Financial Data Analysis : Stock Portfolio

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1. **Portfolio**

- A portfolio is a collection of financial investments like stocks, bonds, commodities, cash, and cash equivalents, including closed-end funds and exchange-traded funds (ETFs). One of the key concepts in portfolio management is the wisdom of diversification—which simply means not to put all your eggs in one basket. Diversification tries to reduce risk by allocating investments among various financial instruments, industries, and other categories. It aims to maximize returns by investing in different areas that would each react differently to the same event.

  Ex) Portfolio consists of two stock items

$$R_p = W_1R_1 + W_2R_2(\text{단}, W_1 + W_2 = 1)$$

$$E(R_p) = W_1\mu_1 + W_2\mu_2$$

$$Var(R_p) = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\rho\sigma_1\sigma_2$$

- Portfolio Effect : An effect that occurs when you build a portfolio by collecting two or more of financial investments. Risk of each financial investment is offset by risk of other financial investments in a portfolio.

1.1 **Loading Data**

- Selected stock items
  - Aaron’s inc (AAN) : Aaron’s, Inc. is a lease-to-own retailer. The company focuses on leases and retail sales of furniture, electronics, appliances, and computers.
  - Advanced Semiconductor Engineering (ASE) : Provider of independent semiconductor assembling and test manufacturing services.
  - IBM (IBM) : IBM is an American cloud platform and cognitive solutions multinational technology and consulting company.
  - Actuant corp (ATU) : Actuant Corporation is a diversified industrial company.
  - Exxon Mobil corp (XOM) : Exxon Mobil Corporation is an American multinational oil and gas corporation.

```
setwd('C:\\Users\\sungi\\Documents\\GitHub\\Financial-Data-Analysis-Stock-Portfolio\\data')

AAN<-read.csv("AAN(Aaron's_Inc).csv")
ATU<-read.csv("ATU(Actuant_Corp).csv")
ASE<-read.csv("ASE(Advanced_Semiconductor_Engineering_Inc.).csv")
IBM<-read.csv("IBM(International_Business_Machines).csv")
XOM<-read.csv("XOM(Exxon_Mobile_corp).csv")
dat.BM <- read.csv('NYSE2.csv')
```

1.2 **Data Handling**

- We selected the stock price when the market is closed.

```
AANc<-AAN[,5]
ATUc<-ATU[,5]
ASEc<-ASE[,5]
IBMc<-IBM[,5]
XOMc<-XOM[,5]
BM<-dat.BM[,5]/100

day<-as.factor(AAN[,1])

dat<-data.frame(day,AANc,ATUc,ASEc,IBMc,XOMc,BM)
colnames(dat)<-c("Date","AAN","ATU","ASE","IBM","XOM","BM")

head(dat)
```

```
##         Date   AAN   ATU  ASE    IBM   XOM       BM
## 1 2015-12-01 22.39 23.96 5.67 137.62 77.95 101.4342
## 2 2015-11-02 24.27 24.76 5.39 139.42 81.66 104.0959
## 3 2015-10-01 24.67 22.80 5.74 140.08 82.74 104.6096
## 4 2015-09-01 36.11 18.39 5.49 144.97 74.35  97.9969
## 5 2015-08-03 37.65 21.44 4.92 147.89 75.24 101.7650
## 6 2015-07-01 36.98 23.06 5.60 161.99 79.21 108.8228
```

```
datm<-as.matrix(dat[,-c(1,7)])

head(datm)
```

```
##        AAN   ATU  ASE    IBM   XOM
## [1,] 22.39 23.96 5.67 137.62 77.95
## [2,] 24.27 24.76 5.39 139.42 81.66
## [3,] 24.67 22.80 5.74 140.08 82.74
## [4,] 36.11 18.39 5.49 144.97 74.35
## [5,] 37.65 21.44 4.92 147.89 75.24
## [6,] 36.98 23.06 5.60 161.99 79.21
```

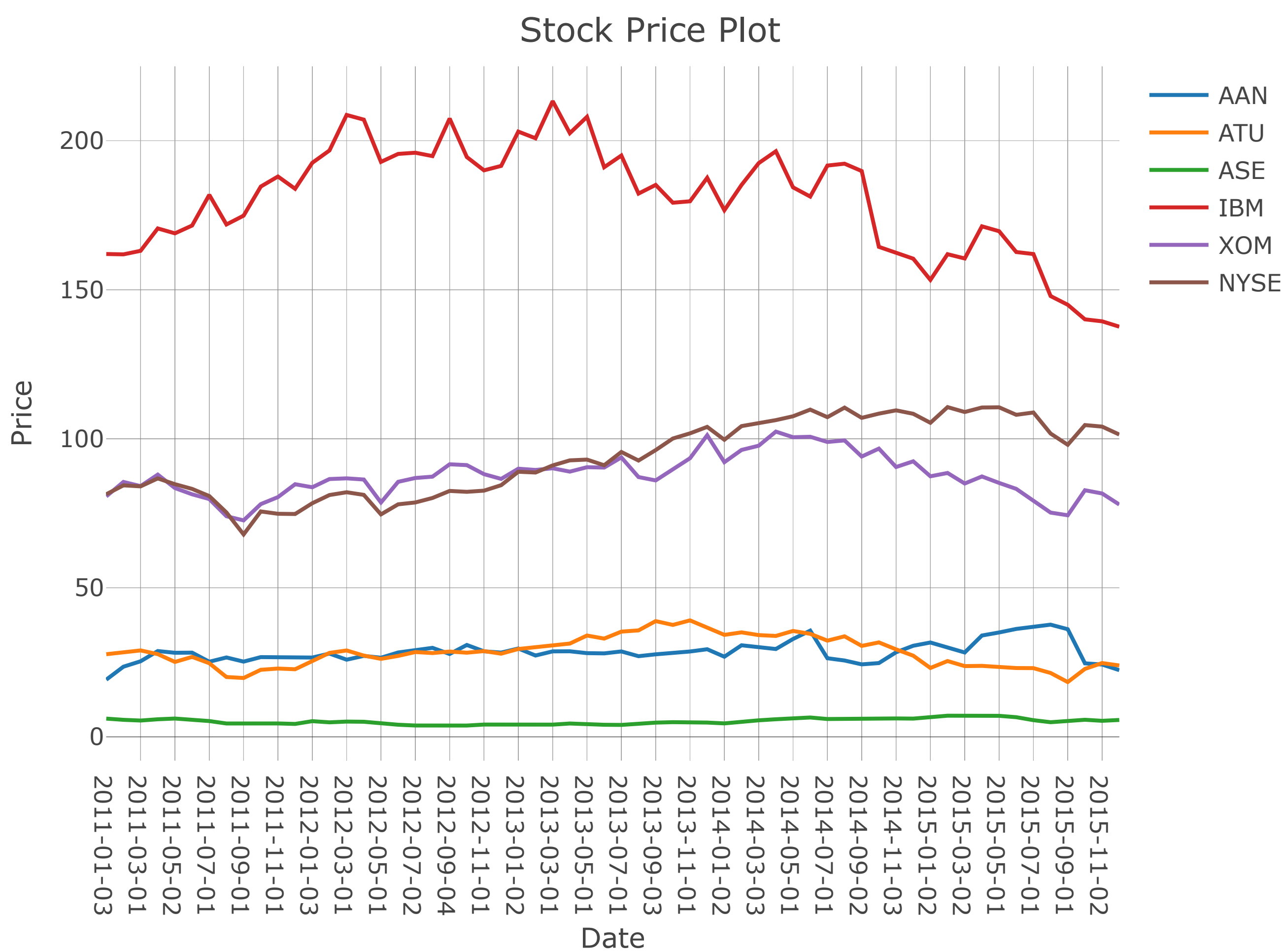
1.3 **Stock Price Plot**

- Plot of the price of each stock item.
- NYSE index has a much higher price compared to other stock items. To see the whole trend of each stock price more easily, we divided each stock price by 100.

```
library(ggplot2)
library(reshape)
library(plotly)
library(grid)
library(gridExtra)
```

```
p <- plot_ly(dat, x = ~Date, y = ~AAN, name="AAN", type='scatter',mode='lines')
p %>% add_trace(y = ~ATU, x = ~Date, name="ATU") %>%
  add_trace(y = ~ASE,x= ~Date, name="ASE") %>%
  add_trace(y = ~IBM, x = ~Date, name="IBM") %>%
  add_trace(y = ~XOM, x = ~Date, name="XOM") %>%
  add_trace(y = ~BM, x = ~Date, name="NYSE") %>%
  layout(title = "Stock Price Plot",yaxis=list(title="Price"),showlegend = TRUE)
```

```
## Warning: `arrange_()` is deprecated as of dplyr 0.7.0.
## Please use `arrange()` instead.
## See vignette('programming') for more help
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_warnings()` to see where this warning was generated.
```



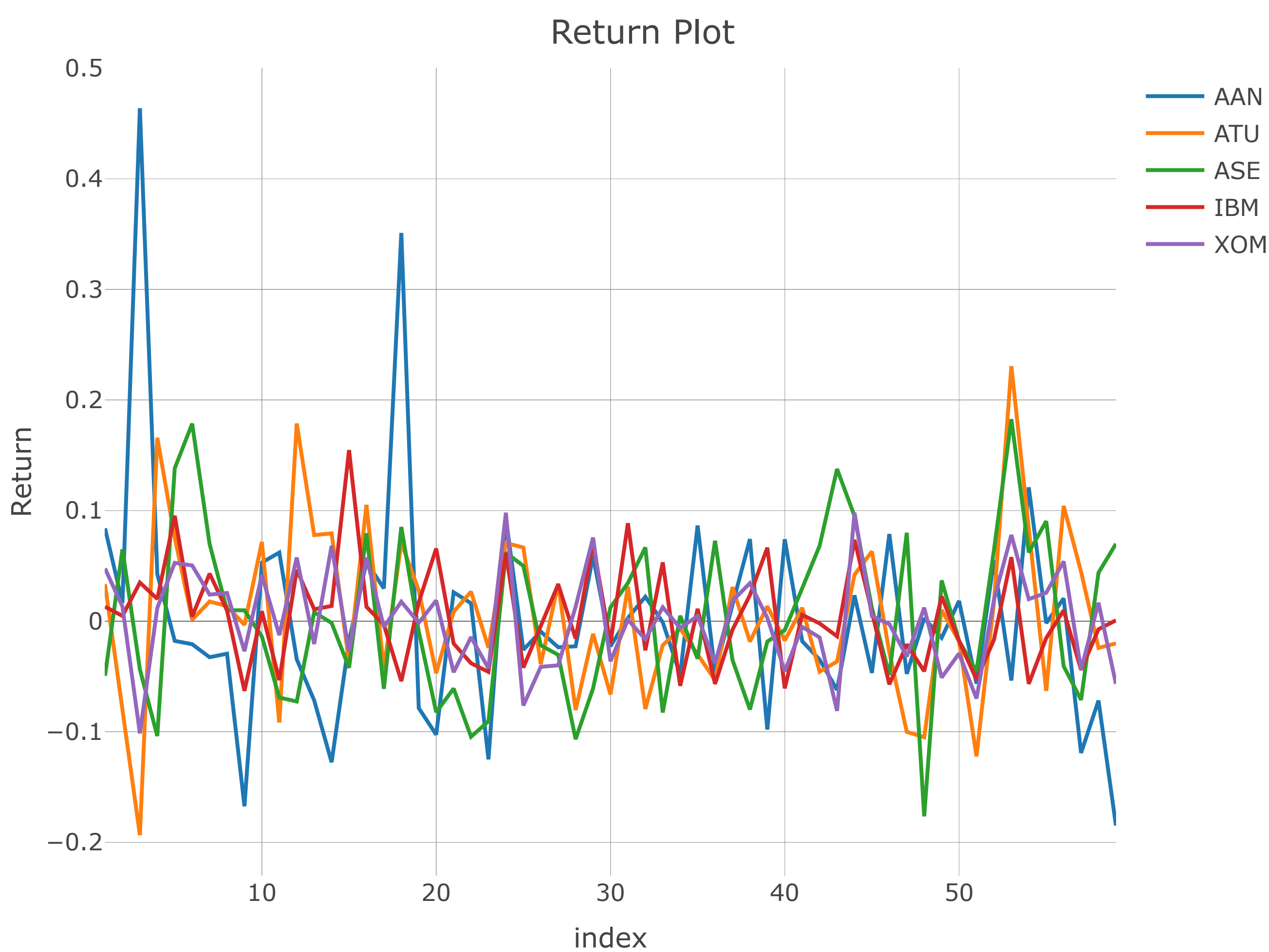
1.4 Returns

```
basket<-1:5
dat.diff <- diff(datm)
Returns <- dat.diff/datm[-nrow(datm),]
Returnsn<-data.frame(index=1:nrow(Returns),Returns)
head(Returns)
```

```
##              AAN          ATU         ASE         IBM         XOM
## [1,]  0.08396610  0.033389025 -0.04938272 0.013079517  0.04759470
## [2,]  0.01648125 -0.079159976  0.06493506 0.004733926  0.01322550
## [3,]  0.46372116 -0.193421061 -0.04355401 0.034908616 -0.10140198
## [4,]  0.04264749  0.165851124 -0.10382514 0.020142084  0.01197041
## [5,] -0.01779554  0.075559605  0.13821138 0.095341173  0.05276450
## [6,] -0.02082209  0.001300997  0.17857143 0.004136051  0.05037240
```

1.5 **Plot Returns**

```
p <- plot_ly(Returnsn, x=~index, y = ~AAN, name="AAN", type='scatter', mode='lines')
p %>% add_trace(x=~index,y = ~ATU, name="ATU") %>%
  add_trace(x=~index, y = ~ASE, name="ASE") %>%
  add_trace(x=~index,y = ~IBM, name="IBM") %>%
  add_trace(x=~index, y = ~XOM, name="XOM") %>%
  layout(title = "Return Plot",yaxis=list(title="Return"),showlegend = TRUE)
```



2. Portfolios

2.1 Minimum Variance Portfolio (MVP)

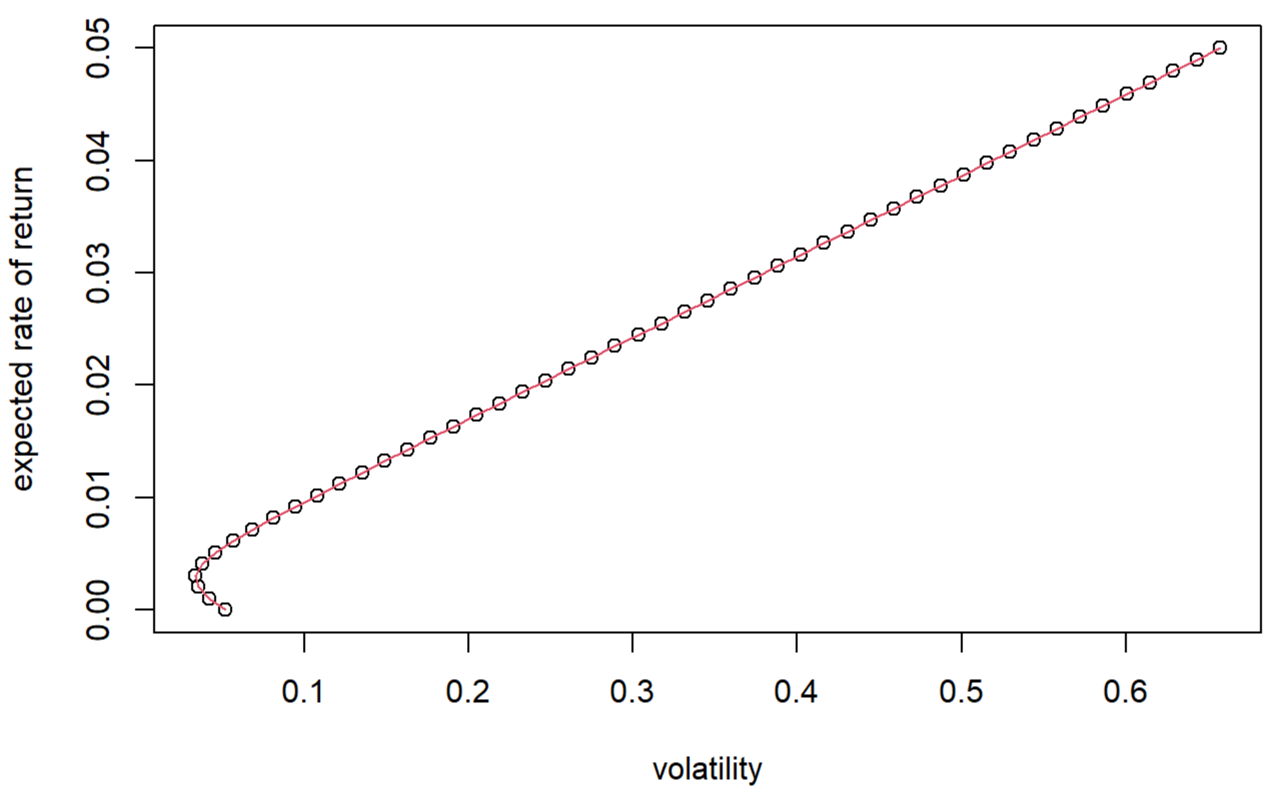
- To minimize $\mathbf{w}'\mathbf{\Sigma}\mathbf{w}$ subject to $\mathbf{w}'\mu = \mu_p$, $\mathbf{w}'\mathbf{1} = 1$.

$$\mathbf{w_p} = \mathbf{g} + \mathbf{h}\mu_p$$

$\mathbf{g} = \frac{B(\mathbf{\Sigma}^{-1}\mathbf{1}) - A(\mathbf{\Sigma}^{-1}\mu)}{D}$ $\mathbf{h} = \frac{C(\mathbf{\Sigma}^{-1}\mu) - A(\mathbf{\Sigma}^{-1}\mathbf{1})}{D}$ $A = \mathbf{1}'\Sigma^{-1}\mu$, $B = \mu'\Sigma^{-1}\mu$, $C = \mathbf{1}'\Sigma^{-1}\mathbf{1}$, $D = BC - A^2$

- MVP function

```
mu.p <- seq(from=0, to=0.05, length=21)
res.mvp <- Minimum.variance.portfolio(Returns=Returns[,basket],
                                      mu.p=seq(from=0, to=0.05, length=50),
                                      do.plot=T, prt=F)
```




```
## Min.sigma.p    Min.mu.p
## 0.033446029 0.003061224
```

- The minimum volatility is **0.033** and corresponding expected rate of return to that value is **0.003**. Therefore, we set expected rate of return to be larger than **0.003** and checked the portfolio result.

```
MVP.prediction <- Minimum.variance.portfolio(Returns=Returns[,basket],
                                             mu.p=c(0.005,0.01),
                                             do.plot=F, prt=F)$w.p
colnames(MVP.prediction) <- c("ERR: 0.005", "ERR: 0.01")
MVP.prediction
```

```
##     ERR: 0.005  ERR: 0.01
## AAN  0.1224416  0.1420905
## ATU  0.3703707  1.1736297
## ASE  0.2139527  0.3670784
## IBM  0.6984054  1.4655150
## XOM -0.4051705 -2.1483137
```

- Result above shows the predicted weights for each stock item in order to achieve the given expected rate of return when MVP portfolio is used.

2.2 **Tangency Portfolio**

- To minimize $\mathbf{w}'\mathbf{\Sigma}\mathbf{w}$ subject to $\mathbf{w}'\mu$ + $(1-\mathbf{w}'\mathbf{1})R_f$ = μ_p.

$$\mathbf{w_q} = \frac{\mathbf{w_p}}{\mathbf{1}'\mathbf{w_p}} = \frac{c_p\bar{\mathbf{w}}}{c_p\mathbf{1}'\bar{\mathbf{w}}} = \frac{1}{\mathbf{1}'\mathbf{\Sigma}^{-1}(\mu - R_f\mathbf{1})} \cdot \mathbf{\Sigma^{-1}}(\mu - R_f\mathbf{1})$$

- Tangency function

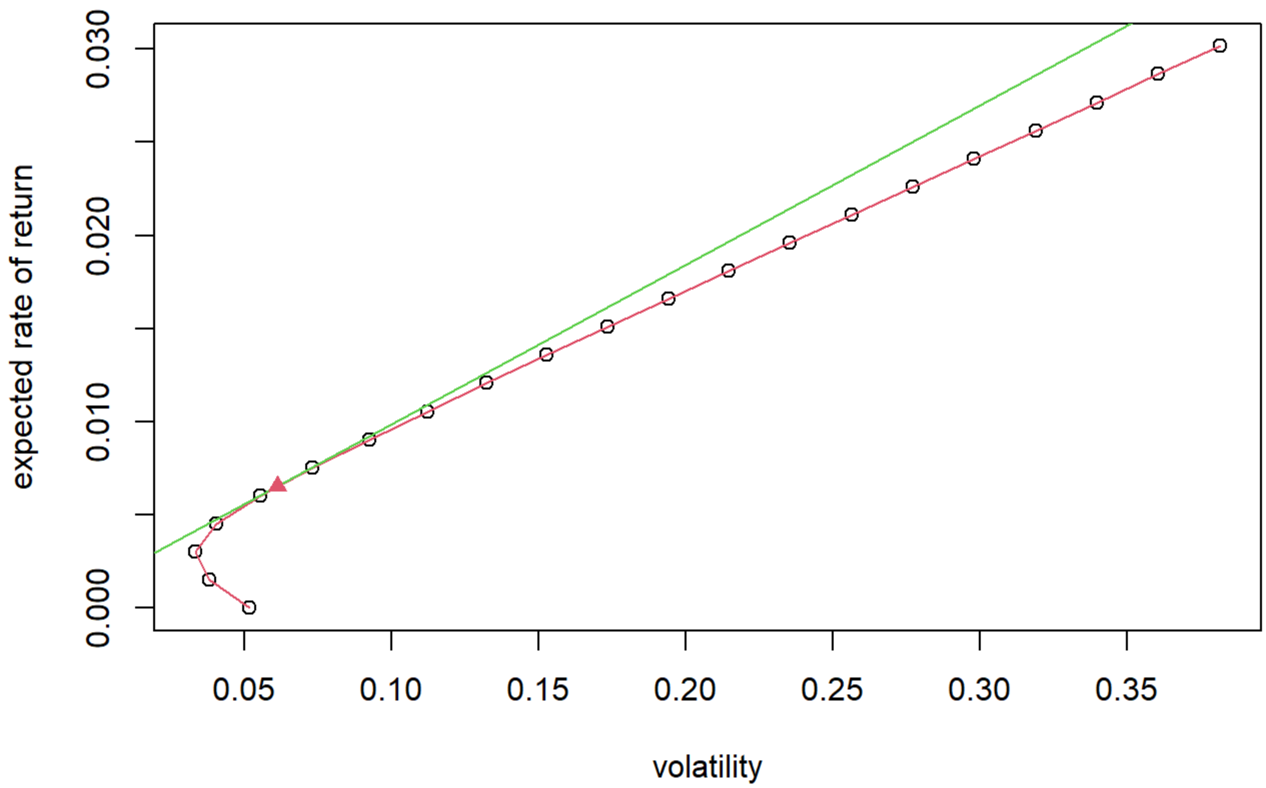
```
Tangency.portfolio <- function(Returns, r.f, do.plot=F)
{
  mu <- apply(Returns, 2, mean)
  Sigma <- cov(Returns)
  Sigma.inv <- solve(Sigma)
  ones <- rep(1, dim(Returns)[2])
  tmp <- mu-r.f*ones
  w.bar <- Sigma.inv%*%tmp

  w.q <- w.bar/sum(w.bar)
  mu.q <- crossprod(w.q, mu)
  sigma.q <- sqrt((t(w.q)%*%Sigma%*%w.q)[1,1])

  lb <- max(0, mean(mu)-0.025)
  ub <- max(mu)+0.025
  mvp <- Minimum.variance.portfolio(Returns, mu.p=seq(from=lb, to=ub, len=21), do.plot=do.plot,
 prt=F)

  if (do.plot) {
    slope <- (mu.q-r.f)/sigma.q
    abline(r.f, slope, col=3)
    points(sigma.q, mu.q, pch=17, col=2)
  }

  return(list(mu.q=mu.q, sigma.q=sigma.q, w.q=w.q))
}
```

- CD(Negotiable Certificate of Deposit) Interest Rate : 1.56%

$$r.f = \frac{CD}{12 * 100} = 0.0013$$

- The point of contact in this graph represents the portfolio which has the maximum Sharpe ratio. It is the most optimized portfolio when a risk free asset exists.

```
res.tan <- Tangency.portfolio(Returns[,basket], r.f=0.0013, do.plot=T)
```




```
res.tan
```

```
## $mu.q
##               [,1]
## [1,] 0.006542412
##
## $sigma.q
## [1] 0.06127035
##
## $w.q
##             [,1]
## AAN  0.1285030
## ATU  0.6181620
## ASE  0.2611893
## IBM  0.9350452
## XOM -0.9428994
```

2.3 **Sharp Ratio**

- Sharp ratio is used to help investors understand the return of an investment compared to its risk. Generally, the greater the value of the Sharpe ratio, the more attractive the risk-adjusted return.

$$\frac{\mu_p - R_f}{\sigma_p}$$

```
(res.tan$mu.q-0.0013)/res.tan$sigma.q
```

```
##            [,1]
## [1,] 0.08556198
```

3. **CAPM measure**

- CAPM Model(Capital Asset Pricing Model)
  - Capital Market Line :

$$\mu_p = R_f + \frac{\mu_A - R_f}{\sigma_A}\sigma_p$$

  - Capital asset pricing model (CAPM), depicts the trade-off between risk and return for efficient portfolios. It is a theoretical concept that represents all the portfolios that optimally combine the risk-free rate of return and the market portfolio of risky assets.
  - SCL(Security Characteristic Line)

$$R_i = \alpha_i + \beta_i R_m$$

  - Equilibrium & SML(Security market Line)

$$\frac{E(R_m) - R_f}{\sigma_m} = \frac{E(R_i) - R_f}{\sigma_{im}}$$

- CAPM function

```
CAPM.measures <- function(R.p, R.BM, r.f, do.plot=T)
{
  x <- R.BM - r.f
  y <- R.p - r.f
  res.lm <- lm(y~x)
  plot(x, y, xlab='Benchmark', ylab='Portfolio')
  abline(res.lm$coef, col=2, lwd=2)
  abline(v=0, lty=2)
  abline(h=0, lty=2)
  expected.excess.return <- mean(y)
  vol <- sd(y)
  alpha <- res.lm$coef[1]
  beta <- res.lm$coef[2]
  Sharpe <- expected.excess.return/vol
  Treynor <- expected.excess.return/beta

  return(list(Expected.excess.return=expected.excess.return,
              Volatility=vol,
              Jensen.alpha=alpha,
              Beta=beta,
              Sharpe=Sharpe,
              Treynor=Treynor))
}
```

Expected excess return

- Bigger the better

Volatility

- Represents a volatility of expected return. It is used as an indicator of risks.

Jensen's alpha

- Represents a excess return relative to the current market economy. The higher the value of Jensen's alpha, the better the performance of the fund compared to the market performance. If it has a negative value, it means the performance of the fund is lower than the market performance.

Beta

$$\beta = \frac{\sigma_{im}}{\sigma_{m^2}}$$

- Measure of the volatility of a portfolio compared to the market as a whole.
- If a stock has a beta of 1.0, it indicates that its price activity is strongly correlated with the market.A beta value that is less than 1.0 means that the security is theoretically less volatile than the market. A beta that is greater than 1.0 indicates that the security's price is theoretically more volatile than the market.

Treynor ratio

$$\frac{\mu_p - R_f}{\beta_p}$$

- The Treynor ratio, also known as the reward-to-volatility ratio, is a performance metric for determining how much excess return was generated for each unit of risk taken on by a portfolio.

3.1 **Benchmark Data Handling**

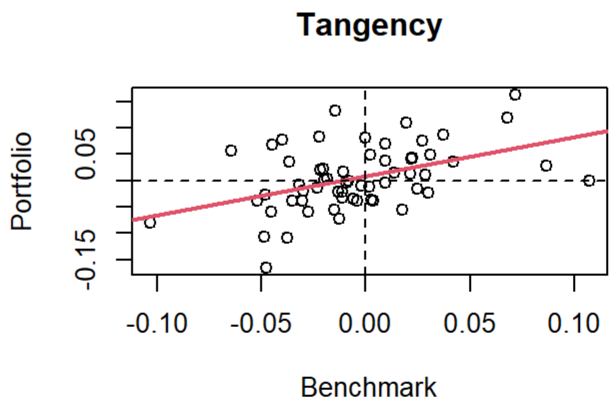
```
dat.BM <- as.matrix(dat.BM[,-1])
BM.diff <- diff(dat.BM)
R.BM <- BM.diff/dat.BM[-nrow(dat.BM),]
head(R.BM)
```

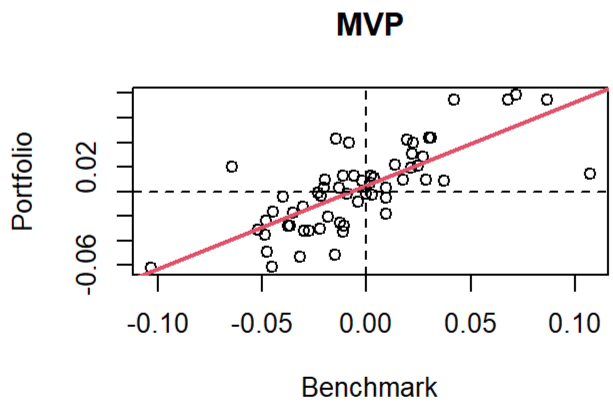
```
##              Open         High          Low        Close      Volume
## [1,]  0.002634283  0.011196544  0.026429027  0.026240649  0.02166620
## [2,] -0.061315107 -0.009716602 -0.043807927  0.004934884  0.02185005
## [3,]  0.035003902 -0.016697314 -0.013615100 -0.063213083 -0.01733789
## [4,]  0.069470813  0.053459378 -0.005817981  0.038451171  0.04765398
## [5,] -0.003584313  0.010662173  0.117011405  0.069353930 -0.12027231
## [6,]  0.021873838  0.012527403  0.013772867 -0.007083081 -0.05307191
##         Adj.Close
## [1,]  0.026240649
## [2,]  0.004934884
## [3,] -0.063213083
## [4,]  0.038451171
## [5,]  0.069353930
## [6,] -0.007083081
```

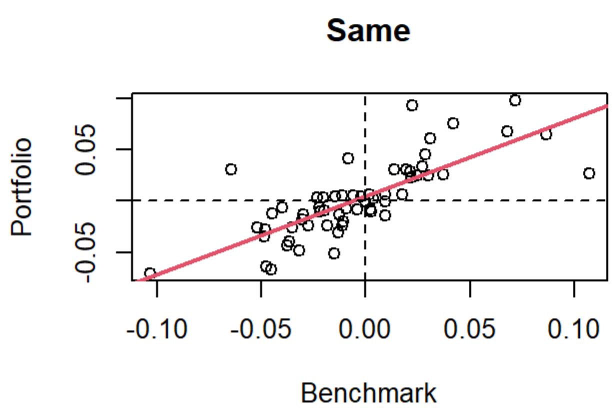
3.2 **Feature of Each Portfolio by CAPM**

3.2.1 **Diversified Investment Portfolio**

- In the above graph of the efficient frontier, the minimum volatility was **0.033** and the corresponding expected rate of return was **0.003**.
  - We set expected rate of return to **0.003** and created a portfolio using MVP method.

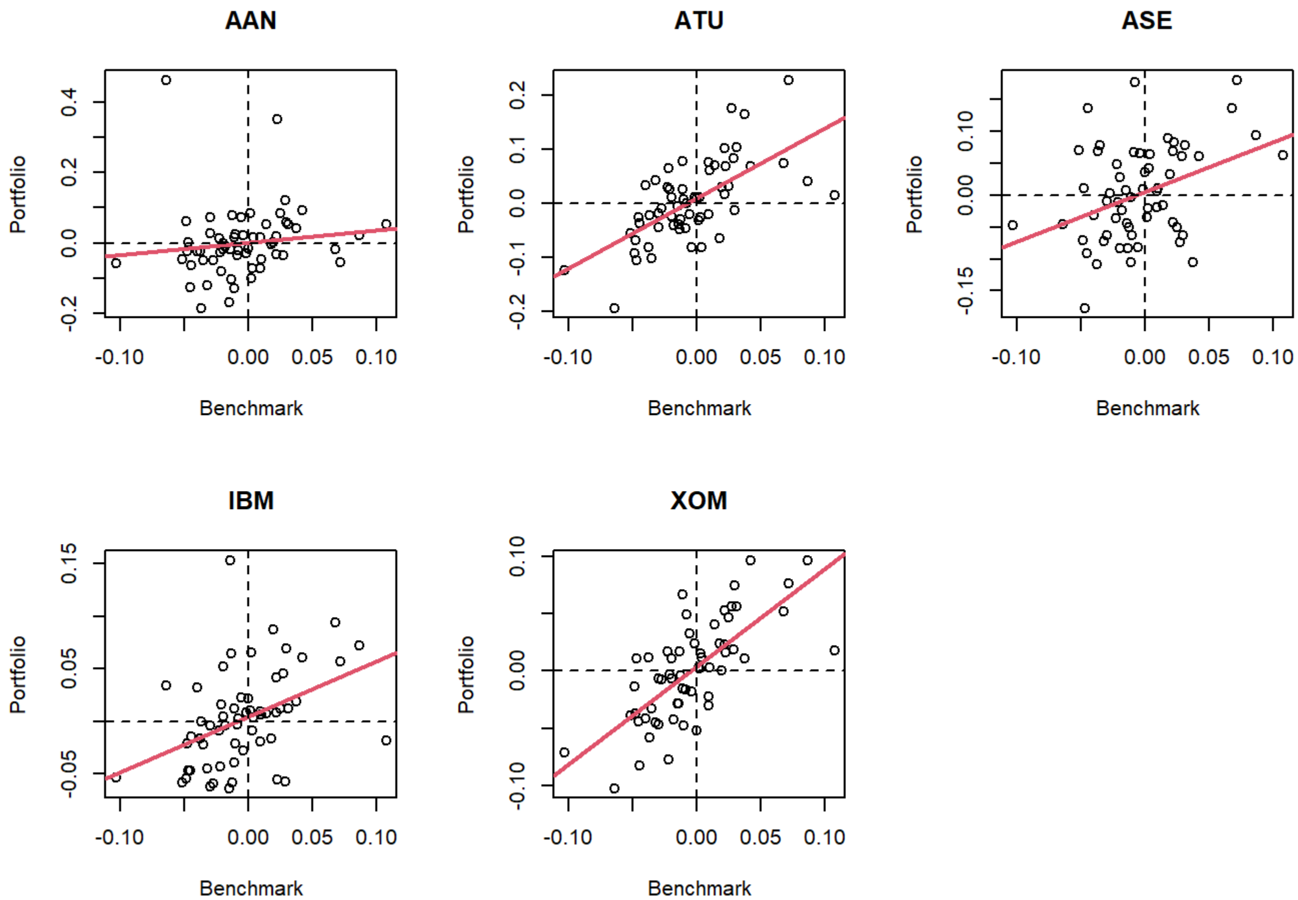







```
##          Jensen_Alpha   Beta
## Tangency       0.0085 0.7460
## MVP            0.0047 0.6790
## Same           0.0053 0.7619
```

3.2.2 **Investment without creating a Portfolio (Individual stock items)**




```
##      Jensen_Alpha   Beta
## AAN        0.0021 0.3484
## ATU        0.0095 1.3029
## ASE        0.0061 0.7806
## IBM        0.0048 0.5276
## XOM        0.0039 0.8503
```

	Tangency	MVP	Same	AAN	ATU	ASE	IBM	XOM
Expected Excess Return	0.0052	0.0018	0.002	0.0005	0.0039	0.0027	0.0025	0.0002
Volatility	0.0613	0.0334	0.0367	0.1006	0.0743	0.0747	0.0455	0.044
Jensen Alpha	0.0085	0.0047	0.0053	0.0021	0.0095	0.0061	0.0048	0.0039
Beta	0.7456	0.6791	0.7619	0.3484	1.3029	0.7806	0.5276	0.8503
Sharpe	0.0856	0.0527	0.0536	0.0055	0.0518	0.0363	0.0541	0.0053
Treynor	0.007	0.0026	0.0026	0.0016	0.003	0.0035	0.0047	0.0002

3.3 **Portfolio Comparison**

- We compared each portfolio’s feature values using a bar-plot.

3.3.1 **Data Handling**

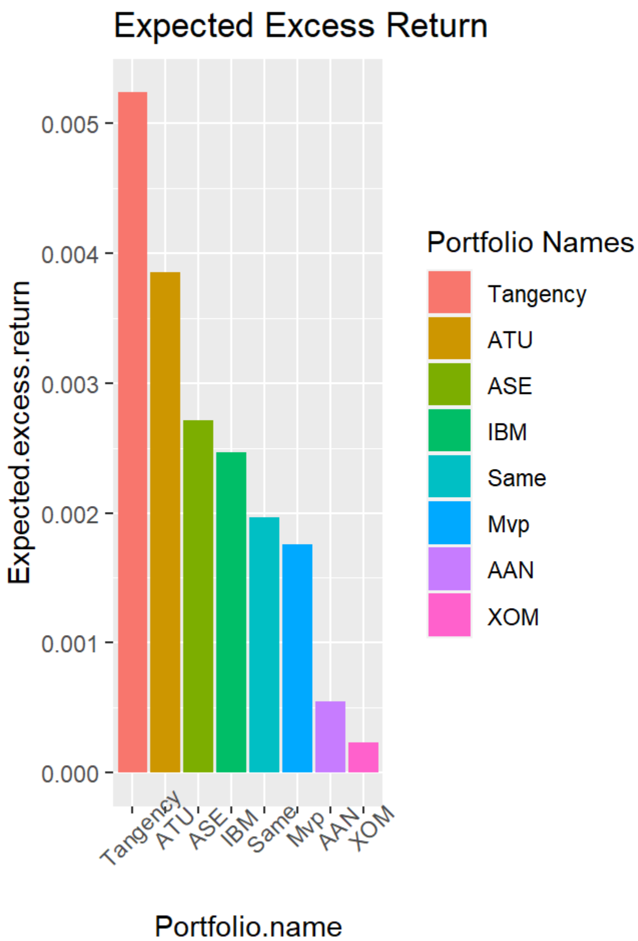
```
CAPM<-data.frame(expect.r,vol,alpha,beta,sharpe,treynor,name)
colnames(CAPM)<-c("Expected.excess.return",
                  "Volatility","Jensen.alpha","Beta",
                  "Sharpe","Treynor","Portfolio.name")
CAPM
```

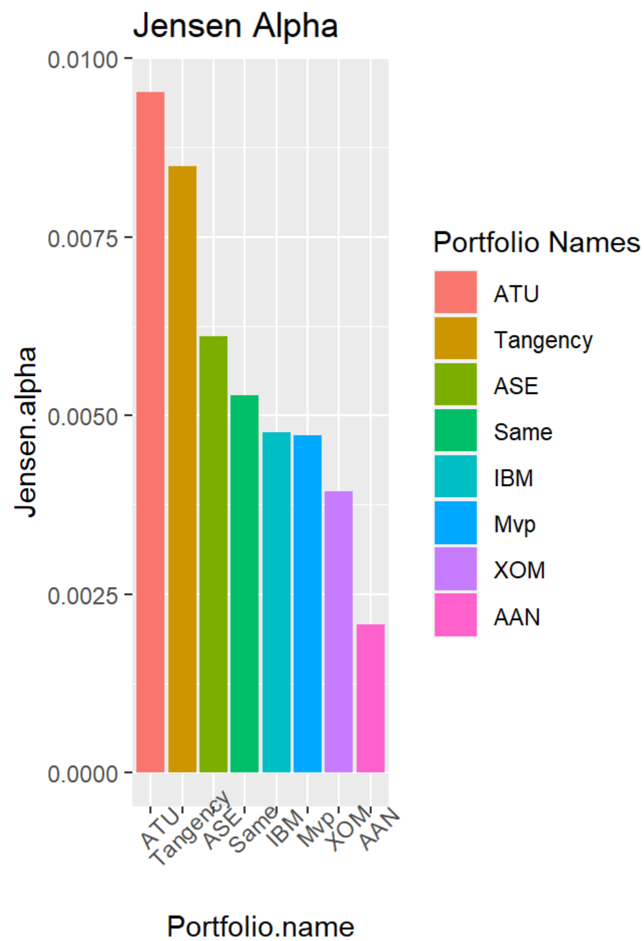
```
##   Expected.excess.return Volatility Jensen.alpha      Beta      Sharpe
## 1           0.0052424118 0.06127035  0.008492308 0.7456456 0.085561976
## 2           0.0019627466 0.03665043  0.005283638 0.7619345 0.053553165
## 3           0.0017612245 0.03344603  0.004721250 0.6791387 0.052658702
## 4           0.0005499687 0.10060481  0.002068381 0.3483796 0.005466624
## 5           0.0038511703 0.07427546  0.009529783 1.3028825 0.051849836
## 6           0.0027126135 0.07470554  0.006114790 0.7805842 0.036310739
## 7           0.0024646076 0.04553918  0.004764043 0.5275749 0.054120596
## 8           0.0002353729 0.04403471  0.003941192 0.8502511 0.005345166
##        Treynor Portfolio.name
## 1 0.0070307018       Tangency
## 2 0.0025760044           Same
## 3 0.0025933206            Mvp
## 4 0.0015786476            AAN
## 5 0.0029558846            ATU
## 6 0.0034751066            ASE
## 7 0.0046715785            IBM
## 8 0.0002768275            XOM
```

```
CAPM.m<-melt(CAPM,id=c("Portfolio.name"))
head(CAPM.m)
```

```
##   Portfolio.name               variable        value
## 1       Tangency Expected.excess.return 0.0052424118
## 2           Same Expected.excess.return 0.0019627466
## 3            Mvp Expected.excess.return 0.0017612245
## 4            AAN Expected.excess.return 0.0005499687
## 5            ATU Expected.excess.return 0.0038511703
## 6            ASE Expected.excess.return 0.0027126135
```

3.3.2 **Expected.excess.return & Alpha**





Expected Excess Return

- Expected excess return of tangency portfolio has the highest value.

Jensen’s alpha

- Jensen’s alpha of ATU (individual stock item) has the highest value. Hence, it has the highest expected excess return relative to the current market economy.
- Tangency portfolio has the highest value among diversified investments.

3.3.3 **Volatility & Beta**

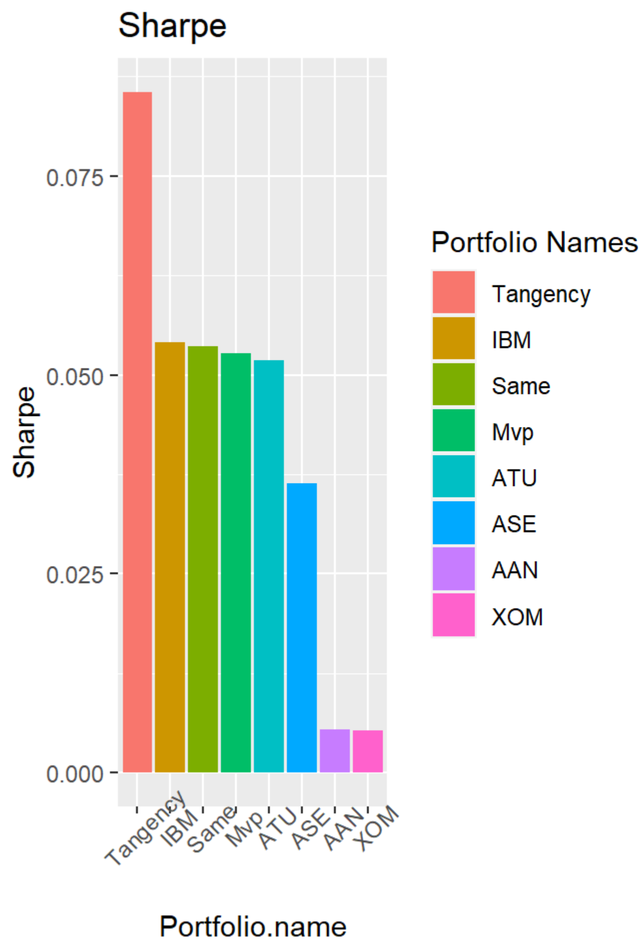
Volatility

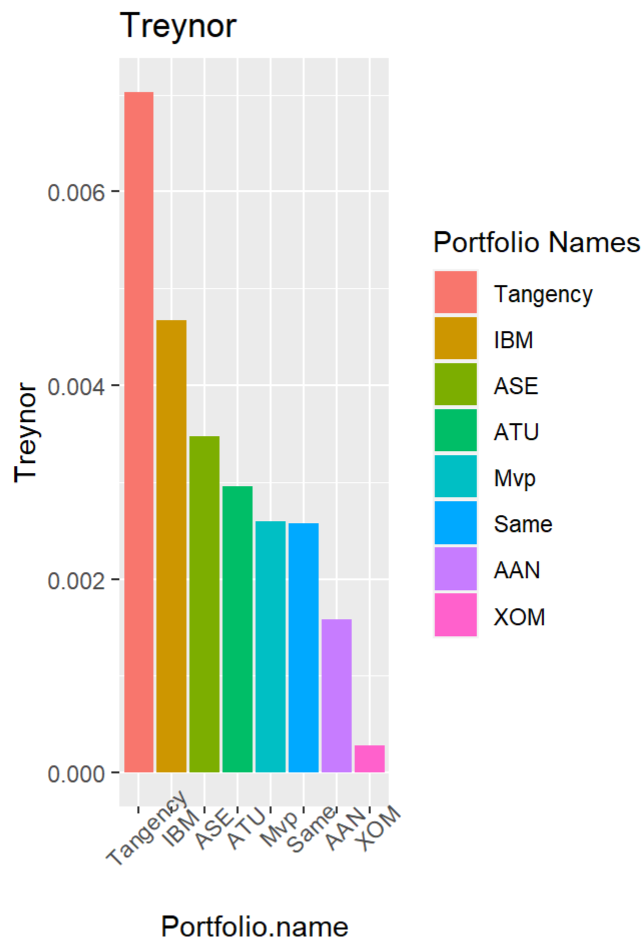
- As all of us expected, most of the diversified investments showed lower volatility than the investments on only one item.
- Mvp portfolio has the lowest volatility.
- Tangency portfolio was the only diversified investment that had higher volatility than IBM and XOM. But as we can see, it greatly reduced the volatility of AAN, ASE, ATU, and therefore it's hard to say that it failed to reduce the entire volatility.

Beta

- Stock prices of the industrial company (ATU) has the highest beta value.
- Oil and gas company (XOM), Semiconductor assembling and test manufacturing company (ASE), and diversely invested portfolios have moderate beta values.
- IT company (IBM) and retailing company (AAN) have the lowest beta values.

3.3.4 **Sharpe & Treynor**





Sharpe & Trynor ratio

- Tangency portfolio has the highest values for both ratios.

3.4 **Efficient Frontier**

```
## null device
##           1
```

---

4. **Evaluation of each Portfolios**

4.1 **Data Handling**

- Stock Price Data from Dec. 2016 to April. 2017 is used to calculate how much returns would have been made, if we invested according to portfolios we created above.

```
setwd('C:\\Users\\sungi\\Documents\\GitHub\\Financial-Data-Analysis-Stock-Portfolio\\data')
AAN16<-read.csv("AAN(2016).csv")
ATU16<-read.csv("ATU(2016).csv")
ASE16<-read.csv("ASE(2016).csv")
IBM16<-read.csv("IBM(2016).csv")
XOM16<-read.csv("XOM(2016).csv")

p16<-data.frame(AAN16[,5],ATU16[,5],ASE16[,5],IBM16[,5],XOM16[,5])
colnames(p16)<-c("AAN","ATU","ASE","IBM","XOM")
p16[1:5,]<-p16[5:1,]
rownames(p16)<-c("Dec","Jan","Feb","Mar","Apr")

dat16<-as.matrix(p16)
diff16 <- diff(dat16)
Returns16 <- diff16/dat16[-nrow(dat16),]
rownames(Returns16)<-c("Jan","Feb","Mar","Apr")
Returns16
```

Tangency Portfolio

```
##         Jan         Feb         Mar         Apr
## -0.11543486  0.03615016  0.16206149 -0.06470580
```

Minimum Variance Portfolio

```
##         Jan         Feb         Mar         Apr
## -0.01429559  0.03699036  0.06382549  0.01715929
```

Equal Weight Portfolio

```
##          Jan          Feb          Mar          Apr
## -0.031488744  0.028081442  0.077399538  0.004649254
```

AAN Portfolio

```
##         Jan         Feb         Mar         Apr
## 0.021884771 0.004807736 0.091779034 0.044223068
```

ATU Portfolio

```
##          Jan          Feb          Mar          Apr
## -0.028380552  0.005584149  0.055531781  0.080938894
```

ASE Portfolio

```
##         Jan         Feb         Mar         Apr
## -0.05643739  0.05046729  0.04092527 -0.12307692
```

IBM Portfolio

```
##         Jan         Feb         Mar         Apr
## -0.09322769  0.05000399  0.15584216 -0.03638161
```

XOM Portfolio

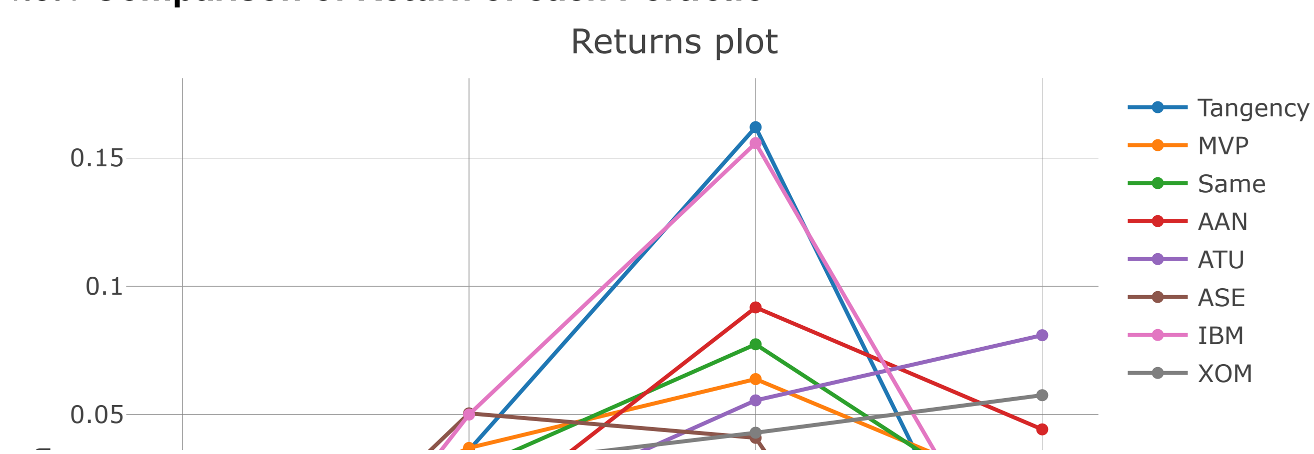
```
##          Jan          Feb          Mar          Apr
## -0.001282861  0.029544047  0.042919450  0.057542843
```

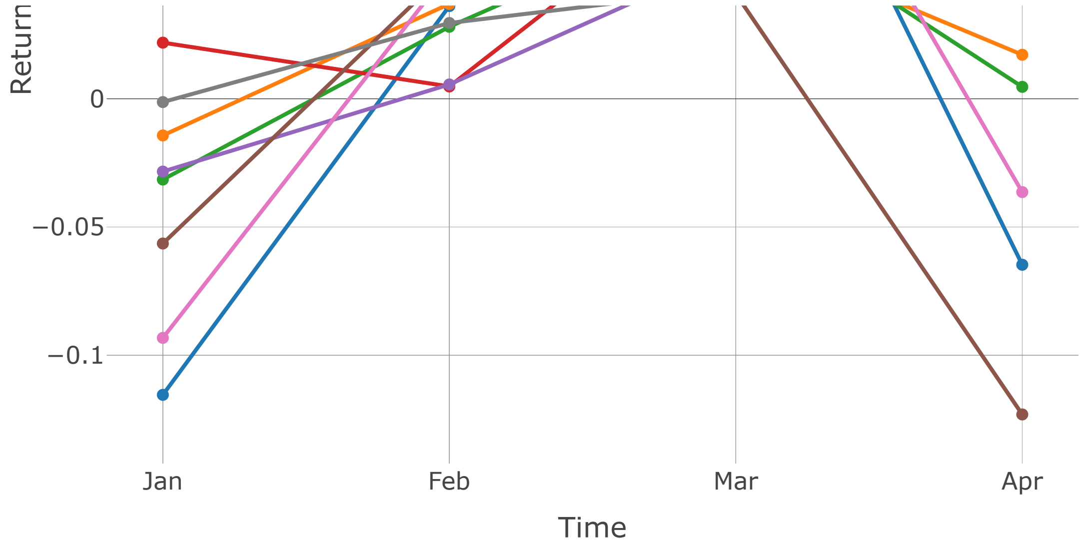
Final Result of Portfolios

```
##             tan         mvp         same          AAN          ATU         ASE
## Jan -0.11543486 -0.01429559 -0.031488744 0.021884771 -0.028380552 -0.05643739
## Feb  0.03615016  0.03699036  0.028081442 0.004807736  0.005584149  0.05046729
## Mar  0.16206149  0.06382549  0.077399538 0.091779034  0.055531781  0.04092527
## Apr -0.06470580  0.01715929  0.004649254 0.044223068  0.080938894 -0.12307692
##             IBM          XOM
## Jan -0.09322769 -0.001282861
## Feb  0.05000399  0.029544047
## Mar  0.15584216  0.042919450
## Apr -0.03638161  0.057542843
```

4.3 **Return Plot**

4.3.1 **Comparison of Return of each Portfolio**





- **Result**
  - Jan : Except for AAN, all had negative values. Bad start.
  - Feb : All the portfolios' return have increased and became positive. Nice.
  - March : Most of the portfolios' return has reached its maximum point. Best time to make money.
  - April : Stock price of IBM and ASE has decreased significantly. All the portfolios' return has decreased as they also got affected.

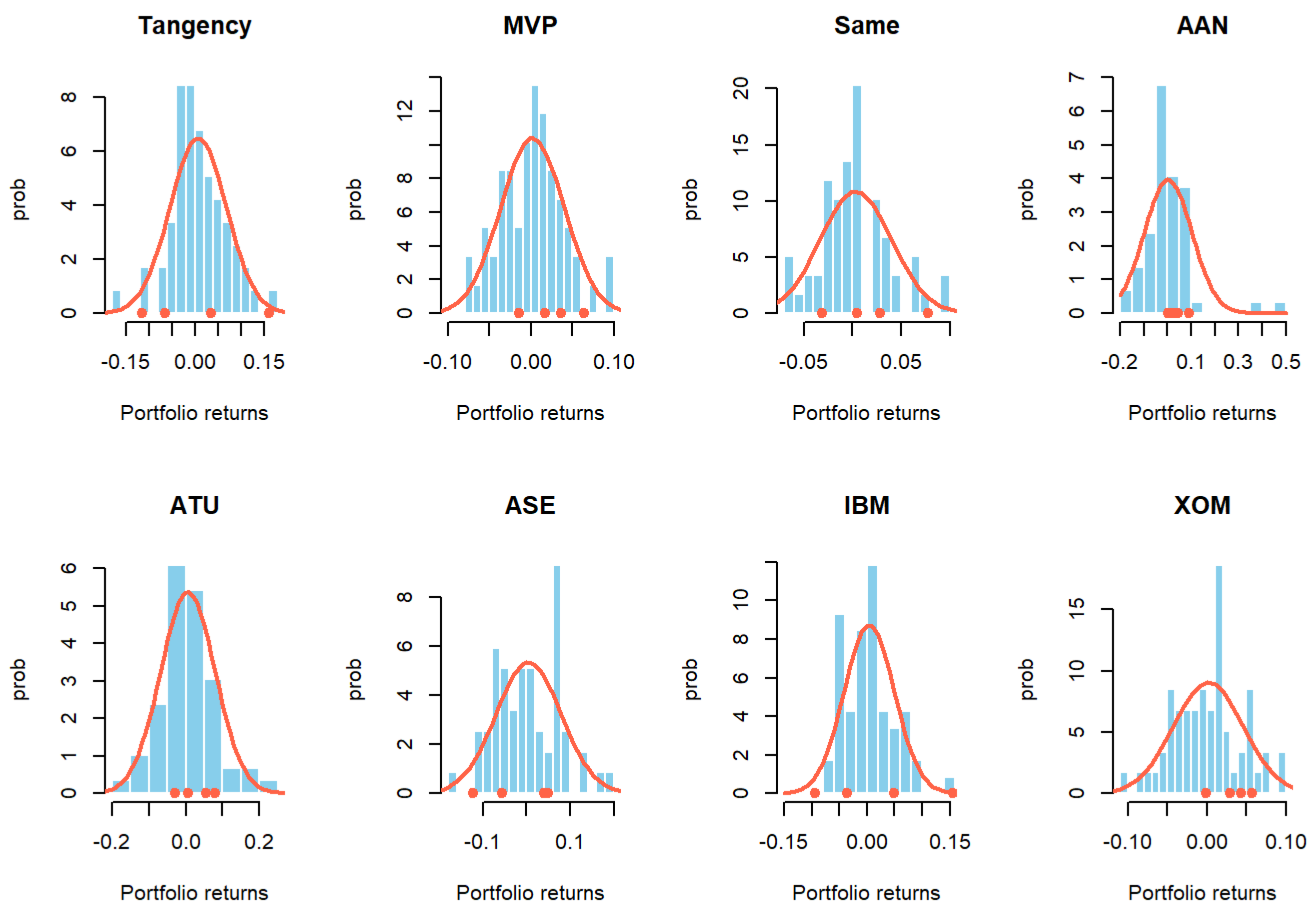
- Especially, Tangency portfolio which assigned large weight on IBM and negative weight; short stock buying, on XOM also showed a sharp drop.

```
##           [,1]
## AAN  0.1285030
## ATU  0.6181620
## ASE  0.2611893
## IBM  0.9350452
## XOM -0.9428994
```

4.3.2 **Aggregational Gaussian**

Aggregational Gaussian

- We can see that aggregated expected rate of return (R_p) follows Gaussian distribution ($N(\mu_p, \sigma_p^2)$).



- Considering **Aggregational Gaussian**'s property, **ASE** and **IBM** showed a rate of return that is hard to happen from January to April.

4.3.3 **Aggregational Gaussian Comparison**

```
## null device
##           1
```

```
##          Volatility Expected_returns
## Tangency 0.06127035      0.006542412
## MVP      0.03344603      0.003061224
## Same     0.03665043      0.003262747
## AAN      0.10060481      0.001849969
## ATU      0.07427546      0.005151170
## ASE      0.07470554      0.004012613
## IBM      0.04553918      0.003764608
## XOM      0.04403471      0.001535373
```

Result

- The portfolio with the highest expected rate of return is **Tangency** portfolio.

- The portfolios with the lowest volatility are **MVP** and **Same** portfolios. However, they have the expected rate of return that is only a half of **Tangency** portfolio.
- Therefore, if you want a portfolio with the highest expected rate of return, choose **Tangency** portfolio. If you want a portfolio with the lowest volatility, choose **MVP** or **Same** portfolio.