Discrete vs continuous return

1 Data

Data

`dfitfool`

2 Discrete vs continuous return

Returns of one asset - Four plots

(Discrete/Continuous) Return

(Discrete/Continuous) Expected return and volatility

Example - Difference between discrete mean return and continuous mean return

Data

We get data from yahoo. It is free. It also provide the adjusted close, which is adjusted for dividends and splits. Here is steps you follow to get data.

[**Step 1**] Go to `http://finance.yahoo.com/`.

[**Step 2**] Type `WMT` or any other ticker symbol at **Get Quotes** window and return.

[**Step 3**] Click **Historical Prices**.

[**Step 4**] Click **Get Prices**. You can get Daily, Weekly, or Monthly data.

[**Step 5**] Click **Download to Spreadsheet**.

[**Step 6**] Save the downloaded file as a **csv file**.

[**Step 7**] Convert the numeric part of the csv file into a **mat file**.

[**Step 8**] Play with the **Adj Close**, not Close.

[**Step 9**] Arrange data as old one first and new one last. You may need `flipud`.

[**Step 10**] Arrange data as benchmark last column.

Date	Stock 1	Stock 2	Stock 3	⋯	Benchmark
2012-06-07	34.28	48.72	126.38	⋯	12457.28
2012-06-10	35.53	49.66	126.41	⋯	12546.77
2012-06-11	36.35	49.52	127.24	⋯	12634.43
2012-06-12	35.27	48.35	131.30	⋯	12593.65
2012-06-13	36.88	47.78	129.36	⋯	12626.24
⋮	⋮	⋮	⋮	⋮	⋮

Benchmark Index VFINX instead of S&P 500

S&P 500 data from yahoo is not adjusted for dividends and splits. So we use Vanguard 500 Index Fund Inv (VFINX) as our benchmark index instead. VFINX is the industry's first index fund for individual investors. The 500 Index Fund is a low cost way to gain diversified exposure to the U.S. equity market. The fund invests in 500 of the largest U.S. companies, which span many different industries and account for about three-fourths of the U.S. stock markets value. The key risk for the fund is the volatility that comes with its full exposure to the stock market. Because the 500 Index Fund is broadly diversified within the large-capitalization market, it may be considered a core equity holding in a portfolio. You can get `VFINX` data from yahoo in the same manner.

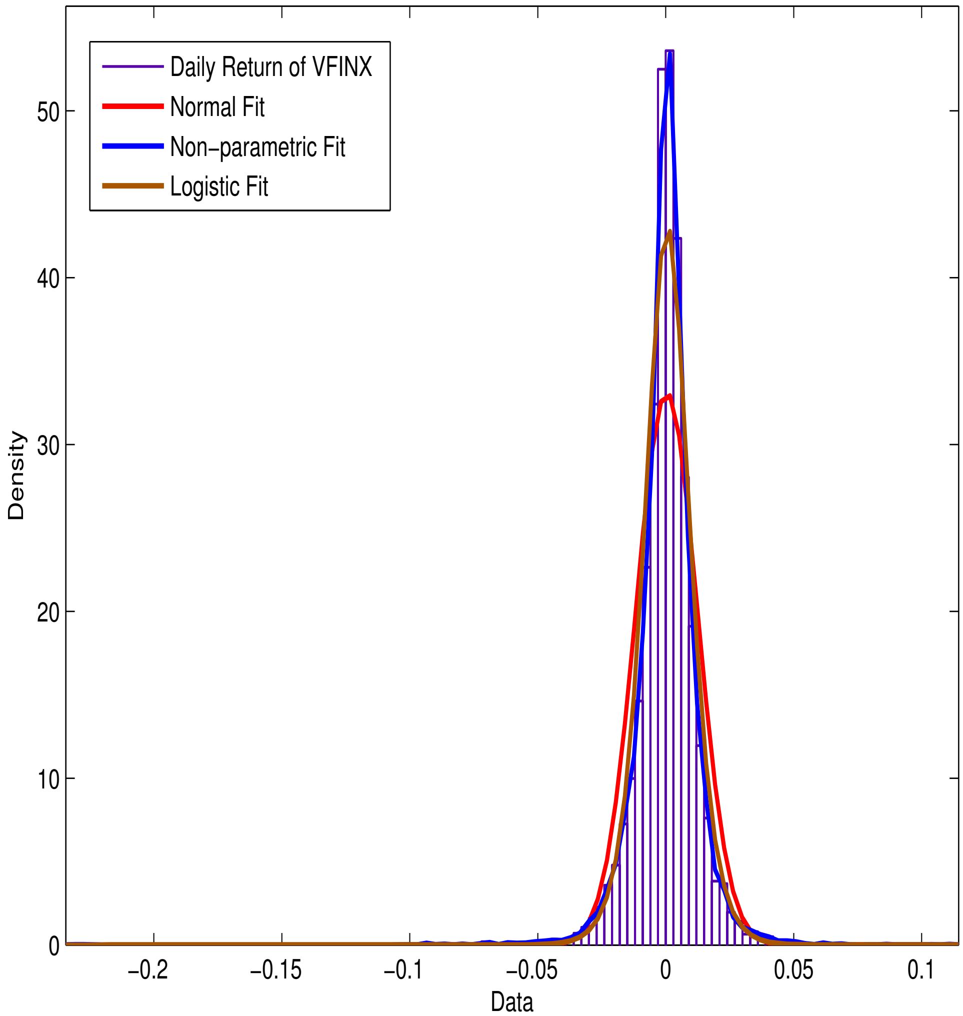
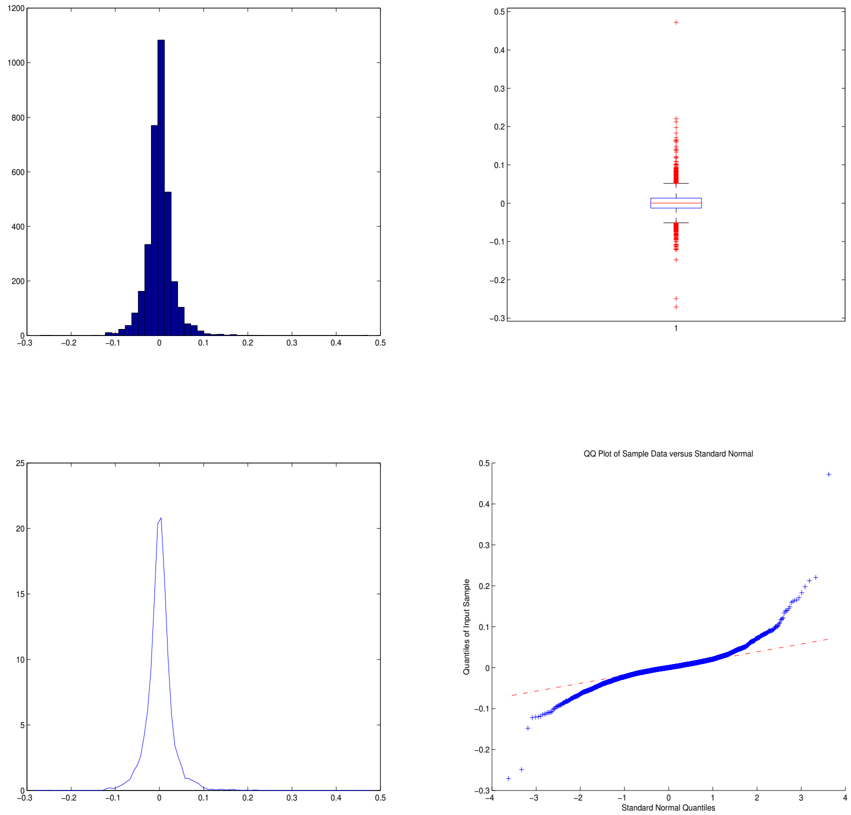


Figure 1: [dfittool] Daily Close, Return, Histogram of Return of VFINX.

```
%% Daily Close, Return, Histogram of Return of VFINX (dfittool) %%%%%%%%%%

load('VFINXAdjClose.mat')
A=VFINXAdjClose(:,end);
A=flipud(A);
r=log(A(2:end)./A(1:end-1));

dfittool
```

```
%% Output from Normal dfittool %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Distribution:    Normal
Log likelihood:  20015.5
Domain:          -Inf < y < Inf
Mean:            0.000342022
Variance:        0.000144264

Parameter  Estimate     Std. Err.
mu         0.000342022  0.000147123
sigma         0.012011  0.000104043

Estimated covariance of parameter estimates:
       mu           sigma
mu      2.1645e-08  -3.8155e-25
sigma  -3.8155e-25   1.0825e-08

%% Output from Non-parametric dfittool %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Kernel:     normal
Bandwidth:  0.0014156
Domain:     -Inf < y < Inf

%% Output from Logistic dfittool %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Distribution:    Logistic
Log likelihood:  20805.8
Domain:          -Inf < y < Inf
Mean:            0.000557427
Variance:        0.000109708

Parameter  Estimate     Std. Err.
mu         0.000557427   0.00011976
sigma       0.00577472  6.04331e-05

Estimated covariance of parameter estimates:
       mu            sigma
mu      1.43425e-08  -8.01255e-11
sigma  -8.01255e-11   3.65216e-09

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Returns of one asset - Four plots



	Description
`mean`	Mean for each column
`std`	Standard deviation for each column
`cov`	Covariance matrix between columns
`corr`	Correlation matrix between columns

	Description
`hist`	Histogram
`boxplot`	Boxplot
`ksdensity`	Kernel smooth density
`qqplot`	Qqplot

```
clear all; close all; clc;

load('SP500AdjClose.mat');
A=SP500AdjClose(:,1);
[~,ind]=min(A);
n=ind-1;
A=A(1:n,:);
A=flipud(A);

r=(A(2:end,:)-A(1:end-1,:))./A(1:end-1,:);
mu=mean(r); si=std(r);
mu=mu*252, si=si*sqrt(252)

subplot(2,2,1); hist(r,50)
subplot(2,2,2); boxplot(r)
subplot(2,2,3); ksdensity(r)
subplot(2,2,4); qqplot(r)
```

(Discrete/Continuous) Return

$$\text{Discrete return} \qquad S_T = S_0(1 + R^{(D)})$$

$$\text{Continuous return} \qquad S_T = S_0 e^{R^{(C)}}$$

$\boxed{\text{Discrete return is user-friendly for portfolio computation}}$

$$\text{Discrete return } R^{(D)} \qquad \sum_{i=1}^{n} x_i(1 + R_i^{(D)}) = 1 + R^{(D)} \quad \Rightarrow \quad R^{(D)} = \sum_{i=1}^{n} x_i R_i^{(D)}$$

$$\text{Continuous return } R^{(C)} \qquad \sum_{i=1}^{n} x_i e^{R_i^{(C)}} = e^{R^{(C)}} \quad \Rightarrow \quad R^{(C)} \neq \sum_{i=1}^{n} x_i R_i^{(C)}$$

$\boxed{\text{Continuous return is user-friendly for holding period return computation}}$

$$\text{Discrete return} \qquad (1 + R_1^{(D)})(1 + R_2^{(D)}) = 1 + R^{(D)} \quad \Rightarrow \quad R^{(D)} \neq R_1^{(D)} + R_2^{(D)}$$

$$\text{Continuous return} \qquad e^{R_1^{(C)}} e^{R_2^{(C)}} = e^{R^{(C)}} \quad \Rightarrow \quad R^{(C)} = R_1^{(C)} + R_2^{(C)}$$

(Discrete/Continuous) Expected return and volatility

$$\begin{array}{ccccccccc}
\frac{dS}{S} & = & \color{red}{\mu} & * & dt & + & \sigma & * & db \\
\uparrow & & \color{red}{\uparrow} & & & & \uparrow & & \\
\text{Discrete return} & & \color{red}{\text{Expected discrete return}} & & & & \text{Volatility} & &
\end{array}$$

$$\begin{array}{ccccccccc}
d\log S & = & \color{red}{\left(\mu - \frac{1}{2}\sigma^2\right)} & * & dt & + & \sigma & * & db \\
\uparrow & & \color{red}{\uparrow} & & & & \uparrow & & \\
\text{Continuous return} & & \color{red}{\text{Expected continuous return}} & & & & \text{Volatility} & &
\end{array}$$

$$dt = \frac{1}{252} \qquad (1 \text{ month} = 21 \text{ days, } 1 \text{ year} = 252 \text{ days})$$

Discrete and continuous expected return

$$\mu_d = \mu \quad \text{and} \quad \mu_c = \mu - \frac{1}{2}\sigma^2 \quad \Rightarrow \quad \color{red}{\mu_c = \mu_d - \frac{1}{2}\sigma^2}$$

Discrete and continuous volatility

$$\sigma_d = \sigma_c = \sigma$$

Example - Difference between discrete expected return and continuous expected return

The first two columns of `SP500AdjClose.mat` are daily adjusted close of A (Agilent Technologies Inc (NYSE)) and AA (Alcoa Inc (NYSE)). We calculate the annualized mean and volatility of discrete/continuous return of these two.

	Mean (A)	Vol (A)	Mean (AA)	Vol (AA)
Discrete return $\frac{dS}{S}$	11.37%	49.71%	2.56%	37.54%
Continuous return $d\log S$	−0.96%	49.71%	−4.43%	37.33%

Can you rationalize these discrepancies in mean?

Let's check whether this relation holds for A;

$$\mu \text{ from discrete mean return} \qquad \mu = \mu_d = 11.37\%$$

$$\mu \text{ from continuous mean return} \qquad \mu = \mu_c + \frac{1}{2}\sigma^2 = -0.96\% + \frac{1}{2}(49.71\%)^2 = 11.40\%$$

How about AA?

$$\mu \text{ from discrete mean return} \qquad \mu = \mu_d = 2.56\%$$

$$\mu \text{ from continuous mean return} \qquad \mu = \mu_c + \frac{1}{2}\sigma^2 = -4.43\% + \frac{1}{2}(37.33\%)^2 = 2.54\%$$