 2$

4. $5x - 5\arctan x + C$

5. $x^2 - 2x - 2\ln|x + 1| + C$

6. $1 + 2\ln 2$

Completing the Square:

Example $\frac{1}{2}\tan^{-1}\frac{x+2}{2} + C$

1. $\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{t-5}{\sqrt{7}}\right) + C$

2. $\frac{\pi}{6}$

3. $\arctan\frac{t-2}{4} + C$

4. $\frac{\pi}{6}$

5. $\tan^{-1}\left(\frac{x-3}{3}\right) + C$