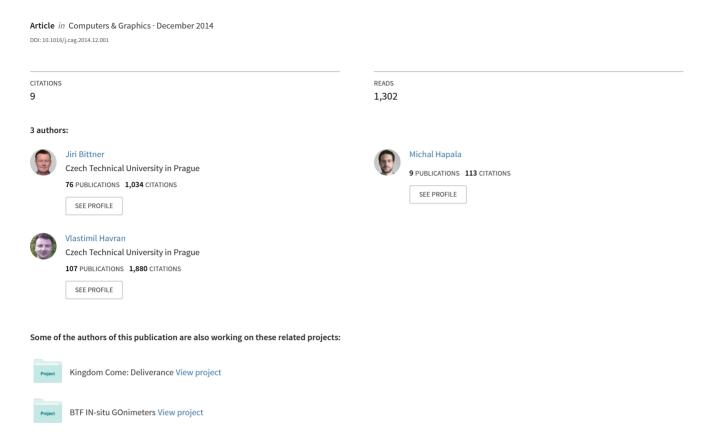
Incremental BVH construction for ray tracing



Incremental BVH Construction for Ray Tracing

Jiří Bittner, Michal Hapala, Vlastimil Havran

Abstract

We propose a new method for incremental construction of Bounding Volume Hierarchies (BVH). Despite the wide belief that the incremental construction of BVH is inefficient we show that our method incrementally constructs a BVH with quality comparable to the best SAH builders. We illustrate the versatility of the proposed method using a flexible parallelization scheme that opens new possibilities for combining different BVH construction heuristics. We demonstrate the usage of the method in a proof-of-concept application for real-time preview of data streamed over the network. We believe that our method will renew the interest in incremental BVH construction and it will find its applications in ray tracing based remote visualizations and fast previews or in interactive scene editing applications handling very large data sets.

Kevwords:

bounding volume hierarchies, ray tracing

1. Introduction

Interactive ray tracing becomes an increasingly popular alternative to rasterization mainly because ray tracing based algotrithms allow computing accurate global illumination and thus
cachieving high degree of realism. One of the main obstacles
for their interactive usage is the necessity to organize the scene
in an acceleration data structure in order to efficiently compute
the ray-object intersection queries. The most commonly used
methods involve spatial subdivisions (uniform grids, octrees,
kd-trees) and bounding volume hierarchies (BVH). In particular
BVHs became a vivid choice for many recent implementations
they have predictable memory footprint, allow relatively easy
dynamic updates, and perform well in GPU ray tracing implementations [1].

Practically all currently used BVH build methods require 16 that the whole scene is known in advance. While this is of-17 ten the case, there are also applications, in which accessing the 18 scene data takes significant amount of time. Waiting for all the 19 data to be present in memory introduces significant latency in 20 the whole rendering process. Another use case when the whole 21 scene is not known in advance is for example an interactive 22 modeling session of complex data assemblies for which high 23 quality preview is required. A natural solution in these appli-24 cations could be an incremental BVH construction, which in-25 serts pieces of the scene geometry into the BVH as soon as they 26 become available. It is however widely believed that the in-27 cremental BVH construction is inefficient particularly in terms 28 of ray tracing performance of the resulting BVH. In this pa-29 per we show that using a careful optimization of the incremen-30 tal BVH construction combined with global structural updates 31 leads to efficient BVHs. In particular we aim at three main 32 contributions: (1) We present an incremental construction al-33 gorithm, which produces high quality BVH. We are the first 34 to show that the insertion based incremental BVH construc35 tion can lead to efficient BVHs, which directly contradicts the
36 state of the art results [2, 3]. (2) We propose two parallelization
37 schemes of the incremental BVH construction, which are actu38 ally the first parallel schemes of incremental BVH construction
39 we are aware of. (3) We test the proposed method in a proof40 of-concept application which performs GPU ray tracing of the
41 data streamed over the network while using different data prior42 itization schemes. An illustration of the incremental BVH con43 struction combined with data streaming is shown in Figure 1.

44 2. Related Work

Bounding volume hierarchies provide an efficient way of 46 organizing scene primitives and they have a long tradition in 47 the context of ray tracing. Already in the early 80s Rubin and 48 Whitted [4] used a manually created BVH, while Weghorst et 49 al. [5] proposed to build the BVH using the modeling hier-50 archy. Kay and Kajiya [6] designed a top down BVH con-51 struction algorithm using spatial median splits. Goldsmith and 52 Salmon [7] proposed the measure currently known as the sur-53 face area heuristic (SAH) which predicts the efficiency of the 54 hierarchy already during the BVH construction. In this highly 55 influential work Goldsmith and Salmon proposed to build BVH 56 incrementally by insertion. However the algorithm they pro-57 vided was limited to greedy decisions during the insertion pro-58 cess and did not properly explore the space of all possible in-59 sertion positions. This insertion based method thus generally 60 results in a poor quality BVH as was shown in performance 61 studies by Havran [2] and later by Masso et al. [3]. This led 62 to a belief that the incremental construction of a BVH by in-63 sertion is inefficient and these methods were practically disre-64 garded by the research community. In our paper we revisit the 65 idea of incremental BVH construction and show that it can ac-66 tually lead to trees of higher quality than the nowadays used 67 top-down SAH construction methods.











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Figure 1: Snapshots showing ray traced images of the Power Plant scene (12.7M triangles) during data streaming. A high quality BVH is constructed incrementally on the CPU, while the scene is being ray traced on the GPU at real-time (60FPS). The data is sent by prioritizing the geometry based on its estimated projected area. By streaming a fraction of the scene geometry we already obtain a good overview of the visible part of the scene.

69 in the context of collision detection, for which Omohundro [8] 113 in the case of massive model visualizations [25, 26]. These 70 designed an efficient method using a priority queue based search 114 methods typically use specialized scene representations (such 71 for construction of a hierarchy of bounding spheres. A similar 72 search strategy was recently used by Bittner et al. [9] in an al-₇₃ gorithm, which optimizes the BVH in a postprocess. This work 74 however gives no indication if the proposed optimization meth-75 ods can also be used for the actual construction of high quality 76 BVHs.

The vast majority of currently used methods for BVH con-78 struction use a top-down approach together with the surface 79 area heuristic [10]. These methods require sorting and thus gen-80 erally exhibit O(NlogN) complexity (N is the number of scene 81 triangles). Several techniques have been proposed to reduce 82 the constants behind the asymptotic complexity. For example 83 Havran et al. [11], Wald et al. [10], and Ize et al. [12] used 84 approximate SAH cost function evaluation based on binning. 85 Hunt et al. [13] suggested to use the structure of the scene graph 86 to speed up the BVH construction process. Dammertz et al. [14] 87 proposed to use a higher branching factor of the BVH to better 88 exploit SIMD units in modern CPUs. More recently, the par-89 allel build-up of a BVH has been demonstrated also on a GPU 90 by Lauterbach et al. [15], using a 3D space-filling curve. Aila 91 and Laine [1] targeted optimization of BVH traversal on the 92 GPU. Wald studied the possibility of fast rebuilds from scratch 93 on an upcoming Intel architecture with many cores [16]. Pan-94 taleoni and Luebke [17], Garanzha et al. [18], and Karras [19] 95 proposed GPU based methods for parallel BVH construction. 96 These methods achieve impressive performance, but generally construct a BVH of lower quality than the full SAH builders.

Recently more interest has been devoted to methods, which 99 are not limited to the top-down BVH construction. Walter et 100 al. [20] propose to use bottom-up agglomerative clustering for 101 constructing a high quality BVH. Gu et al. [21] propose a paral-102 lel approximative agglomerative clustering for accelerating the bottom BVH construction. Kensler [22], Bittner et al. [9], and Karras and Aila [23] propose to optimize the BVH by perform-105 ing topological modifications of the existing tree. These ap-106 proaches allow to decrease the expected cost of a BVH beyond 107 the cost achieved by the traditional top down approach. The 108 comparison of different BVH construction methods and new quality metrics have been studied recently by Aila et al. [24].

Our paper makes use of the incremental BVH construction in an application, which receives streamed scene data over the

Bounding volume hierarchy construction was also studied 112 network. This area has been thoroughly researched particularly as LODs, point clouds, or voxels) and work usually with the 116 rasterization paradigm rather than ray tracing. In our paper the 117 streaming component is used only as a particular use case of the 118 proposed incremental BVH construction and thus for more de-119 tails about the remote and out-of-core visualization techniques we direct an interested reader to the survey of Gobetti et al. [27].

> The paper is further structured as follows: The overview 122 of the algorithm is given in Section 3. The incremental BVH 123 construction algorithm is described in Section 4 and its par-124 allelization in Section 5. Section 6 presents the framework, 125 which exploits the proposed BVH construction for ray tracing 126 data streamed over the network. Section 7 presents the results which are discussed in Section 8. Finally, Section 9 concludes 128 the paper.

129 3. Algorithm Overview

The core of our method is the incremental insertion of scene 131 geometry into the BVH. In the sequential version of the algo-132 rithm we construct a new leaf node for each geometric primitive 133 (triangle), which is then inserted at an appropriate position into 134 the BVH. We use a branch and bound search to find a posi-135 tion in the tree which minimizes the increase of the tree cost 136 evaluated using SAH. The new leaf is then linked to the tree 137 and the process continues with the next geometric primitive. 138 Apart from the sequential algorithm we propose two methods 139 of parallelization of the algorithm. The first method searches 140 for the best positions of the triangles in the BVH for a batch 141 of triangles in parallel. The second method subdivides the in-142 put triangle stream into chunks for which small local BVHs are 143 constructed in parallel and then sequentially inserted into the 144 global BVH.

The final BVH quality depends on the order of inserted 146 primitives - for some orders the tree might get globally imbalanced with respect to the SAH cost metric. We compensate for 148 that by performing global tree updates by re-inserting selected 149 nodes at better positions in the BVH so the global BVH cost 150 is minimized. The selection of nodes for re-insertion is driven 151 by tracking the history of BVH modifications performed for the 152 inserted geometry.

The BVH construction can handle input geometry provided in arbitrary order. We also discuss view dependent prioritization schemes which change the order in which the data is streamed. These methods are based on evaluating the importance of scene primitives for the current camera view and using either a deterministic or a stochastic approach for prioritizing the data according to their importance.

160 4. Incremental BVH Construction

In this section we recall the SAH cost model and then we present the incremental BVH construction, which forms a core contribution of our paper.

164 4.1. SAH Cost Model

The quality of the BVH for ray tracing purposes is commonly measured using the SAH cost model, which expresses the expected number of operations to process a ray intersecting the scene. This cost can be expressed as:

$$C(T) = \frac{1}{S(T)} \left[c_T \cdot \sum_{N \in inner \ nodes} S(N) + c_I \cdot \sum_{N \in leaves} S(N) \cdot t_N \right], \tag{1}$$

where S(T) is the surface area of the bounding box of node N, t_N is the number of triangles in leaf N, and c_T and c_I are constants representing the traversal and intersection costs. Note that the given scene supposed there is a single primitive per leaf. Thus given scene supposed there is a single primitive per leaf. Thus the cost term which should be minimized when inserting new which corresponds to the traversal overhead of the interior part of the tree $(c_T \cdot \sum S(N))$.

180 4.2. Inserting Primitives

The geometric primitives are inserted into the BVH incrementally, one by one. For each primitive we first create a new
leaf node containing this primitive. Then we need to find an
appropriate position in the BVH where the node should be inserted. For this purpose we use the branch and bound algorithm
proposed by Bittner et al. [9], which was originally designed for
BVH optimization by repositioning its subtrees. This algorithm
searches for a node in the tree which will become the sibling of
the inserted node, such that the global cost increase given by
Eq. 1 is minimized.

191 4.3. Global BVH Updates

The primitive insertion step of the algorithm finds an optimal position of the node with respect to the current BVH topology, but without reflecting primitives that will be inserted later.
Therefore, in general, the tree might get imbalanced with respect to the SAH metric, since the order of insertions is also
miproriant. We solve this problem by interleaving the primitive
misertion with a small number of global updates of the BVH. In
propagation particular we perform a batch of insertion operations followed

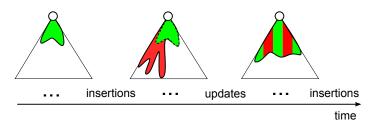


Figure 2: Illustration of the interleaving of insertion and update operations. The incremental insertion of nodes is searching for the best position of inserted nodes, however the overall structure of the tree might get imbalanced. This is corrected by BVH updates, which aim to globally optimize the current tree. Note that unlike this illustration, the tree optimized according to SAH will typically not be balanced in terms of depths.

₂₀₀ by a batch of tree update operations. The process of interleav-₂₀₁ ing insertions and updates is illustrated in Figure 2.

The global updates work by selecting a number of nodes whose children are removed from the tree and then reinserted at better positions in the tree. The nodes are selected using a metric which aims to identify those nodes that cause a cost overhead and thus the re-insertion procedure applied on these nodes has a higher chance for reducing the tree cost. Bittner et al. [9] proposed to use a combined inefficiency measure. We observed that this measure also works well for the optimization during incremental BVH construction. As an alternative approach we can use the surface area of the node as its inefficiency measure, which gives only marginally worse results.

Node update cache. During the incremental construction it 214 is often the case that only some branches of the tree are modi-215 fied by subsequent insertion operations. We exploit this obser-218 operations. These nodes correspond to the union of paths in the 219 BVH from the inserted leaves towards the root (see Figure 3). 220 The update procedure then uses only the cached nodes when 221 selecting the nodes to be updated. We use two constants in our 222 algorithm: the first constant N_u gives the number of modified 223 nodes, reaching which the batch of update operations is applied. 224 The second constant k_u is a fraction of nodes to be updated in $k_u.N_u$ nodes with the highest inefficiency metric are 226 updated in the given batch. Setting larger N_u increases the size 227 of the length of the insertion batch, while the length of the up-228 date batch is given by both constants. We used $N_u = 8000$ and $k_u = 1\%$, which works well for the tested scenes. We observed 230 that the proposed algorithm is generally not very sensitive to 231 these two constants.

232 4.4. Optimizations

Clustering subsequent primitives. Although the algorithm stated above assumes no particular order of scene primitives, it is often the case that these are already ordered in a spatially coherent way. We can use a simple optimization which makes use of such coherence to reduce the number of insertion operations. In particular we check whether two consecutively inserted primitives are spatially coherent and if this is the case we connect the leaves representing these primitives to form a small

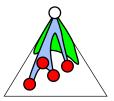


Figure 3: Selecting candidate nodes for topological updates. Several new leaves were added to the tree (shown in red). The part of the tree for which the bounding boxes have been modified corresponds to the candidate nodes for the update (shown in blue). Note the unmodified part of the tree which does not serve for candidate selection (shown in green).

²⁴¹ subtree with a single inner node. Then this subtree is inserted ²⁴² into the BVH using a single insertion operation. The coherence ²⁴³ of two primitives x, y is measured using the ratio of the surface ²⁴⁴ area of the union of their bounding boxes and the sum of surface ²⁴⁵ areas of the bounding boxes:

$$R_{coh}(x, y) = \frac{S(x \cup y)}{S(x) + S(y)}$$

If $R_{coh}(x,y) < R_{max}$ (we used $R_{max} = 1.5$), the primitives are assumed to be coherent and they are connected to form a subtree which is inserted into the BVH as a whole. This simple optimization brings up to 30% speedup for some scenes, while reaching a very similar BVH cost.

BVH postprocessing. Another possible optimization is to perform a larger batch of update operations after the incremental BVH construction has been finalized [9]. Note that we did not use this optimization in order to present the raw results for the incremental BVH construction for the streamed triangle gle data.

257 5. Parallel Incremental BVH Construction

The incremental BVH construction processing individual triangles is inherently sequential, i.e. the BVH is constructed by subsequently extending the current BVH one triangle at a time. The amount of parallelism exploitable while inserting a single triangle into the BVH is limited, since the branch-and-bound search procedure performs localized search and thus does not visit too many nodes of the tree.

However if we subdivide the input stream into batches of triangles of a given size, we can exploit parallelism while inserting the triangle batch into the BVH. We propose two conceptually different ways of parallelizing the incremental BVH construction, parallel search and block parallel construction. Later in the results section we will show that the choice of the method depends on the properties of the input triangle stream and also on the desired BVH quality. Note that both methods have been designed to exploit multi core CPUs rather than GPUs. This matches our target application that will be described in Section 6, in which we aim to fully utilize the GPU for rendering in order to maximize ray tracing performance.

277 5.1. Parallel Search

The most costly operation in the BVH construction is the search for the position of the currently inserted node in the tree.

Thus by parallelizing this operation we can speed up the whole BVH construction process. We execute the branch-and-bound search algorithm in a number of threads for all nodes corresponding to the triangles in the batch. As a result of this parallel operation each node is assigned a node in the BVH to be connected with. Then the nodes are inserted into the BVH sequentially. Using sequential linking into the tree prevents conflicts of threads inserting a node into the same position in the tree. The algorithm based on parallel search is illustrated in Figure 4.

For implementing the method we have used Intel's Thread Building Blocks (TBB) library, which is extremely simple to use and also handles efficient scheduling of the threads. Note that it is beneficial to use a small batch size roughly corresponding to the number of threads used for the search. Larger batch sizes decrease the quality of the constructed BVH as the results of the search do not reflect the positions of the triangles from the same batch.

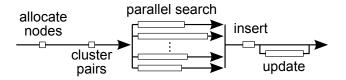


Figure 4: Illustration of parallelization of the search phase of the BVH construction algorithm. Note that the length of the white rectangles roughly corresponds to the costs of the individual steps of the algorithm.

298 5.2. Block Parallel Construction

The parallelization scheme described above does not provide a linear speedup. This is mainly due to the sequential
insertion phase and the associated need of synchronizing the
search threads. We can improve the scalability of the algorithm
by using a different parallelization scheme in which the CPU
cores will get better utilized.

The idea of this parallelization scheme is to create a number of larger triangle batches for which we invoke parallel construction of small BVHs representing the triangles in the batch. We denote these small trees *bBVH* (batch BVH). The bBVHs are fed to a thread which inserts them into the global BVH. In both cases we use the insertion based method. Note that in the case of the bBVHs they can be constructed by any existing method size since all triangles in the batch are known when the construction of the bBVH is invoked. Apart from the input triangle buffer the method uses two work queues: the first queue contains the dueue contains the already constructed bBVHs which should be inserted into the global BVH. The overview of this parallelization method is shown in Figure 5.

If the input triangle stream is coherent, we can create batches of triangles just by grouping the consecutive triangles in the input stream. However for incoherent streams such method would lead to a low quality BVH as the bBVHs might contain incoherent geometry and in turn the bBVHs would have significant spatial overlaps. We handle this issue by creating the triangle

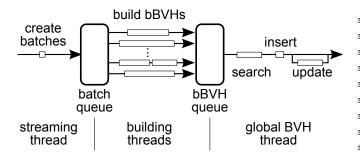


Figure 5: Illustration of the block parallel BVH construction algorithm. Streaming thread creates coherent triangle batches, building threads construct bBVHs for the batches in parallel, and the global BVH thread inserts the constructed bBVHs into the global BVH.

325 batches by spatial sorting the buffered input stream. The trian-326 gles currently available in the buffer are sorted using a quicksort like approach corresponding to spatial median splits.

Initially all currently buffered triangles form one batch. We evaluate whether the triangles in the batch B are sufficiently co-330 herent using an extension of the above defined coherency mea-331 Sure:

$$R_{coh}^{*}(B) = \frac{S(B)}{\sum_{i \in B} S(i)} \sqrt[3]{|B| - 1},$$

where S(B) is the surface area of the bounding box of the 333 triangle batch, |B| is the number of triangles in the batch, S(i) is 334 the surface area of the bounding box of the triangle i. Note that 376 of the view direction and the vector from the camera position to-335 the extension is derived so that for two triangles $R_{coh}^*(\{x,y\}) =$ $_{336}$ $R_{coh}(x,y)$ and for larger batches $R_{coh}^*(B) \approx 1$ if the bounding $_{378}$ a deterministic algorithm, which at each step selects a batch of boxes of the triangles form cells of a regular 3D grid.

If $R_{coh}^*(B)$ is smaller than a threshold R_{max} , we consider the 380 339 batch to be coherent and send it for processing without further 381 jected area of the object as its priority. For this scheme we 340 subdivision. Otherwise, if $R_{coh}^*(B) \ge R_{max}$, the batch is incoher-341 ent and needs to be subdivided. We use a cycling axis spatial 342 median pivot (center of the bounding box of the batch in the 343 current axis) to sort the triangles into two groups according to the pivot. This process repeats until the coherency criterion is 345 met or we have a single triangle in the batch.

346 6. Ray Tracing Streamed Data

Our method is capable of adding new primitives to an al-348 ready built BVH without reducing its quality and therefore its 349 possible application lies for example in rendering scenes that are received in parts. This may involve either very large data sets, for which it is impractical to wait until the storage medium provides the whole set, or data streamed over a network, where 353 it may take a long time untill the next part arrives. In these cases 354 our method can provide an interactive ray traced visualization 396 355 of the data set even when it is not complete.

6.1. Application Architecture

We designed and implemented a pilot application, which is 358 capable of real-time ray tracing of data streamed over a net-359 work. The application contains client and server parts. For

360 each connected client the server provides the client the objects representing the scene data using a certain data prioritization scheme. The client application inserts all received objects into 363 the BVH using the proposed incremental algorithm and renders 364 them using the GPU based ray-tracer by Karras et al. [28]. The 365 client also informs the server of any camera changes, since this 366 is necessary for the computation of some of the prioritization metrics. The overview of the application framework is shown 368 in Figure 6.

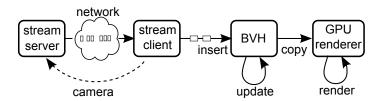


Figure 6: Overview of the application framework for ray tracing streaming data with the incremental BVH construction at its core.

369 6.2. Data Prioritization

In the early stages of the rendering session the visualized 371 scene data are incomplete. In order to evaluate our incremen-372 tal construction we used different prioritization schemes for the 373 streamed data. In particular we have tested the following four 374 prioritization schemes:

The view direction prioritization scheme uses a dot product 377 wards the object (triangle) as the priority of the object. We used 379 k untrasferred objects with highest priorities using a partial sort.

The projected area prioritization uses the estimated pro-382 used stochastic sampling algorithm that constructs a cumula-383 tive distribution function (CDF) and uses it to randomly draw 384 the objects to be sent with probability proportional to the pri-385 orities. To select an object to be sent we generate a uniformly 386 distributed random number which is mapped to a particular object index by using a binary search in the CDF.

The as is scheme involves no prioritization and is suitable 389 for the case when the camera parameters are not available at the 390 server side or when the server could get overloaded by evaluating the view dependent client prioritization schemes.

The random scheme sends the scene objects in a random or-393 der. This allows to test how the incremental construction han-394 dles incoherent data both in terms of speed and BVH quality.

395 7. Results

We have implemented the proposed incremental BVH con-397 struction method in C++. The GPU ray tracing part is imple-398 mented using CUDA. The results were evaluated on a PC with 399 Intel Xeon E5-1620/3.60GHz CPU (4 cores) with 16GBytes 400 RAM, equipped with NVIDIA GeForce GTX 580 GPU with 401 3GBytes RAM. For measurements we used nine test scenes 402 which are summarized in Figure 7.

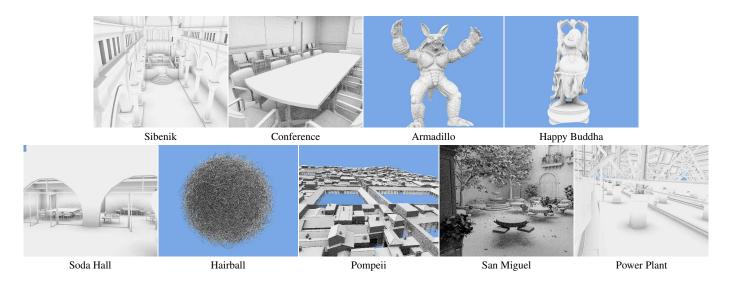


Figure 7: Snapshots of the tested scenes.

403 7.1. Incremental BVH Construction

First we evaluated the proposed incremental BVH construc-405 tion algorithms. We focused on the construction time and the 406 resulting quality of the BVH. The quality was expressed using 407 the SAH cost of the BVH and also by measuring the GPU ray 408 casting performance. As reference methods we used a BVH 409 constructed by a high quality sweep-based SAH builder (de-410 noted SAH) and by spatial median splits (denoted Median). For our proposed algorithm we evaluated four versions: the first one 412 (Incr) uses only insertion operations and performs no global 413 updates, the second one (IncrU) uses the global updates, the 414 third one (IncrUP) uses parallel search and global updates, and 415 the fourth one (IncrUPB) uses block parallel construction with 416 global updates. The parameters for the global updates were $_{417}$ $N_u = 8000$ and $k_u = 1\%$. We have used three different stream 418 ordering methods: as is, view direction prioritization, and ran-419 dom. Note that the random order represents an extreme case 420 for the incremental insertion build as there is almost no coherence among consecutive triangles in the stream. The measured results are summarized in Table 1. 422

Build time. The results show that even the sequential imple-424 mentations of the proposed methods (Incr, IncrU) are always significantly faster than the full sweep SAH builder (SAH) in 426 terms of BVH construction speed. For coherent stream orders 427 they are about twice slower than the spatial median algorithm (Median), but this gap gets larger for random ordering. We can 429 also observe that the IncrU method is faster than Incr for all 430 cases except for the random stream order. This is due to the 431 fact that the method continuously works with a slightly more 432 optimized BVH, which also reduces the cost of insertion opera-433 tions. The parallel search based implementation of the method 434 IncrUP is about 15 – 50% faster than IncrU, while the block 435 parallel method IncrUPB is up to 5 times faster than IncrU. 436 However for random stream order the speed benefit of the In-437 crUPB method reduces and it can even get slower than the In-438 crUP method.

BVH cost. Regarding the quality of the constructed BVH we can observe that in most cases both incremental construction methods construct a BVH with even lower cost than the full top-down SAH builder. In particular the BVH constructed with IncrU method has usually about 10% lower cost than the BVH constructed with full SAH. An exception when the BVH cost for the incremental construction is higher than SAH is the Happy Buddha scene. An interesting observation is that the random stream order leads to higher quality BVH for the incremental methods. This is however paid by significantly longer construction times.

Streaming speed. We also expressed the average streaming throughput for the incremental BVH construction expressed in millions of triangles per second inserted into the BVH (MTris/s). This throughput varies among the tested scenes in the range of 0.1 - 0.8 MTris/s for the sequential implementation and 0.1 - 2.9 MTris/s for parallel implementation. When comparing the speed versus quality of the different incremental construction methods we can observe that the IncrUP would be the method of choice when the BVH quality is important. On the other hand the IncrUPB method is a good choice when maximum streaming throughput is desired.

Ray tracing speed. Table 1 also shows the measured GPU ray tracing performance for the final BVH constructed by the different methods expressed in millions of rays per second (MRays/s) for two different ray types (primary rays and ambient occlusion rays). For all the proposed methods the measured performance varies between 25-294 MRays/s and allows real-time ray tracting of the tested scenes. We can observe that the highest rendess dering performance is mostly obtained using the IncrU or Independent of the tested scenes are the block parallel IncrUPB method usudally achieves slightly lower ray tracing performance.

Progress of the computation. To evaluate the progress of the incremental BVH construction we show the number of pro-473 cessed triangles as a function of time (Figure 8-left). We ob-474 served that the triangle insertion throughput slightly decreases 475 as the BVH contains more nodes, but this dependence is very 530 lion triangles, the rendering speed is sufficient for interactive 476 weak. This conforms with the theoretic logarithmic decay of 531 ray tracing of the scene as shown in Table 1. 477 the triangle insertion throughput. Figure 8-middle shows that 478 the BVH cost has generally non uniform evolution as we can 479 observe also the sudden reductions of the BVH cost in time 480 which are caused by a successful batch of update operations. 481 Note that for the case of random triangle order the cost evo-482 lution curve is much smoother (see Figure 8-right). Figure 9 483 shows a detailed comparison of the BVH cost evolution for dif-484 ferent streaming strategies on three selected scenes. To give an 485 idea how frequent the global BVH updates are we measured the 486 relative number of update operations expressed as the number ⁴⁸⁷ of update operations with respect to the number of triangles in 488 the scene. This value varies among 0.6-1.7%, so a relatively 489 low number of update operations is able to keep the tree well 490 balanced.

We also tested the influence of changing the number of up-492 dated nodes per batch (k_u) . When increasing k_u from 1% to 5%, 493 we observed a marginal increase of build time in order of 1% 494 to 5% and also a reduction of the BVH cost in order of few 495 percent for vast majority of tests. In some cases the reduction 496 of the BVH cost was even more significant (e.g. 20% lower 497 cost for IncrU on Happy Buddha at 5% increase of build time). 498 However, in some other cases the time increase was higher, but 499 it was not reflected in the higher cost reduction (e.g. 30% in-500 crease of build time with 2% cost reduction for IncrU at San 501 Miguel).

502 7.2. Ray Tracing Streaming Data

In order to evaluate the sample application using network 504 streaming we captured several videos showing the behavior of 505 the application depending on the data prioritization method and 506 network bandwidth (the videos are provided as a supplementary 507 material for the paper). Several snapshots showing the applica-508 tion at different stages of data streaming are shown in Figure 1.

The projected area based prioritization provides a very fast global overview of the scene structure, however due to its inherent stochastic nature the scene contains a lot of noise appearing as cluttered geometry. The view direction prioritization on the other hand quickly reveals the details in the area of camera focus, while it takes longer to give the global scene structure. In 515 our tests we generally found the view direction method more 516 pleasant to use and very intuitive - when the user moves the 517 camera the method automatically streams the part of the scene in the new camera focus.

We also measured the GPU ray tracing performance in de-520 pendence on the number of received triangles for the different 521 streaming strategies (see Figure 10). We observed that for the 522 projected area based prioritization the ray tracing speed reduces 523 faster than for the other two methods. This follows from the 524 fact that this prioritization technique is designed to fill the ren-525 dered image with objects as fast as possible (most rays intersect 526 some visible objects at early stages of the computation). The 527 other two methods fill the image more gradually, which as a 528 side product is reflected in the slower reduction of the render-529 ing speed. Note that even for the final BVH with several mil-

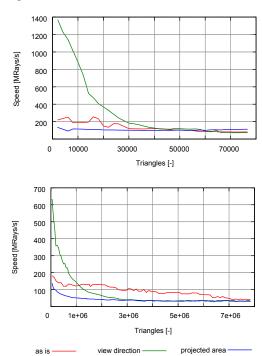


Figure 10: Performance of the GPU ray tracing depending on the number of triangles inserted into the BVH. The graph shows different streaming prioritization methods measured on the Sibenik (top) and San Miguel scene (bottom).

532 8. Discussion and Limitations

BVH cost. The results show that the proposed method con-534 structs a very high quality BVH for most tested scenes. How-535 ever we have observed that for some scenes with a simpler 536 and more regular structure the methods performs slightly worse 537 than the top-down SAH (e.g. HappyBuddha, Armadillo). This 538 can be compensated by subsequent update passes applied on 539 such scenes [9].

Comparison to Goldsmith and Salmon. The only previously 541 proposed and evaluated incremental BVH construction method 542 for ray tracing is the technique proposed in the highly influ-543 ential paper of Goldsmith and Salmon [7]. This paper contains rather vague description of the actual algorithm, however the results obtained by different implementations of the method [2, 3] 546 show that our technique creates more than an order of mag-547 nitude better BVH in terms of its cost, particularly for larger 548 scenes for which the method of Goldsmith and Salmon fails to 549 construct a BVH comparable with the top-down SAH builders.

Construction Speed. The proposed methods achieve con-551 struction speeds of 0.1-2.9MTris/s. This is on one hand much 552 higher than the equivalent speed of the reference full SAH builder, on the other hand lower than the speed of the fast GPU builders [17, 554 18]. A benefit of the proposed method is that by performing the 555 construction on the CPU, the GPU can ray trace the scene in 556 real-time without being forced to offload its resources to the 557 BVH construction. Another important benefit is the reduced

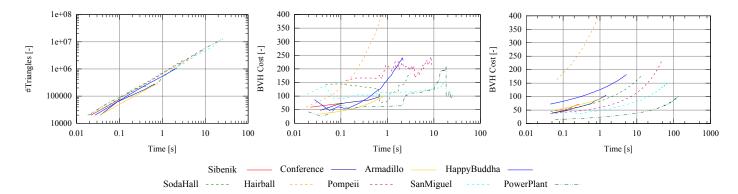


Figure 8: (left) The number of inserted triangles as a function of time for all tested scenes using as is triangle order. (middle) The evolution of the BVH cost during the BVH construction using as is triangle order. We can observe moments when the cost was decreased due to the global BVH updates. (right) The evolution of the BVH cost during the BVH construction using random triangle order. Note the logarithmic scales of the graphs.

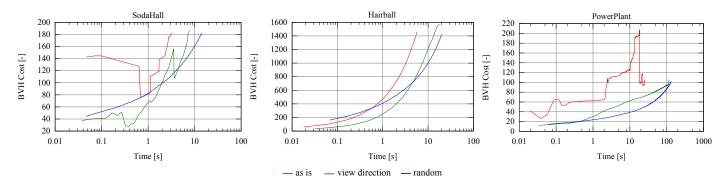


Figure 9: The evolution of the BVH cost during the BVH construction for the IncrU method measured on Soda Hall, Hairball, and Power Plant scenes. We can observe moments when the cost was decreased due to the global BVH updates, especially in the case of *as is* stream order. Note that the random stream order causes smooth BVH cost evolution and leads to slightly lower final BVH cost at the expense of higher computational time.

solution speed in MTris/s is higher or comparable to the streaming throughput our method leads to minimal latency in the appearance of the data on the screen regardless of the scene complexity. The latency is caused only by inserting either a single trisangle or a batch of triangles into the tree. Note that the latency reduction is useful also for loading large data sets from the disk. It is often the case that the data is stored in a format which needs decompression and parsing and thus the streaming throughput of the parser in MTris/s is similar to the speed of our incremental construction algorithm. That means that as soon as the parsing of the scene is finished, the BVH is already available and can be used for rendering.

Latency Analysis and Comparison. We conducted a comparison, which aims at defining a use case for which the inparison, which aims at defining a use case for which the inparison is based on the recent reparison is based on the recent reparison is based on the recent reparison is based on the recent re-

For the comparison we use the San Miguel scene with building times and ray traversal performance reported in the original papers. For the method of Gu et al. we scaled the reported building performance to four core CPU to make the results comparable to the ones measured on our hardware. We evaluate the latency of appearance of a batch of triangles once the batch is received by the test application. For the non-incremental meth-

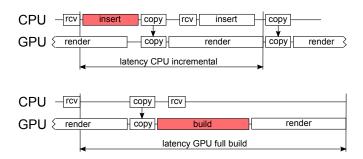
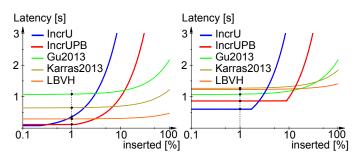


Figure 11: The main components of the latency of appearance of newly received geometry. (top) Latency for CPU incremantal construction. Note that if the newly inserted geometry is small enough the insertion time is completelly hidden by the rendering time and thus the latency is given only by copy and rendering times. (bottom) Latency for full build on the GPU.

583 ods we assume that the BVH is rebuilt from scratch when the 584 batch of triangles is received. The latency has three main com-585 ponents: time for copying the new data to the GPU, time for 586 building/updating the BVH, and time for rendering the frame 587 (see Figure 11). For the GPU builders (denoted Karras 2013 and 588 LBVH) the latency can be approximated as: $t_l = 2(pN_T/s_C + 589 (1+p)N_T/s_B + N_R/s_R)$, where p is the relative number of newly 590 inserted triangles, N_T is the number of scene triangles, s_C is 591 the speed of copying the triangles from CPU to GPU, s_B is 592 the construction speed, N_R is the number of rays cast for one

594 BVH. For the CPU builder proposed by Gu et al. [21] (de- 624 until the batch size of 12% of scene size. For a short interval ₅₉₅ noted Gu2013) the latency is expressed as $t_l = 2(pN_T/s_C + 625)$ of batch sizes (12%-17%) the method of Gu et al. provides $_{596}$ max $((1 + p)N_T/s_B, N_R/s_R))$ since the CPU building and GPU $_{626}$ the best results as it is relatively fast and provides a high qual-597 rendering can run in parallel. For the proposed incremental 627 ity BVH, while for the even larger batches again the LBVH methods (IncrU and IncrUPB) the latency is expressed as t_l = 599 $2(pN_T/s_C + max(pN_T/s_B, N_R/s_R))$ since the insertion and GPU 600 rendering runs in parallel and furthemore we only insert the new 601 triangles in the tree. Note that in the latency models we assume 602 that the triangle insertion speed and the ray tracing speed are constant for the given method, which does not hold especially 604 when p is large as both are influenced by the newly inserted tri-605 angles. However, we target at the use case when p is small for 606 which this approximation is sufficient.



			4M	30M	
Method	s_B	s_R	t _l [ms]		
	[MTris/s]	[MRays/s]			
IncrU	0.44	99	359	605	
IncrUPB	1.60	69	116	866	
LBVH	107.0	55	294	1081	
Karras2013	29.0	84	650	1272	
Gu2013	14.8	92	1081	1234	

Figure 12: The comparison of rendering latency for different BVH construction methods when inserting a batch of new triangles in the scene. The plots and the table show the latency in dependence on the size of the inserted batch for the SanMiguel scene. (top) Casting 4M rays per frame. (middle) Casting 30M rays per frame. (bottom) The table showing parameters used for compared methods and the evaluated latency for the case of inserting 1% of new scene triangles and tracing either 4M or 30M rays. s_B is the construction speed, and s_R is the ray tracing speed, t_l is the evaluated latency. Note that the CPU to GPU transfer speed was set to $s_C = 500MTris/s$ for all methods.

The results of the comparison for small number of rays 608 per frame (4M) and larger number of rays per frame (30M) 609 are shown in Figure 12. We can observe that with 4M rays 610 per frame the incremental construction (methods IncrU and In-611 crUPB) lead to significantly lower latency for small values of 612 p. Observe that for the incremental methods the latency is con-613 stant for small batches as it is solely given by copy and ren-614 dering times. Therefore the benefit of the incremental con-615 struction would become even more apparent if lower number 616 of rays would be cast. For larger batches (> 3% of scene size) 617 the slower triangle throughput of the incremental insertion be-618 comes more apparent and the LBVH method leads to the small-619 est latency among compared methods. For higher number of 620 rays shown in the second plot the situation is similar for small 621 batches of inserted triangles although the latency reduction is

 $_{593}$ frame, and s_R is the speed of tracing the rays with the given $_{623}$ significant. The incremental methods provide the best results 628 method leads to the smallest latency. To summarize the latency analysis, we conclude, that our method significantly reduces the 630 latency compared to the state of the art full-build methods for 631 the case of incrementally inserting batches of triangles forming 632 only a fraction of the scene size.

> *Implementation*. The implementation of the method is straight-634 forward and particularly in its sequential version it is much sim-635 pler than that of the other high quality BVH builders. This 636 makes the method a good choice for rapid prototyping of appli-637 cations requiring high quality BVH. In more complex projects 638 the method can coexist with other BVH construction / update 639 implementations (running either on CPU or GPU) and the one 640 most efficient for target application should be used.

Limitations. As the main limitation of the method we see 642 the need for synchronization of the insertion and update opera-643 tions. The proposed parallelization methods are able to partially 644 remove this limitation. However, the parallel search method 645 does not scale well to larger number of threads. The block par-646 allel construction scales well except for the random triangle or-647 der and generally leads to trees of slightly lower quality. The 648 scalability of the method might be improved by a combination 649 of insertion based construction with a different build strategy, 650 but we leave this as a topic for future work. Additional issue which would have to be addressed in the actual streaming based 652 application is handling materials and particularly textures. As 653 textures are typically defined over larger geometric groups the 654 streaming should take texture information into account when 655 determining a geometry order providing the fastest visual feed-

Data Prioritization. We used three basic strategies for data 658 prioritization in order to demonstrate the possibilities of the 659 proposed incremental BVH construction. There are numerous 660 alternatives how to prioritize the data and also how to incor-661 porate scalable geometric representation by using LOD tech-662 niques. A deeper evaluation of the different streaming strategies and associated LOD methods goes out of the scope of our paper, in which the core contribution is the incremental BVH 665 construction algorithm and its evaluation.

666 9. Conclusion

We have proposed an incremental BVH construction algo-668 rithm, which constructs a BVH with better or comparable qual-669 ity than the traditional SAH based top-down BVH construction 670 methods. The proposed method debunks the myth of insertion 671 based BVH construction not being competitive with the top-672 down BVH construction. The sequential implementation of the algorithm achieves construction speeds up to 0.8 million trian-₆₇₄ gles per second, and the parallel algorithm achieves speeds up 675 to 2.9 million triangles per second on a 4 core CPU. This makes 676 the proposed method significantly faster compared with the ref-622 not that significant anymore as the tracing time becomes more 677 erence implementation of the precise top-down SAH build.

We have shown a possible application of the method for real-time ray tracing of scenes which are streamed over a net880 work. This application uses GPU ray tracing, while the net881 working layer and the incremental BVH construction is imple882 mented on the CPU. We have used several simple prioritization
883 schemes allowing fast previewing of large data sets even in the
884 case of low network bandwidth. We believe that our method has
885 a prospective use in mobile setups when streaming data over the
886 network. In the future we would like to study other possible ap887 plications of the incremental BVH construction such as LOD
888 methods or handling large scale online virtual worlds.

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	Build	BVH	Stream.	GPU	GPU	Build	BVH	Stream.	GPU	GPU	Build	BVH	Stream.	GPU	GPU
Method	time	cost	speed	primary	AO	time	cost	speed	primary	AO	time	cost	speed	primary	AO
	[s]	[-]	$\left[\frac{\dot{M}Tris}{}\right]$	$\lceil \frac{MRays}{3} \rceil$	$\left[\frac{MRays}{a}\right]$	[s]	[-]	$\left[\frac{MTris}{a}\right]$	[MRays]	$\lceil \frac{MRays}{r} \rceil$	[s]	[-]	$\left[\frac{MTris}{a}\right]$	$\left[\frac{MRays}{a}\right]$	$\left[\frac{MRays}{s}\right]$
Sibenik, 80k triangles					Conference, 283k triangles					Armadillo, 307k triangles					
SAH	0.44	82.3	n/a	137	191	1.93	130	n/a	124	198	1.98	86.3	n/a	159	86.5
Median	0.06	391	n/a	18.7	27.3	0.20	842	n/a	14.4	30.8	0.24	144	n/a	93.8	61.1
as is															
Incr	0.11	80.2	0.68	105	140	0.68	119	0.41	113	192	0.77	101	0.39	125	74.4
IncrU	0.12	73.8	0.66	127	176	0.68	109	0.41	123	215	0.87	97.3	0.35	130	75.5
IncrUP	0.10	82.0	0.78	99.8	144	0.49	121	0.56	97.6	178	0.70	98.3	0.43	132	75.5
IncrUPB	0.04	83.7	1.96	93.2	136	0.13	133	2.11	116	216	0.58	111	0.52	111	68.3
								direction							
Incr	0.24	85.8	0.33	74.6	114	1.00	132	0.28	96.1	179	1.62	273	0.18	56.8	38.8
IncrU	0.22	73.9	0.35	125	156	0.92	109	0.30	103	203	1.17	134	0.26	91.1	61.2
IncrUP	0.18	74.8	0.42	101	142	0.72	108	0.39	112	216	0.88	133	0.34	90.0	61.9
IncrUPB	0.04	96.0	1.67	58.3	116	0.19	128	1.42	72.4	143	0.22	126	1.38	102	64.6
T.,	0.24	70.4	0.22	116	1.40	1.20		ndom	125	221	0.06	94.9	0.21	122	77.0
Incr IncrU	0.24 0.29	79.4 71.7	0.33 0.27	116 136	148 181	1.30 1.45	116 105	0.21 0.19	125 132	221 237	0.96	94.9 94.1	0.31 0.27	133 138	77.9 78.2
IncrUP	0.23	71.7	0.27	125	179	1.13	105	0.19	132	238	0.93	94.3	0.27	136	77.9
IncrUPB	0.25	91.6	0.32	100	133	1.13	139	0.20	68.0	117	1.08	105	0.32	114	73.1
					133				oo.o Ok triangle		1.00	1	l .		
SAH	HappyBuddha, 1,087k triangles SAH 8.91 165 n/a 355 82.6				82.6	22.5	217	n/a	113	156	Hairball, 2,880k triangles 24.1 1415 n/a 13.6 36.9				
Median	0.82	165 276	n/a n/a	355 203	44.9	1.88	1396	n/a	8.11	8.96	2.42	2447	n/a n/a	13.6 8.18	21.2
Wicdian	0.02	270	11/4	203	77.7	1.00		as is	0.11	0.70	2.72	2771	11/α	0.10	21.2
Incr	2.63	346	0.41	162	42.5	3.84	204	0.56	84.2	116	6.69	1517	0.43	9.23	25.6
IncrU	2.35	230	0.46	227	56.2	3.55	183	0.61	75.1	157	6.19	1460	0.46	11.2	29.7
IncrUP	1.76	242	0.61	210	52.2	2.90	224	0.74	67.2	95.9	5.18	1908	0.55	7.60	22.8
IncrUPB	1.56	271	0.69	170	49.9	0.76	229	2.85	86.2	113	1.08	2115	2.65	7.03	16.1
							view	direction	I.		1		I		
Incr	6.13	457	0.17	120	36.0	8.52	220	0.25	102	134	18.8	1772	0.15	8.68	22.7
IncrU	4.49	243	0.24	233	55.0	8.16	188	0.26	121	155	18.0	1571	0.15	10.4	27.1
IncrUP	3.39	240	0.32	226	55.0	6.54	189	0.33	81.8	158	14.1	1569	0.20	10.7	27.6
IncrUPB	1.39	289	0.77	148	49.6	2.05	238	1.05	38.0	87.8	8.08	2601	0.35	4.43	18.8
								ndom							
Incr	4.53	184	0.24	298	72.1	12.5	198	0.17	112	135	17.7	1431	0.16	11.8	31.5
IncrU	5.49	181	0.19	294	73.1	14.5	183	0.14	115	175	20.3	1424	0.14	12.0	31.7
IncrUP	4.21	183	0.25	291	72.4	11.4	185	0.19	107	157	15.8	1424	0.18	11.7	31.2
IncrUPB	4.85	194	0.22	266	67.8	13.1	229	0.16	62.0	101	21.8	1853	0.13	9.71	26.4
Pompeii, 5,646k triangles				SanMiguel, 7,881k triangles				PowerPlant, 12,749k triangles							
SAH	46.7	253	n/a	24.7	36.4	107	181	n/a	44.0	95.5	209	116	n/a	141	75.1
Median	4.27	767	n/a	8.59	12.9	7.96	1278	n/a	4.32	8.62	14.6	661	n/a	8.82	9.44
Inon	11 /	266	0.40	20.8	26.0	20.2		is is	40.2	90.6	247	120	0.26	25.4	74.2
Incr IncrU	11.4 10.6	266 231	0.49 0.53	20.8 24.5	36.0 42.4	20.3	177 158	0.38 0.44	40.3 48.6	80.6 99.3	34.7 27.5	120 104	0.36 0.46	35.4 139	74.3 82.0
IncrUP	7.97	258	0.33	20.8	34.3	13.3	172	0.59	34.4	84.2	20.3	118	0.40	101	61.4
IncrUPB	2.13	272	2.64	20.2	35.3	4.92	192	1.59	34.0	69.3	4.63	117	2.75	87.6	64.6
					22.5	,2		direction		07.0		1 /		1 00	00
Incr	27.7	274	0.20	19.7	34.2	49.7	212	0.15	25.4	58.9	121	132	0.10	96.9	59.5
IncrU	25.8	240	0.21	23.0	38.4	46.7	165	0.16	38.9	84.1	114	107	0.11	126	78.3
IncrUP	19.3	240	0.29	22.5	36.4	36.3	166	0.21	41.7	87.2	93.3	108	0.13	118	77.2
IncrUPB	4.53	348	1.24	15.7	27.4	16.3	205	0.48	26.5	58.4	44.8	149	0.28	60.5	40.7
random															
Incr	41.1	241	0.13	23.5	38.7	58.8	169	0.13	33.5	86.9	115	107	0.11	131	76.6
IncrU	48.8	234	0.11	24.1	37.6	69.2	154	0.11	43.5	101	136	102	0.09	149	85.8
IncrUP	35.7	233	0.15	24.9	40.2	52.3	153	0.15	45.6	102	93.5	103	0.13	140	86.5
IncrUPB	46.6	313	0.12	17.4	32.2	64.6	178	0.12	35.1	78.7	128	130	0.09	60.0	56.0

Table 1: Results of the incremental BVH build. The lowest BVH costs and the highest streaming and rendering speeds for the given scene and the stream order are highlighted.