Asymptotic Notations

Asymptotic notations give us an idea about how good a given algorithm is compared to some other algorithm
Let us see the mathematical definition of "order of" now.

Primarily there are three types of widely used asymptotic notations.

1) Big oh notation (O)
2) Big omega notation (Ω)
3) Big theta notation (Θ) ⟶ Widely used one!

Big oh notation

Big oh notation is used to describe asymptotic upper bound
Mathematically, if $f(n)$ describes running time of an algorithm; $f(n)$ is $O(g(n))$ iff there exist positive constants C and n_0 such that

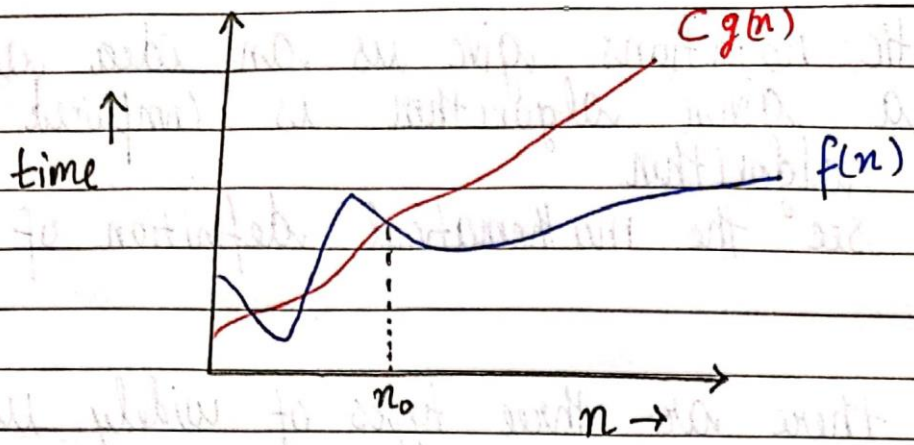
$$0 \leq f(n) \leq C\,g(n) \quad \text{for all } n \geq n_0$$

⇓
used to give upper bound on a function.

if a function is $O(n)$, it is automatically $O(n^2)$ as well!

represents the upper bound or the worst-case scenario of an algorithm's time complexity

Graphic example for Big oh (O)



Big Omega notation

Just Like O notation provides an asymptotic upper bound, Ω notation provides asymptotic lower bound. Let f(n) define running time of an algorithm;

f(n) is said to be $\Omega(g\,n)$ if there exists positive constants C and n_0 such that

$$0 \leq C\,g(n) \leq f(n) \quad \text{for all } n \geq n_0$$

⇓ used to give lower bound on a function

if a function is $O(n^2)$ it is automatically $O(n)$ as well

represents the lower bound or the best-case scenario of an algorithm's time complexity

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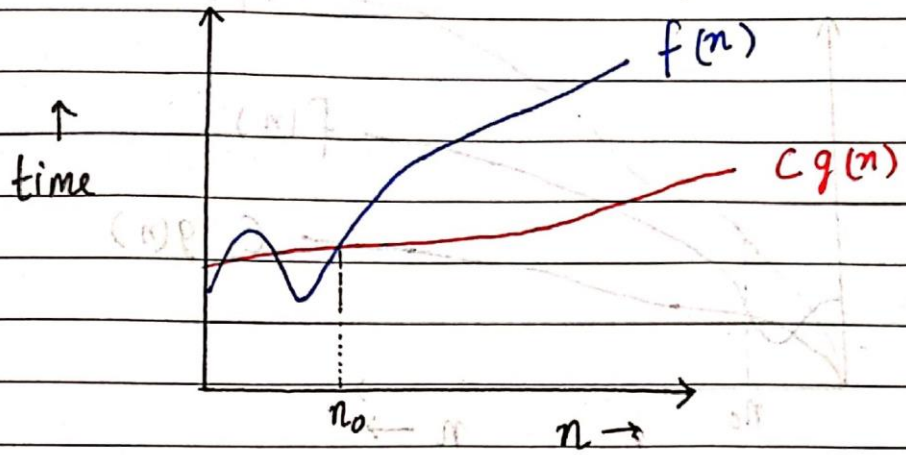
Graphic example for Big omega (Ω)



Big theta notation

Let $f(n)$ define running time of an algorithm

$f(n)$ is said to be $\Theta(gn)$ iff $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$

Mathematically,

$$0 \leq f(n) \leq C_1 g(n) \quad \forall\ n \geq n_0$$ → Sufficiently large value of n

$$0 \leq C_2 g(n) \leq f(n) \quad \forall\ n \geq n_0$$

Merging both the equations, we get:

$$0 \leq C_2 g(n) \leq f(n) \leq C_1 g(n) \quad \forall\ n \geq n_0$$

The equation simply means there exist positive constants C_1 and C_2 such that $f(n)$ is sandwiched between $C_2 g(n)$ and $C_1 g(n)$

Theta notation represents both the upper and lower bounds, providing a tight bound on an algorithm's time complexity

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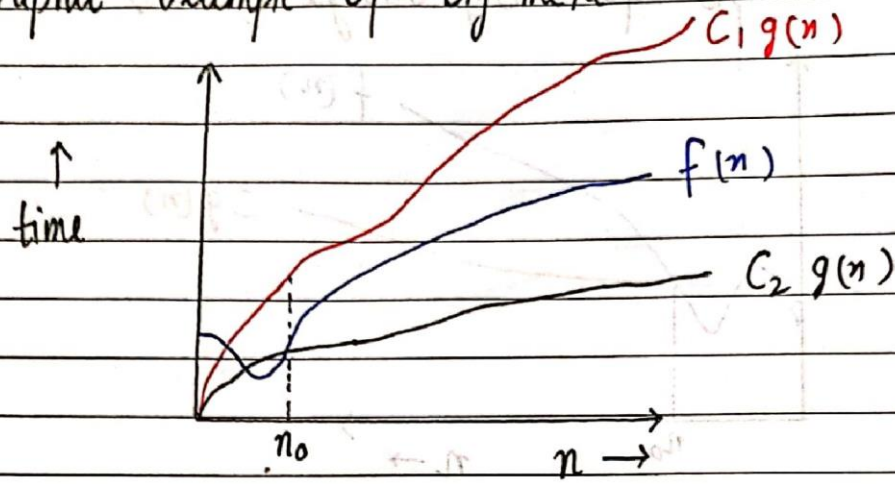
Graphic example of Big theta



Which one of these to use ?
Since Big theta gives a better picture of runtime for a given algorithm, most of the interviewers expect you to provide an answer in terms of Big theta when they say "Order of".

Quick Quiz :- Prove that n^2+n+1 is $O(n^3)$, $\Omega(n^2)$ and $\Theta(n^2)$ using respective definitions.

Increasing order of common runtimes

$$1 < \log n < n < n\log n < n^2 < n^3 < 2^n < n^n$$

Better ... Worse

Common runtimes from better to worse →