
Financial
Data
Analysis

Part 2:
Portfolio Theory

Portfolio?

두 가지 주식으로 구성한 포트폴리오: (w_1, w_2)

$$R_p = w_1 R_1 + w_2 R_2 \quad with \quad w_1 + w_2 = 1.$$

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2) = w_1 \mu_1 + w_2 \mu_2$$

$$Var(R_p) = w_1^2 Var(R_1) + w_2^2 Var(R_2) + 2 w_1 w_2 Cov(R_1, R_2)$$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho \sigma_1 \sigma_2$$

$$\le (w_1 \sigma_1 + w_2 \sigma_2)^2$$: 포트폴리오 효과

다수의 주식으로 구성한 포트폴리오: $\left(w_1, w_2, \cdots, w_n\right)$

$$R_p = \sum_{i=1}^{n} w_i R_i = \mathbf{w}'\mathbf{R}$$

$$E\left(R_p\right) = \sum_{i=1}^{n} w_i E\left(R_i\right) = \mathbf{w}' E(\mathbf{R}) = \mathbf{w}'\boldsymbol{\mu}$$

$$Var\left(R_p\right) = \mathbf{w}' Var(\mathbf{R})\mathbf{w} = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$$

Optimal portfolio

Minimum variance portfolio

두 가지 주식으로 구성한 포트폴리오의 경우:

$$R_p = wR_1 + (1-w)R_2.$$

$$\mu_p = E(R_p) = w\mu_1 + (1-w)\mu_2$$

$$\sigma_p^2 = Var(R_p) = w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_{12}$$

$$\boxed{w^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}}$$

예제

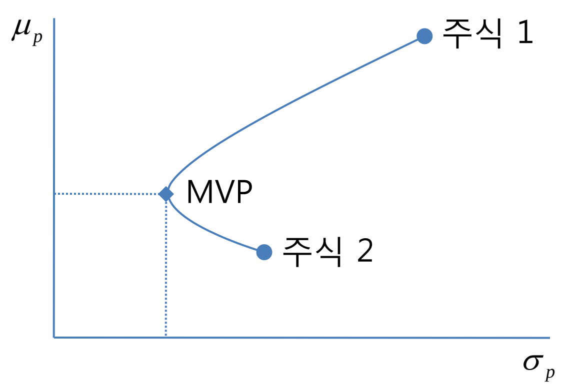
	기대수익률	표준편차
주식 1	0.1	0.12
주식 2	0.05	0.07

$$\rho = 0.5 \longrightarrow w^* = \frac{0.07^2 - 0.5 \times 0.12 \times 0.07}{0.12^2 + 0.07^2 - 2 \times 0.5 \times 0.12 \times 0.07} = 0.064$$

즉, 주식 1에 6.4%를 투자하고 나머지 93.6%를 주식 2에 투자.

$$\mu_p^* = 0.0532,$$

$$\sigma_p^{*2} = 0.486 \left(\sigma_p^* = 0.0697\right)$$



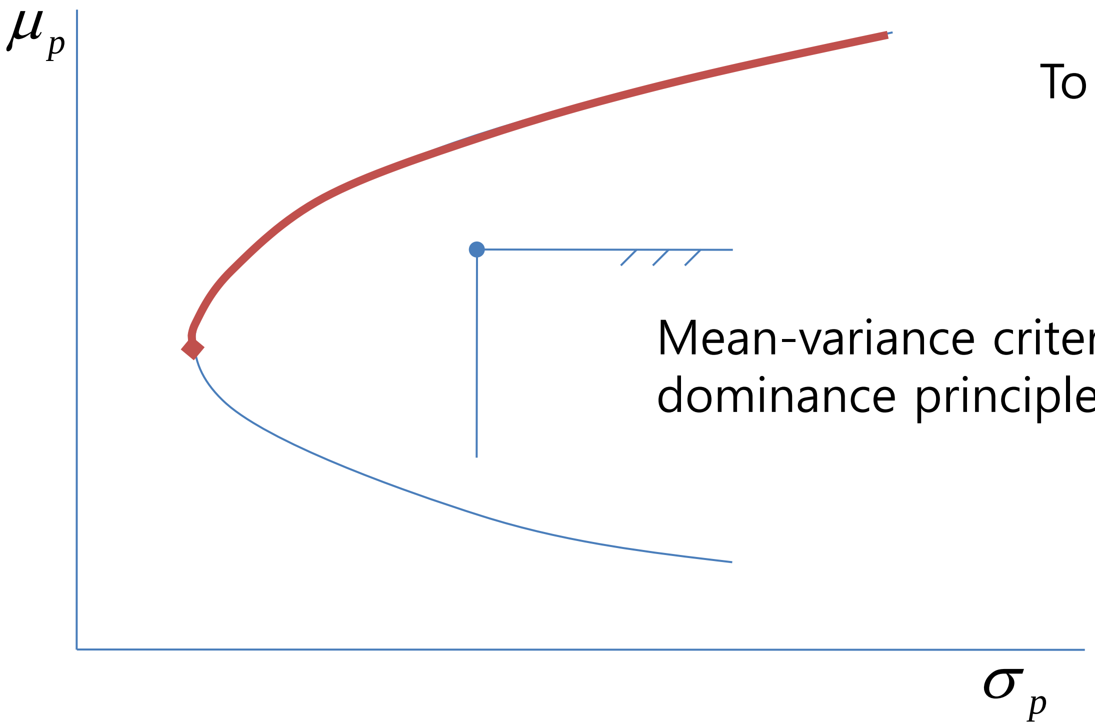
$$\rho = 0.8 \longrightarrow w^* = \frac{0.07^2 - 0.5 \times 0.12 \times 0.07}{0.12^2 + 0.07^2 - 2 \times 0.8 \times 0.12 \times 0.07} = -0.311$$

즉, 주식 1을 보유자금의 31.1%만큼 공매하여 자금을 조달하고 이를 보유자금과 합쳐 모두 주식 2에 투자.

$$\mu_p^* = 0.0345,$$

$$\sigma_p^{*2} = 0.4335 \left(\sigma_p^* = 0.0658 \right)$$

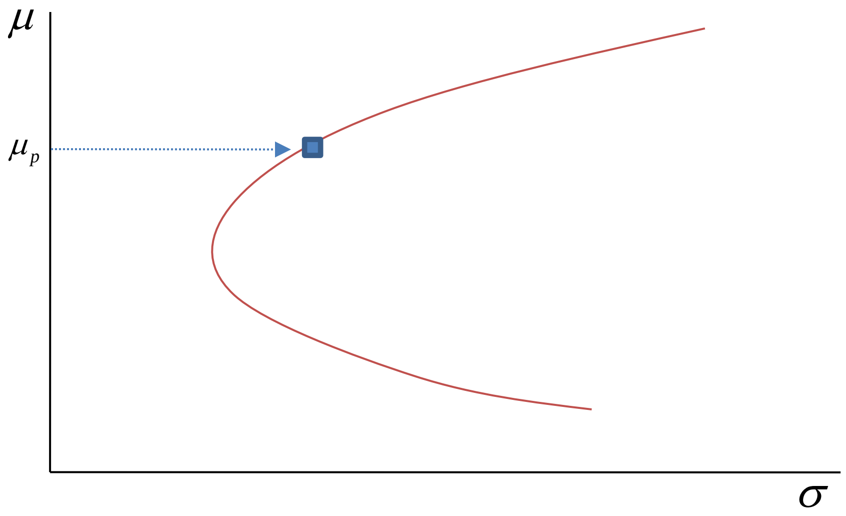
Efficient frontier



To maximize $\mathbf{w}'\boldsymbol{\mu}$ wrt $\mathbf{w}$

s.t. $\sigma^2 = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}, \quad 1 = \mathbf{1}'\mathbf{w}.$

Minimum variance portfolio



To minimize $\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$ subject to

$$\mathbf{w}'\boldsymbol{\mu} = \mu_p$$
$$\mathbf{w}'\mathbf{1} = 1.$$

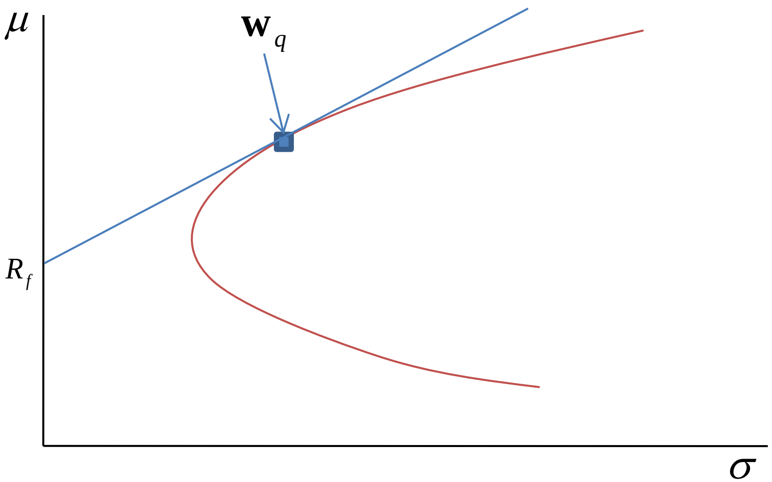
$$\boxed{\mathbf{w}_p = \mathbf{g} + \mathbf{h}\mu_p}$$

$$\mathbf{g} = \frac{B(\boldsymbol{\Sigma}^{-1}\mathbf{1}) - A(\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu})}{D}$$

$$\mathbf{h} = \frac{C(\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}) - A(\boldsymbol{\Sigma}^{-1}\mathbf{1})}{D}$$

$$A = \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \quad B = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$$
$$C = \mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}, \quad D = BC - A^2.$$

Optimal portfolio with a risk-free asset: tangency portfolio



To minimize $\mathbf{w}'\mathbf{\Sigma}\mathbf{w}$ subject to

$$\mathbf{w}'\boldsymbol{\mu} + (1 - \mathbf{w}'\mathbf{1})R_f = \mu_p.$$

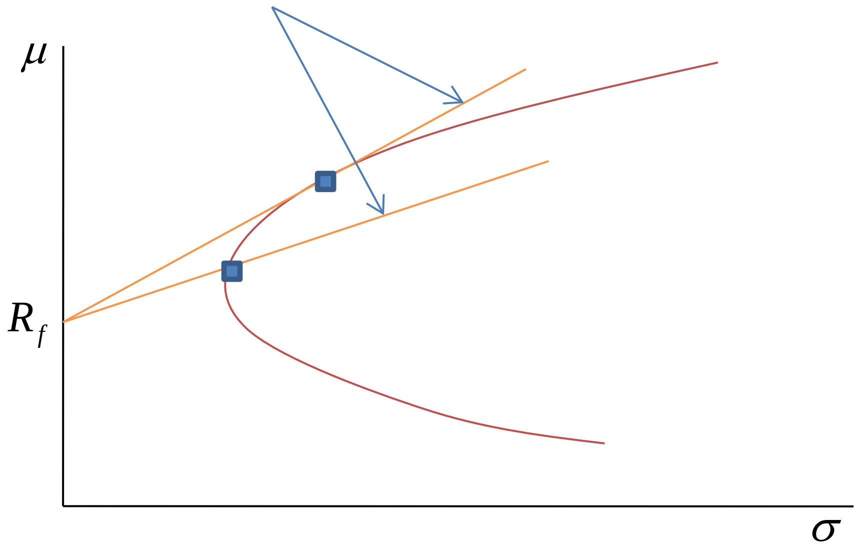
$$\mathbf{w}_p = \underbrace{\frac{\mu_p - R_f}{(\boldsymbol{\mu} - R_f\mathbf{1})'\mathbf{\Sigma}^{-1}(\boldsymbol{\mu} - R_f\mathbf{1})}}_{c_p} \cdot \underbrace{\mathbf{\Sigma}^{-1}(\boldsymbol{\mu} - R_f\mathbf{1})}_{\overline{\mathbf{w}}}$$

$$\mathbf{w}_q = \frac{\mathbf{w}_p}{\mathbf{1}'\mathbf{w}_p} = \frac{c_p\overline{\mathbf{w}}}{c_p\mathbf{1}'\overline{\mathbf{w}}} = \frac{1}{\mathbf{1}'\mathbf{\Sigma}^{-1}(\boldsymbol{\mu} - R_f\mathbf{1})} \cdot \mathbf{\Sigma}^{-1}(\boldsymbol{\mu} - R_f\mathbf{1})$$

CAPM

capital asset pricing model

CML: capital market line



무위험자산을 포함하는 포트폴리오:

$$R_p = wR_A + (1-w)R_f$$

$$\mu_p = w\mu_A + (1-w)R_f$$

$$\sigma_p^2 = w^2\sigma_A^2$$

$$\therefore \quad \underset{\text{(CML)}}{\mu_p = R_f + \frac{\mu_A - R_f}{\sigma_A} \cdot \sigma_p}$$

Market portfolio $\mathbf{w} = (w_1, w_2, \cdots, w_n)'$

Beta

$$\sigma_m^2 = \mathbf{w}'\Sigma\mathbf{w} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \sum_{i=1}^{n} w_i \sum_{j=1}^{n} w_j \sigma_{ij} = \sum_{i=1}^{n} w_i \sigma_{im}$$

주식 i을 보유함으로써 부담하게 되는 위험: $w_i \sum_{j=1}^{n} w_j \sigma_{ij} = w_i \sigma_{im}$

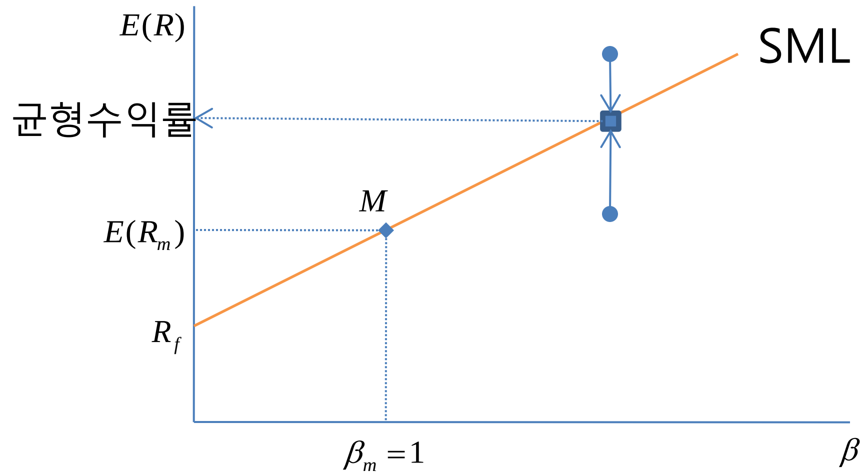
$$\frac{w_i \sigma_{im}}{\sigma_m^2} = w_i \beta_i$$

SCL: Security characteristic line $R_i = \alpha_i + \beta_i R_m$

균형 & Security market line

$$\frac{E(R_m)-R_f}{\sigma_m^2}=\frac{E(R_i)-R_f}{\sigma_{im}}$$

$$E(R_i)=R_f+\underbrace{\left(E(R_m)-R_f\right)\beta_i}_{\text{(Risk premium)}}$$



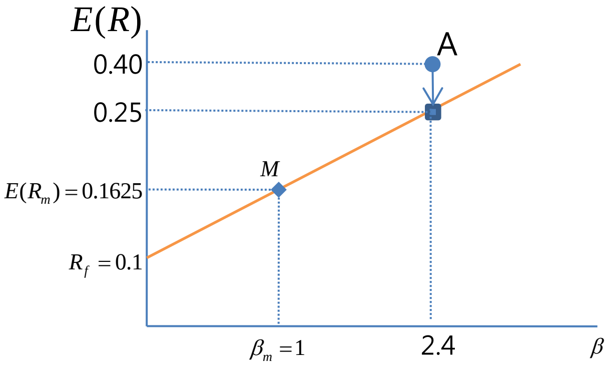
의미 자본시장의 균형상태에서는 개별 주식의 기대수익률이 베타와 선형 관계.

SML 활용 예제

A 기업 현재 주가 20,000원, 연말 1주당 배당액 2,000원
연말 예상 주가 26,000원, 베타 2.4
따라서 기대수익률은 {(26,000+2,000)-20,000}/20,000=0.4.

시장포트폴리오 기대수익률 0.1625, 무위험이자율 0.1

1. 균형수익률 0.1+(0.1625-0.1)*2.4=0.25. (by SML)
   따라서 균형가격은 (26,000+2,000)/(1+0.25)=22,400원
2. 현재 주가 20,000원은 균형가격 22,400원에 비해 저평가.
   따라서 주가가 상승할 것으로 예상.



Empirical Analysis

1. Fit $R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$ to get $\hat{\beta}_i$.

2. Fit $\overline{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \varepsilon_i$ and compare it with the theoretical security market line.

Sharpe Ratio $\dfrac{\mu_a - R_f}{\sigma_a}$

: the expected return per unit risk