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CSE 6363 - Machine Learning

Homework-1 Spring 2019

SUNDESH RAJ
1001633297

①

Q1 :- MLE and MAP

Given:- for a poisson process probability for the first event to occur is described by an exponential distribution

$$p_{\lambda}(x) = \lambda e^{-\lambda x}$$

① MLE for λ of the model given $D = \{k_1, \dots, k_n\}$

i) likelihood function:-

$$L(\lambda; k_1, \dots, k_n) = \lambda^n e^{-\lambda \sum_{i=1}^n k_i}$$

Proof:- $L(\lambda; k_1, \dots, k_n) = \prod_{i=1}^n f(k_i; \lambda)$

$$= \prod_{i=1}^n \lambda e^{-\lambda k_i}$$

$$= \underline{\underline{\lambda^n e^{-\lambda \sum_{i=1}^n k_i}}}$$

ii) log likelihood function:-

$$l(\lambda; k_1, \dots, k_n) = \ln(L(\lambda; k_1, \dots, k_n))$$

$$= \ln\left(\lambda^n \exp\left(-\lambda \sum_{i=1}^n k_i\right)\right)$$

$$\Rightarrow \ln(\lambda^n) + \ln\left(\exp\left(-\lambda \sum_{i=1}^n k_i\right)\right)$$

$$= n \ln(\lambda) - \lambda \sum_{i=1}^n k_i //$$

(iii) Maximum likelihood estimator

$$\hat{\lambda} = \operatorname{argmax}_{\lambda} l(\lambda; k_1, \dots, k_n)$$

$$\Rightarrow \frac{d}{d\lambda} l(\lambda; k_1, \dots, k_n) = 0 \quad \text{—— first order condition for a maximum}$$

$$\begin{aligned} \Rightarrow \frac{d}{d\lambda} l(\lambda; k_1, \dots, k_n) &= \frac{d}{d\lambda} \left(n \ln(\lambda) - \lambda \sum_{i=1}^n k_i \right) \\ &= \frac{n}{\lambda} - \sum_{i=1}^n k_i \end{aligned}$$

By setting the value equal to 0, we get

$$\boxed{\lambda = \frac{n}{\sum_{i=1}^n k_i}}$$

(b) given data $D = \{1.5, 3, 2.5, 2.75, 2.9, 3\}$

$$\lambda = \frac{n}{\sum_{i=1}^n k_i}$$

$$\Rightarrow \frac{6}{1.5 + 3 + 2.5 + 2.75 + 2.9 + 3}$$

$$\Rightarrow \underline{\underline{0.38338}}$$

$$\begin{aligned}
 1(c) \quad \hat{\lambda}_{\text{map}} &= \operatorname{argmax} P(D|\lambda) P(\lambda) \\
 &= \lambda^n e^{-\lambda \sum x_i} \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta \lambda} \\
 &= \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot \lambda^{n+\alpha-1} e^{-\lambda (\sum x_i + \beta)} \\
 &= \text{Gamma}(\alpha+n, \sum x_i + \beta)
 \end{aligned}$$

log posterior,

$$\log P(\lambda|D) = (n+\alpha-1) \log \lambda - \lambda (\sum x_i + \beta)$$

$$\frac{\partial}{\partial \lambda} \log(P(\lambda|D)) = \frac{(n+\alpha-1)}{\lambda} - \sum x_i - \beta = 0$$

$$\Rightarrow \lambda = \frac{n+\alpha-1}{\sum x_i + \beta}$$

Given, $\alpha = 5$, $\beta = 10$, $n = 6$

$$\sum x_i = \frac{1.5 + 3 + 2.5 + 2.75 + 2.9 + 3}{6} = 2.60833$$

$$\lambda = \frac{6+5-1}{2.60833+10} \Rightarrow \underline{\underline{0.79312}}$$

②

given

$$D = \{ ((170, 57, 32), W), ((192, 95, 28), M), ((150, 45, 30), W), ((170, 65, 29), M), ((175, 78, 35), M), ((185, 90, 32), M), ((170, 65, 28), W), ((155, 48, 31), W), ((160, 55, 30), W), ((182, 80, 30), M), ((175, 69, 28), W), ((180, 80, 27), M), ((160, 50, 31), W), ((175, 72, 30), M) \}$$

(a)

(i) Input $\rightarrow (155, 40, 35)$

Height	Weight	Age		Distance	
170	57	32	W	22.869	5
192	95	28	M	66.655	
150	45	30	W	8.660	1
170	65	29	M	29.765	
175	78	35	M	42.941	
185	90	32	M	58.386	
170	65	28	W	29.983	
155	48	31	W	8.944	2
160	55	30	W	16.583	4

182	80	30	M	48.518
175	69	28	W	35.916
180	80	27	M	47.843
160	50	31	W	11.874
175	72	30	M	38.065

3

If $K=1$

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

where x = height, y = weight and z = age

$$\text{distance} = 8.660 \quad \text{Gender} = \underline{\text{Woman}}$$

If $K=3$

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$\begin{array}{ll} \text{distance 1} = 8.660 & \text{Gender} = \text{Woman} \\ 2 = 8.944 & \text{Gender} = \text{Woman} \\ 3 = 11.874 & \text{Gender} = \text{Woman} \end{array}$$

Woman

If $K=5$

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$\begin{array}{ll} \text{distance 1} = 8.660 & \text{Gender} = \text{Woman} \\ 2 = 8.944 & \text{Gender} = \text{Woman} \\ 3 = 11.874 & \text{Gender} = \text{Woman} \\ 4 = 16.583 & \text{Gender} = \text{Woman} \\ 5 = 22.869 & \text{Gender} = \text{Woman} \end{array}$$

$$\therefore (155, 40, 35) \Rightarrow \text{Woman}$$

(ii) Input = (170, 70, 32)

(4)

Height	Weight	Age		Distance	
170	57	32	W	13	
192	95	28	M	33.541	
150	45	30	W	32.078	
170	65	29	M	5.830	2
175	78	35	M	9.899	5
185	90	32	M	25	
170	65	28	W	6.403	3
155	48	31	W	26.645	
160	55	30	W	18.138	
182	80	30	M	15.748	
175	69	28	W	6.480	4
180	80	27	M	15	
160	50	31	W	22.383	
175	72	30	M	5.744	1

If $k=1$, distance = 5.744, Gender = Man

If $k=3$, distance \rightarrow 1. 5.744, Gender = Man
 2. 5.830, Gender = Man
 3. 6.403, Gender = woman } Man

If $k=5$, distance \rightarrow 1. 5.744, Gender = Man
 2. 5.830, Gender = Man
 3. 6.403, Gender = Woman
 4. 6.480, Gender = woman
 5. 9.899, Gender = Man } Man

(iii) Input = (175, 70, 35)

Height	Weight	Age		Distance	
170	57	32	W	14.247	
192	95	28	M	31.032	
150	45	30	W	35.707	
170	65	29	M	9.273	4
175	78	35	M	8	3
185	90	32	M	22.561	
170	65	28	W	9.949	5
155	48	31	W	30	
160	55	30	W	21.794	
182	80	30	M	13.190	
175	69	28	W	7.071	2
180	80	27	M	13.747	
160	50	31	W	25.317	
175	72	30	M	5.385	1

If $K=1$, distance = 5.385, Gender = Male

If $K=3$, distance 1 = 5.385 Gender = Male
 2 = 7.071 Gender = woman
 3 = 8 Gender = male } Man

If $K=5$ distance 1 = 5.385 Gender = Man
 2 = 7.071 Gender = woman
 3 = 8 Gender = man
 4 = 9.273 Gender = Man
 5 = 9.949 Gender = woman } Man

(iv) Input :- (180, 90, 20)

Height	Weight	Age		Distance	
170	57	32	W	36.510	
192	95	28	M	15.264	4
150	45	30	W	55	
170	65	29	M	28.390	
175	78	35	M	19.849	5
185	90	32	M	13	2
170	65	28	W	28.089	
155	48	31	W	50.099	
160	55	30	W	41.533	
182	80	30	M	14.282	3
175	69	28	W	23.021	
180	80	27	M	12.206	1
160	50	31	W	46.054	
175	72	30	M	21.1897	

If $k=1$, distance = 12.206, Gender = Man

If $k=3$, distance 1 = 12.206, Gender = Man
 2 = 13, Gender = man
 3 = 14.282, Gender = Man } Man

If $k=5$, distance 1 = 12.206, Gender = Man
 2 = 13, Gender = Man
 3 = 14.282, Gender = Man
 4 = 15.264, Gender = Man
 5 = 19.849, Gender = Man } Man

20 Repeat KNN prediction removing age data.

Given:-

(i) Input = (155, 40)

Height	Weight		Distance	
170	57	W	22.671	5
192	95	M	66.287	
150	45	W	7.071	1
170	65	M	29.154	
175	78	M	42.941	
185	90	M	58.309	
170	65	W	29.154	
155	48	W	8	2
160	55	W	15.811	4
182	80	M	48.259	
175	69	W	35.227	
180	80	M	47.169	
160	50	W	11.180	3
175	72	M	37.735	

If $K=1$, distance = 7.071, Gender = woman

If $K=3$, distance \rightarrow 1) 7.071, Gender = Woman
 2) 8, Gender = woman
 3) 11.180, Gender = woman } Woman

If $K=5$, distance \rightarrow 1) 7.071, Gender = woman
 2) 8, Gender = woman
 3) 11.180, Gender = woman
 4) 15.811, Gender = woman
 5) 22.671, Gender = woman } Woman

(ii) Input:- (170, 70)

Height	Weight		Distance	
170	57	W	13	5
192	95	M	33.301	
150	45	W	32.015	
170	65	M	5	1
175	78	M	9.433	4
185	90	M	25	
170	65	W	5	2.1
155	48	W	26.627	
160	55	W	18.027	
182	80	M	15.620	
175	69	W	5.099	2
180	80	M	14.142	
160	50	W	22.360	
175	72	M	5.385	3

If $K=1$, distance = 5, Gender = Male/Woman

If $K=3$, distance \rightarrow 1) 5, Gender = Man
 2) 5, Gender = Woman
 3) 5.099, Gender = Woman } Woman

If $K=5$, distance = 1) 5, Gender = Man
 2) 5, Gender = Woman
 3) 5.099, Gender = Woman
 4) 5.385, Gender = Man
 5) 9.433, Gender = Man } Man

(ii) Input $\rightarrow (175, 70)$

Height	Weight		Distance	
170	57	W	13.928	
192	95	M	30.232	
150	45	W	35.355	
170	65	M	7.071	3
175	78	M	8	5
185	90	M	22.360	
170	65	W	7.071	4
155	48	W	29.732	
160	55	W	21.213	
182	80	M	12.206	
175	69	W	1	1
180	80	M	11.180	
160	50	W	25	
175	72	M	2	2

If $k=1$, Distance = 1, gender = woman

If $k=3$, Distance =

- 1) 1, Gender = woman
- 2) 2, Gender = man
- 3) 7.071, Gender = Man/woman

} Man/woman

If $k=5$, distance,

- 1) 1, Gender = woman
- 2) 2, Gender = man
- 3) 7.071, Gender = man
- 4) 7.071, Gender = woman
- 5) 8, Gender = man

} Man

(iv) Input $\rightarrow (180, 90)$

Height	Weight		Distance	
170	57	W	34.481	5
192	95	M	13	
150	45	W	54.083	
170	65	M	26.925	
175	78	M	13	
185	90	M	5	4
170	65	W	26.925	
155	48	W	48.877	1
160	55	W	40.311	
182	80	M	10.198	
175	69	W	21.587	3
180	80	M	10	
160	50	W	44.721	
175	72	M	18.681	

If $k=1$, distance = 5, Gender = Man

If $k=3$, distance

1) 5	, Gender = Man	} <u>Man</u>
2) 10	, Gender = Man	
3) 10.198	, Gender = Man	

If $k=5$, distance

1) 5	, Gender = Man	} Man
2) 10	, Gender = Man	
3) 10.198	, Gender = Man	
4) 13	, Gender = Man	
5) 13	, Gender = Man	

Question 2Conclusion:-

From the calculations and predictions seen in 2a and 2c we can clearly see that 2a provides a better prediction. With the age data present the prediction is more accurate.

If we remove the age data we can see that there are redundant predictions made which can either result in Man or Woman.

Q3

(a) Given: data from Q2

Probabilistic classifier

$$\text{posterior} = \frac{\text{prior} \times \text{likelihood}}{\text{evidence}}$$

$$P(C_k, x_1, \dots, x_n)$$

$$P(x_i | x_{i+1}, \dots, x_n, C_k) = P(x_i | C_k)$$

$$= P(C_k) \prod_{i=1}^n P(x_i | C_k)$$

$$\therefore \hat{y} = \underset{K \in (1, \dots, K)}{\text{argmax}} P(C_k) \prod_{i=1}^n P(x_i | C_k)$$

Gaussian Naive Bayes:-

$$P(x=v | C_k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

Determine:-

$$\text{posterior}(\text{male}) = \frac{P(\text{male}) P(\text{height} | \text{male}) P(\text{weight} | \text{male}) P(\text{age} | \text{male})}{\text{evidence}}$$

$$\text{posterior}(\text{woman}) = \frac{P(\text{woman}) P(\text{height} | \text{woman}) P(\text{weight} | \text{woman}) P(\text{age} | \text{woman})}{\text{evidence}}$$

$$P(\text{male}) = 0.5$$

$$P(\text{woman}) = 0.5$$

~~Man~~ Man :-

$$\text{Mean}(\text{height}) = 179.857$$

$$\text{Var}(\text{height}) = 53.809$$

$$\text{Mean}(\text{age}) = 30.142$$

$$\text{Var}(\text{age}) = 7.142$$

$$\text{Mean}(\text{Weight}) = 80$$

$$\text{Var}(\text{weight}) = 103$$

woman :-

$$\text{Mean}(\text{height}) = 162.857$$

$$\text{Var}(\text{height}) = 82.142$$

$$\text{Mean}(\text{age}) = 30$$

$$\text{Var}(\text{age}) = 2.333$$

$$\text{Mean}(\text{weight}) = 55.571$$

$$\text{Var}(\text{weight}) = 78.619$$

~~$P(\text{height}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(155-179.857)^2}{2\sigma^2}\right)$~~

i) Input $\rightarrow (155, 40, 35)$

$$P(\text{height}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(155-179.857)^2}{2\sigma^2}\right) = \underline{0.00017}$$

$$P(\text{weight}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(40-80)^2}{2\sigma^2}\right) = \underline{1.664 \times 10^{-5}}$$

$$P(\text{age}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(35-30.142)^2}{2\sigma^2}\right) = \underline{0.02863}$$

$$P(\text{height}|\text{woman}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(155-162.857)^2}{2\sigma^2}\right) = \underline{0.03023}$$

$$P(\text{weight}|\text{woman}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(40-55.571)^2}{2\sigma^2}\right) = \underline{0.00962}$$

$$P(\text{age}|\text{woman}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(35-30)^2}{2\sigma^2}\right) = \underline{0.00123}$$

(10)

$$\text{posterior}(\text{Man}) = P(M) P(\text{Height}|M) P(\text{weight}|M) P(\text{Age}|M)$$

considering only the numerator

$$\Rightarrow (0.5)(0.0017)(1.664 \times 10^{-5})(0.02863)$$

$$\Rightarrow \underline{\underline{4.1602 \times 10^{-11}}}$$

$$\text{Posterior}(W) = P(W) P(\text{height}|W) P(\text{weight}|W) P(\text{Age}|W)$$

considering the numerator:-

$$\Rightarrow (0.5)(0.0302)(0.0096)(0.00123)$$

$$\Rightarrow \underline{\underline{1.7914 \times 10^{-7}}}$$

As the posterior(W) is greater than posterior(M)
the prediction for (155, 40, 35) \Rightarrow woman.

(ii) input:- (170, 70, 32)

$$\rightarrow P(\text{height} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(170 - 179.86)^2}{2\sigma^2}\right) = \underline{0.02205}$$

$$\rightarrow P(\text{weight} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70 - 80)^2}{2\sigma^2}\right) = \underline{0.0241}$$

$$\rightarrow P(\text{age} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(32 - 30.14)^2}{2\sigma^2}\right) = \underline{0.11725}$$

$$\rightarrow P(\text{height} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(170 - 162.857)^2}{2\sigma^2}\right) = \underline{0.03227}$$

$$\rightarrow P(\text{weight} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70 - 55.57)^2}{2\sigma^2}\right) = \underline{0.01197}$$

$$\rightarrow P(\text{age} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(32 - 30)^2}{2\sigma^2}\right) = \underline{0.11083}$$

$$\text{Posterior}(\text{man}) = (0.5)(0.02205)(0.0241)(0.11725)$$

$$\Rightarrow \underline{0.00003127}$$

$$\text{Posterior}(\text{woman}) = (0.5)(0.032)(0.0119)(0.1108)$$

$$\Rightarrow \underline{0.000021406}$$

since $\text{posterior}(M)$ is greater than $\text{posterior}(W)$
given input (170, 70, 32) \rightarrow Man

(11)

(iii) Input $\rightarrow (175, 70, 35)$

$$\rightarrow P(\text{height} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(175-179.86)^2}{2\sigma^2}\right) = \underline{0.04368}$$

$$\rightarrow P(\text{weight} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70-80)^2}{2\sigma^2}\right) = \underline{0.0241}$$

$$\rightarrow P(\text{age} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(35-30.14)^2}{2\sigma^2}\right) = \underline{0.02863}$$

$$\rightarrow P(\text{height} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(175-162.857)^2}{2\sigma^2}\right) = \underline{0.0179}$$

$$\rightarrow P(\text{weight} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70-55.57)^2}{2\sigma^2}\right) = \underline{0.01197}$$

$$\rightarrow P(\text{age} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(35-30)^2}{2\sigma^2}\right) = \underline{0.00123}$$

$$\text{Posterior}(\text{man}) = 0.5(0.0436)(0.0241)(0.0286)$$

$$\Rightarrow 1.512 \times 10^{-5} \Rightarrow \underline{\underline{0.00001512}}$$

$$\text{Posterior}(W) = 0.5(0.0179)(0.0119)(0.00123)$$

$$\Rightarrow 1.322 \times 10^{-7} \Rightarrow \underline{\underline{0.0000001322}}$$

Since $\text{posterior}(W) < \text{posterior}(M)$

$(175, 70, 35) \rightarrow \underline{\underline{\text{man}}}$

(iv) Input $\rightarrow (180, 90, 20)$

$$\rightarrow P(\text{height} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(180 - 179.86)^2}{2\sigma^2}\right) = \underline{0.0543}$$

$$\rightarrow P(\text{weight} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(90 - 80)^2}{2\sigma^2}\right) = \underline{0.02419}$$

$$\rightarrow P(\text{age} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(20 - 30.14)^2}{2\sigma^2}\right) = \underline{0.00011}$$

$$\rightarrow P(\text{height} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(180 - 162.857)^2}{2\sigma^2}\right) = \underline{0.00736}$$

$$\rightarrow P(\text{weight} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(90 - 55.57)^2}{2\sigma^2}\right) = \underline{2.39 \times 10^{-5}}$$

$$\rightarrow P(\text{age} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(20 - 30)^2}{2\sigma^2}\right) = \underline{1.3 \times 10^{-10}}$$

$$\text{Posterior}(\text{man}) = 0.5(0.0543)(0.0241)(0.00011)$$

$$\Rightarrow \underline{7.319 \times 10^{-6}}$$

$$\text{Posterior}(W) = 0.5(0.00736)(2.39 \times 10^{-5})(1.3 \times 10^{-10})$$

$$\Rightarrow \underline{1.136 \times 10^{-11}}$$

Since $\text{posterior}(M) > \text{posterior}(W)$

$(180, 90, 20) \rightarrow \underline{\text{Man}}$

Q3

(12)

(c) Removing the age parameter.

(i) Input $\rightarrow (155, 40)$

$$P(\text{height} | m) = \frac{1}{\sqrt{(2\pi\sigma^2)}} \exp\left(-\frac{(155 - 179.86)^2}{2\sigma^2}\right) = \underline{0.00017}$$

$$P(\text{weight} | m) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(40 - 80)^2}{2\sigma^2}\right) = \underline{1.664 \times 10^{-5}}$$

$$P(\text{height} | w) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(155 - 162.857)^2}{2\sigma^2}\right) = \underline{0.03023}$$

$$P(\text{weight} | w) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(40 - 55.57)^2}{2\sigma^2}\right) = \underline{\underline{0.00962}}$$

$$\text{Posterior}(m) = 0.5(0.00017)(1.664 \times 10^{-5})$$

$$\Rightarrow \underline{\underline{1.4533 \times 10^{-9}}}$$

$$\text{Posterior}(w) = 0.5(0.03023)(0.00962)$$

$$\Rightarrow \underline{\underline{0.000145}}$$

Since $\text{posterior}(w) > \text{posterior}(m)$
 $(155, 40) \rightarrow \underline{\underline{\text{Woman}}}$

(ii) Input $\rightarrow (170, 70)$

$$P(\text{height} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(170 - 179.86)^2}{2\sigma^2}\right) = \underline{0.02205}$$

$$P(\text{weight} | M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70 - 80)^2}{2\sigma^2}\right) = \underline{0.02419}$$

$$P(\text{height} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(170 - 162.857)^2}{2\sigma^2}\right) = \underline{0.03227}$$

$$P(\text{weight} | W) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70 - 55.57)^2}{2\sigma^2}\right) = \underline{0.01197}$$

$$\text{Posterior}(M) = 0.5(0.02205)(0.02419)$$

$$\Rightarrow \underline{\underline{0.0002667}}$$

$$\text{Posterior}(W) = 0.5(0.03227)(0.01197)$$

$$\Rightarrow \underline{\underline{0.000193}}$$

Since $\text{posterior}(M) > \text{posterior}(W)$

$(170, 70) \rightarrow \underline{\underline{\text{Man}}}$

(ii) Input $\rightarrow (175, 70)$

(13)

$$P(\text{height}|\text{man}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(175-179.86)^2}{2\sigma^2}\right) = \underline{0.04368}$$

$$P(\text{weight}|\text{m}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70-80)^2}{2\sigma^2}\right) = \underline{0.02419}$$

$$P(\text{height}|\text{w}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(175-162.857)^2}{2\sigma^2}\right) = \underline{0.01794}$$

$$P(\text{weight}|\text{w}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(70-55.57)^2}{2\sigma^2}\right) = \underline{0.01197}$$

$$\text{Posterior}(\text{m}) = 0.5(0.04368)(0.02419) \\ \Rightarrow 5.2 \times 10^{-4}$$

$$\Rightarrow \underline{\underline{0.000528}}$$

$$\text{Posterior}(\text{w}) = 0.5(0.0179)(0.01197) \\ \Rightarrow 1.07 \times 10^{-4}$$

$$\Rightarrow \underline{\underline{0.000107}}$$

Since $\text{posterior}(\text{m}) > \text{posterior}(\text{w})$
 $(175, 70) \rightarrow \underline{\underline{\text{man}}}$

(iv) Input $\rightarrow (180, 90)$

$$P(\text{height} | m) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(180 - 179.86)^2}{2\sigma^2} \right) = \underline{0.05437}$$

$$P(\text{weight} | m) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(90 - 80)^2}{2\sigma^2} \right) = \underline{0.02419}$$

$$P(\text{height} | w) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(180 - 162.857)^2}{2\sigma^2} \right) = \underline{0.00736}$$

$$P(\text{weight} | w) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(90 - 55.57)^2}{2\sigma^2} \right) = \underline{2.39 \times 10^{-5}}$$

$$\text{Posterior}(m) = 0.5(0.05437)(0.02419)$$

$$\Rightarrow 6.5 \times 10^{-4}$$

$$\Rightarrow \underline{0.000657}$$

$$\text{Posterior}(w) = 0.5(0.00736)(2.39 \times 10^{-5})$$

$$\Rightarrow \underline{8.809 \times 10^{-8}}$$

Since $\text{posterior}(m) > \text{posterior}(w)$

$(180, 90) \rightarrow \underline{\text{Man}}$

Q3d)Conclusions:-

Based on the 2 classifier test performed in 2b,c and 3b,c we can see that there are both pro's and cons in both the classifiers.

In KNN the age parameter removed makes redundant predictions which are not justifiable. Similarly in Naive Bayes classifiers removing the age parameter causes changes in the probabilistic values and will result in inconclusive predictions.

KNN classifiers assign category based on selected nearest neighbours.

The Naive Bayes classifier chooses the probability based on the prior knowledge of the value & its probability.

Hence Naive Bayes are comparatively better than KNN considering large data sets.