

# CS-E5740 Complex Networks, Answers to exercise set 3

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## Problem 1

- a) Write down degree of the node with the highest degree: 35  
Write down the total number of links in your generated network: 200.  
Visualize the network.

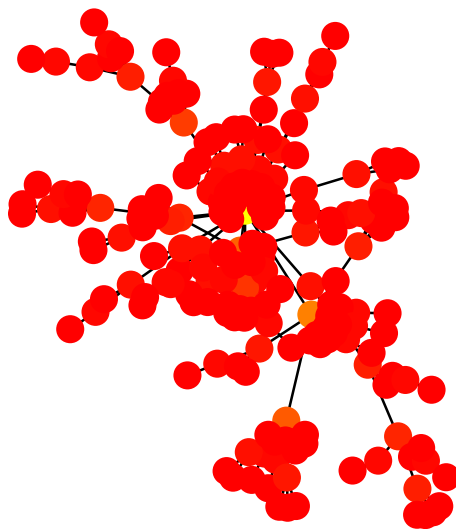
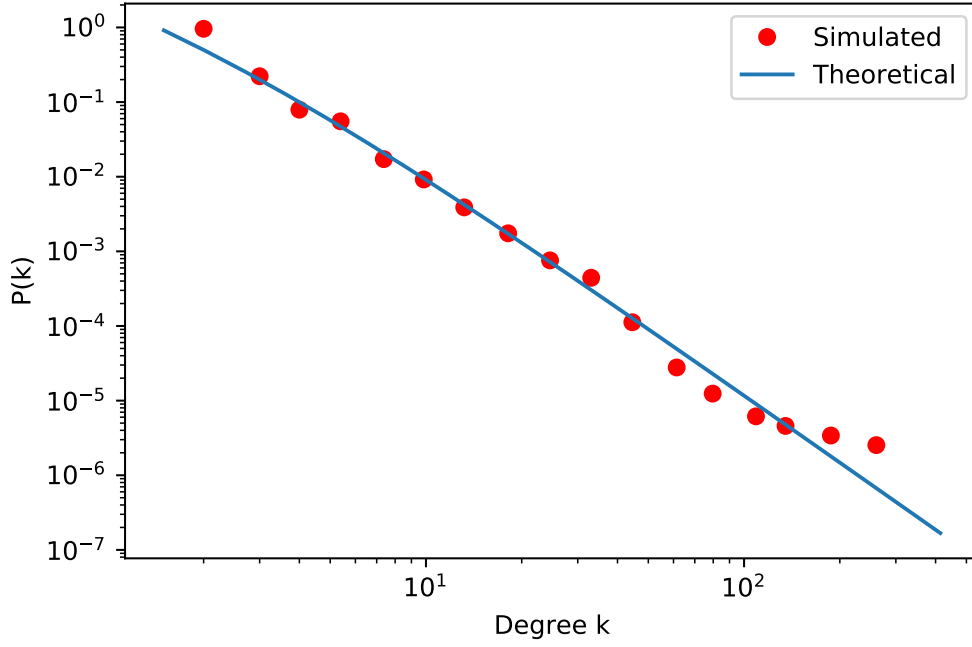


Figure 1: BA Visualized



b)

Figure 2: BA Degree Distribution

## Problem 2

a)

$$\begin{aligned}
 \Pi(k) &= Np_{k,N} * \prod_i \\
 &= Np_{k,N} * \frac{k}{\sum_{j=1}^N k_j} \\
 &= Np_{k,N} * \frac{k}{2 * m * t} \\
 &= Np_{k,N} * \frac{k}{2 * m * N} \\
 &= \frac{kp_{k,N}}{2m}
 \end{aligned} \tag{1}$$

b) According to how this network will generate, when  $k = m$ ,  $n_k^+ = 1$  because the new adding node will have degree m, so there will be one more node with degree m.

$$\begin{aligned}
 (N+1)p_{k,N+1} - Np_{k,N} &= n_k^+ - n_k^- \\
 &= \begin{cases} \frac{1}{2}(k-1)p_{k-1,N} - \frac{1}{2}k * p_{k,N} & k > m \\ 1 - \frac{1}{2}k * p_{k,N} & k = m \end{cases}
 \end{aligned} \tag{2}$$

c)

$$\begin{aligned}
 p_k &= (N+1)p_k - N * p_k \\
 &= (N+1)p_{k,N+1} - N * p_{k,N} \\
 &= \frac{1}{2}(k-1)p_{k-1} - \frac{1}{2}kp_k
 \end{aligned} \tag{3}$$

Hence

$$\begin{aligned}
 p_k &= \frac{k-1}{k+2}p_{k-1} \\
 p_m &= (N+1)p_m - Np_m \\
 &= 1 - \frac{1}{2}mp_m
 \end{aligned} \tag{4}$$

Hence

$$p_m = \frac{2}{m+2}$$

d) when  $k = m$ ,

$$p_k = p_m = \frac{2}{m+2}$$

When  $k = m+1$ ,

$$p_{m+1} = p_k = \frac{1}{2}m * p_m - \frac{1}{2}(m+1)p_k = \frac{1}{2}m \frac{2}{m+2} - \frac{1}{2}(m+1)k$$

Then,

$$p_k = \frac{2m}{(k+1)(k+2)}$$

When  $k = m+2$ ,

$$p_{m+2} = p_k = \frac{1}{2}(m+1)p_{m+1} - \frac{1}{2}kp_k = \frac{1}{2}(m+1) \frac{2m}{k(k+1)} - \frac{1}{2}kp_k$$

Then

$$p_k(1 + \frac{1}{2}k) = \frac{1}{2}(m+1) \frac{2m}{k(k+1)}$$

Then

$$(2+k)p_k = \frac{2m(m+1)}{k(k+1)}$$

Then

$$p_k = \frac{2m(m+1)}{k(k+1)(k+2)}$$