CS-E5740 Complex Networks, Answers to exercise set 2

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Problem 1

a) Based on the question, we could calculate the table below.

Ensemble i	Probability π	Average Degree k	Clustering Coefficient c	Diameter d^*
1	$(1-p)^3$	0	0	0
2	$p(1-p)^2$	$\frac{2}{3}$	0	1
3	$p(1-p)^2$	$\frac{2}{3}$	0	1
4	$p(1-p)^2$	$\frac{2}{3}$	0	1
5	$p^2(1-p)$	$\frac{4}{3}$	0	2
6	$p^2(1-p)$	$\frac{4}{3}$	0	2
7	$p^2(1-p)$	$\frac{4}{3}$	0	2
8	p^3	2	1	1

According to the table, we could calculate that

$$\langle k \rangle = \sum_i \pi_i k(G_i) = \frac{2}{3}$$

$$\langle c \rangle = \sum_i \pi_i c(G_i) = p^3 = \frac{1}{27}$$

$$\langle d^* \rangle = \sum_i \pi_i d^*(G_i) = \frac{25}{27}$$

b)

$$\langle k \rangle = \sum_i \pi_i k(G_i) = \frac{2}{3} * p(1-p)^2 * 3 + \frac{4}{3} * p^2 * (1-p) * 3 + p^3 * 2 = 2p$$

$$\langle c \rangle = \sum_i \pi_i c(G_i) = p^3$$

$$\langle d^* \rangle = \sum_i \pi_i d^*(G_i) = 3p(1-p)^2 + 6p^2(1-p) + p^3 = -2p^3 + 3p$$

Problem 2

a) Formula explanation in detail:
$\binom{N-1}{k}$: For a node with degree k, select k nodes out of $N-1$ nodes to connect with.
p^k: Probability of having k edges.
$(1-p)^{(N-1)-k}$: Probability of missing the rest of the $(N-1-k)$ edges.

b) Since

$$C_i = \frac{2e_i}{k_i(k_i-1)}$$

, where e_i is the number of edges between i's neighbors, k_i is the degree of node i. Edges in graph appear i.i.d with probability of p. So

$$e_i = p * \frac{k_i(k_i-1)}{2}$$

. So

$$C_i = \frac{p * k_i * (k_i-1)}{k_i(k_i-1)} = p$$

Hence $\langle c \rangle$ = p

c)

$$C_i = \frac{p * k_i * (k_i - 1)}{k_i(k_i - 1)} = p = \frac{\langle k \rangle}{N - 1} \approx \frac{\langle k \rangle}{N}$$

So if $N \to \infty$ with $\langle k \rangle$ bounded, $\langle c \rangle$ goes to zero.

d) The result is plotted below.

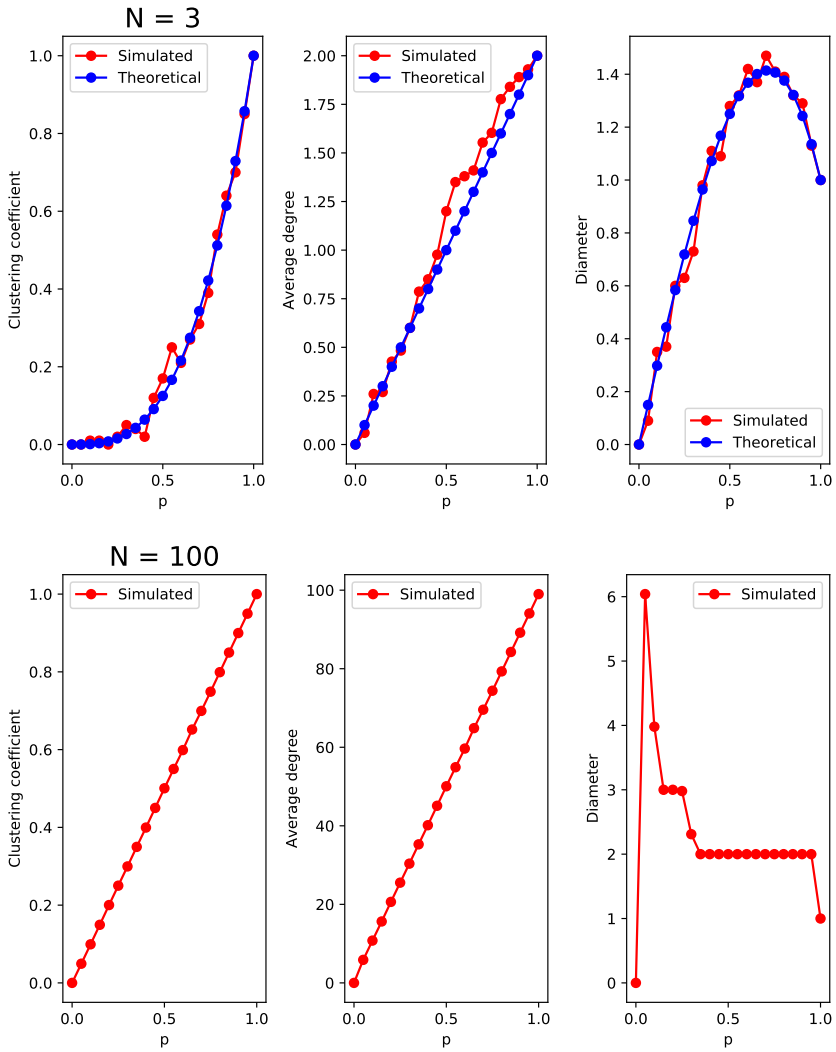

Figure 1: ER properties of 3 and 100 nodes

Problem 3

a) The visulization figure is shown below.
The total number of links is 30 and number of rewired links is 3.
Network with N = 100, the total number of links is 200 and number of rewired links is 104.

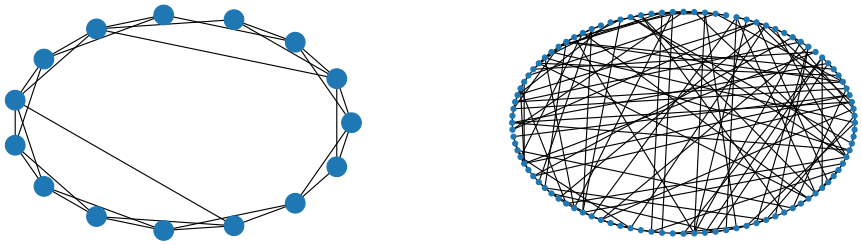

Figure 2: Watts Strogatz small world of 15 and 100 nodes

b) The plotted picture is shown as below.

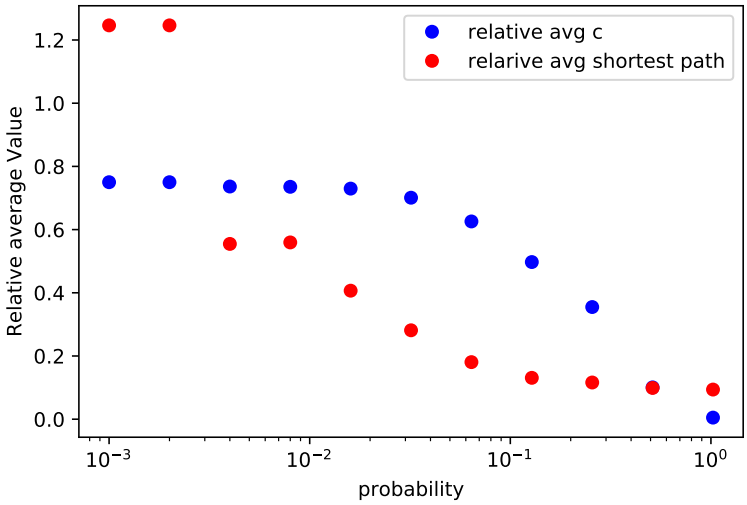

Figure 3: Relative average cc and spl