CS-E5740 Complex Networks, Answers to exercise set X

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Compile with `pdflatex ex_template.tex`

Problem 1

a) If $ij \in E$, then $a(ij) = 1$ and vice versa. So the adjacency matrix A of the graph is as below:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

b) According to graph, we could easily find the edge is 9 and node is 8. The edge density of the graph is

$$rho = \frac{m}{\binom{N}{2}} = \frac{2m}{N(N-1)} = \frac{2*9}{8*7} = \frac{18}{56} = 0.3214$$

c) The degree of k_i is the number of the edges it is incident to. So it should be as:

$$k_1 = 1 \qquad k_2 = 1 \qquad k_3 = 2 \qquad k_4 = 5$$

$$k_5 = 3 \qquad k_6 = 3 \qquad k_7 = 2 \qquad k_8 = 1$$

The degree distribution is $P(K) = \frac{N_k}{N}$ where N_k represents the number of nodes of degree k. So the degree distribution should be:

$$P_1 = \frac{3}{8} = 0.375 \quad P_2 = \frac{2}{8} = 0.25$$

$$P_3 = \frac{2}{8} = 0.25 \quad P_5 = \frac{1}{8} = 0.125$$

d) The mean degree $\langle k \rangle$ of the graph should be:

$$\langle k \rangle = \sum_i \frac{k_i}{N} = \frac{2m}{N} = \frac{2 * 9}{8} = 2.25$$

e) The diameter d of the graph is the maximum distance. In this graph the maximum distance is 4, hence the diameter

$$d = max(d_{ij}) = 4$$

f) Firstly, the clustering coefficient for node whose degree > 1 is calculated using the formula

$$C_i = \frac{E_i}{\binom{k_i}{2}} = \frac{2E_i}{k_i(k_i - 1)}$$

$$C_3 = \frac{2 * 1}{2 * (2 - 1)} = 1 \qquad C_4 = \frac{2 * 2}{5 * (5 - 1)} = 0.2$$

$$C_5 = \frac{2 * 2}{3 * (3 - 1)} = \frac{2}{3} = 0.667 \qquad C_6 = \frac{2 * 1}{3 * (3 - 1)} = \frac{1}{3} = 0.333$$

$$C_7 = \frac{2 * 0}{2 * (2 - 1)} = 0$$

Then we calculate the average clustering coefficient below:

$$c = \frac{1}{N} \sum_i C_i = \frac{1}{8} * (0 + 0 + 1 + 0.2 + \frac{2}{3} + \frac{1}{3} + 0 + 0) = 0.275$$

Problem 2

Please check the code in the jupyterbook.

a) Load the edge list and see Figure 1.

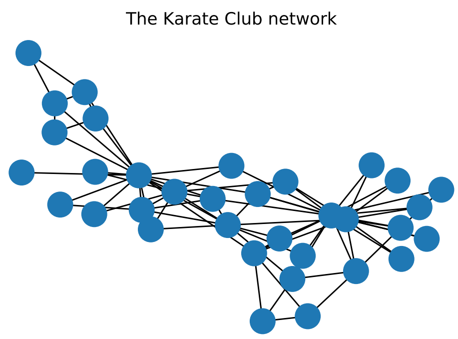


Figure 1: The network visualization.

b) Calculate the edge density.

```
D from self-written algorithm: 0.13903743315508021
D from NetworkX function: 0.13903743315508021
```

Figure 2: Edge Density.

c) Compare the clustering coefficient. Code in jupyter notebook.

```
C from self-written algorithm: 0.5706384782076824
C from NetworkX function: 0.5706384782076824
```

Figure 3: Clustering Coefficient.

d) Calculate the degree distribution and 1-CDF. Code in Jupyter notebook.

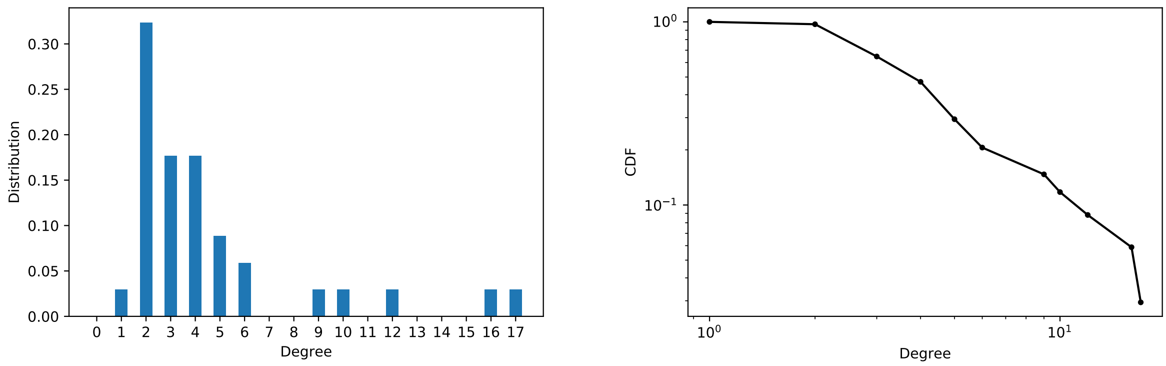


Figure 4: degree distribution and 1-CDF.

e) Calculate the average shortest path length.

<l> from NetworkX function: 2.408199643493761

Figure 5: Average Shortest Path Length

f) Scatter plot.

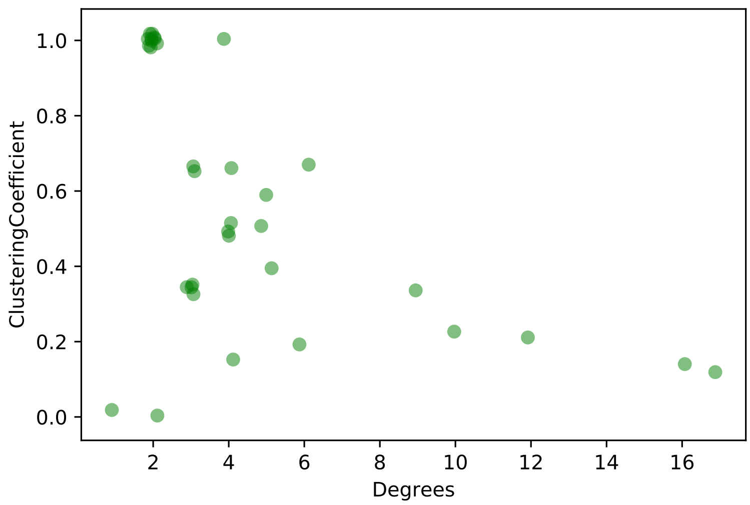
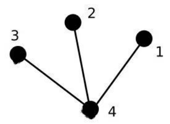


Figure 6: Scatter Plot.

Problem 3

a) The subgraph is as below.



Calculate the number of walks between all pair node is 12.
Detail:

$$\{1,1\}: 1-4-1 \quad \{1,2\}: 1-4-2 \quad \{1,3\}: 1-4-3$$

$$\{2,1\}: 2-4-1 \quad \{2,2\}: 2-4-2 \quad \{2,3\}: 2-4-3$$

$$\{3,1\}: 3-4-1 \quad \{3,2\}: 3-4-2 \quad \{3,3\}: 3-4-3$$

$$\{4,4\}: 4-1-4, 4-2-4, 4-3-4$$

Compute matrix A^2

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Compare the result, we could easily find that the matrix A^2 represents the number of 2-length-walk in the graph. For example, from node 4 to node 4 there are 3 walks of length 2, and we could also see from the matrix $A_{44} = 3$

b) Compute the number of walks of length three from node 3 to node 4 is 3.
Detail:

$$3-4-3-4, \quad 3-4-2-4, \quad 3-4-1-4$$

$$A^3 = A^2 * A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \\ 3 & 3 & 3 & 0 \end{bmatrix}$$

From the matrix,

$$(A^3)_{34} = 3$$

c) By the definition of A^1, which is the adjacency matrix, it is obvious that the element $A^1_{i,j}$ corresponds to the number of walks of length 1 between node i and node j.
We assume that this statement holds for a general m. Then for (m+1), we compute

$$a^{(m+1)}_{i,j} = \sum (a^{(m)}_{i,k} * a^{(1)}_{k,j}), (k \in V(G))$$

where $a^{(m)}_{i,j}$ represents the number of the walks of length m. It means that for the number of walks of length m+1, it equals to the sum of all the possible walk with number of walks of length m from node i to node k, while node k will reach j in another length 1 walk.Since $(A^m)_{i,j} = a^{(m)}_{i,j}$, so the formula above represents exactly $(A^{(m+1)})_{i,j}$
Hence, the element $A^m_{i,j}, m \in N$ corresponds to the number of walks of length m between nodes i and j.