

# UE20CS312 - Data Analytics

## Worksheet 2b :Multiple Linear Regression

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### Multiple Linear Regression

Multiple Linear Regression (mlr) is a statistical technique that uses several explanatory variables to predict the outcome of response variable. The goal of mlr is to model a **linear relationship** between explanatory (independent) variables and response (dependent) variables.

### Data Dictionary

The data required for this worksheet can be downloaded [from this GitHub Link](#). The data was obtained from [this](#) dataset from Kaggle. The dataset contains features of songs on Spotify collected using Spotify API. The features are as follows :

**-acousticness** : A confidence measure from 0.0 to 1.0 of whether the track is acoustic. 1.0 represents high confidence the track is acoustic.

**-danceability** : Danceability describes how suitable a track is for dancing based on a combination of musical elements including tempo, rhythm stability, beat strength, and overall regularity. A value of 0.0 is least danceable and 1.0 is most danceable.

**-duration\_ms** : The duration of track in milliseconds.

**-energy** : Energy is a measure from 0.0 to 1.0 and represents a perceptual measure of intensity and activity. Perceptual features contributing to this attribute include dynamic range, perceived loudness, timbre, onset rate, and general entropy.

**-instrumentalness** : Predicts whether a track contains no vocals. The closer the instrumentalness value is to 1.0, the greater likelihood the track contains no vocal content. Values above 0.5 are intended to represent instrumental tracks, but confidence is higher as the value approaches 1.0.

**-key** : The key the track is in. Integers map to pitches using standard Pitch Class notation

**-liveness** : Detects the presence of an audience in the recording. Higher liveness values represent an increased probability that the track was performed live. A value above 0.8 provides strong likelihood that the track is live.

**-loudness** : The overall loudness of a track in decibels (dB). Loudness values are averaged across the entire track and are useful for comparing relative loudness of tracks. Loudness is the quality of a sound that is the primary psychological correlate of physical strength (amplitude). Values typical range between -60 and 0 db.

**-mode** : Mode indicates the modality (major or minor) of a track, the type of scale from which its melodic content is derived. Major is represented by 1 and minor is 0.

**-speechiness** : Speechiness detects the presence of spoken words in a track. The more exclusively speech-like the recording (e.g. talk show, audio book, poetry), the closer to 1.0 the attribute value. Values above 0.66

describe tracks that are probably made entirely of spoken words. Values between 0.33 and 0.66 describe tracks that may contain both music and speech, either in sections or layered, including such cases as rap music. Values below 0.33 most likely represent music and other non-speech-like tracks.

**-tempo** : The overall estimated tempo of a track in beats per minute (BPM). In musical terminology, tempo is the speed or pace of a given piece and derives directly from the average beat duration.

**-time\_signature** : An estimated overall time signature of a track. The time signature (meter) is a notational convention to specify how many beats are in each bar (or measure).

**-valence** : A measure from 0.0 to 1.0 describing the musical positiveness conveyed by a track. Tracks with high valence sound more positive (e.g. happy, cheerful, euphoric), while tracks with low valence sound more negative (e.g. sad, depressed, angry).

Throughout the course of this worksheet , our response variable is energy. We shall try and apply the concepts learnt in class to predict the energy of a song using the other features of a song.

## Libraries used

-tidyverse

-corrplot

-olsrr : [documentation](#)

## Points

The problems for this worksheet is for a total of 10 points and the weightage is not uniformly distributed.

- *Problem 1* : 0.5 points
- *Problem 2* : 2 points
- *Problem 3* : 2 points
- *Problem 4* : 1 point
- *Problem 5* : 1.5 points
- *Problem 6* : 1 point
- *Problem 7* : 2 points

## Loading the Dataset

After downloading the dataset and ensuring the working directory is right , we read the csv into the dataframe.

```
library(tidyverse)
spotify_df <- read_csv('spotify.csv')
```

## Problem-1 (0.5 Points)

Check for missing values in the dataset and normalize the dataset.

```
colSums((is.na(spotify_df)))#displays the number of missing values in each column.
```

```
##      danceability      energy      key      loudness
##           0           0           0           0
##      mode      speechiness      acousticness      instrumentality
##           0           0           0           0
##      liveness      valence      tempo      duration_ms
##           0           0           0           0
##      time_signature
##           0
```

```
#Since there are No Missing Values, We move on to Normalizing the dataset.
```

```
#Normalizing using Min-Max Scaling to Normalize all the data from 0 to 1  
library(caret)
```

```
## Loading required package: lattice
```

```
##
```

```
## Attaching package: 'caret'
```

```
## The following object is masked from 'package:purrr':
```

```
##
```

```
## lift
```

```
normalized <- preprocess(as.data.frame(spotify_df), method=c("range"))
```

```
spotify_df <- predict(normalized, as.data.frame(spotify_df))
```

```
head(spotify_df)
```

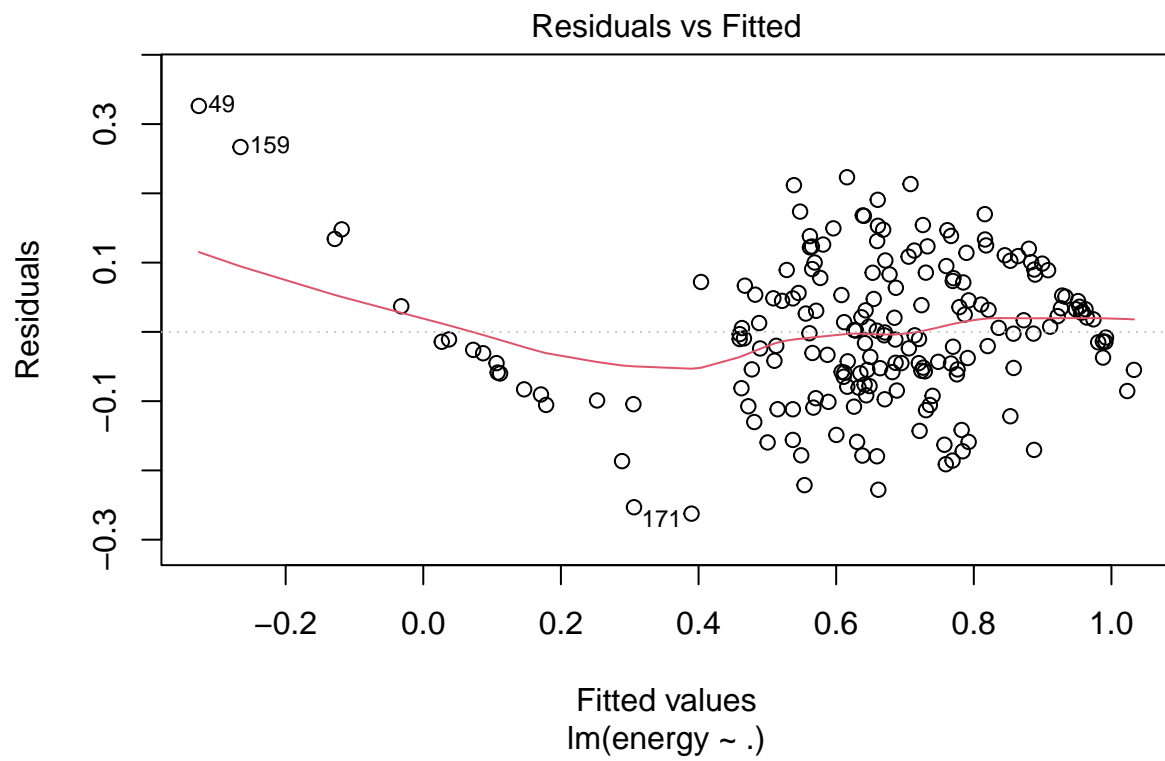
```
## danceability energy key loudness mode speechiness acousticness  
## 1 0.8247549 0.62560386 0.63636364 0.8890920 0 0.03885201 0.45326466  
## 2 0.7745098 0.70511272 0.90909091 0.8593613 0 0.54314721 0.20703275  
## 3 0.1605392 0.01258052 0.09090909 0.3690169 1 0.02752831 0.99698492  
## 4 0.7254902 0.73832528 0.27272727 0.8833312 0 0.05993752 0.43316409  
## 5 0.8051471 0.57326892 0.09090909 0.8702567 1 0.37914877 0.14572602  
## 6 0.7941176 0.63365539 0.72727273 0.8978334 1 0.18976962 0.04060007  
## instrumentalness liveness valence tempo duration_ms time_signature  
## 1 7.574819e-04 0.11151859 0.627394940 0.2986443 0.3932821 0.75  
## 2 0.000000e+00 0.09684947 0.512014396 0.7605056 0.2940693 0.75  
## 3 9.256966e-01 0.11485248 0.003069758 0.1261836 0.3629418 0.75  
## 4 1.217750e-06 0.14985831 0.578702234 0.2476870 0.2278801 0.75  
## 5 0.000000e+00 0.07034506 0.647507145 0.7921078 0.1768309 0.75  
## 6 0.000000e+00 0.09684947 0.838043823 0.6739248 0.2540198 0.75
```

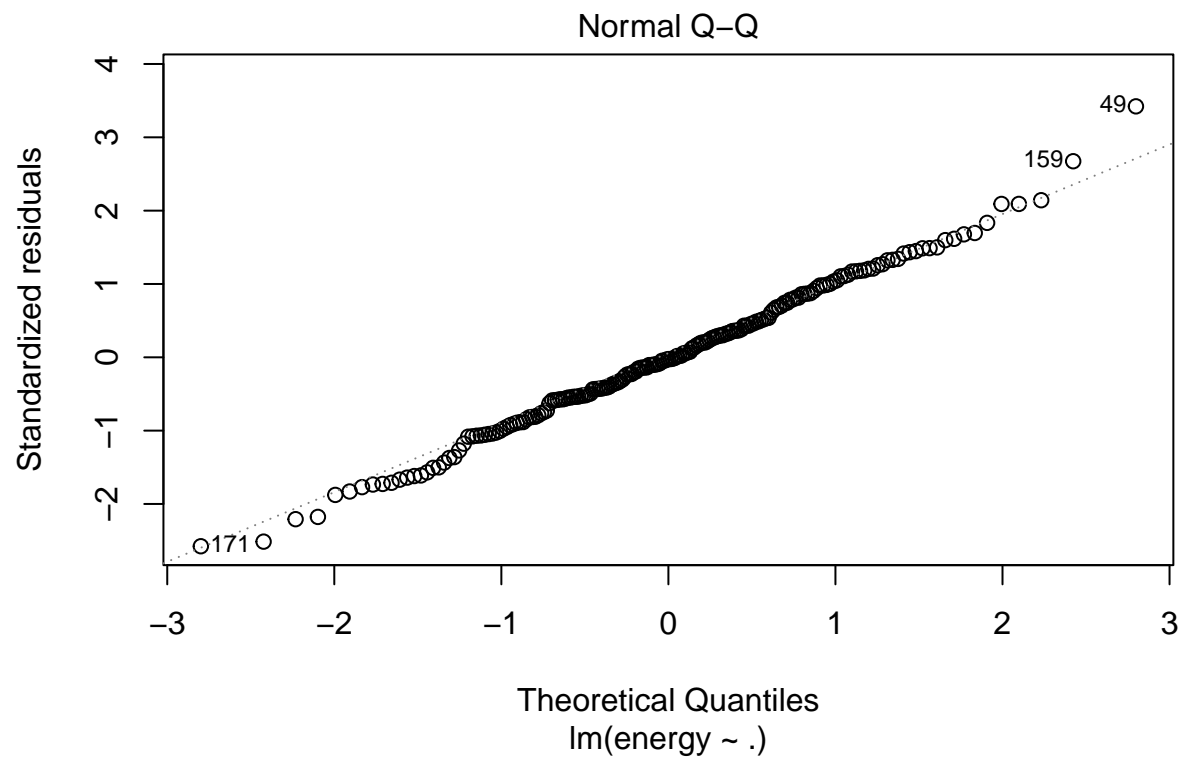
## Problem-2 (2 Points)

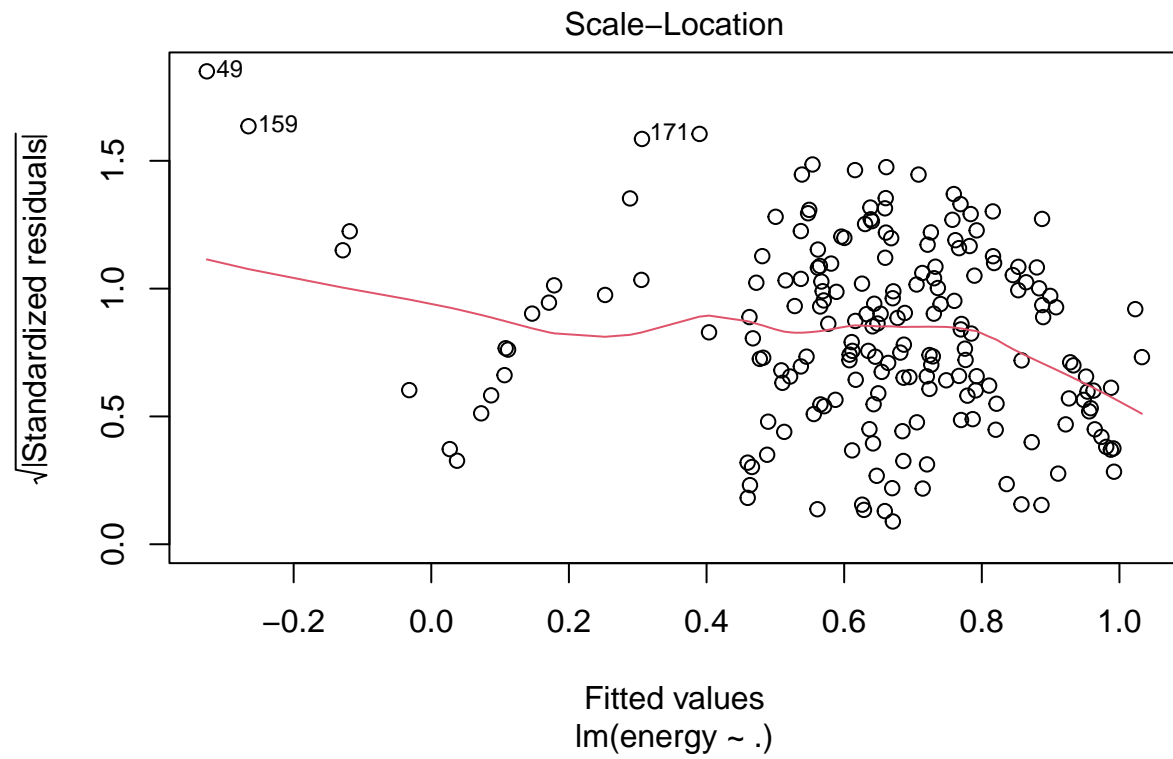
Fit a linear model to predict the *energy* rating using *all* other attributes. Get the summary of the model and explain the results in detail. [Hint : Use the `lm()` function. [Click here](#) To get the documentation of the same.]

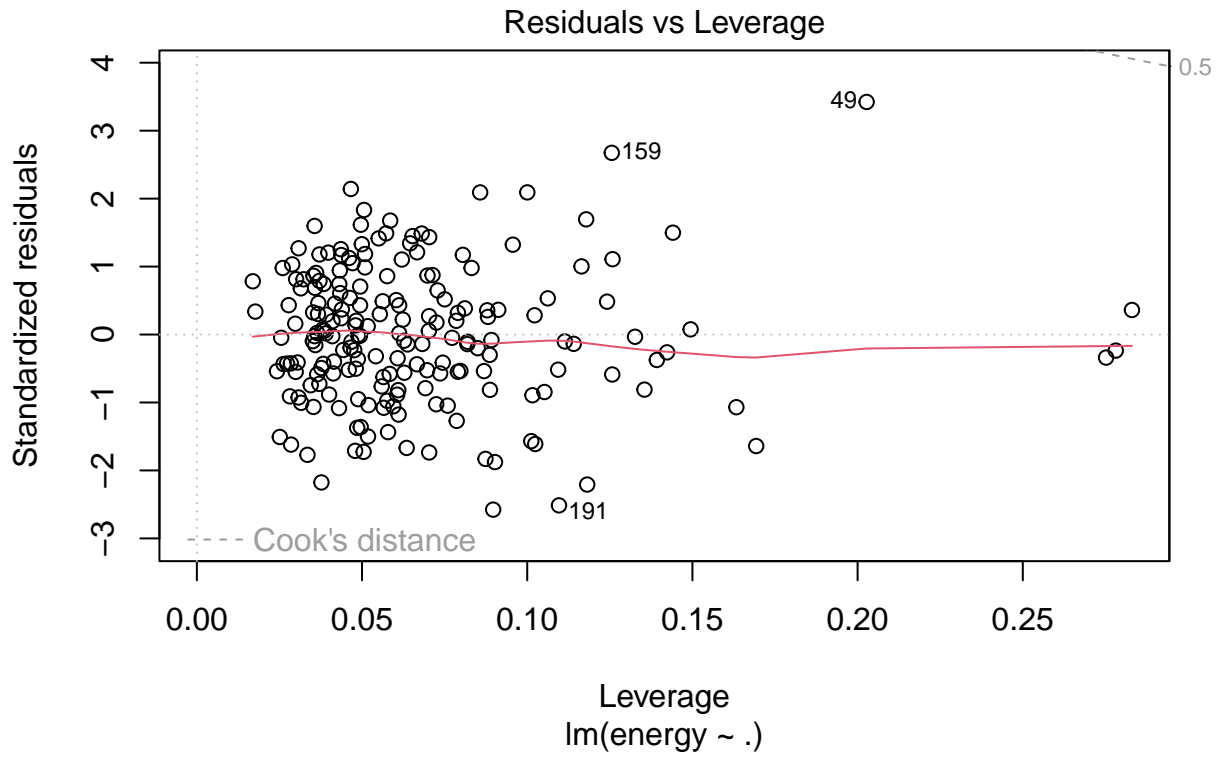
```
lm_pred <- lm(formula=energy ~ ., data=spotify_df)
```

```
plot(lm_pred)
```









```
summary(lm_pred)
```

```
##
## Call:
## lm(formula = energy ~ ., data = spotify_df)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-0.26238	-0.05992	-0.00255	0.07276	0.32616

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.17513	0.10422	-1.680	0.09459 .
danceability	-0.27128	0.05478	-4.952	1.67e-06 ***
key	0.04190	0.02537	1.652	0.10030
loudness	1.12359	0.07305	15.381	< 2e-16 ***
mode	-0.02511	0.01589	-1.580	0.11582
speechiness	0.02627	0.03918	0.670	0.50343
acousticness	-0.27894	0.03358	-8.306	2.21e-14 ***
instrumentalness	0.10937	0.04086	2.677	0.00811 **
liveness	0.02970	0.04594	0.646	0.51880
valence	0.18905	0.03588	5.269	3.85e-07 ***
tempo	-0.02676	0.03681	-0.727	0.46817
duration_ms	-0.03911	0.06926	-0.565	0.57298
time_signature	0.05589	0.07471	0.748	0.45535

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1067 on 182 degrees of freedom
## Multiple R-squared:  0.844, Adjusted R-squared:  0.8338
## F-statistic: 82.08 on 12 and 182 DF,  p-value: < 2.2e-16
```

### Analysis:

The min value of the residuals is -.26070 and the max value is 0.32407. The median is -0.00253. The residual standard error is 0.106 on 182 degrees of freedom. Multiple R-squared value is 0.844. Adjusted R-squared value is 0.8338. P-value is less than 2.2e-16.

Since the F-statistic is 82.08.[high], we reject the Null Hypothesis

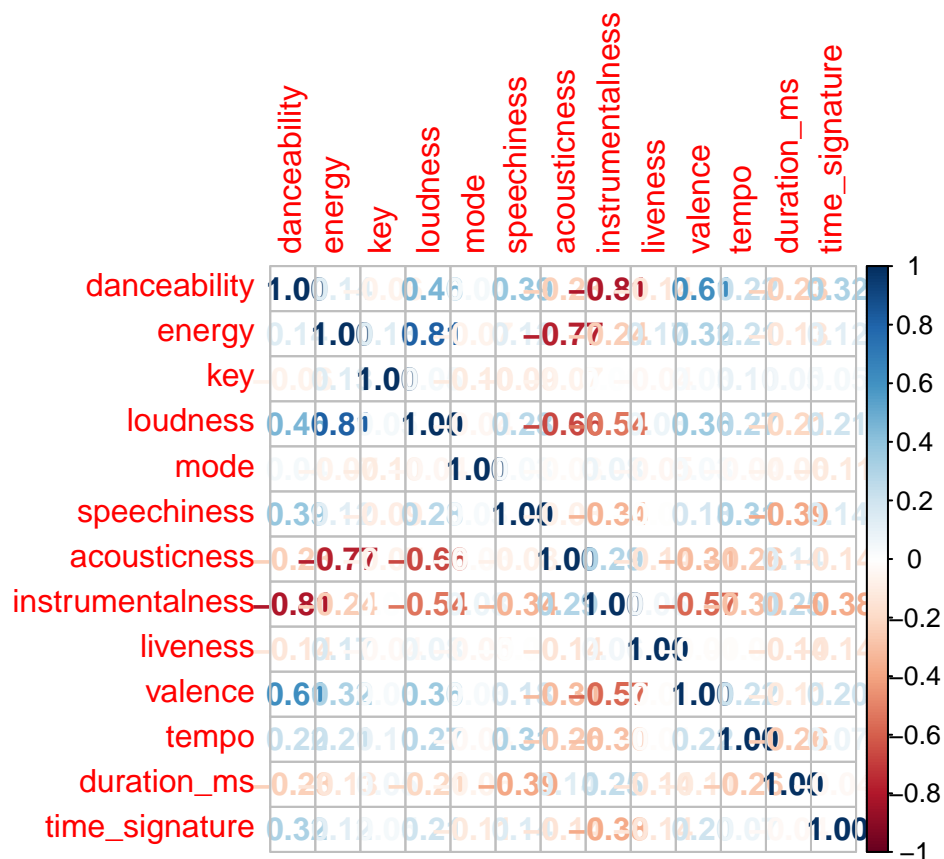
### Problem-3 (2 points)

With the help of a correlogram and scatter plots, choose the features you think are important and model an mlr. Justify your choice and explain the new findings.

```
library(corrplot)
```

```
## corrplot 0.92 loaded
```

```
data<-cor(spotify_df)
#finding correlogram
corrplot(data,method="number")
```

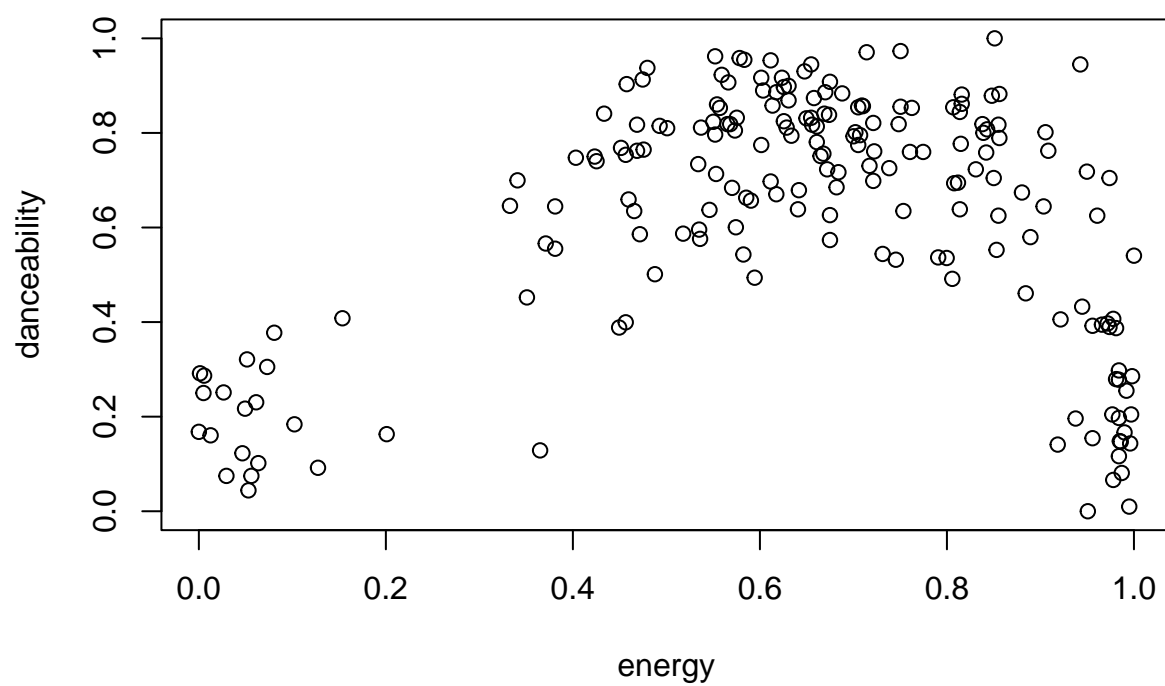


```
#scatter plots
```

```
plot(x=spotify_df$energy , y=spotify_df$danceability,xlab="energy",ylab = "danceability",main="energy vs danceability")
```

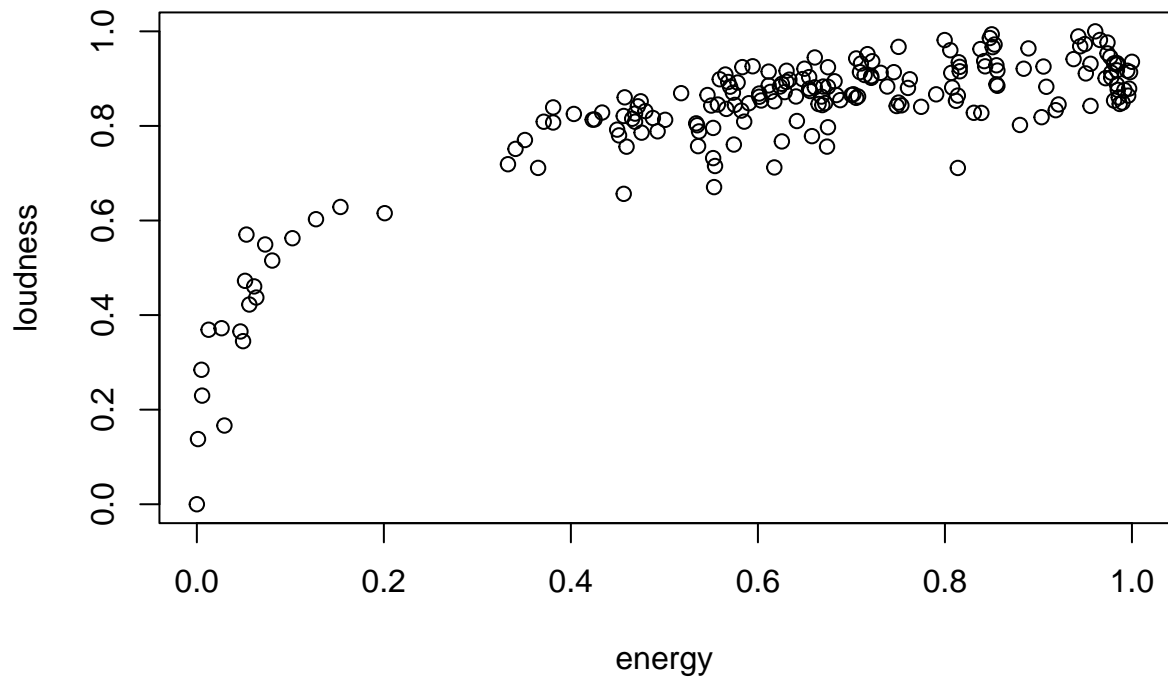


**energy vs danceability**



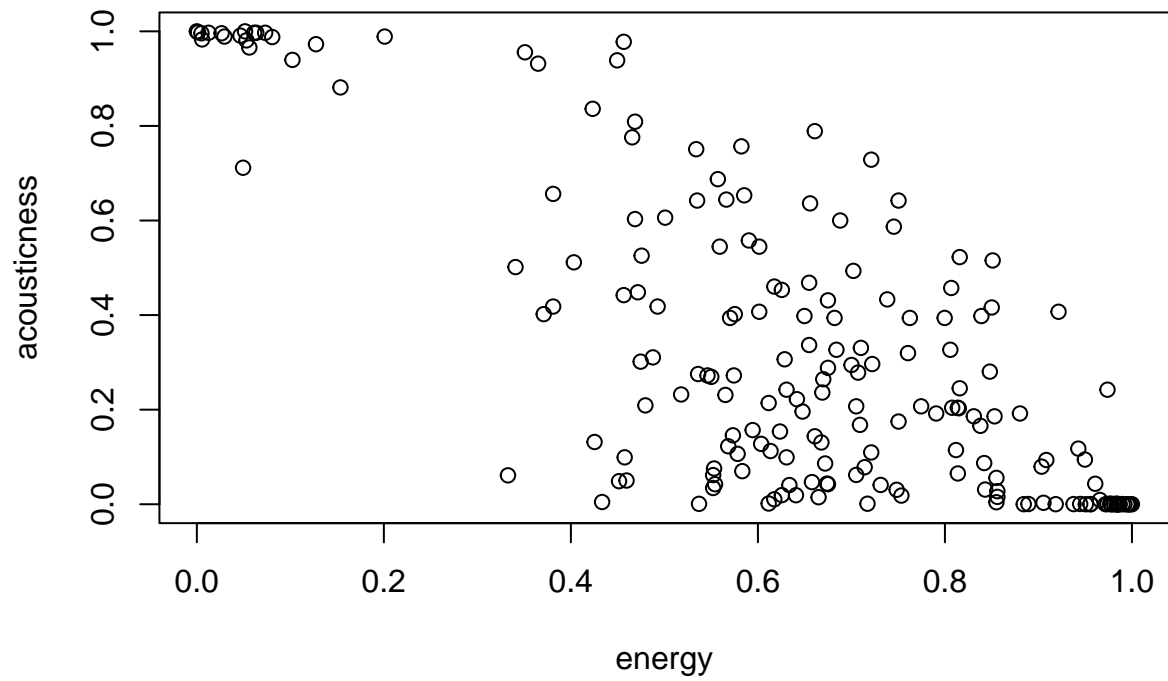
```
plot(x=spotify_df$energy , y=spotify_df$loudness, xlab="energy", ylab="loudness", main="energy vs loudness")
```

**energy vs loudness**



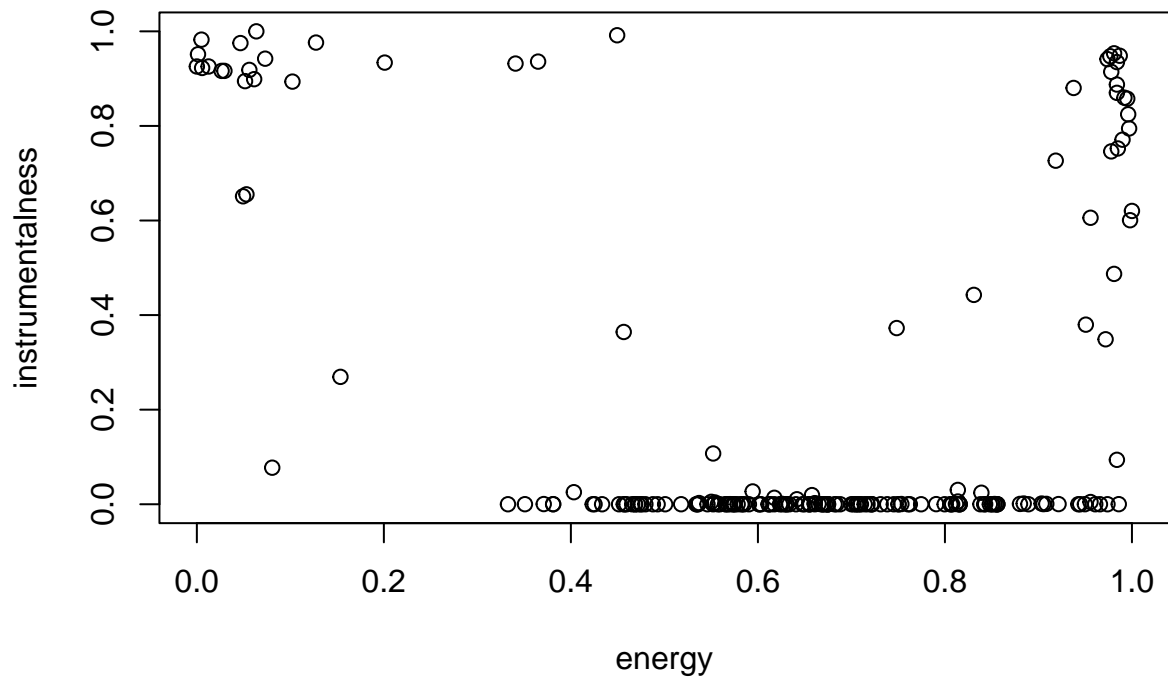
```
plot(x=spotify_df$energy , y=spotify_df$acousticness, xlab="energy", ylab="acousticness", main="energy vs acousticness")
```

**energy vs acousticness**



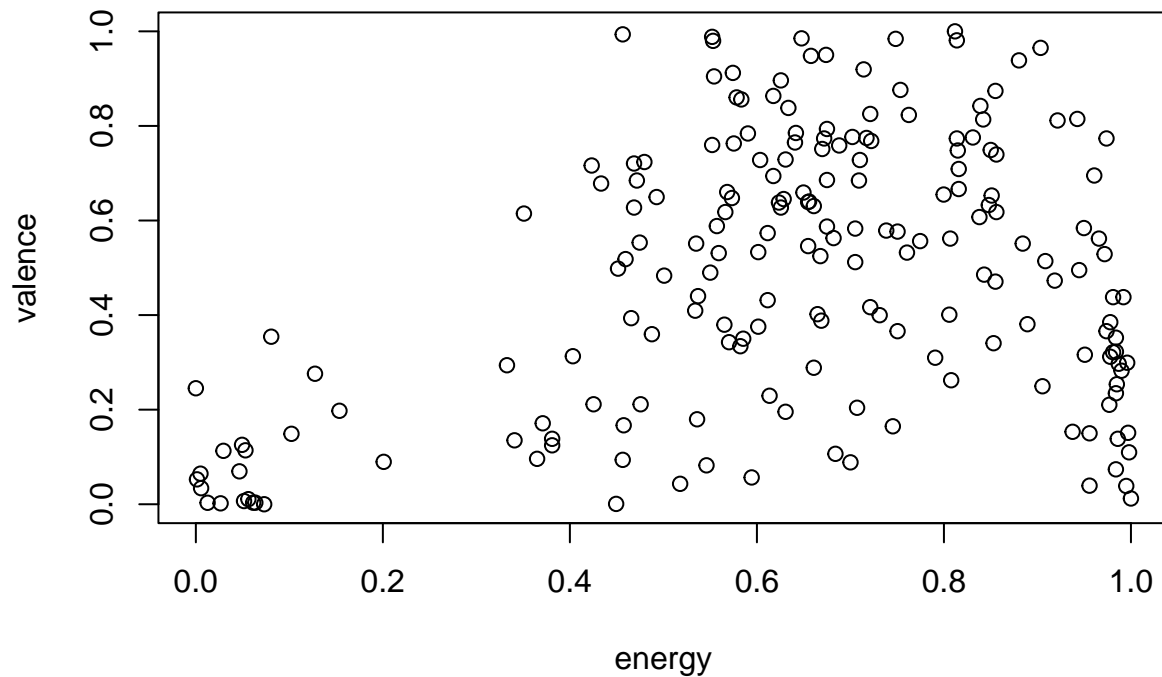
```
plot(x=spotify_df$energy , y=spotify_df$instrumentalness, xlab="energy",ylab="instrumentalness",main="energy vs instrumentalness")
```

## energy vs instrumentalness



```
plot(x=spotify_df$energy , y=spotify_df$valence, xlab="energy", ylab="valence", main=" energy vs valence")
```

## energy vs valence



```
#Finding the Relative Importance of variables  
library(relaimpo)
```

```
## Loading required package: MASS  
##  
## Attaching package: 'MASS'  
## The following object is masked from 'package:dplyr':  
##  
##   select  
## Loading required package: boot  
##  
## Attaching package: 'boot'  
## The following object is masked from 'package:lattice':  
##  
##   melanoma  
## Loading required package: survey  
## Loading required package: grid  
## Loading required package: Matrix  
##  
## Attaching package: 'Matrix'  
## The following objects are masked from 'package:tidyr':
```

```
##
##      expand, pack, unpack
## Loading required package: survival
##
## Attaching package: 'survival'
## The following object is masked from 'package:boot':
##
##      aml
## The following object is masked from 'package:caret':
##
##      cluster
##
## Attaching package: 'survey'
## The following object is masked from 'package:graphics':
##
##      dotchart
## Loading required package: mitools
## This is the global version of package relaimpo.
## If you are a non-US user, a version with the interesting additional metric pmvd is available
## from Ulrike Groempings web site at prof.beuth-hochschule.de/groemping.
regressor <- lm(energy ~ . , data = spotify_df) # fit lm() model
relImportance <- calc.relimp(regressor, type = "lmg", rela = TRUE)
sort(relImportance$lmg, decreasing=TRUE) # relative importance

##      loudness      acousticness      valence      danceability
##      0.488260623      0.355156971      0.047325139      0.028356838
## instrumentalness      liveness      tempo      key
##      0.027574268      0.013432729      0.012597091      0.009575742
##      speechiness      duration_ms      time_signature      mode
##      0.005128007      0.004408885      0.004255113      0.003928594

#From the RelImportance we can find the Importance of each variable and select that variable that contr
reduced_data<-lm(energy ~ danceability+instrumentalness+loudness+acousticness+valence, data=spotify_df)
reduced_data

##
## Call:
## lm(formula = energy ~ danceability + instrumentalness + loudness +
##      acousticness + valence, data = spotify_df)
##
## Coefficients:
##      (Intercept)      danceability instrumentalness      loudness
##      -0.13939      -0.28784      0.08986      1.14236
##      acousticness      valence
##      -0.27539      0.18718
```

### Reasoning:

After finding the Relative Importance of each column. We selected that Columns that contributed the most.

## Problem-4 (1 Point)

Conduct a partial F-test to determine if the attributes not chosen by you in *Problem-3* are significant to predict the energy. What are the null and alternate hypotheses? [ *Hint* : Use the anova function between models created in *Problem-2* and *Problem-3*]

```
anova(reduced_data, lm_pred)

## Analysis of Variance Table
##
## Model 1: energy ~ danceability + instrumentalness + loudness + acousticness +
##   valence
## Model 2: energy ~ danceability + key + loudness + mode + speechiness +
##   acousticness + instrumentalness + liveness + valence + tempo +
##   duration_ms + time_signature
##   Res.Df    RSS Df Sum of Sq    F Pr(>F)
## 1      189 2.1622
## 2      182 2.0733  7   0.08892 1.1151 0.3554
```

### Analysis:

Null Hypothesis: The attributes not chosen are not Significant in predicting the energy value

Alternate Hypothesis : The attributes not chosen are Significant in predicting the energy value.

Since the P-value is 0.3554, which is not less than ( $\alpha=0.05$ ).

**we fail to reject the null hypothesis.**

## Problem-5 (1.5 Points)

AIC - Akaike Information Criterion is used to compare different models and determine the best fit for the data. The best-fit model according to AIC is the one that explains greatest amount of variation using the fewest number of attributes. Check [this](#) resource to learn more about AIC.

Build a model based on AIC using Stepwise AIC regression. Elucidate your observations from the new model. ( *Hint* : Use an appropriate function in [olsrr](#) package.)

```
library(olsrr)

##
## Attaching package: 'olsrr'
##
## The following object is masked from 'package:MASS':
##
##   cement
##
## The following object is masked from 'package:datasets':
##
##   rivers

aic_stepwise <- lm(energy ~ ., data=spotify_df)
ols_step_both_aic(aic_stepwise)

##
##
##                               Stepwise Summary
## -----
## Variable           Method      AIC      RSS      Sum Sq      R-Sq      Adj. R-Sq
## -----
## loudness           addition    -175.784    4.495      8.799      0.66189      0.66014
```

```
## acousticness      addition    -239.021    3.217    10.077    0.75803    0.75551
## danceability      addition    -285.619    2.507    10.787    0.81141    0.80844
## valence           addition    -307.057    2.223    11.071    0.83276    0.82924
## instrumentalness  addition    -310.477    2.162    11.131    0.83735    0.83305
## mode              addition    -311.706    2.127    11.167    0.84002    0.83491
## key               addition    -312.104    2.101    11.193    0.84198    0.83606
## -----
```

```
summary(aic_stepwise)
```

```
##
## Call:
## lm(formula = energy ~ ., data = spotify_df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.26238 -0.05992 -0.00255  0.07276  0.32616
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -0.17513    0.10422  -1.680  0.09459 .
## danceability  -0.27128    0.05478  -4.952 1.67e-06 ***
## key           0.04190    0.02537   1.652  0.10030
## loudness      1.12359    0.07305  15.381 < 2e-16 ***
## mode         -0.02511    0.01589  -1.580  0.11582
## speechiness   0.02627    0.03918   0.670  0.50343
## acousticness -0.27894    0.03358  -8.306 2.21e-14 ***
## instrumentalness 0.10937    0.04086   2.677  0.00811 **
## liveness      0.02970    0.04594   0.646  0.51880
## valence       0.18905    0.03588   5.269 3.85e-07 ***
## tempo        -0.02676    0.03681  -0.727  0.46817
## duration_ms  -0.03911    0.06926  -0.565  0.57298
## time_signature 0.05589    0.07471   0.748  0.45535
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1067 on 182 degrees of freedom
## Multiple R-squared:  0.844, Adjusted R-squared:  0.8338
## F-statistic: 82.08 on 12 and 182 DF,  p-value: < 2.2e-16
```

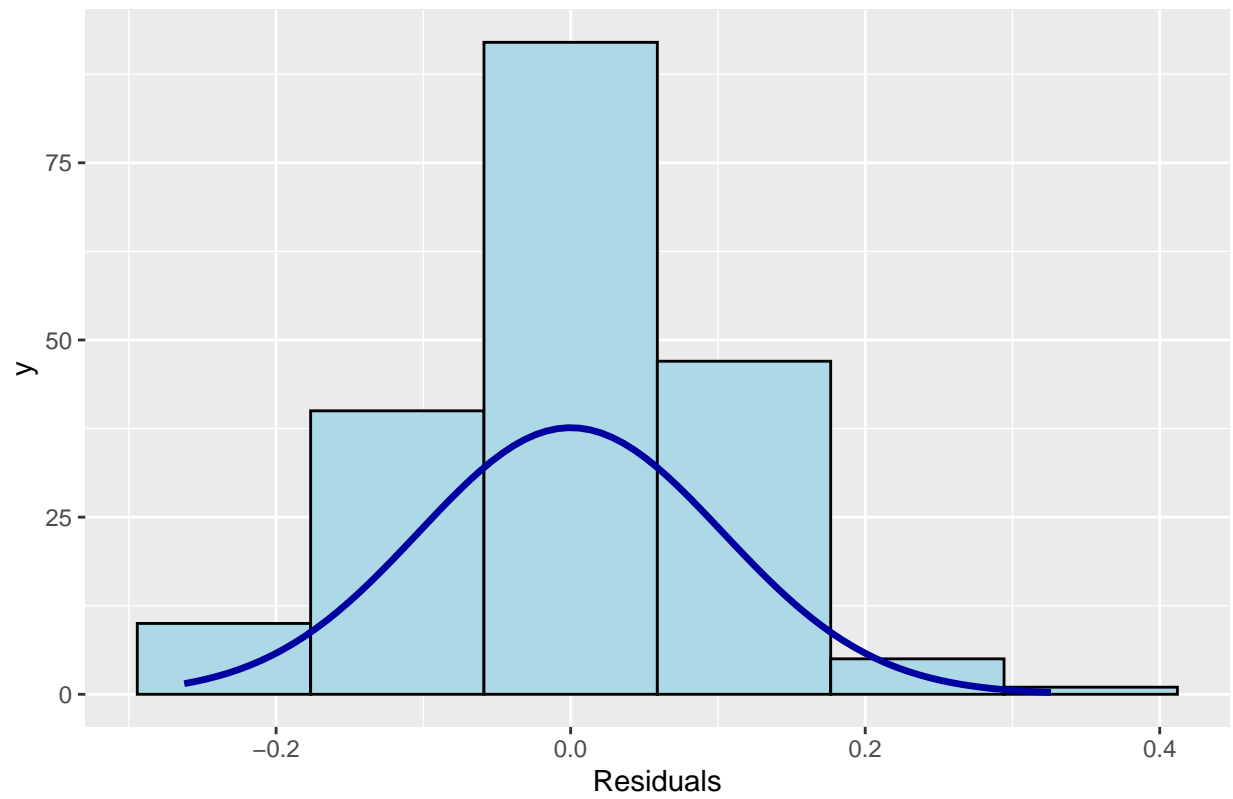
## Problem-6 (1 Point)

Plot the residuals of the models built till now and comment on it satisfying the assumptions of mlr.

```
ols_plot_resid_hist(lm_pred)
```

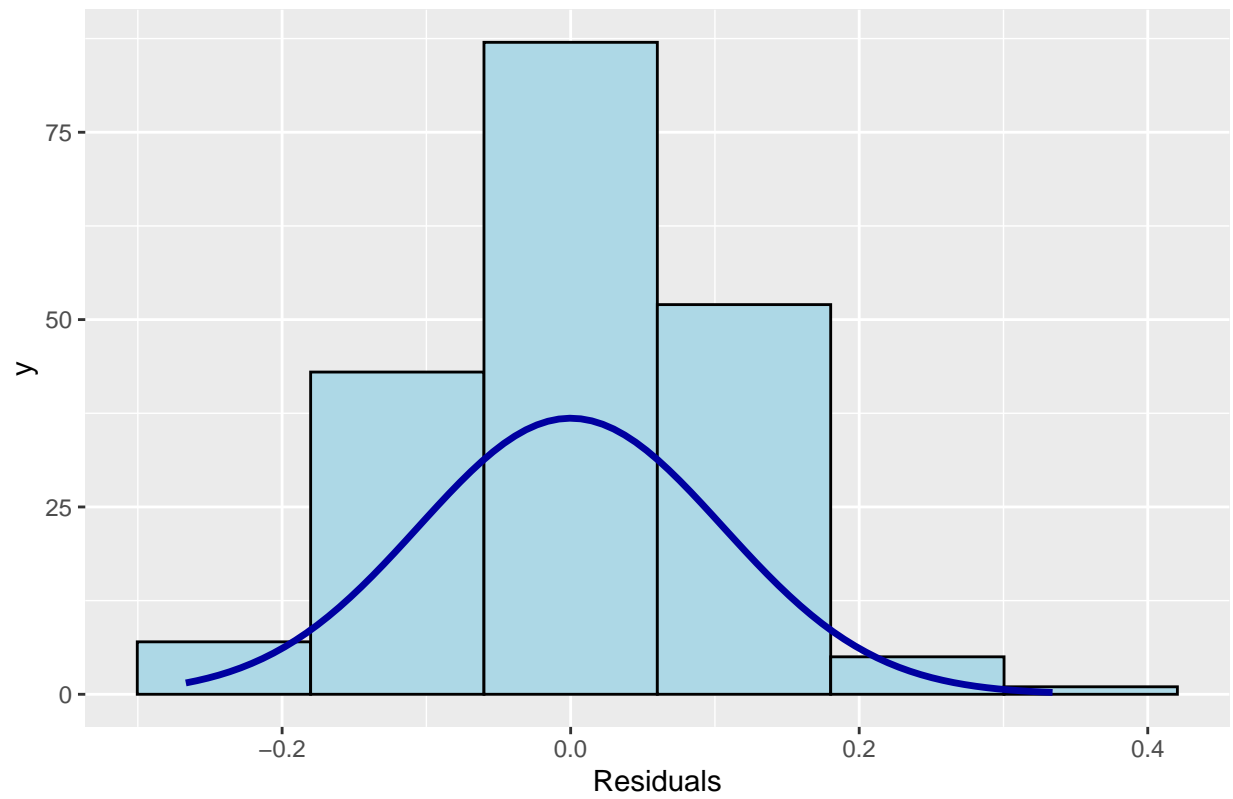


Residual Histogram



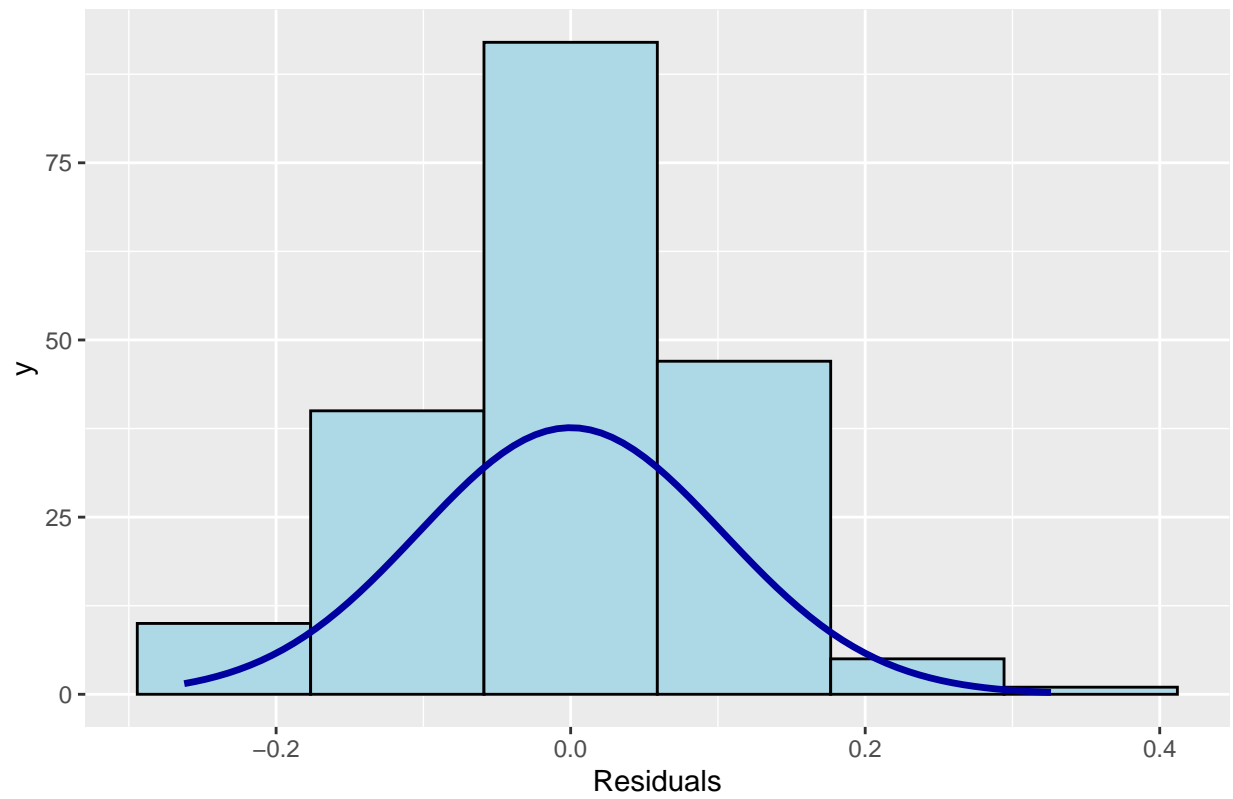
```
ols_plot_resid_hist(reduced_data)
```

Residual Histogram

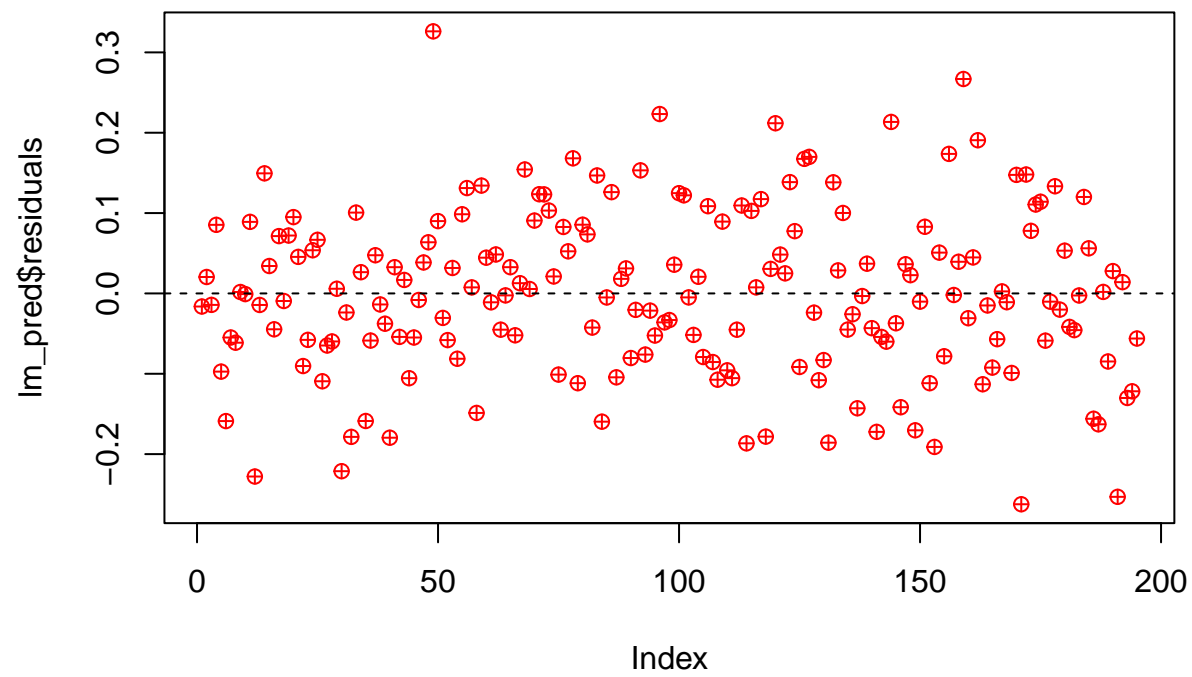


```
ols_plot_resid_hist(aic_stepwise)
```

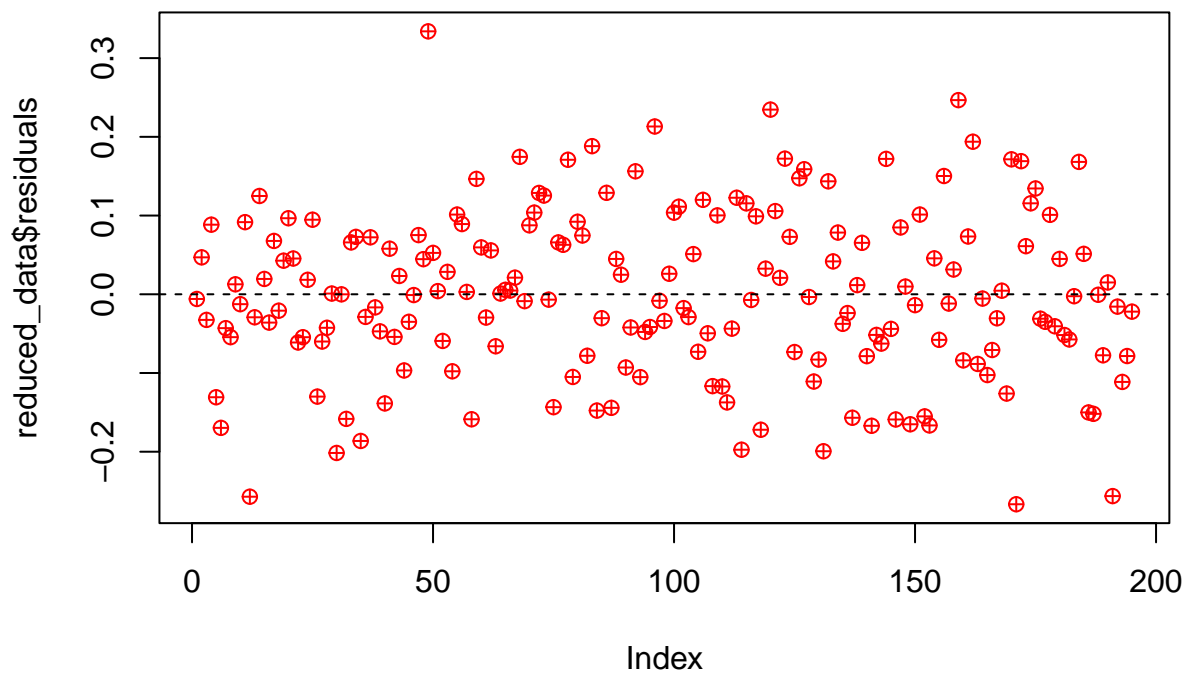
Residual Histogram



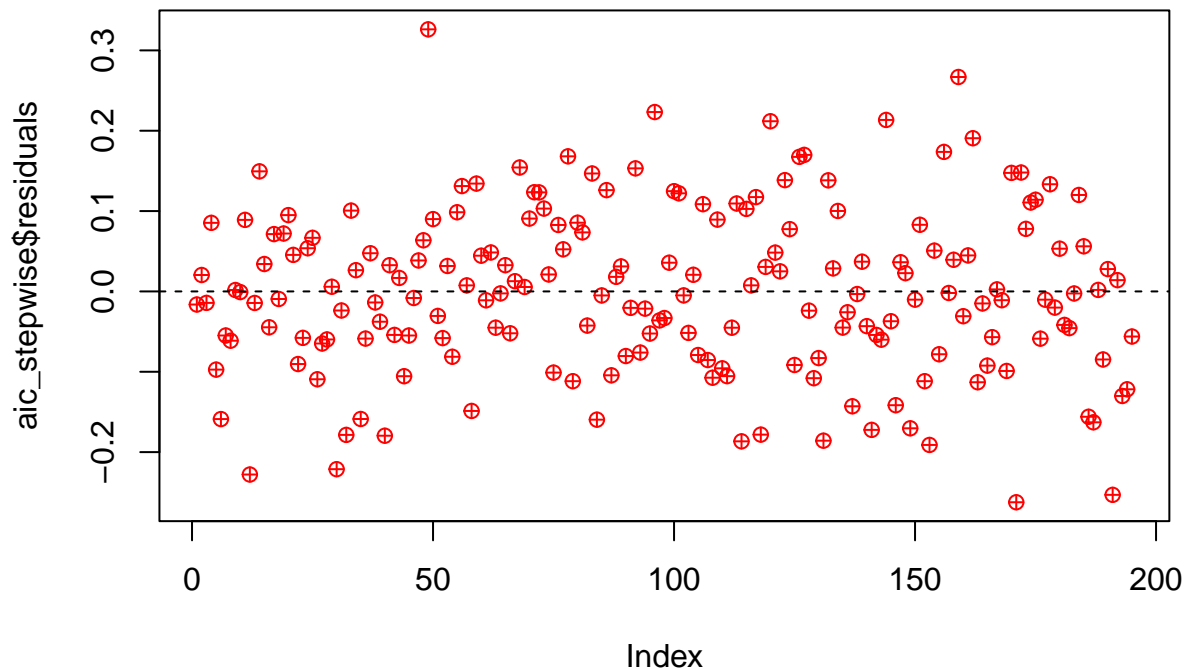
```
plot(lm_pred$residuals,pch =10 ,col="red")  
abline(h=0,lty=2)
```



```
plot(reduced_data$residuals,pch =10 ,col="red")  
abline(h=0,lty=2)
```



```
plot(aic_stepwise$residuals,pch =10 ,col="red")
abline(h=0,lty=2)
```



### Problem-7 (2 Points)

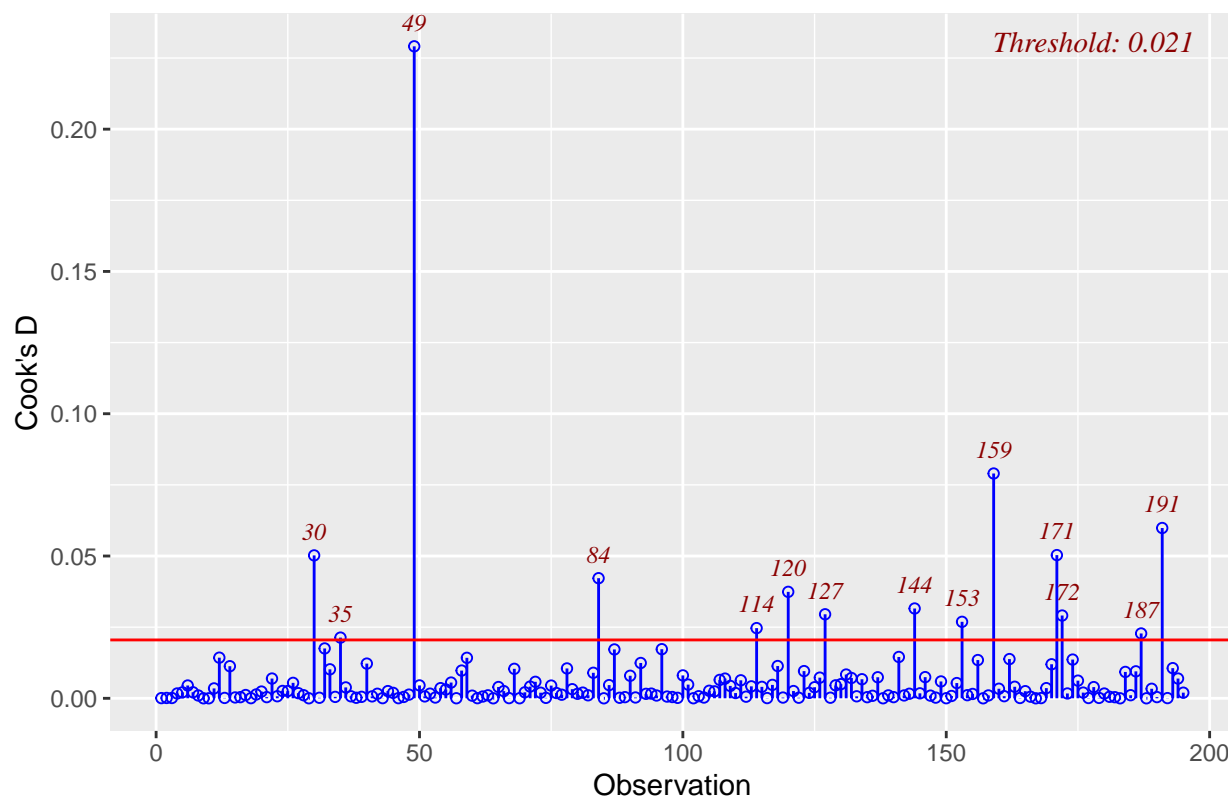
For the model built in **Problem-2**, determine the presence of multicollinearity using VIF. Determine if there are outliers in the data using [Cook's Distance](#). If you find any, remove the outliers and fit the model for **Problem-2** and see if the fit improves. [ *Hint* : All the relevant functions can be found in *olsrr* package. An observation can be termed as an outlier if it has a Cook's distance of more than  $4/n$  where  $n$  is the number of records.]

```
#determining multicollinearity using VIF
ols_vif_tol(lm_pred)
```

```
##           Variables Tolerance      VIF
## 1  danceability 0.2776703 3.601393
## 2           key 0.9467671 1.056226
## 3    loudness 0.4119898 2.427245
## 4           mode 0.9308390 1.074300
## 5  speechiness 0.6921660 1.444740
## 6  acousticness 0.5009458 1.996224
## 7 instrumentality 0.2755568 3.629016
## 8    liveness 0.8914397 1.121781
## 9       valence 0.5680642 1.760364
## 10        tempo 0.7892957 1.266952
## 11 duration_ms 0.7855373 1.273014
## 12 time_signature 0.8262918 1.210226
```

```
#representing Outliers
cookd <- ols_plot_cooksd_chart(lm_pred);
```

## Cook's D Chart



*#Yes there are outliers in the dataset.*

*#Removing Outliers*

```
fit<- lm(energy ~ ., data = spotify_df);
spotify_df$cooks_d <- cooks.distance(fit); # Defining outliers based on 4/n criteria
#Creating a new column called outlier which tells if that row of data is an outlier or not.
spotify_df$outlier <- ifelse(spotify_df$cooks_d < 4/nrow(spotify_df), "keep", "delete");
#Now we are trying to delete all the rows that contain the data "outlier = delete "
spotify_df=spotify_df[!grepl("delete", spotify_df$outlier),];
head(spotify_df)
```

```
##  danceability    energy      key  loudness mode  speechiness  acousticness
## 1    0.8247549  0.62560386 0.63636364 0.8890920  0      0.03885201   0.45326466
## 2    0.7745098  0.70511272 0.90909091 0.8593613  0      0.54314721   0.20703275
## 3    0.1605392  0.01258052 0.09090909 0.3690169  1      0.02752831   0.99698492
## 4    0.7254902  0.73832528 0.27272727 0.8833312  0      0.05993752   0.43316409
## 5    0.8051471  0.57326892 0.09090909 0.8702567  1      0.37914877   0.14572602
## 6    0.7941176  0.63365539 0.72727273 0.8978334  1      0.18976962   0.04060007
##  instrumentalness  liveness  valence  tempo  duration_ms  time_signature
## 1    7.574819e-04  0.11151859 0.627394940 0.2986443  0.3932821      0.75
## 2    0.000000e+00  0.09684947 0.512014396 0.7605056  0.2940693      0.75
## 3    9.256966e-01  0.11485248 0.003069758 0.1261836  0.3629418      0.75
## 4    1.217750e-06  0.14985831 0.578702234 0.2476870  0.2278801      0.75
## 5    0.000000e+00  0.07034506 0.647507145 0.7921078  0.1768309      0.75
## 6    0.000000e+00  0.09684947 0.838043823 0.6739248  0.2540198      0.75
##      cooks_d outlier
```

```
## 1 6.922961e-05 keep
## 2 1.258874e-04 keep
## 3 1.080694e-04 keep
## 4 1.695989e-03 keep
## 5 2.096273e-03 keep
## 6 4.502237e-03 keep
```

*#Now this spotify\_df is free of outliers..*

```
summary(spotify_df)
```

```
##  danceability      energy      key      loudness
##  Min.   :0.0000    Min.   :0.004952    Min.   :0.0000    Min.   :0.2297
##  1st Qu.:0.4914    1st Qu.:0.552134    1st Qu.:0.1818    1st Qu.:0.8137
##  Median :0.7230    Median :0.669887    Median :0.5455    Median :0.8668
##  Mean   :0.6385    Mean   :0.660475    Mean   :0.5073    Mean   :0.8363
##  3rd Qu.:0.8248    3rd Qu.:0.842995    3rd Qu.:0.7273    3rd Qu.:0.9137
##  Max.   :1.0000    Max.   :1.000000    Max.   :1.0000    Max.   :1.0000
##    mode      speechiness    acousticness    instrumentalness
##  Min.   :0.0000    Min.   :0.000000    Min.   :0.0000    Min.   :0.0000000
##  1st Qu.:0.0000    1st Qu.:0.05857    1st Qu.:0.0406    1st Qu.:0.0000000
##  Median :1.0000    Median :0.14096    Median :0.2070    Median :0.0000043
##  Mean   :0.5359    Mean   :0.23576    Mean   :0.3001    Mean   :0.1791565
##  3rd Qu.:1.0000    3rd Qu.:0.39672    3rd Qu.:0.4573    3rd Qu.:0.0254902
##  Max.   :1.0000    Max.   :1.00000    Max.   :1.0000    Max.   :1.0000000
##    liveness      valence      tempo      duration_ms
##  Min.   :0.00000    Min.   :0.0000    Min.   :0.0000    Min.   :0.0000
##  1st Qu.:0.08235    1st Qu.:0.2887    1st Qu.:0.3408    1st Qu.:0.1768
##  Median :0.11985    Median :0.5311    Median :0.5245    Median :0.2230
##  Mean   :0.18734    Mean   :0.4989    Mean   :0.5104    Mean   :0.2380
##  3rd Qu.:0.23654    3rd Qu.:0.7280    3rd Qu.:0.6901    3rd Qu.:0.2851
##  Max.   :1.00000    Max.   :1.0000    Max.   :1.0000    Max.   :1.0000
##  time_signature    cooksd      outlier
##  Min.   :0.000    Min.   :2.490e-07    Length:181
##  1st Qu.:0.750    1st Qu.:4.061e-04    Class :character
##  Median :0.750    Median :1.675e-03    Mode  :character
##  Mean   :0.732    Mean   :3.365e-03
##  3rd Qu.:0.750    3rd Qu.:4.634e-03
##  Max.   :1.000    Max.   :1.752e-02
```