The Hybrid Production Planning Algorithm

The production planning problem is inherently a multi-objective optimization problem. The objectives we concentrate on are the four performance indicators suggested by domain experts. The first objective is to minimize the order delay rate:

$$A_{delay} = \sum_{i \in I_D} \sum_{t \in T} \frac{m_{i,t}}{D_{i,t} + 1} \tag{1}$$

where I_D is the products with demand, T represents the period that the production plan targets at, $m_{i,t}$ means the number of delayed orders, and $D_{i,t}$ denotes the total order demand. The timestep of the algorithm is one day. The denominator is added by one to avoid the error caused when the demand is 0. The second objective, minimizing the production cost, is formulated as follows:

$$A_{cost_p} = \sum_{i \in I_{AI}} \sum_{p \in P_i} PC_{i,p} \cdot \left(\sum_{t \in T} \hat{x}_{i,p,t} \right) \tag{2}$$

where I_{AI} encodes the products whose production will consume other products or raw materials, P_i denotes the plants which can produce product i, $PC_{i,p}$ represents the unit production cost, and $\hat{x}_{i,p,t}$ indicates the planned production output. Minimizing the inventory cost is the third objective:

$$A_{cost_i} = \sum_{i \in I_{AI}} \sum_{p \in P_i} HC_{i,p} \cdot \sum_{t \in T} inv_{i,p,t} + \sum_{i \in I_{AI}} \sum_{p \in P_i} HC_{i,p} \cdot PT_{i,p} \cdot \sum_{t \in T} \hat{x}_{i,p,t}, \tag{3}$$

where $HC_{i,p}$ indicates the unit inventory cost, $inv_{i,p,t}$ denotes the inventory of a product at the end of a day, and $PT_{i,p}$ represents the processing cycle of the product.

The last objective is to minimize the weekly smoothing rate of production capacity use. Eqn. 4 is used to compute the smoothing rate of a capacity set c in week T:

$$A_{smooth_{c,T}} = \frac{\left| \sum_{t \in T} \sum_{i \in I_{p,c}} U_{i,p,c} \cdot \hat{x}_{i,p,t} - \sum_{t \in \overline{T-1}} \sum_{i \in I_{p,c}} U_{i,p,c} \cdot \hat{x}_{i,p,t} \right|}{\sum_{t \in \overline{T-1}} \sum_{i \in I_{p,c}} U_{i,p,c} \cdot \hat{x}_{i,p,t} + 1}. \tag{4}$$

Here, $\overline{T-1}$ means the week before week T and $U_{i,p,c}$ represents unit production capacity. The objective is obtained by averaging the smoothing rate of all the capacity sets.

Then, we employ the weighted sum model (WSM) [4] to address the multi-objective production planning problem:

$$WSM_{score}^* = w_1 A_{delay} + w_2 A_{cost_p} + w_3 A_{cost_i} + w_4 A_{smooth_{c,T}} \tag{5}$$

The optimized plan is reached by minimizing WSM_{score}^*, and the decision variables are $\hat{x}_{i,p,t}$. The weights w_j are predefined by domain experts according to the importance of the four performance indicators.

The optimization model also contains a huge number of constraints which can be grouped into more than 20 categories. For example, the maximum production output of the product is constrained by the production capacity of the factory, as illustrated in the following inequality:

$$\sum_{i \in I_{p,c}} U_{i,p,c} \cdot \hat{x}_{i,p,t} \leq CAP_{p,t,c}, \forall p \in P, \forall t \in T, \forall c \in C, \tag{6}$$

where $CAP_{p,t,c}$ denotes the maximum production capacity available. Due to the limited space, other types of constraints will not be described in detail.

The optimization problem defined in Eqn. 5 can be solved by integer programming. However, directly employing integer programming to solve such a large scale production planning problem is time-consuming. Therefore, in this paper, we adopt a hybrid production planning algorithm which combines linear programming [3] and heuristic algorithms [1]. It is adapted from an approach proposed by Sahling et al [2].

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