

Definite Integral \leftrightarrow Limit of a Riemann Sum

Concept: $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n (f(a + k\Delta x) \Delta x) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \underbrace{\left(f\left(a + k \frac{b-a}{n}\right) \right)}_{\text{right-hand heights}} \overbrace{\frac{b-a}{n}}^{\text{width}}$

where n is the number of subdivisions.

Example: $\int_1^4 x^5 dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(1 + k \cdot \frac{3}{n} \right)^5 \frac{3}{n} \right)$

Write each Riemann Sum as a definite integral and each definite integral as a right Riemann Sum.
Do not evaluate.

1. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(f\left(2 + k \cdot \frac{3}{n} \right) \frac{3}{n} \right)$

2. $\int_1^5 f(x) dx$

3. $\int_2^4 \sin x dx$

4. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\cos\left(0 + \frac{k\pi}{n} \right) \frac{\pi}{n} \right)$

5. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\sqrt{3 + \frac{2k}{n}} \cdot \frac{2}{n} \right)$

6. $\int_4^5 (x+1)^2 dx$

7. $\int_0^5 (x^2 + 1) dx$

8. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(3\left(\frac{k}{n} + 4 \right) + 2 \right) \frac{1}{n} \right)$

9. $\int_2^7 (2x^2 + 5x) dx$

10. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(\cos\left(\frac{k\pi}{3n} \right) \right) \frac{\pi}{3n} \right)$

Solutions:

1. $\int_2^5 f(x) dx$

2. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(f \left(1 + k \cdot \frac{4}{n} \right) \frac{4}{n} \right)$

3. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\sin \left(2 + \frac{2k}{n} \right) \frac{2}{n} \right)$

4. $\int_0^\pi \cos x dx$

5. $\int_3^5 \sqrt{x} dx$

6. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(4 + \frac{k}{n} + 1 \right)^2 \frac{1}{n} \right)$

7. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(\left(\frac{5k}{n} \right)^2 + 1 \right) \frac{5}{n} \right)$

8. $\int_4^5 (3x + 2) dx$

9. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(2 \left(2 + \frac{5k}{n} \right)^2 + 5 \left(2 + \frac{5k}{n} \right) \right) \frac{5}{n} \right)$

10. $\int_0^{\frac{\pi}{3}} \cos x dx$