CSE215: Lecture 03
Foundations of Computer Science

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Course materials and Info available here:
https://github.com/zhoulaifu/22_cse215_spring

News

- Attendance check —> From next week
- hybrid —> In person only, unless covid
- Homework —> Per week instead of per chapter

Today’s objectives

To understand

- the scope of propositional logic
- a mathematical way to check if a proposition is true or false

Why?

SBU’s answer (selection)

Remove ambiguity.

English is ambiguous:

- “The cat chased the mouse until it fell.”
- “My mother never made cake, which we hated.”

Artificial Intelligence 47 (1991) 31–56
Elsevier

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Logic and artificial intelligence

Nils J. Nilsson

Computer Science Department, Stanford University, Stanford, CA 94305, USA

Received February 1989

Abstract

Nilsson, N.J., Logic and artificial intelligence, Artificial Intelligence 47 (1990) 31–56.

The theoretical foundations of the logical approach to artificial intelligence are presented.

- 1989 paper from a founding AI researcher at Stanford CS

- Quote: “Logic provides the vocabulary and many of the techniques needed both for analyzing the processes of representation and reasoning and for synthesizing machines that represent and reason.”

Scope

Example: Intelligence in software analysis used by Facebook, Microsoft, or Google

```
int x = 0;
while (x < 10){
    x = x + 1;
}
```

http://cpp.sh/5p7zo

- Without executing the code, what will the value of x become after the loop ends?
- Answer: x must equals to 10. Since x>=10 here after the loop, and x<11 as an invariant. Combined with the fact that x is an integer, we can be certain that x = 10.

How do we know if a proposition is true or false

- There exists life in other planets
- The next President of Korea will be Mr / Ms ___.
- The earth is round.
- If earth is round, I can return to where I am by traveling toward a certain direction.
- To check truthfulness of these propositions require physics or geography knowledge beyond computer scientists / mathematicians.
- But they may tell you whether you can rightfully argue that, based on some assumptions, whether a statement is true.
- Namely, they may tell if your argument makes sense. For example, …

Scope of propositional logic

- To check if a proposition like (p -> q) -> (q -> p) is true of false

- We call such propositions compound statement.

- Scope: This class is not to decide truthfulness of unit proposition, but their combination and connection, and the truthfulness of the resulted compound statements.

How?

Can we check (p -> q) -> (q -> p) with a truth table talked yesterday?

Let us try!

A more systematic study
(based on SBU’s materials)

Truth in the sense of Logic / Mathematics

Rigor	Truth type	Field	Truth teller
0	Word of God	Religion	God/Priests
1	Authoritative truth	Business	Boss
2	Legal truth	Judiciary	Law/Judge/Law makers
3	Philosophical truth	Philosophy	Plausible argument
4	Scientific truth	Physical sciences	Experiments/observation
5	Statistical truth	Statistics	Data sampling
6	Mathematical truth	Mathematics	Logical deduction

Proposition

Definition

- A statement or proposition is a sentence for which a truth value (either true or false) can be assigned

Classes of statements

- True statements. "The atomic number of Oxygen is 8."
- False statements. "$1 + 1 = 3$."
- Truth value currently unknown.
  - Goldbach's conjecture ▷ Why?
- Truth values change with time/scenarios.
  - "Today is Sunday." written on a paper slip ▷ Why?

Compound statement

Definition

- A compound statement is a complex sentence that is obtained by joining propositional variables using logical connectives

Logical operator	Notation	Read as
Negation	$\sim p$	not p
Conjunction	$p \wedge q$	p and q
Disjunction	$p \vee q$	p or q
Conditional	$p \to q$	p implies q if p, then q p only if q q if p q, provided that p
Biconditional	$p \leftrightarrow q$	p if and only if q
Logical equivalence	$p \equiv q$	p logically equivalent to q

Truthfulness of compound statements

Negation $(\sim p)$

Definition

- Negation of a statement p, denoted by $\sim p$, is a statement obtained by changing the truth value of p.

p	$\sim p$
T	F
F	T

Truthfulness of compound statements

Conjunction $(p \wedge q)$

Definition

- Conjunction of statements p and q, denoted by $p \wedge q$, is a statement such that it is true if both p and q are true and it is false, otherwise.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truthfulness of compound statements

Disjunction $(p \vee q)$

Definition

- Disjunction of statements p and q, denoted by $p \vee q$, is a statement such that it is false if both p and q are false and it is true, otherwise.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truthfulness of compound statements

Exclusive or $(p \oplus q)$

Definition

- Exclusive or of statements p and q, denoted by $p \oplus q$, is defined as p or q but not both. It is computed as $(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Do you want Kimchi, or do you want Gimbap?

Truthfulness of compound statements

Definition

- Conditional or implication is a compound statement of the form "if p, then q". It is denoted by $p \to q$ and read as "p implies q". It is false when p is true and q is false, and it is true, otherwise.

p ->q seen as

~p ⋁ q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Examples of from absurd to absurd

- If 1+1 = 3, then 1 = 0
- If the earth is plat, I am walking on the moon

Precedence of Logical Operators

Priority	Operator	Comments
1	$\sim$	Evaluate $\sim$ first
2	$\wedge$ $\vee$	Evaluate $\wedge$ and $\vee$ next; Use parenthesis to avoid ambiguity
3	$\rightarrow$ $\leftrightarrow$	Evaluate $\rightarrow$ and $\leftrightarrow$ next; Use parenthesis to avoid ambiguity
4	$\equiv$	Evaluate $\equiv$ last

- p ⋁ q ⋀ r reads as …
- ~ p -> q reads as …
- p -> q ⋀ q -> p reads as …

Logic equivalence

Definition

- Two statement forms p and q are logically equivalent, denoted by $p \equiv q$, if and only if they have the same truth values for all possible combination of truth values for the propositional variables

Checking logical equivalence

1. Construct and compare truth tables (most powerful)
2. Use logical equivalence laws

Logical equivalence: Example

Problem

- Show that $p \wedge (q \vee r) \not\equiv (p \wedge q) \vee r$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	F	F	T
F	T	F	T	F	F	F
F	F	T	T	F	F	T
F	F	F	F	F	F	F

Useful logical equivalence

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Tautology and contradiction

Definitions

- A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables.
- A contradication is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables.

Examples

- $p \vee \sim p$ $\quad\triangleright$ Tautology
- $p \wedge \sim p$ $\quad\triangleright$ Contradiction

The secret of a fortune teller

Let’s practice - 2021 Final

Problem 2. [5 points]

Check the logical equivalence of $\sim (p \vee (\sim p \wedge q))$ **and** $\sim p \wedge \sim q$.

- Use De Morgan law and $\sim(p \vee q) = \ldots$
- Use distribution law $p \wedge (q \vee r) = \ldots$

See how logic saved Chris Gardner

https://www.youtube.com/watch?v=W2r4BUB-Rsc

What would you say if a person wearing such a T-shirt walking into the interview, and I hired him

—> Get Hired

- Interview's **proposition**: Bad-T-shirt ∧ Get-hired
- Usually we **assume** Bad-T-shirt —> ~Get-hired
- Following this usual assumption, we now know ~Get-hired ∧ Get-hired. That means **contradiction**.
- Never tell the interviewer that what they said was a contradiction.
- As Chris followed CSE215 long time ago, he knows the assumption Bad-T-shirt —> ~Get-hired is **false**
- So, Chris is now thinking from another angle. What can we imply from Get-hired?
- Since Get-hired means there must be some extraordinary quality. Chris thinks of two things: Get-hired -> Nice-T-shirt ∨ Nice-Pants
- He put this one to the interviewer's proposition, he gets Bad-T-Shirt ∧ (Nice-T-shirt ∨ Nice-Pants)
- Because he knows **distributive law**, he gets (Bad-T-shirt ∧ Nice T-shirt) ∨ (Bad-T-shirt and Nice-Pants)
- So he gets an **equivalent** formula: Bad-T-shirt ∧ Nice-pants.
- Bad-T-shirt is kind of **tautology** for the interviewer. So he answers, with confidence from his logic lessons, that he got "nice pants".

Takeaway for today

- What kind of things does Propositional Logic study
- How to evaluate (1) basic logic structures and (2) compound ones
- Practiced with a bit of real-world examples

Thank you for your attention!