CS-E5740 Complex Networks, Answers to exercise set 3

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Problem 1 7/7

a) Write down degree of the node with the highest degree: 35
Write down the total number of links in your generated network: 200.
Visualize the network.

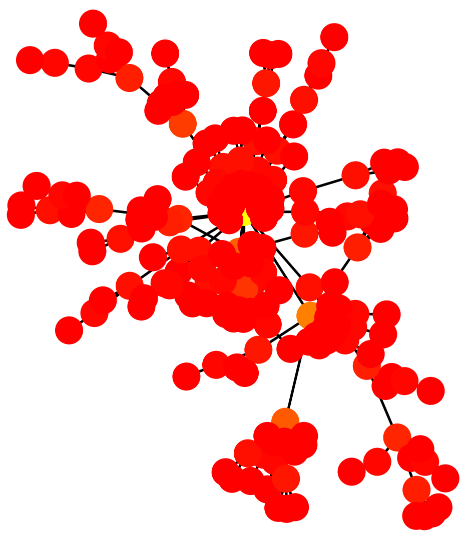
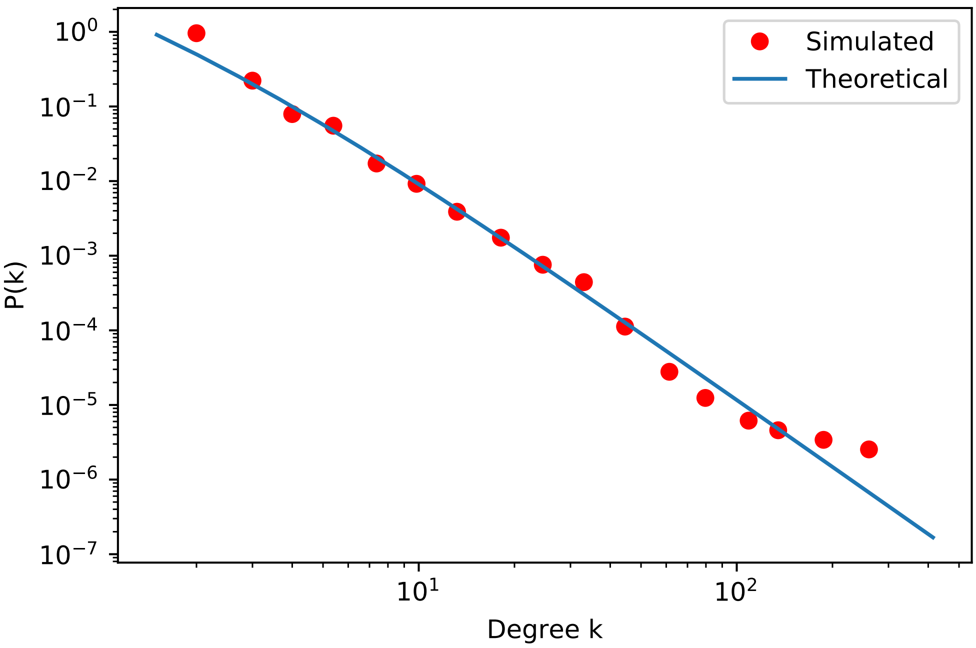


Figure 1: BA Visualized



b)

Figure 2: BA Degree Distribution

Problem 2

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a)

$$\begin{aligned}\prod(k) &= Np_{k,N} * \prod_i \\ &= Np_{k,N} * \frac{k}{\sum_{j=1}^{N} k_j} \\ &= Np_{k,N} * \frac{k}{2*m*t} \\ &= Np_{k,N} * \frac{k}{2*m*N} \\ &= \frac{kp_{k,N}}{2m}\end{aligned} \tag{1}$$

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b) According to how this network will generate, when $k = m$, $n_k^+ = 1$ because the new adding node will have degree m, so there will be one more node with degree m. and there are no nodes with degree <m

$$\begin{aligned}(N+1)p_{k,N+1} - Np_{k,N} &= n_k^+ - n_k^- \\ &= \begin{cases} \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}k * p_{k,N} & k > m \\ 1 - \frac{1}{2}k * p_{k,N} & k = m \end{cases}\end{aligned} \tag{2}$$

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c)

$$
\begin{aligned}
p_k &= (N+1)p_k - N * p_k \\
&= (N+1)p_{k,N+1} - N * p_{k,N} \\
&= \frac{1}{2}(k-1)p_{k-1} - \frac{1}{2}kp_k
\end{aligned} \tag{3}
$$

Hence

$$p_k = \frac{k-1}{k+2}p_{k-1}$$

$$
\begin{aligned}
p_m &= (N+1)p_m - Np_m \\
&= 1 - \frac{1}{2}mp_m
\end{aligned} \tag{4}
$$

Hence

$$p_m = \frac{2}{m+2}$$

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d) when k = m,

$$p_k = p_m = \frac{2}{m+2}$$

When k = m+1,

$$p_{m+1} = p_k = \frac{1}{2}m * p_m - \frac{1}{2}(m+1)p_k = \frac{1}{2}m\frac{2}{m+2} - \frac{1}{2}(m+1)k$$

Then,

$$p_k = \frac{2m}{(k+1)(k+2)}$$

When k = m+2,

$$p_{m+2} = p_k = \frac{1}{2}(m+1)p_{m+1} - \frac{1}{2}kp_k = \frac{1}{2}(m+1)\frac{2m}{k(k+1)} - \frac{1}{2}kp_k$$

Then

$$p_k(1 + \frac{1}{2}k) = \frac{1}{2}(m+1)\frac{2m}{k(k+1)}$$

Then

$$(2+k)p_k = \frac{2m(m+1)}{k(k+1)}$$

Then

$$p_k = \frac{2m(m+1)}{k(k+1)(k+2)}$$

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So this holds for k=m+2.

To see the pattern repeating you would have to calculate at least one more step (k=m+3) or use e.g. induction proof