CS-E5740 Complex Networks, Answers to exercise set 4

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Problem 1

a)

$$n_d = \langle q \rangle n_{d-1}$$
$$n_d = (\frac{\langle k^2 \rangle}{\langle k \rangle} - 1) n_{d-1} \approx \langle k \rangle n_{d-1}$$
$$n_{d-1} = \langle k \rangle n_{d-2}$$
$$\dots$$

Hence

$$n_d = \langle k \rangle^d$$

Since a ER network is a Poisson distribution, when $N \to \infty$ such that $\langle k \rangle$ is constant. The mean and variance is equal to $\langle k \rangle$. Prove as below:

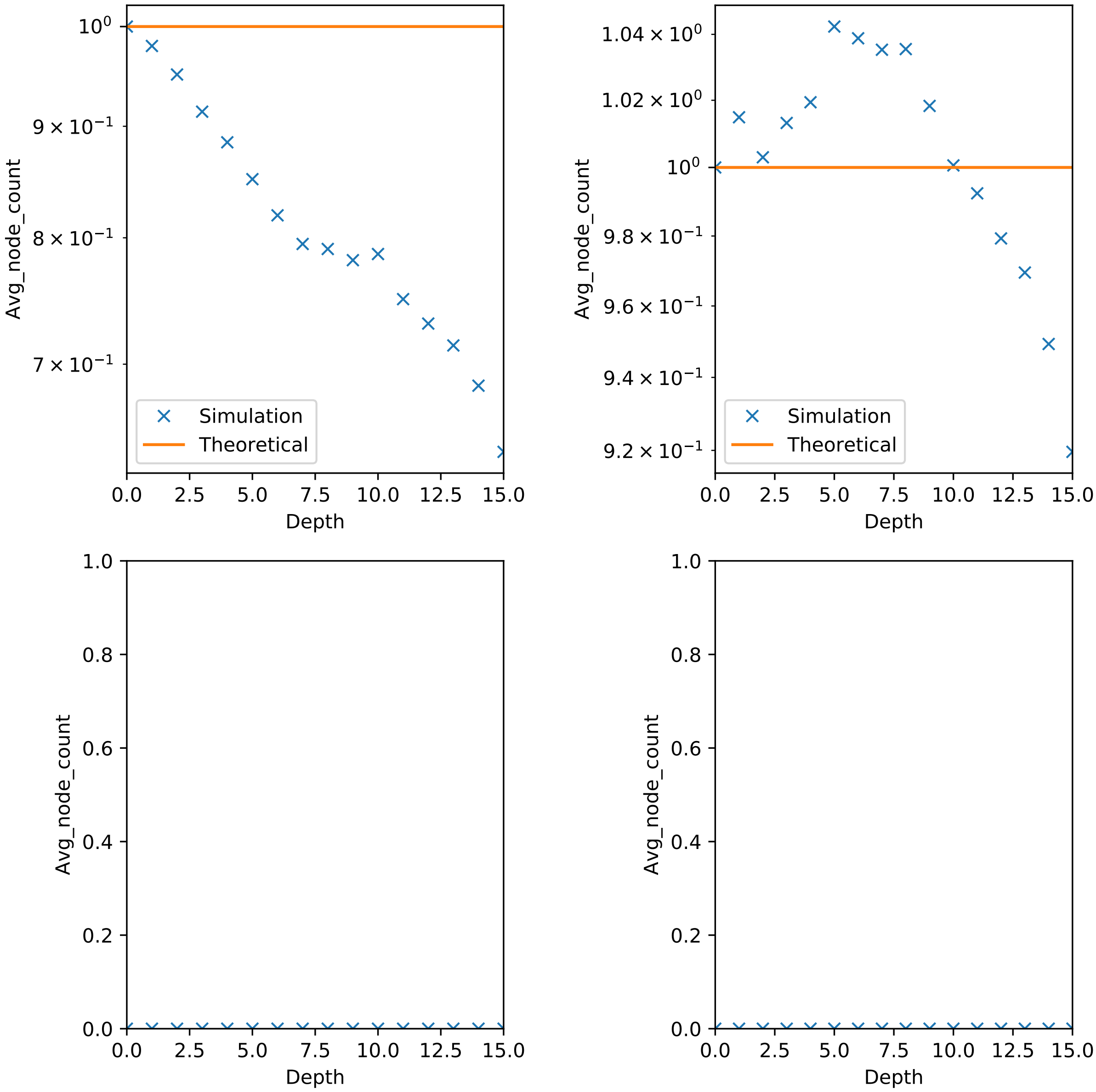
$$\frac{\langle k^2 \rangle}{\langle k \rangle} - 1 > 1$$

$$\frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

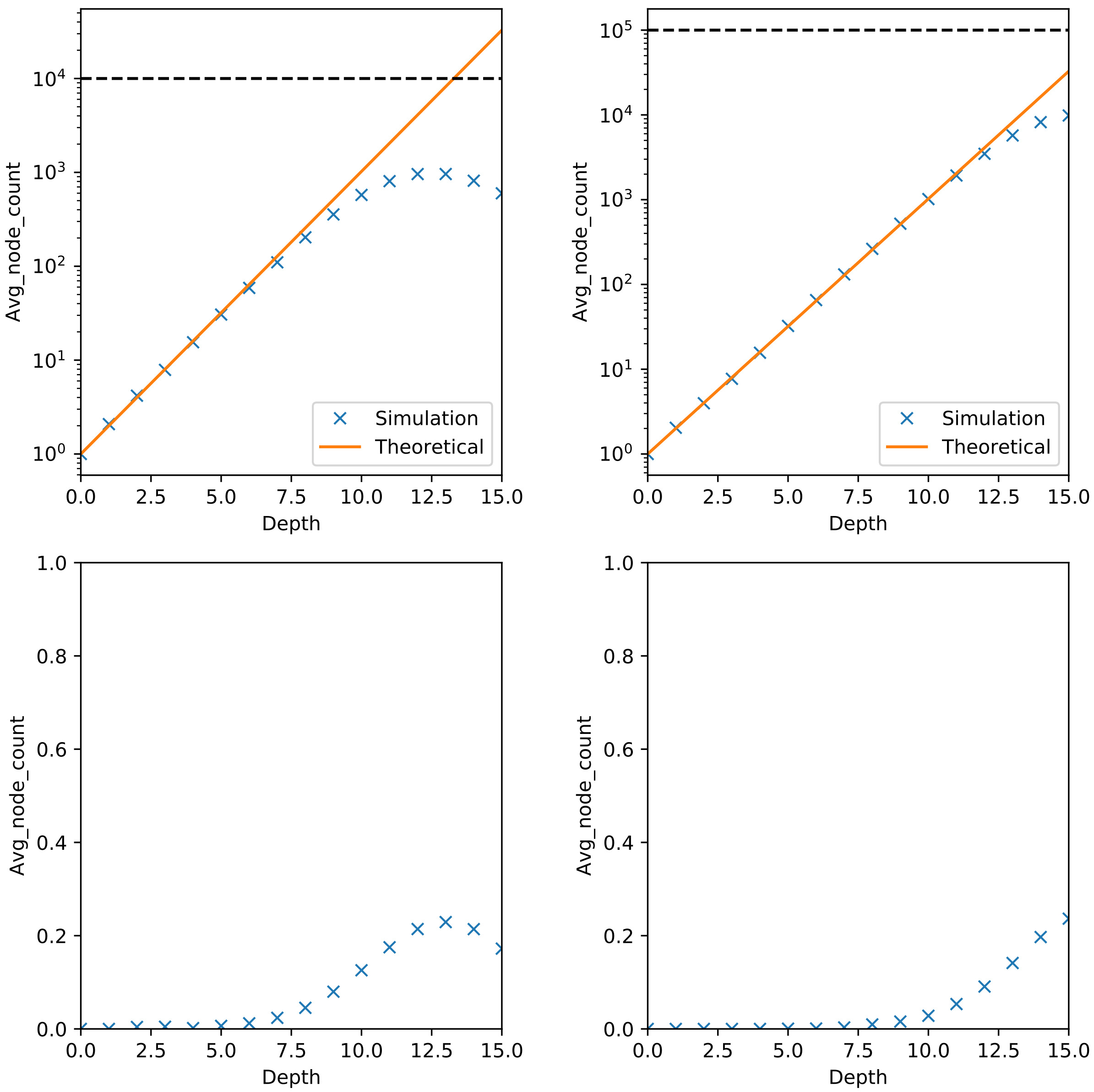
$$\frac{variance + \langle k \rangle^2}{\langle k \rangle} = \frac{\langle k \rangle + \langle k \rangle^2}{\langle k \rangle} > 2$$

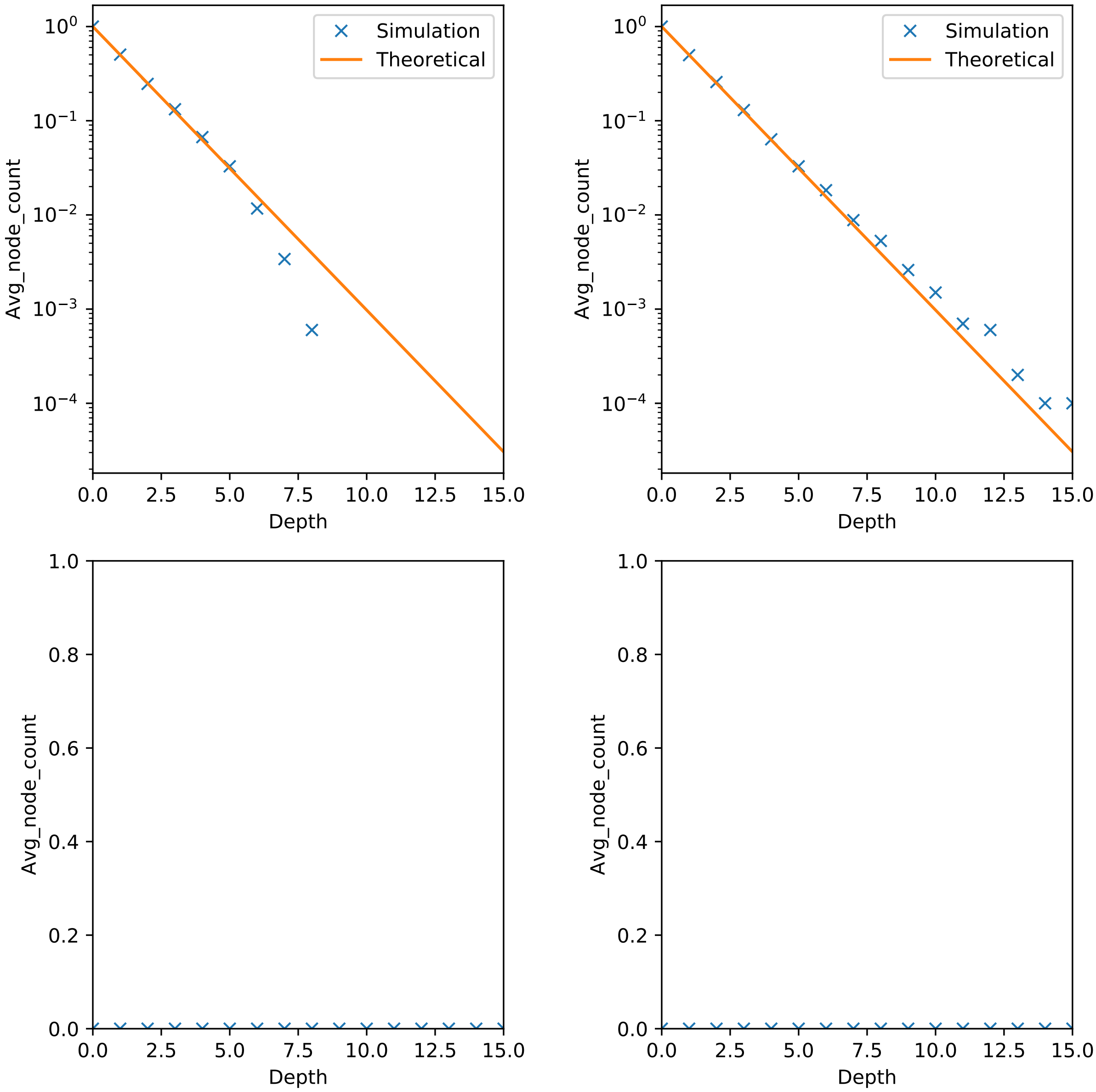
$$\langle k \rangle > 1$$

b) See the plot below.



Avg_node_count
Depth
Simulation
Theoretical
10^0
9×10^{-1}
8×10^{-1}
7×10^{-1}
1.04×10^0
1.02×10^0
9.8×10^{-1}
9.6×10^{-1}
9.4×10^{-1}
9.2×10^{-1}
0.0 2.5 5.0 7.5 10.0 12.5 15.0
0.0 0.2 0.4 0.6 0.8 1.0

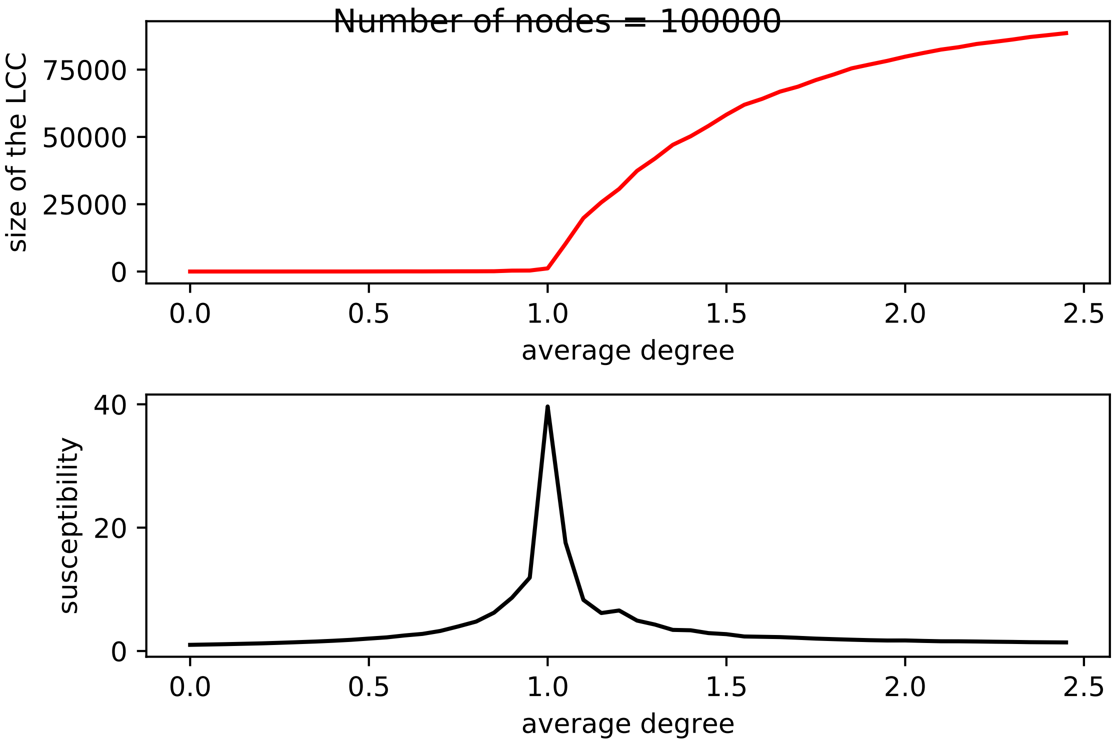

Avg_node_count
Depth
Simulation
Theoretical
0.0
2.5
5.0
7.5
10.0
12.5
15.0
10^0
10^1
10^2
10^3
10^4
10^5
0.0
0.2
0.4
0.6
0.8
1.0



Simulation
Theoretical
Avg_node_count
Depth
Simulation
Theoretical
Avg_node_count
Depth
Avg_node_count
Depth
Avg_node_count
Depth

c) We could observe that the average node count is not the same as the theoretical when depth goes further. The theoritical result is just calculated based on the average degree and the depth. The fact result is based on breadth first search and when the depth is large, the network we generate does not exactly have the average degree of we expect because of some links might be duplicate.

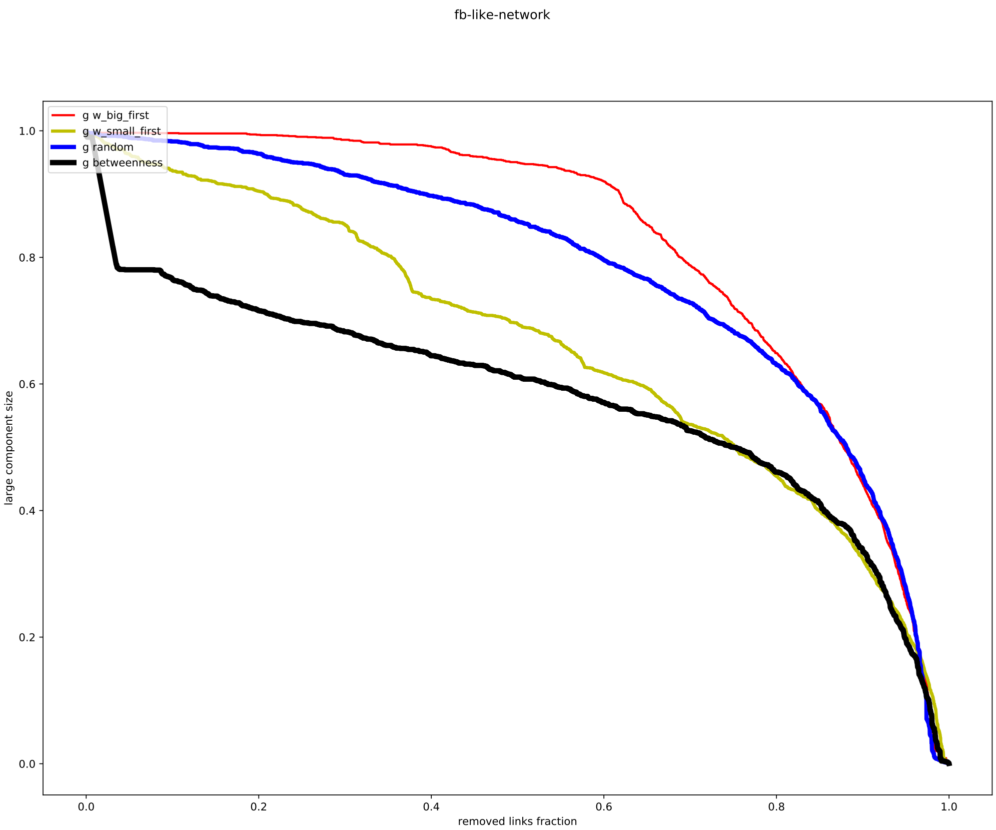
d) The size of the largest component against $\langle k \rangle$ is plotted as below.



e) It is easily to observe that at average degree is approximately 1, the large connected component starts to appear and becomes larger and larger due to the increase of the average degree. This corresponds to the "rich gets richer".
The susceptibility gets very large value at the percolation transition point here as average degree approx. 1.The trend is because even a slightly changed in the average degree will leads to changing of a link of other component.

Problem 2

a) Plot as shown.



b) Most vulnerable: decreasing order of edge-betweenness. Least vulnerable: decreasing order of weight(Weight big first). The vulnerability is not simple to define.

c) Weak links more important for the integrity of the network. The reason is that for the weak links, they normally connected to weakly to the other parts of the network. For example, in the network, there is some link we could called as bridge, which is weak link with lower weight. If we remove those links, the weakly connected component might suddenly became two networks, not connected to each other.

d) Since we removed on descending order of the betweenness centrality, which means we removed links with high betweenness centrality, this will cause very extreme collapse of the large components. This means the most important links to the network is removed firstly. This is easily observed from the plot as well. But if we remove the link randomly, it will not have that much targeted to the betweenness centrality importance. All the links are randomly choose with the same possibility. So that is why the difference between those two removal strategies.