CSE215: Lecture 02
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

February 22, 2022

Course materials and Info available here:
https://github.com/zhoulaifu/22_cse215_spring

Propositional Logic

Predicate Logic

Proof

Why does a computing system fail (or work)?

Sequences

Sets

Functions

Relations

Today's objectives

Know a list of key things that will be covered in the exams

Today's work

book chapter	Topics	Exam problems
2	Propositional logic	2021-final, pb 1
3	Predicate logic	2021-midterm1, pb3
4	Proof	2021-final, pb4
5	Sequences	2021-final, pb7
6	Sets	2021-midterm2, pb2
7	Functions	2021-final, pb9
8	Relations	2021-final, pb11

How we proceed next:

- We first go over the exam problems, emphasizing "key" concepts.
- The instructor will explain the concepts very briefly so that you get an intuition.
- We use the rest of time to do the exercises together. By design, explanation will be time-limited.

Propositional Logic
Final 2021

Problem 1. [5 points]
Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Key: Truth Table

Truth table for p ^ q

p	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

Key: Negation on quantifiers

- $\sim(\forall \boldsymbol{x}, P(\boldsymbol{x})) \equiv \exists \boldsymbol{x}, \sim P(\boldsymbol{x})$
- $\sim(\exists \boldsymbol{x}, P(\boldsymbol{x})) \equiv \forall \boldsymbol{x}, \sim P(\boldsymbol{x})$

Proof — Final 2021

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

Key: Prove things about integers from basic facts

Example of basic facts: an even integer can be written as 2*n; or (x+y)^2= x^2 + 2xy + y^2

Sequences - Final 2021

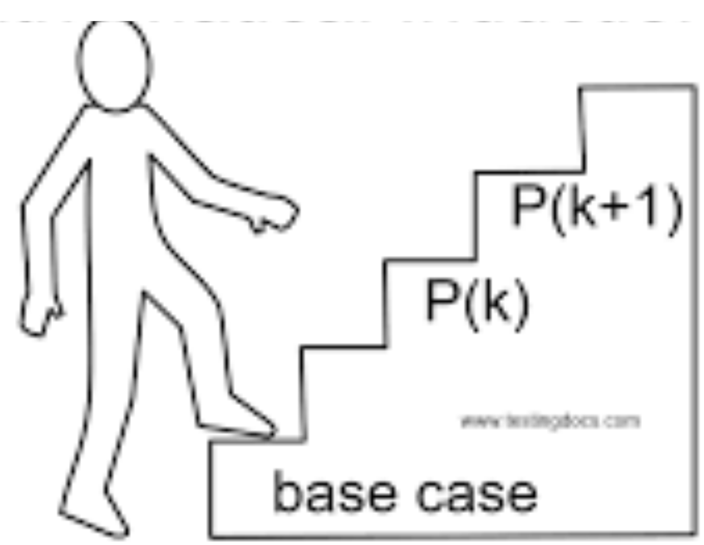
Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

Key: Use Mathematical Induction to show facts about integers



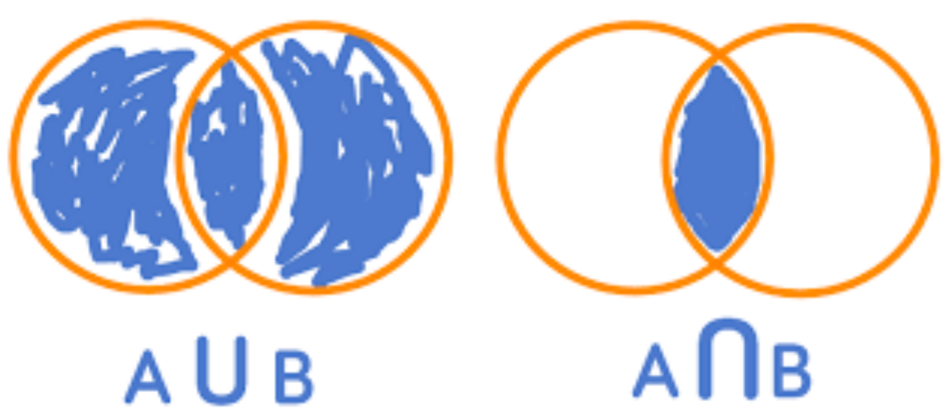
Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U.

(a) **[1 point]** $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

Key: Union and intersection on Sets



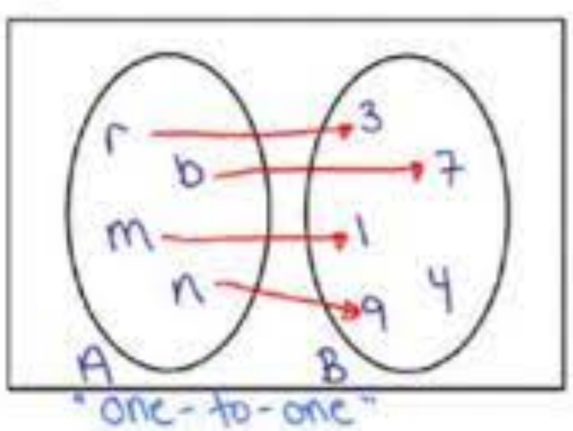
Functions — Final 2021

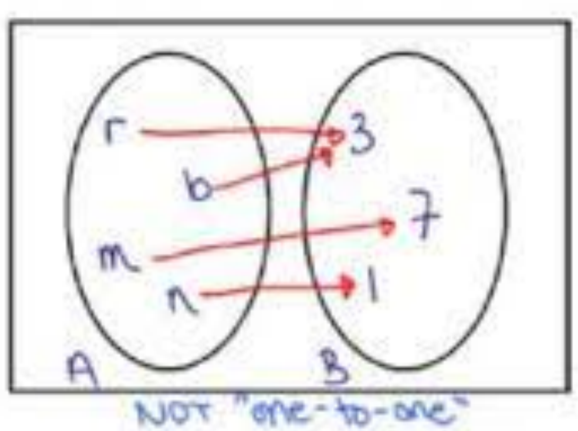
Problem 9. [5 points]

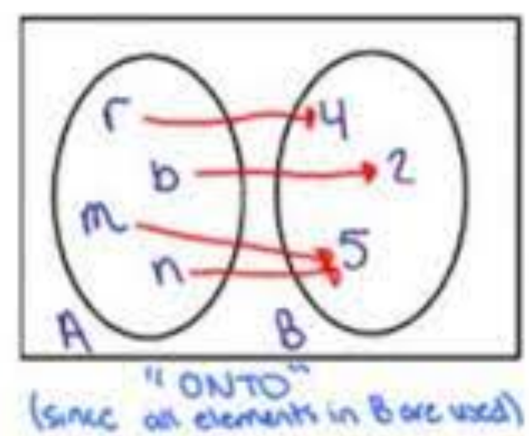
Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

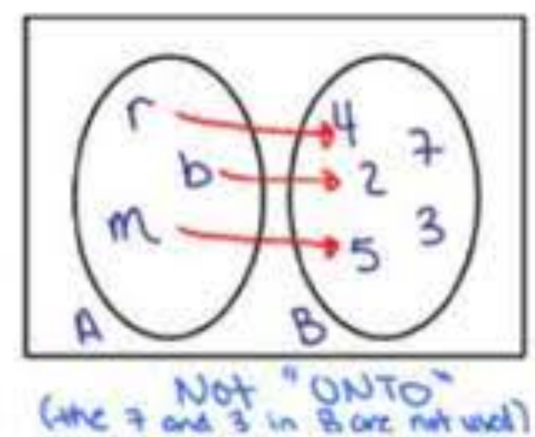
Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \to \mathbb{Z}$		

Key: One-to-one and onto functions







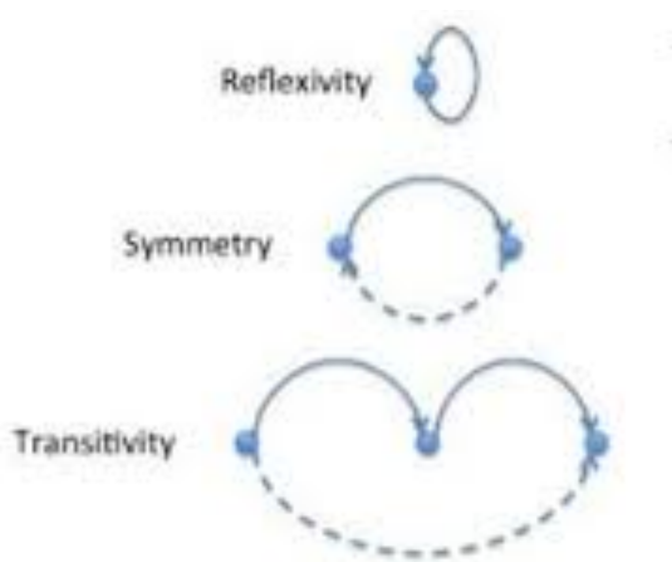


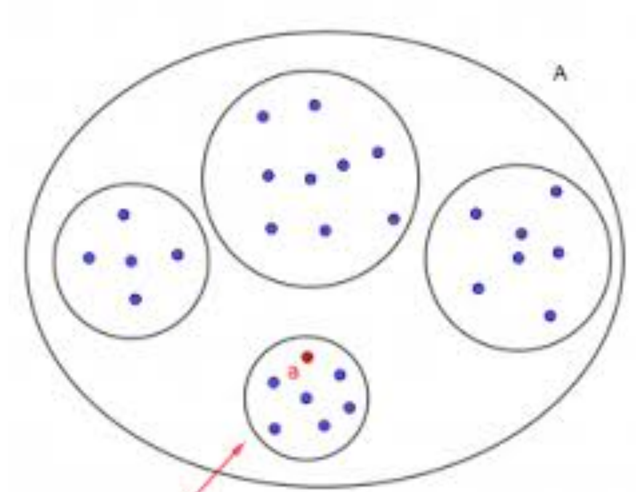
Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A, we have $p\ R\ q \Leftrightarrow p$ has the same birthday as q.
Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

Key: Equivalence relations and Equivalence classes





equivalence class

Today's take-away

book chapter	Topics	Exam problems	Key
2	Propositional logic	2021-final, pb 1	truth table
3	Predicate logic	2021-midterm1, pb3	negation on quantifiers
4	Proof	2021-final, pb4	facts about integers
5	Sequences	2021-final, pb7	math induction
6	Sets	2021-midterm2, pb2	unions and intersections
7	Functions	2021-final, pb9	1-1 and onto
8	Relations	2021-final, pb11	equiv. rel. and classes

Thank you for your attention!