Homework 4.　　徐梓文 2410306105.

一、(1) 考虑 $Ak(R\ T^T) = \begin{pmatrix}0&-1&0&1\\-1&0&0&0\\0&0&1&3\end{pmatrix}$，$P_w$ 的齐次坐标为 $P_w=\begin{pmatrix}4\\3\\-1\\1\end{pmatrix}$.

~~则~~ $\cancel{K(R\ T^T)P_w = \begin{pmatrix}0&-1&0&1\\-1&0&0&0\\0&0&1&3\end{pmatrix}}$ 则 $K(R\ T^T)P_w = \begin{pmatrix}5&0&6\\0&5&4\\0&0&1\end{pmatrix}\begin{pmatrix}0&-1&0&1\\-1&0&0&0\\0&0&1&3\end{pmatrix}\begin{pmatrix}4\\3\\-1\\1\end{pmatrix}$

$= \begin{pmatrix}0&-5&6&23\\-5&0&4&12\\0&0&1&3\end{pmatrix}\begin{pmatrix}4\\3\\-1\\1\end{pmatrix} = \begin{pmatrix}2\\-12\\2\end{pmatrix} = 2\begin{pmatrix}1\\-6\\1\end{pmatrix}$. 故图像坐标系下的坐标为 $(1,-6)$.

(2) 记 $A=K(R\ T)$，则 $A_1 = \begin{pmatrix}5&0&6\\0&5&4\\0&0&1\end{pmatrix}\begin{pmatrix}0&-1&0&1\\-1&0&0&0\\0&0&1&0\end{pmatrix} = \begin{pmatrix}0&-5&6&5\\-5&0&4&0\\0&0&1&0\end{pmatrix}$.

$A_2 = \begin{pmatrix}4&0&1\\0&4&16\\0&0&1\end{pmatrix}\begin{pmatrix}0&1&0&1\\0&0&-1&2\\-1&0&0&2\end{pmatrix} = \begin{pmatrix}-1&4&0&6\\-16&0&-4&40\\-1&0&0&2\end{pmatrix}$.

则有 $\begin{pmatrix}0&-5&6&5\\-5&0&4&0\\0&0&1&0\end{pmatrix}\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix} = w_1\begin{pmatrix}5\\3\\1\end{pmatrix}$，即 $A_2\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix} = w_2\begin{pmatrix}13\\4\\1\end{pmatrix}$.

即 $\begin{pmatrix}0&-5&6&-5&0\\-5&0&4&-3&0\\0&0&1&-1&0\\-1&4&0&0&-13\\4&0&1&0&1\\+1&0&0&0&1\end{pmatrix}\begin{pmatrix}X\\Y\\Z\\w_1\\w_2\end{pmatrix} = \begin{pmatrix}-5\\0\\0\\-6\\10\\2\end{pmatrix}$，解得 $\begin{pmatrix}X\\Y\\Z\\w_1\\w_2\end{pmatrix} = \begin{pmatrix}1\\2\\5\\5\\1\end{pmatrix}$.

故三维坐标 $P_w=(1,2,5)$.

二、绕z轴顺时针旋转α，则有 $\begin{cases}x' = r\cos(\theta-\alpha) = r\cos\theta\cos\alpha + r\sin\theta\sin\alpha = x\cos\alpha + y\sin\alpha\\ y' = r\sin(\theta-\alpha) = r\sin\theta\cos\alpha - r\cos\theta\sin\alpha = y\cos\alpha - x\sin\alpha\\ z' = z.\end{cases}$

则 $R_\alpha = \begin{pmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}$.

~~绕y轴顺时针旋转β，有~~ $\cancel{x' = r\cos(\theta-\beta) = x\cos\beta + z\sin\beta}$

$\cancel{y' = y}$

$\cancel{z' = r\sin(\theta-\beta) = z\cos\beta - x\sin\beta}$.

~~则~~ $\cancel{R_\beta = \begin{pmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{pmatrix}}$

绕y轴顺时针旋转γ，初态有 $\begin{cases} x = r\sin\theta \\ y = y \\ z = r\cos\theta \end{cases}$

旋转后，$\begin{cases} x' = r\sin(\theta-\gamma) = r\sin\theta\cos\gamma - r\cos\theta\sin\gamma = x\cos\gamma - z\sin\gamma \\ y' = y \\ z' = r\cos(\theta-\gamma) = r\cos\theta\cos\gamma + r\sin\theta\sin\gamma = z\cos\gamma + x\sin\gamma \end{cases}$

则 $R_\gamma = \begin{pmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{pmatrix}$.

绕x轴逆时针旋转β，初态有 $\begin{cases} x = x \\ y = r\sin\theta \\ z = r\cos\theta \end{cases}$

旋转后 $\begin{cases} x' = x \\ y' = r\sin(\theta+\beta) = r\sin\theta\cos\beta + r\cos\theta\sin\beta = y\cos\beta + z\sin\beta \\ z' = r\cos(\theta+\beta) = r\cos\theta\cos\beta - r\sin\theta\sin\beta = z\cos\beta - y\sin\beta \end{cases}$

则 $R_\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix}$.

则 $R = R_\beta R_\gamma R_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} \cos\gamma & 0 & -\sin\gamma \\ \sin\beta\sin\gamma & \cos\beta & \sin\beta\cos\gamma \\ \cos\beta\sin\gamma & -\sin\beta & \cos\beta\cos\gamma \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cancel{\cos\alpha\cos\gamma \quad \sin\alpha\cos\gamma} \quad -\sin\gamma \\ \cancel{\cos\alpha\sin\beta\sin\gamma - s} \end{pmatrix}$

$= \begin{pmatrix} \cos\alpha\cos\gamma & \sin\alpha\cos\gamma & -\sin\gamma \\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\beta & \sin\beta\cos\gamma \\ \cos\alpha\cos\beta\sin\gamma + \sin\alpha\sin\beta & \sin\alpha\cos\beta\sin\gamma - \cos\alpha\sin\beta & \cos\beta\cos\gamma \end{pmatrix}$.

三、第1层: D, AN, V, D, A, A, AN

语法G: S → NP | VP

NP → D | AN

AN → A | AN

VP → V | NP.

第二层 ~~D, AN~~ → {D, AN} → NP, {AN, V} → φ, VD → φ, DA → φ, AA → φ, A AN → AN

第三层:
- D AN·V → Dφ → φ
- D·AN V → NP V → VP

为VP.

- AN·V|D → φ|D → φ
- AN|VD → AN|φ → φ

为φ

VDA → φ, DAA → φ,

- A|A·AN → A·AN → AN
- AA|AN → φ

为AN

第四层:
- VP D → φ
- NP φ → φ
- Dφ → φ

为φ.

- φA → φ
- φφ → φ
- ANφ → φ

- Vφ → φ
- φφ → φ
- φA → φ.

- D AN → NP
- φ AN → φ
- φ AN → φ

为NP.

第五层:
- Dφ → φ
- NP φ → φ
- VP φ → φ
- φA → φ

- AN φ → φ
- φ φ → φ
- φ φ → φ
- φA → φ

- V NP → VP
- φ AN → φ
- φ AN → φ
- φ AN → φ

为VP

第六层:
- Dφ → φ
- NPφ → φ
- VPφ → φ
- φφ → φ
- φA → φ

- AN·VP → φ
- φNP → φ
- φAN → φ
- φAN → φ
- φAN → φ

第七层:
- Dφ → φ
- NP VP → S.
- VP NP → S.
- φAN → φ
- φAN → φ
- φAN → φ

得到三角形

S

φ φ

φ φ VP

φ φ φ NP

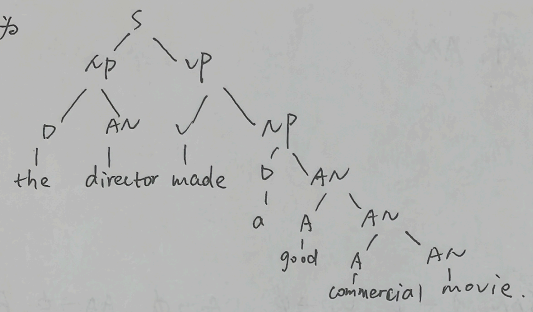
VP φ φ φ AN

NP φ φ φ φ AN

D AN V D A A ~~AN~~ AN.

满足语法

句法分析树为



四. 1#、一、FFNN.

(1) 输入：用户输入的英文影评；通过 Bag-of-Words，将文本映射为10000维向量，每个维度表示单词在影评中出现的次数；若长度不足500词补零，超出500词截断

输出：0-9星分类结果，用10维 one-hot向量表示

(2) Input layer (10000维)

↓

Fully connected layer (512维, ReLU)

↓

Dropout layer

↓

Fully connected layer (256维, ReLU)

↓

Output layer (10维, softmax)

训练参数: fully connected layer 的weight, bias.

(3) 数据集，影评文本对应星级标签，80% training set, 10% validation test, 10% test set

预处理：文本清洗（去掉停用词、标点）.

Loss function: Cross Entropy loss.

训练过程: Batch size=64，早停

(4) 预处理输入影评，转化为词袋向量，forward算出概率，取概率最高的作为星级.

Transformer

输入：影评文本；表示：词嵌入：每个词映射为256维向量，位置编码.

输出：0-9星结果.

) Input layer (500×256) → Transformer编码层(4头注意力，512维隐藏层) → 全局平均Pooling

↓

Fully connected layer(10维, softmax)

数据集，同FFNN，输入为词索引序列

Loss function: CE loss.

训练过程: Batch size=32，梯度裁剪

) 推理过程：影评分词转化为词索引序列；添加位置编码，放入transformer；取池化向量计算概率，输出概率最大的星级.