CSE377 Spring2023 Homework 7: Image Restoration (10 pts)

Due April 18 2023, 11:59PM, via Brightspace

We derived a linear computational model to approximate the Phase Contrast or DIC microscopy imaging process:

✓ ✓ $N \times 1$

$$g = Hf$$

$N \times 1$ $N \times N$

where g is a microscopy image, H is a matrix related to the image formation process and f is the image to be restored.

We formulate the following sparsity-constrained quadratic optimization to restore f:

$$\mathbf{O}(\mathbf{f}) = \|\mathbf{H}\mathbf{f} - \mathbf{g}\|_2^2 + \omega_s \mathbf{f}^T \mathbf{L}\mathbf{f} + \omega_r \|\boldsymbol{\Lambda}\mathbf{f}\|_1$$

$$\Lambda = \begin{bmatrix} \Lambda_{11} & & 0 \\ & \Lambda_{22} & \\ 0 & & \ddots \\ & & & \Lambda_{NN} \end{bmatrix}$$

which can be solved by the following algorithm:

$f^{int} = 1$,
or $f^{int} = 0$,
or $f^{int} = \text{rand}()$

Algorithm I: restoring artifact-free microscopy images

Initialize $\mathbf{f} = \mathbf{f}^{init}$ and $\boldsymbol{\Lambda} = \boldsymbol{\Lambda}^{init}$. $\Lambda^{int} = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & & 1 \end{bmatrix}$ or $\Lambda^{int} = \begin{bmatrix} \text{rand}() & 0 \\ 0 & \ddots \end{bmatrix}$

Repeat the following steps for all pixel j

$$\mathbf{b} = -\mathbf{H}^T\mathbf{g} + \omega_r \text{diag}(\boldsymbol{\Lambda})/2 \quad (1)$$

$\mathbf{b}$: $N \times 1$ $\qquad$ $\text{diag}(\Lambda) = \begin{bmatrix} \Lambda_{11} \\ \Lambda_{22} \\ \vdots \\ \Lambda_{NN} \end{bmatrix}_{N \times 1}$

$$\mathbf{f}_j \leftarrow \mathbf{f}_j \frac{-\mathbf{b}_j + \sqrt{\mathbf{b}_j^2 + 4(\mathbf{Q}^+\mathbf{f})_j(\mathbf{Q}^-\mathbf{f})_j}}{2(\mathbf{Q}^+\mathbf{f})_j} \quad (2)$$

$$\boldsymbol{\Lambda}_{jj} \leftarrow \frac{\boldsymbol{\Lambda}_{jj}^{init}}{\mathbf{f}_j + \gamma} \quad (3)$$

Until convergence. $\|f^{(t)} - f^{(t-1)}\| < \varepsilon$ & iter > maxiter

e.g. $Q = \begin{bmatrix} -2 & 4 & 3 \\ 3 & -4 & 2 \\ 1 & -3 & 5 \end{bmatrix}$

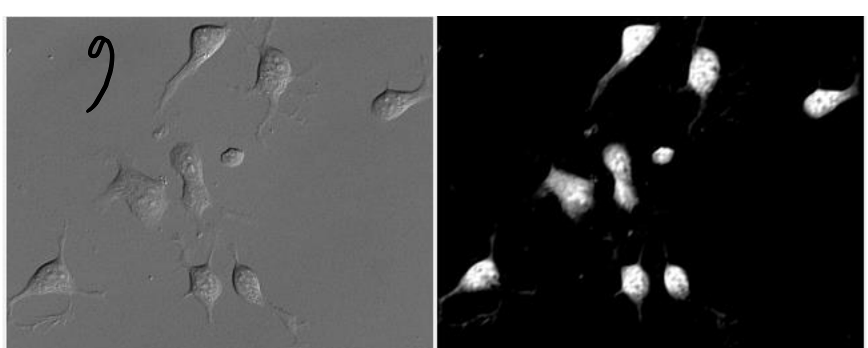
$Q^+ = \begin{bmatrix} 0 & 4 & 3 \\ 3 & 0 & 2 \\ 1 & 0 & 5 \end{bmatrix}$

$Q^- = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 3 & 0 \end{bmatrix}$

where $\mathbf{Q}_{uv}^+ = \begin{cases} \mathbf{Q}_{uv} & \text{if } \mathbf{Q}_{uv} > 0 \\ 0 & \text{otherwise} \end{cases}$ and $\mathbf{Q}_{uv}^- = \begin{cases} |\mathbf{Q}_{uv}| & \text{if } \mathbf{Q}_{uv} < 0 \\ 0 & \text{otherwise} \end{cases}$

$$\mathbf{Q} = \mathbf{H}^T\mathbf{H} + \omega_s \mathbf{L}$$

Starting codes are provided, including the computation of the H matrix, the procedure to flatten the image, and the procedure to compute the Laplacian matrix L of a regular image grid. A testing image is given. Implement Algorithm I and you are expected to achieve the following result:



$W_s = 0$, $W_r = 0$

Try different hyper parameters and see how the constraints affect the restoration processes ad results.

Submit your Jupyter notebook, or .py code with your resultant image in a brief report.