CSE377 Spring2023 Homework 7: Image Restoration (10 pts)

Due April 18 2023, 11:59PM, via Brightspace

We derived a linear computational model to approximate the Phase Contrast or DIC microscopy imaging process:

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f}$$

where $\boldsymbol{g}$ is a microscopy image, $\boldsymbol{H}$ is a matrix related to the image formation process and $\boldsymbol{f}$ is the image to be restored.

We formulate the following sparsity-constrained quadratic optimization to restore $\boldsymbol{f}$:

$$\mathbf{O}(\mathbf{f}) = \|\mathbf{H}\mathbf{f} - \mathbf{g}\|_2^2 + \omega_s \mathbf{f}^T \mathbf{L}\mathbf{f} + \omega_r \|\mathbf{\Lambda}\mathbf{f}\|_1$$

which can be solved by the following algorithm:

Algorithm I: restoring artifact-free microscopy images

Initialize $\mathbf{f} = \mathbf{f}^{init}$ and $\mathbf{\Lambda} = \mathbf{\Lambda}^{init}$.
Repeat the following steps for all pixel j

$$\mathbf{b} = -\mathbf{H}^T\mathbf{g} + \omega_r \mathrm{diag}(\mathbf{\Lambda})/2 \quad (1)$$

$$\mathbf{f}_j \leftarrow \mathbf{f}_j \frac{-\mathbf{b}_j + \sqrt{\mathbf{b}_j^2 + 4(\mathbf{Q}^+\mathbf{f})_j(\mathbf{Q}^-\mathbf{f})_j}}{2(\mathbf{Q}^+\mathbf{f})_j} \quad (2)$$

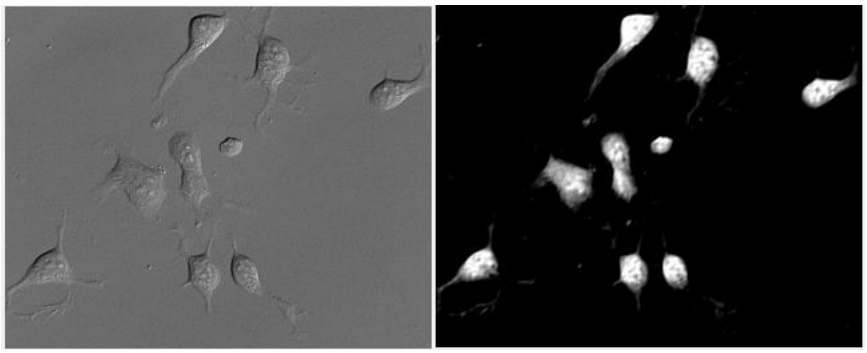
$$\mathbf{\Lambda}_{jj} \leftarrow \frac{\mathbf{\Lambda}_{jj}^{init}}{\mathbf{f}_j + \gamma} \quad (3)$$

Until convergence.

$$\text{where } \mathbf{Q}_{uv}^+ = \begin{cases} \mathbf{Q}_{uv} & \text{if } \mathbf{Q}_{uv} > 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \mathbf{Q}_{uv}^- = \begin{cases} |\mathbf{Q}_{uv}| & \text{if } \mathbf{Q}_{uv} < 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{Q} = \mathbf{H}^T\mathbf{H} + \omega_s\mathbf{L}$$

Starting codes are provided, including the computation of the $\boldsymbol{H}$ matrix, the procedure to flatten the image, and the procedure to compute the Laplacian matrix $\boldsymbol{L}$ of a regular image grid. A testing image is given. Implement Algorithm I and you are expected to achieve the following result:



Try different hyper parameters and see how the constraints affect the restoration processes ad results.

Submit your Jupyter notebook, or .py code with your resultant image in a brief report.