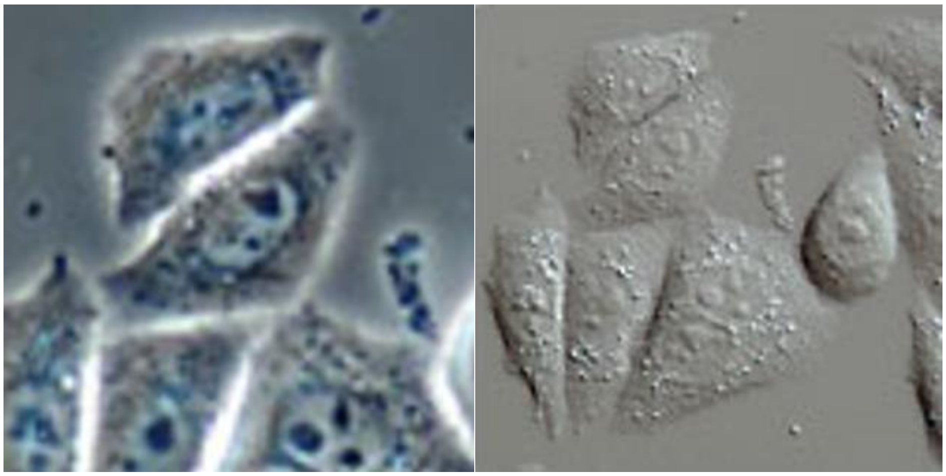
CSE377 HW9: Image Registration (10 pts)

Due May 5 2023, 11:59PM, submitted via Brightspace

Given a source image (left) and a target image (right) below, manually select N pts in the source image (x_n, y_n) and the corresponding N pts in the target image (x'_n, y'_n). $n = 1, \dots, N$ and $N \geq 4$.



1. (2pts) Assume $h_{33} = 1$, estimate the homography transformation matrix

$$h = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

from the source to the target image coordinates by formulating the following linear equation systems:

$$\begin{array}{l} \\ \textbf{Point 1} \\ \\ \textbf{Point 2} \\ \\ \textbf{Point 3} \\ \\ \textbf{Point 4} \\ \\ \textbf{additional points} \end{array}
\begin{array}{c} \textbf{2N x 8} \\ \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4 \end{bmatrix} \\ \vdots \end{array}
\begin{array}{c} \textbf{8 x 1} \\ \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} \\ \\ \end{array}
=
\begin{array}{c} \textbf{2N x 1} \\ \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix} \\ \vdots \end{array}$$

Denote the above linear equation system as $\boldsymbol{Ah} = \boldsymbol{b}$, solve $\boldsymbol{h}$ by pseudo-inverse $\boldsymbol{h} = \left(\boldsymbol{A^T A}\right)^{-1}(\boldsymbol{A^T b})$.

2. (2pts) Without assuming $h_{33} = 1$, estimate the homography transformation matrix from the source to the target image coordinates by formulating the following homogeneous equation systems:

$$\begin{array}{lcccc} & \mathbf{2N \times 9} & \mathbf{9 \times 1} & & \mathbf{2N \times 1} \\ \mathbf{4\ POINTS} & \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 & -y'_1 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} & \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} & = & \begin{bmatrix} 0 \\ 0 \\ \\ \\ \end{bmatrix} \\ \mathbf{additional\ points} & \vdots & & & \vdots \end{array}$$

Denote the above homogeneous equation system as $\boldsymbol{Ah} = \mathbf{0}$, solve $\boldsymbol{h}$ by eigen decomposition (eig) as discussed in the class. Denote the solution as $\boldsymbol{h}_{eig}$.

3. (1pt) Using the same $\boldsymbol{A}$ matrix in part 2, solve $\boldsymbol{h}$ by singular value decomposition (svd) as discussed in the class. Denote the solution as $\boldsymbol{h}_{svd}$.

4. (1pt) Compare $\boldsymbol{h}$, $\boldsymbol{h}_{eig}$ and $\boldsymbol{h}_{svd}$.

Check if $\boldsymbol{h}_{eig}$ is identical to $\boldsymbol{h}_{svd}$.

$\boldsymbol{h}_{eig}$ and $\boldsymbol{h}_{svd}$ are 9×1 vectors. Dividing them by their last element (i.e., $\boldsymbol{h}_{eig} \leftarrow \boldsymbol{h}_{eig}/\boldsymbol{h}_{eig}(9)$ and $\boldsymbol{h}_{svd} \leftarrow \boldsymbol{h}_{svd}/\boldsymbol{h}_{svd}(9)$. Check if the first 8 elements of the new $\boldsymbol{h}_{eig}$ and $\boldsymbol{h}_{svd}$ are the same as $\boldsymbol{h}$ in part 1.

5. (2pts) Implement the <u>forward</u> warping to warp the source image to the target image coordinate, using the estimated $\boldsymbol{h}$ or $\boldsymbol{h}_{eig}$ or $\boldsymbol{h}_{svd}$.

6. (2pts) Implement the <u>backward</u> warping to warp the source image to the target image coordinate, using the estimated $\boldsymbol{h}$ or $\boldsymbol{h}_{eig}$ or $\boldsymbol{h}_{svd}$. You can use the interp2() function.