CS-E5740 Complex Networks, Answers to exercise set 6

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Problem 1

a) 1.Degree centrality

node i	A	B	C	D	E
degree $k(i)$	2	3	4	1	2

2.Betweenness centrality
According to the definition, bc(A) = bc(E) = bc(D) = 0 since there is no shortest paths between other nodes of the network pass through node A, or E or D. For node B, since σ_{AE} includes A-B-E, A-C-E. So,

$$bc(B) = \frac{1}{(5-1)*(5-2)} \frac{\sigma_{ABE}}{\sigma_{AE}} = \frac{1}{24}$$

For node C,

$$bc(C) = \frac{1}{(5-1)*(5-2)} (\frac{\sigma_{ACD}}{\sigma_{AD}} + \frac{\sigma_{ACE}}{\sigma_{AE}} + \frac{\sigma_{BD}}{\sigma_{BCD}} + \frac{\sigma_{DE}}{\sigma_{DCE}}) = \frac{7}{24}$$

node i	A	B	C	D	E
$bc(i)$	0	$\frac{1}{24}$	$\frac{7}{24}$	0	0

3.Closeness centrality

Calculated based on the formula $C(i) = \frac{N-1}{\sum_{v \neq i} d(i,v)}$

C(A) $= \frac{5-1}{1+1+2+2} = \frac{2}{3}$ C(B) $= \frac{5-1}{1+1+1+2} = \frac{4}{5}$ C(C) $= \frac{5-1}{1+1+1+1} = 1$ C(D) $= \frac{5-1}{2+2+1+2} = \frac{4}{7}$
C(E) $= \frac{5-1}{2+1+1+2} = \frac{2}{3}$

node i	A	B	C	D	E
$C(i)$	$\frac{2}{3}$	$\frac{4}{5}$	1	$\frac{4}{7}$	$\frac{2}{3}$

4.k-shell
According to the definition, 1-core includes nodes A,B,C,D,E. Then 2-core includes nodes A,B,C,E. So Node D belongs to 1-shell. 3-core is nothing because when remove A and E, since their degree is less than 3,the degree of node B and C becomes 1 and needs to be removed. So Node A,B,C,E all belong to 2-shell.

node i	A	B	C	D	E
$k_s(i)$	2	2	2	1	2

b) The visualization is shown below.

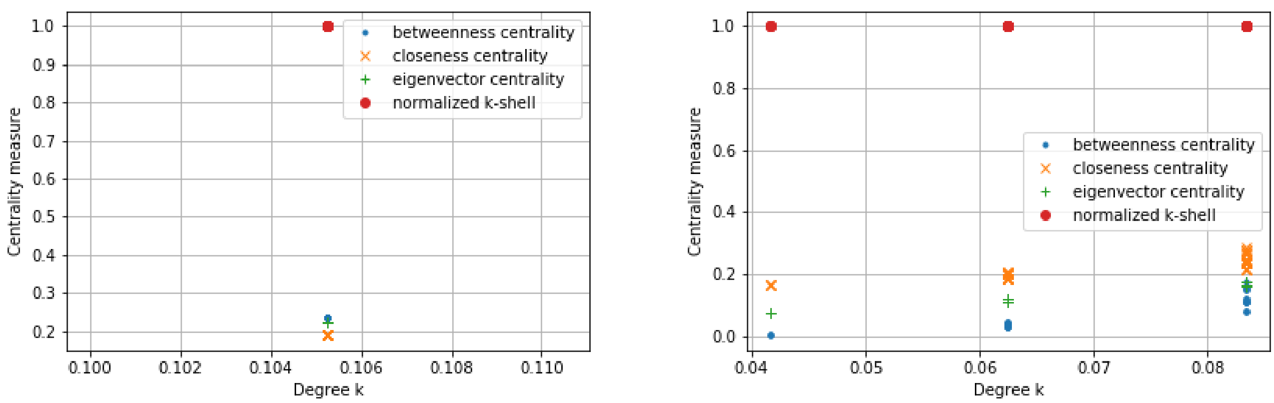

Figure 1: centrality measures of scatter ring(left) and scatter lattice(right).

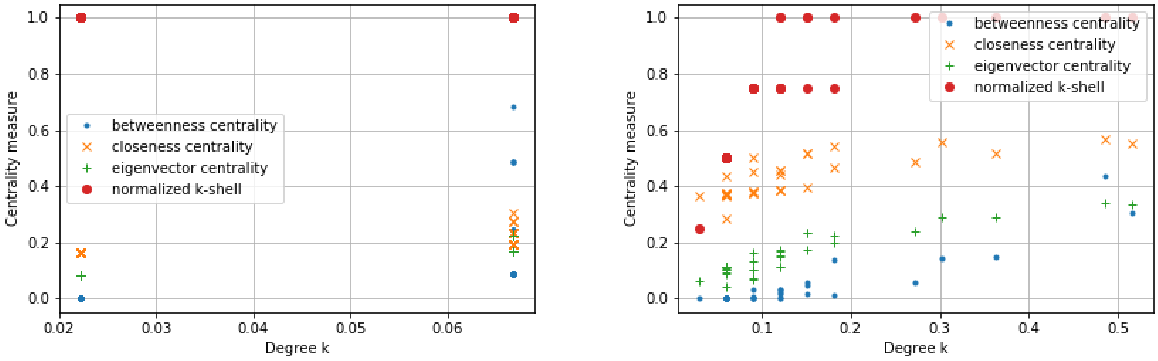

Figure 2: centrality measures of cayley tree(left) and karate(right).

c) The visualization is shown below.

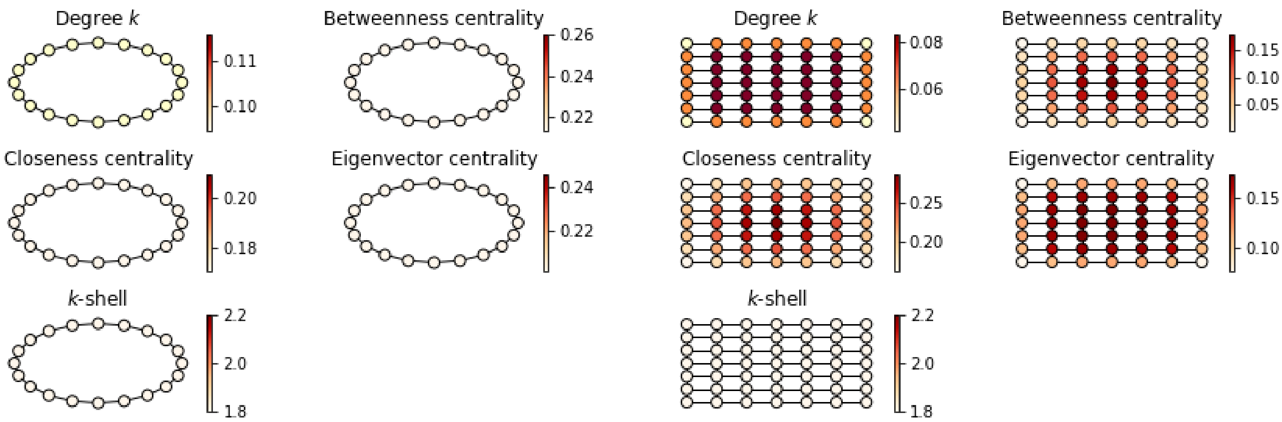

Figure 3: network figure of scatter ring(left) and scatter lattice(right).

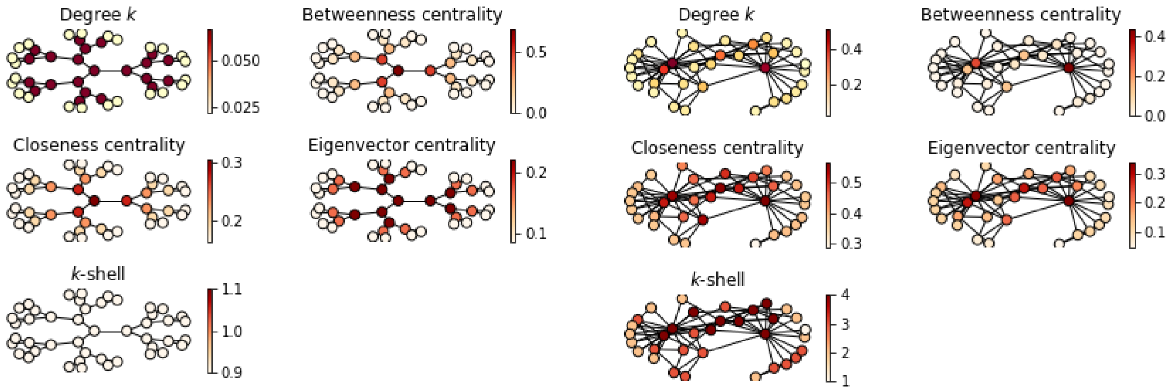

Figure 4: network figure of cayley tree(left) and karate(right).

d) For results of part a, the degree centrality only based on the number of neighbors of each node. For example, node A has degree centrality as 2. But for the betweenness centrality, node A's value is 0, since though it has 2 neighbors, there is no shortest path between other nodes via node A. As for the closeness centrality, node A's value is $\frac{2}{3}$ as the inverse of the average shortest path length. And for k-shell, it belongs to 2-core but not 3-core, so it has the same value as node B,C,E as 2. That's how these cetralities diffe from each other of part a.

For results of part b, I will explain it together with the result of part c, for the rings, every node could be seen as at the same position, so every node has the same betweenness centrality, closeness centrality,eigenvector centrality and normalized k-shell centrality. But the value is different, for example, the k-shell is 1 since every node is in the 1-core not in the 2-core, but the closeness centrality and betweenness centrality is quite low because the network is not well-connected. For the scatter lattice graph, it has nodes with degree 2 and 3 and 4. Even the node with the same degree might have different closeness and betweenness and eigenvector centrality because they are several different positions. But the trend is the node with more central position in the graph most of the time holds the highest centrality score for even several different centrality method. The cayley tree shares the similar properties.And we could observe that with higher degree centrality, more probability that the nodes also holds the higher eigenvector centrality, but for every node in cayley tree, they share the same k-shell centrality since once you remove a node with degree 1, the other nodes will become degree 1 iteratively. The most complex network is karate and this one also similar to network in the real world. We could observe very different distribution of several centrality measures.

Problem 2

a) See the plot.

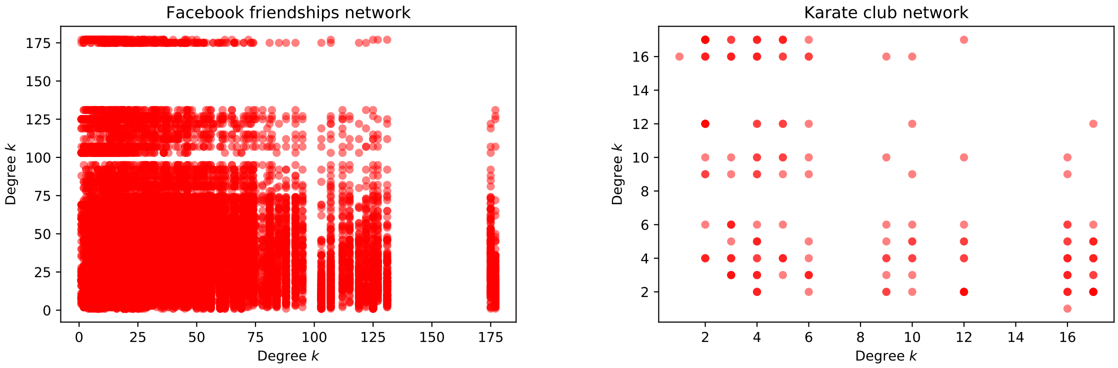

Figure 5: Edge degree correlation scatter plot

b) See the heatmap. And we could find for karate club network, both scatter plot and

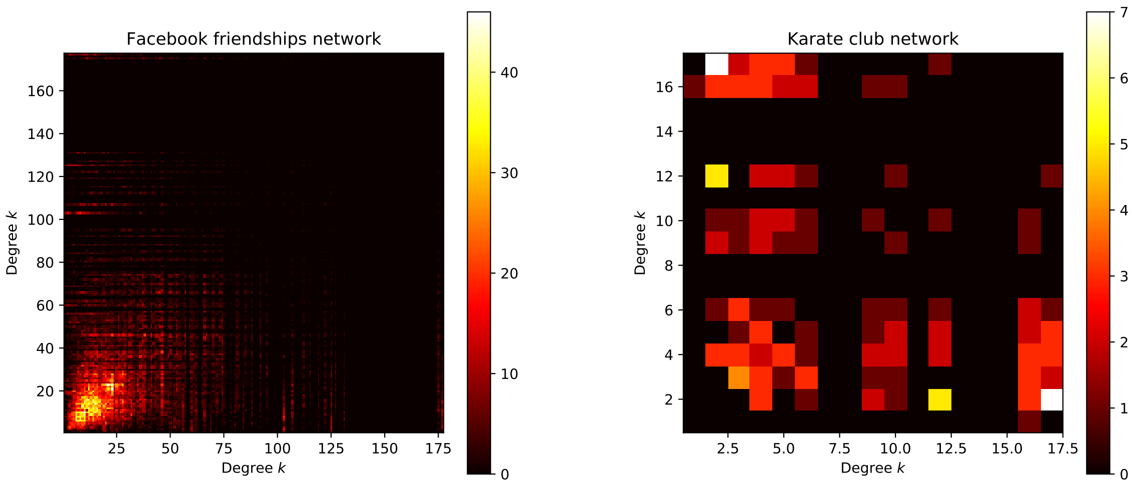

Figure 6: Heat map

heat map give similar information. But for facebook friendships network, we could see from the heat map that there are a large amount of nodes with low degree and most of the nodes have degree lower than 40. Since the scatter plot only plots the point and overlapping happened, we might think there are quite a number of nodes with degree between 100-125, while actually we observe from heatmap that the amount might even be ignored.

c) The assortativity coefficient is calculated and shows the same result. Karate-club network is not assortative. The reason is that the karate-club network is quite sparse, it has only 34 nodes and 78 edges. There are not enough node with high degree to connect to each other, so it performs like disassortativity. For facebook network, it shows assortativity.

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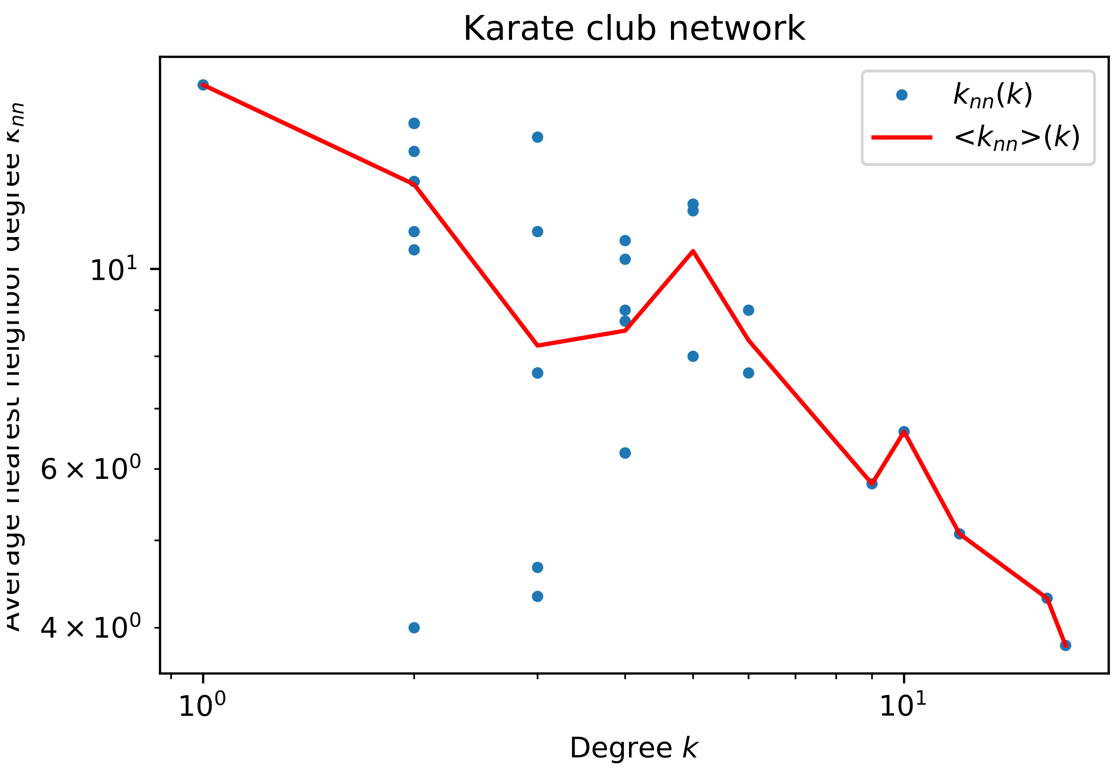
Own assortativity for Karate club network: -0.4756130976846139
NetworkX assortativity for Karate club network: -0.47561309768461457
Own assortativity for Facebook friendships network: 0.05598478476593258
NetworkX assortativity for Facebook friendships network: 0.05598478476593048
```

d) The plot is shown below. This helps us verify what we analyse just now. For the karate club network, the average neighbour degree is on the trend keep decreasing when the degree is increasing. So this means it is disassortativity. As for the facebook friendship networks, the knn shows a fluctuation but still keeps an average of the similar number when degree increases. So it verifies the assortativity of the facebook network.



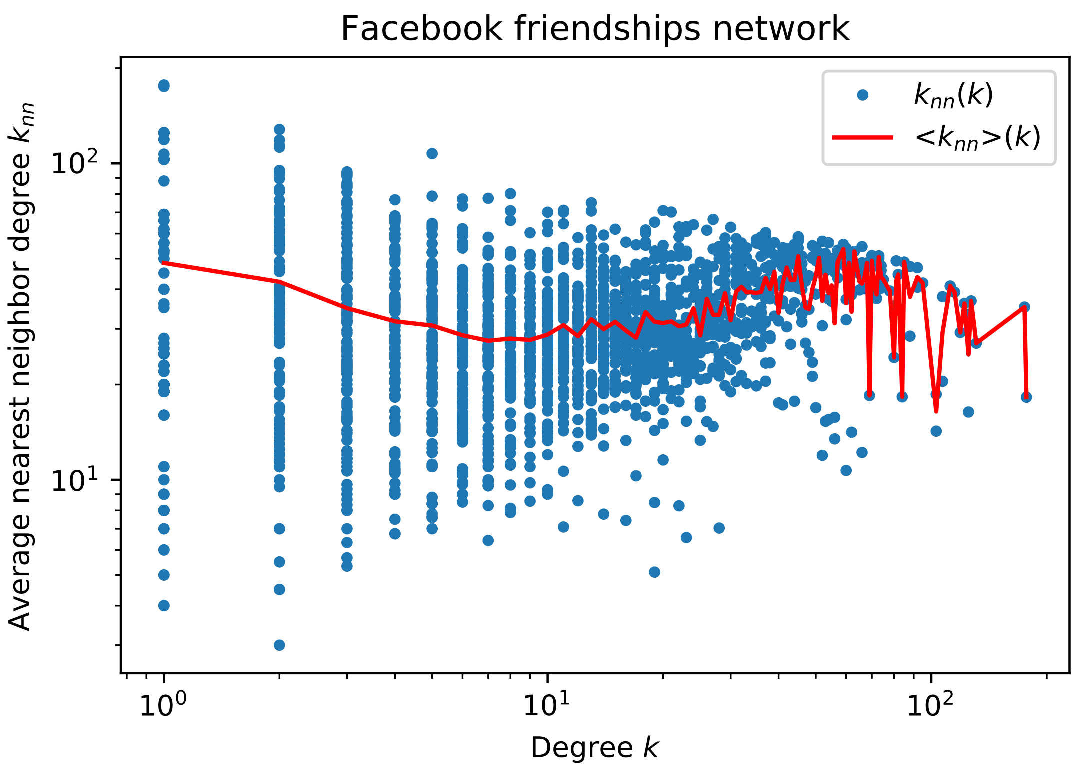
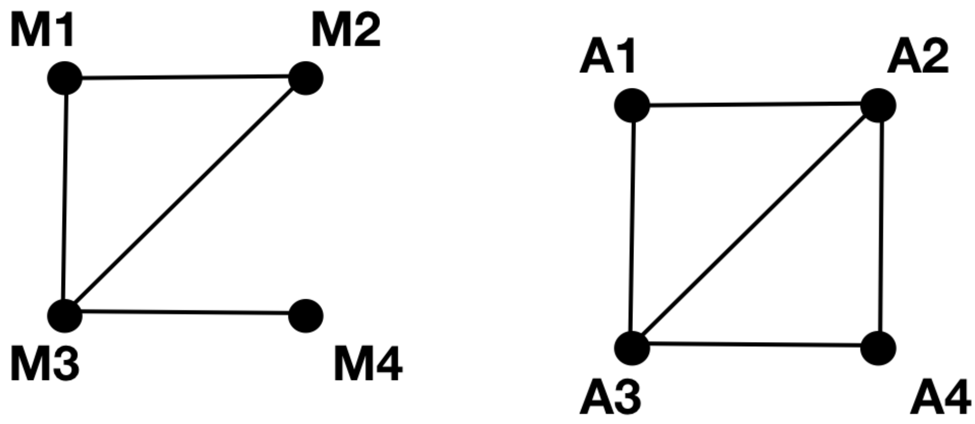


Figure 7: Average Nerest Neighbour degree

Problem 3

a) The two unipartite projections of the network is constructed below.



b) Prove by providing a counterexample.