



BACKPROPAGATION



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Backward Propagation of Errors

Backpropagation, short for “backward propagation of errors”, is a mechanism used to update the **weights** using **gradient descent**. It calculates the gradient of the error function with respect to the neural network’s weights. The calculation proceeds backwards through the network.

Gradient descent is an iterative optimization algorithm for finding the minimum of a function; in our case we want to minimize the error function. To find a local minimum of a function using gradient descent, one takes steps proportional to the negative of the gradient of the function at the current point.



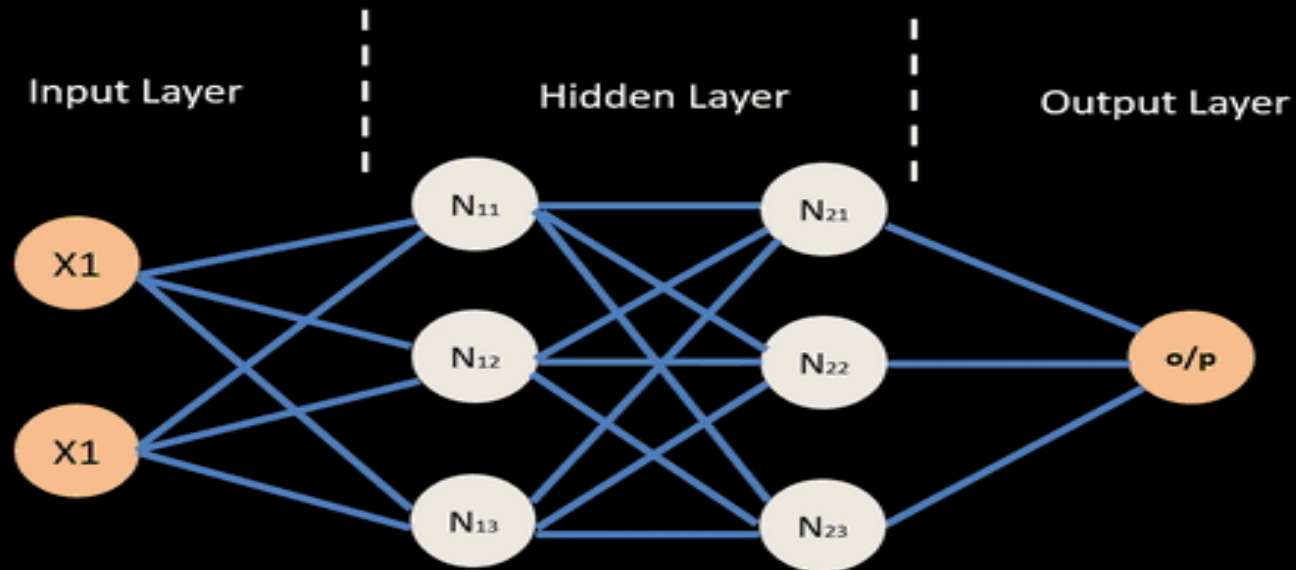
01

01



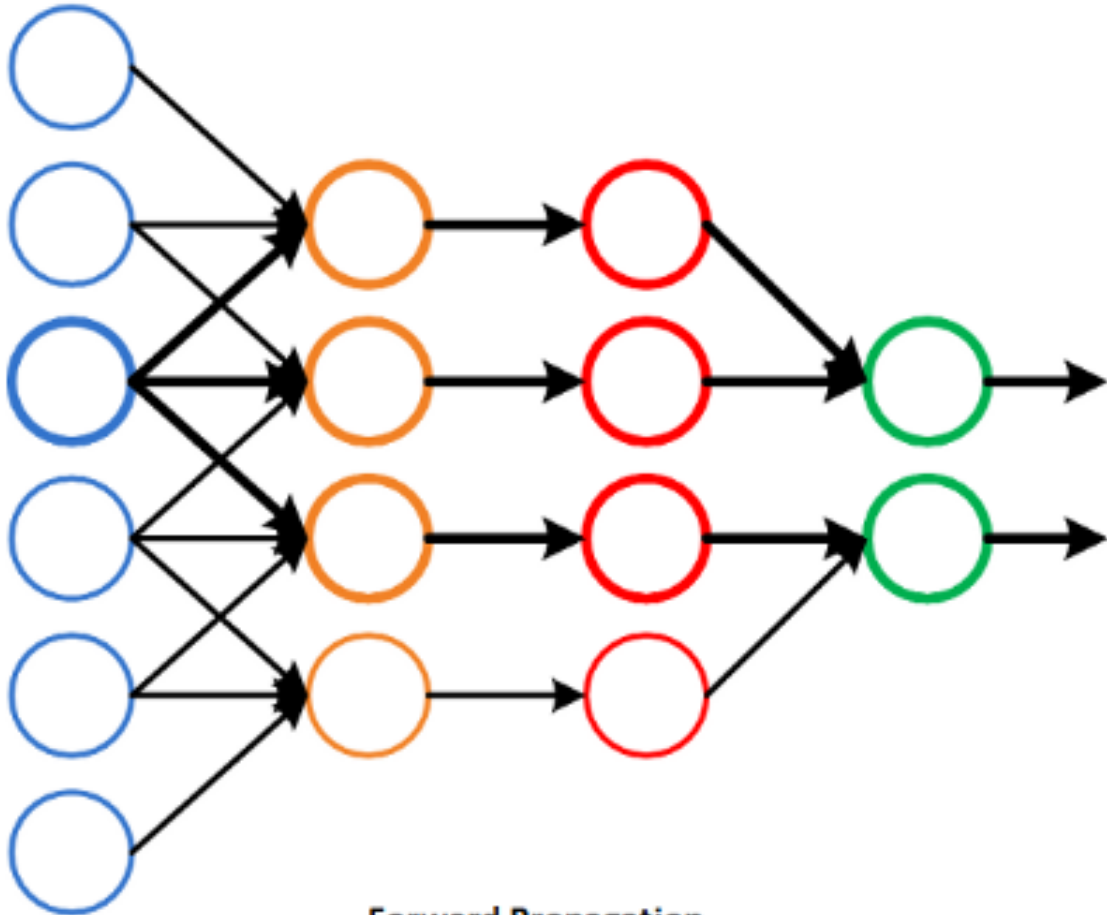
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Neural Network – Backpropagation

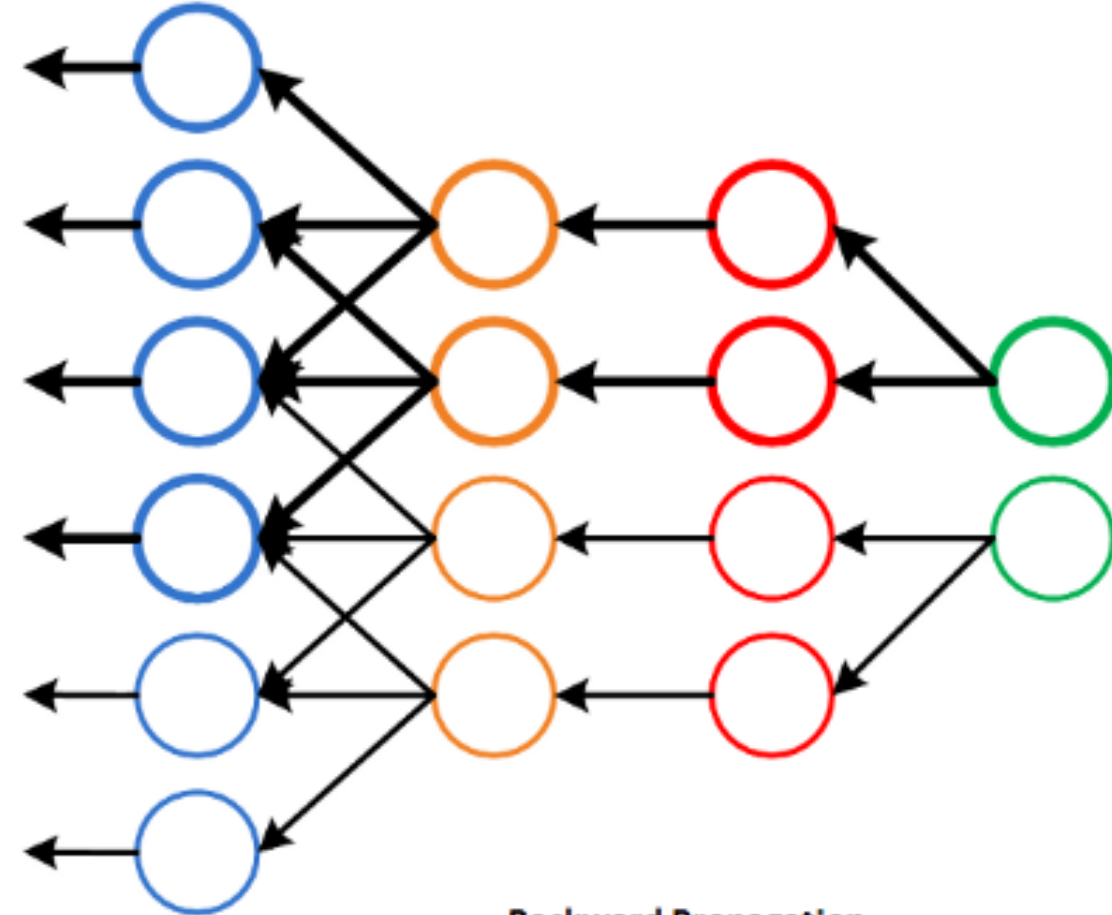




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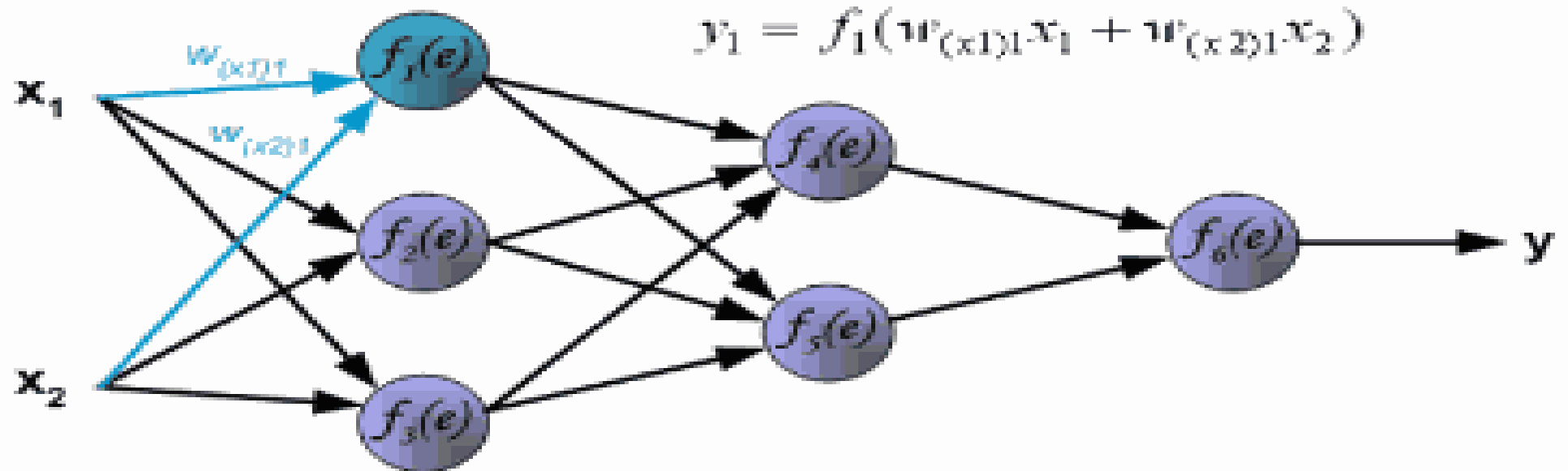
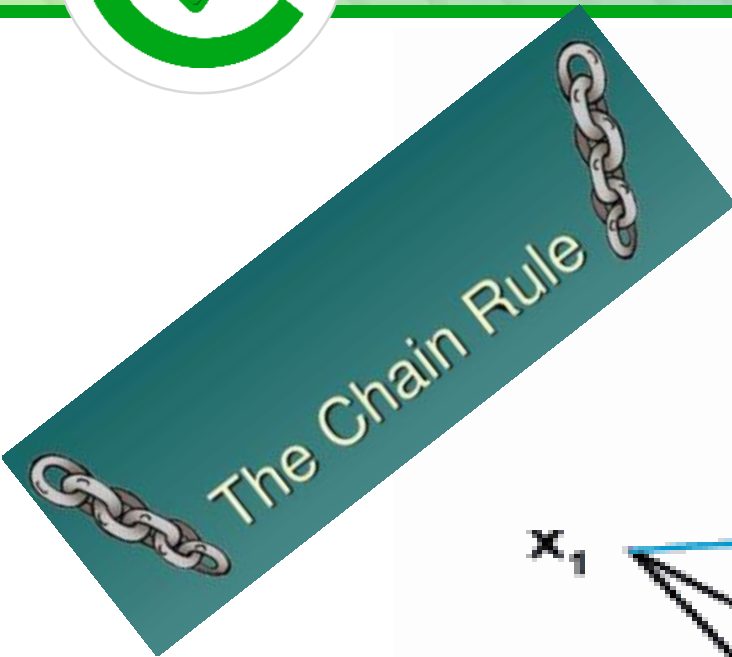


Forward Propagation



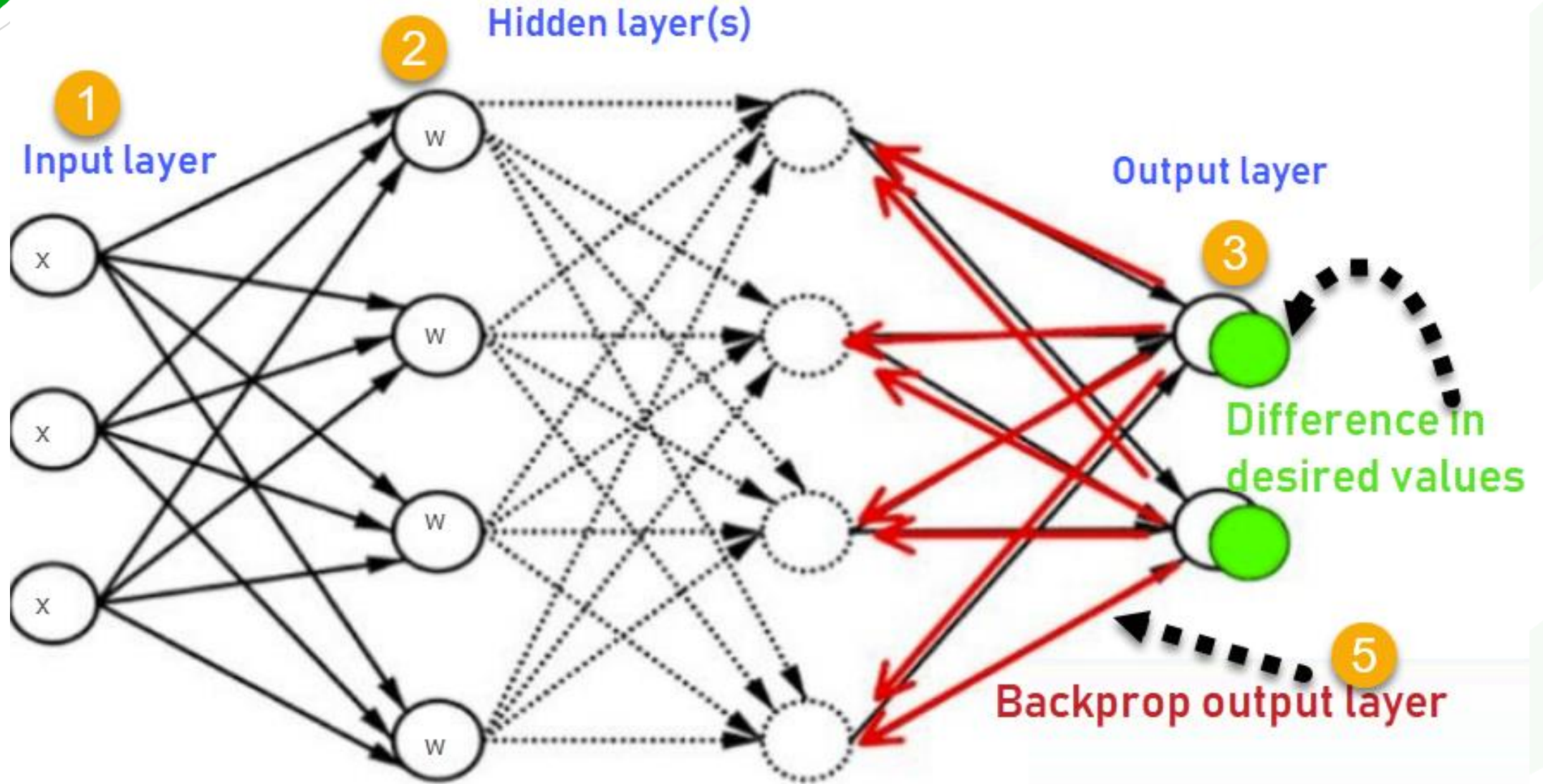
Backward Propagation

BACKPROPAGATION





BACKPROPAGATION





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Derivative of Error with respect to weight

Old weight

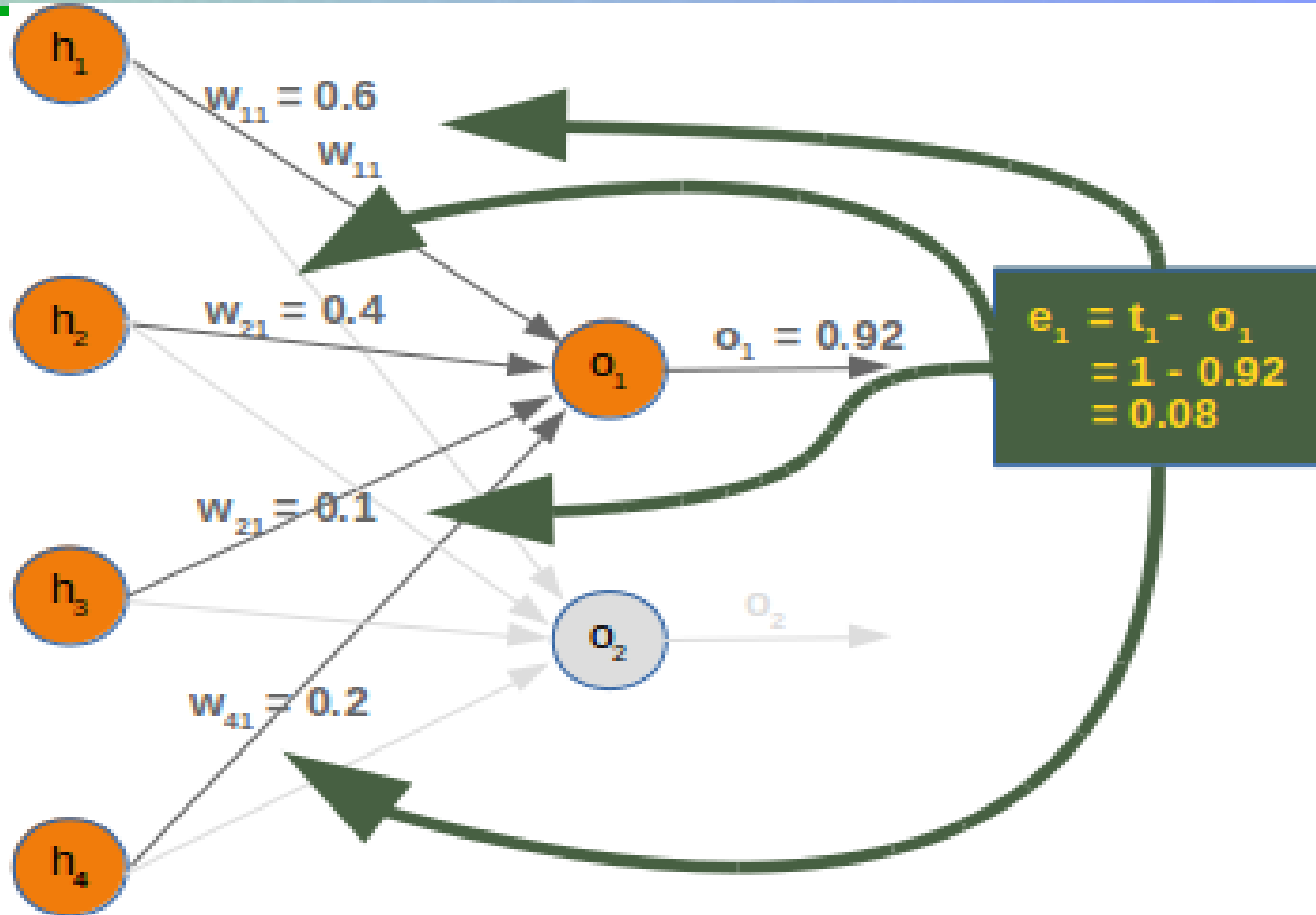
New weight

Learning rate

$$W_x = W_x - a \left(\frac{\partial \text{Error}}{\partial W_x} \right)$$

Step Size = slope x learning rate

New Parameter = Old Parameter - Step Size



$$\begin{bmatrix} w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a \Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a h_1 \Delta \\ a h_2 \Delta \end{bmatrix}$$

$$\begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a \Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \quad w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a i_1 \Delta w_5 & a i_1 \Delta w_6 \\ a i_2 \Delta w_5 & a i_2 \Delta w_6 \end{bmatrix}$$

$$*W_x = W_x - a \left(\frac{\partial \text{Error}}{\partial W_x} \right)$$

Old weight \downarrow
 it \uparrow
 Learning rate \uparrow
 Derivative of Error with respect to weight \downarrow

$$\Delta = 0.191 - 1 = -0.809$$

Delta = prediction - actual

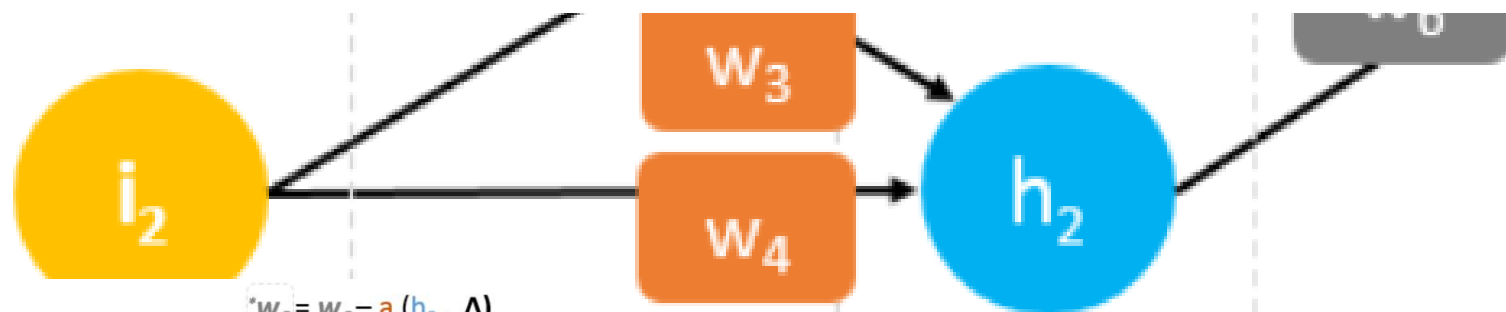
$$a = 0.05$$

Learning rate, we smartly guess this number

$$\begin{bmatrix} w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} - 0.05(-0.809) \begin{bmatrix} 0.85 \\ 0.48 \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} - \begin{bmatrix} -0.034 \\ -0.019 \end{bmatrix} = \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix}$$

$$\begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} .11 & .12 \\ .21 & .08 \end{bmatrix} - 0.05(-0.809) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.14 \quad 0.15] = \begin{bmatrix} .11 & .12 \\ .21 & .08 \end{bmatrix} - \begin{bmatrix} -0.011 & -0.012 \\ -0.017 & -0.018 \end{bmatrix} = \begin{bmatrix} .12 & .13 \\ .23 & .10 \end{bmatrix}$$

prediction



updated weights

$$\begin{aligned}
 *w_6 &= w_6 - a (h_2 \cdot \Delta) \\
 *w_5 &= w_5 - a (h_1 \cdot \Delta) \\
 *w_4 &= w_4 - a (i_2 \cdot \Delta w_6) \\
 *w_3 &= w_3 - a (i_1 \cdot \Delta w_6) \\
 *w_2 &= w_2 - a (i_2 \cdot \Delta w_5) \\
 *w_1 &= w_1 - a (i_1 \cdot \Delta w_5)
 \end{aligned}$$

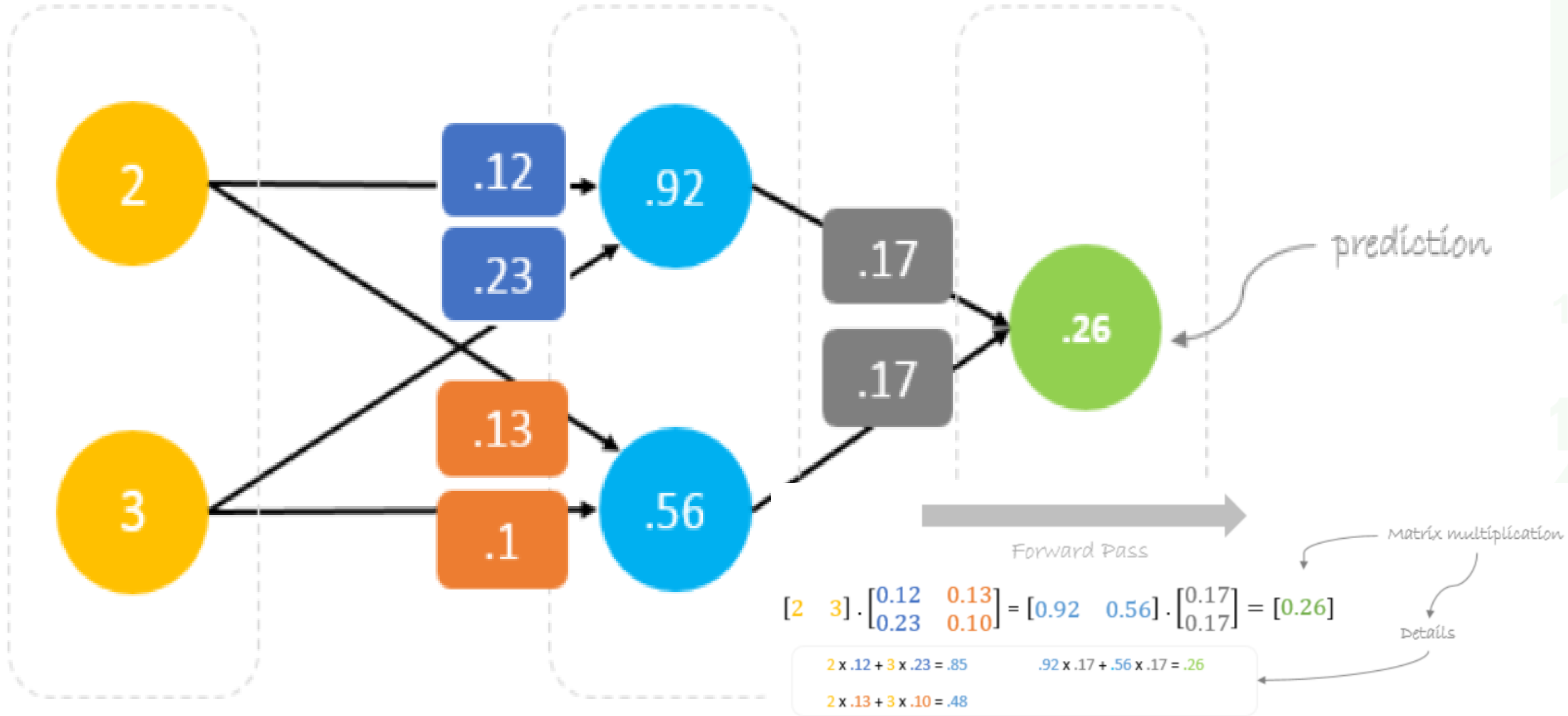
$$*W_6 = W_6 - a \left(\frac{\partial \text{Error}}{\partial W_6} \right)$$

$$*W_6 = W_6 - a \Delta h_2$$

Input layer

Hidden layer

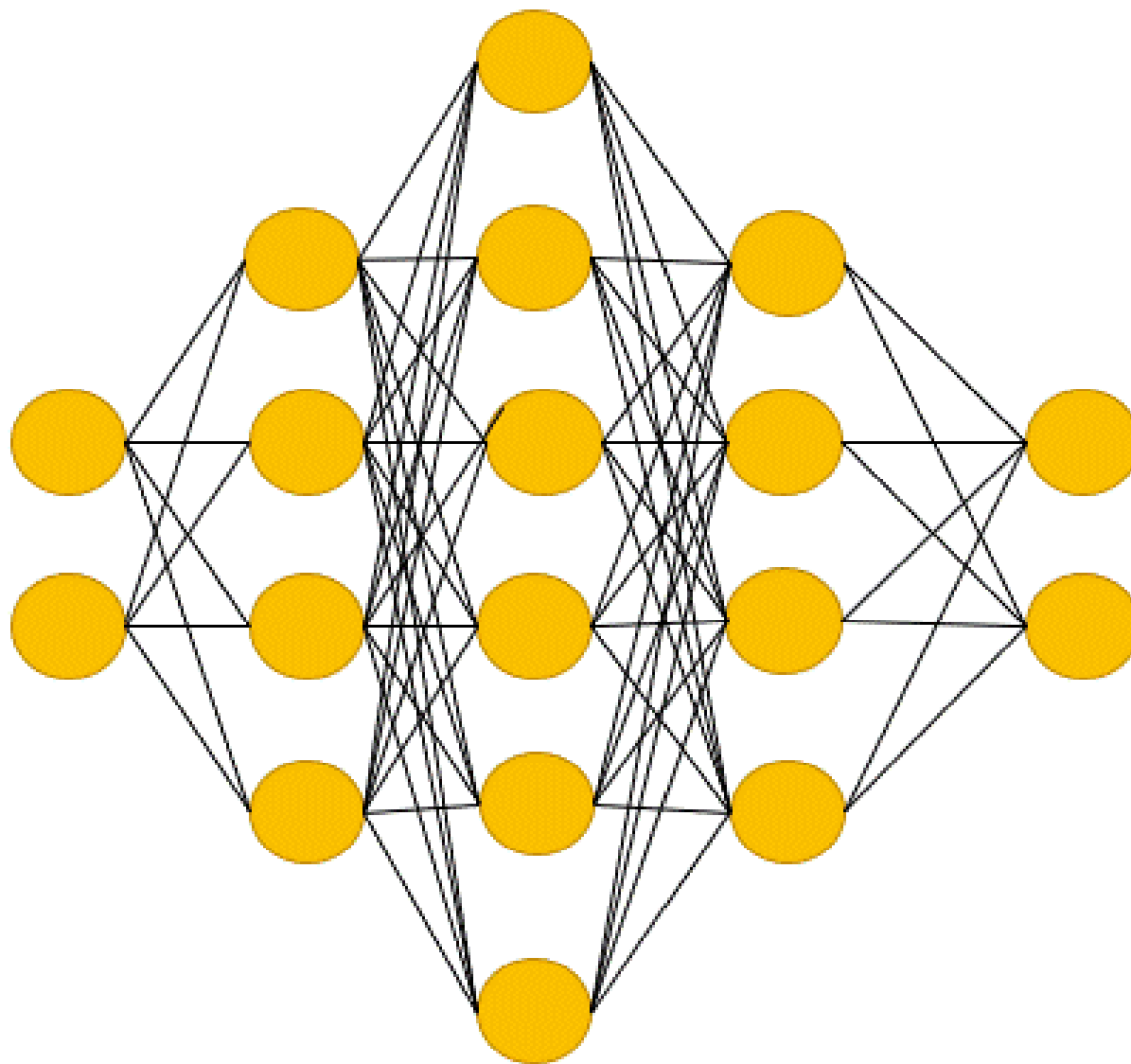
Output layer



Let me ask,
Gru

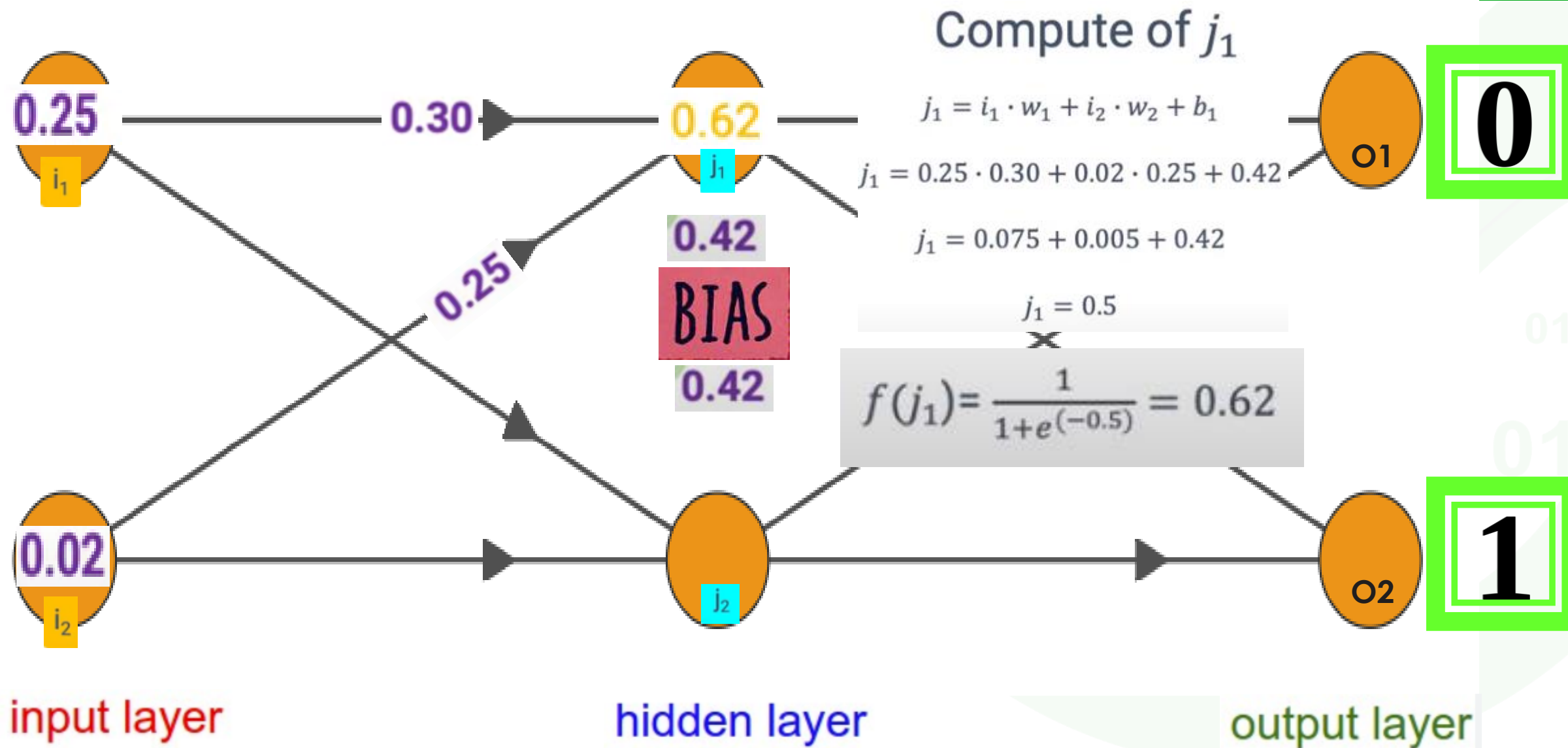


What is the Price
of the Jet, Bob?
It must be 50K



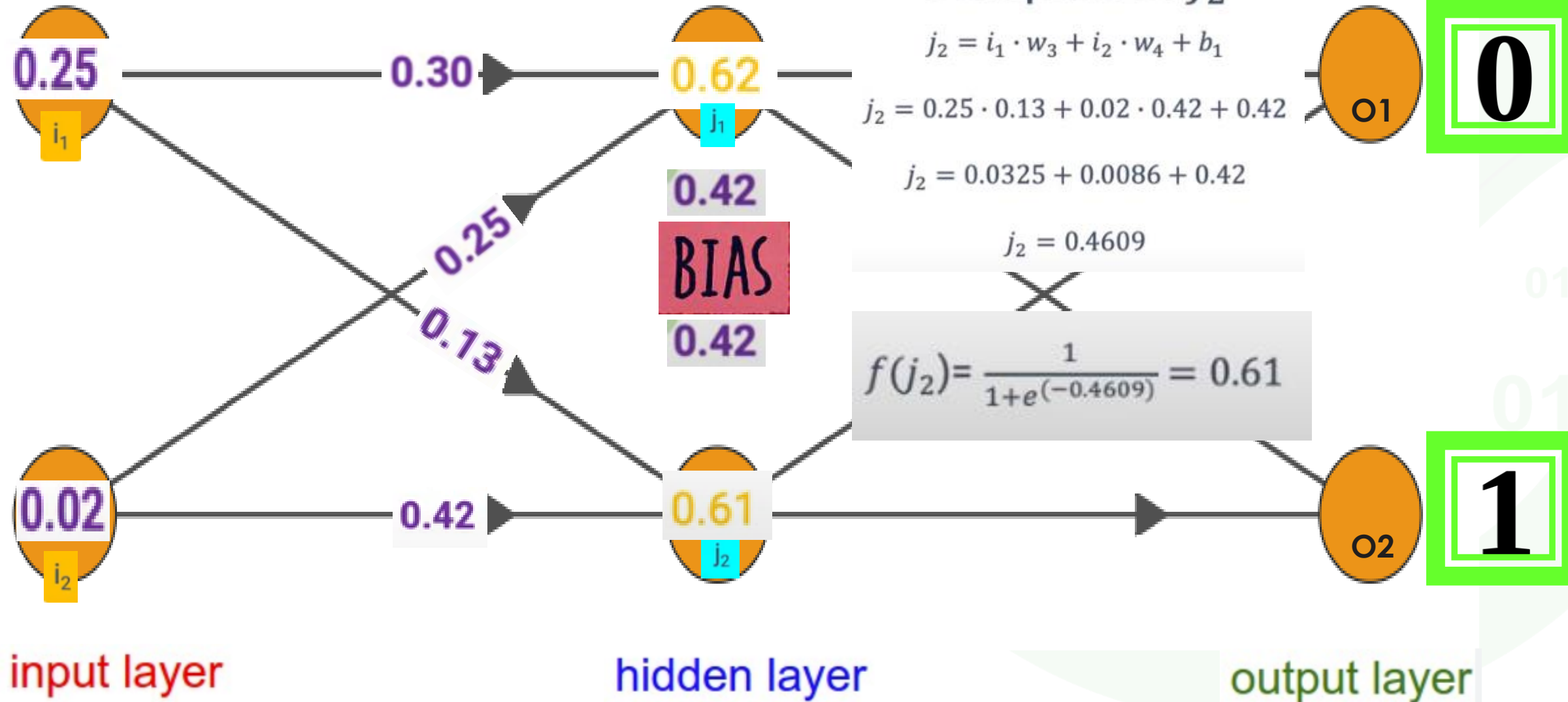
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Forward propagation



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Compute of o_1

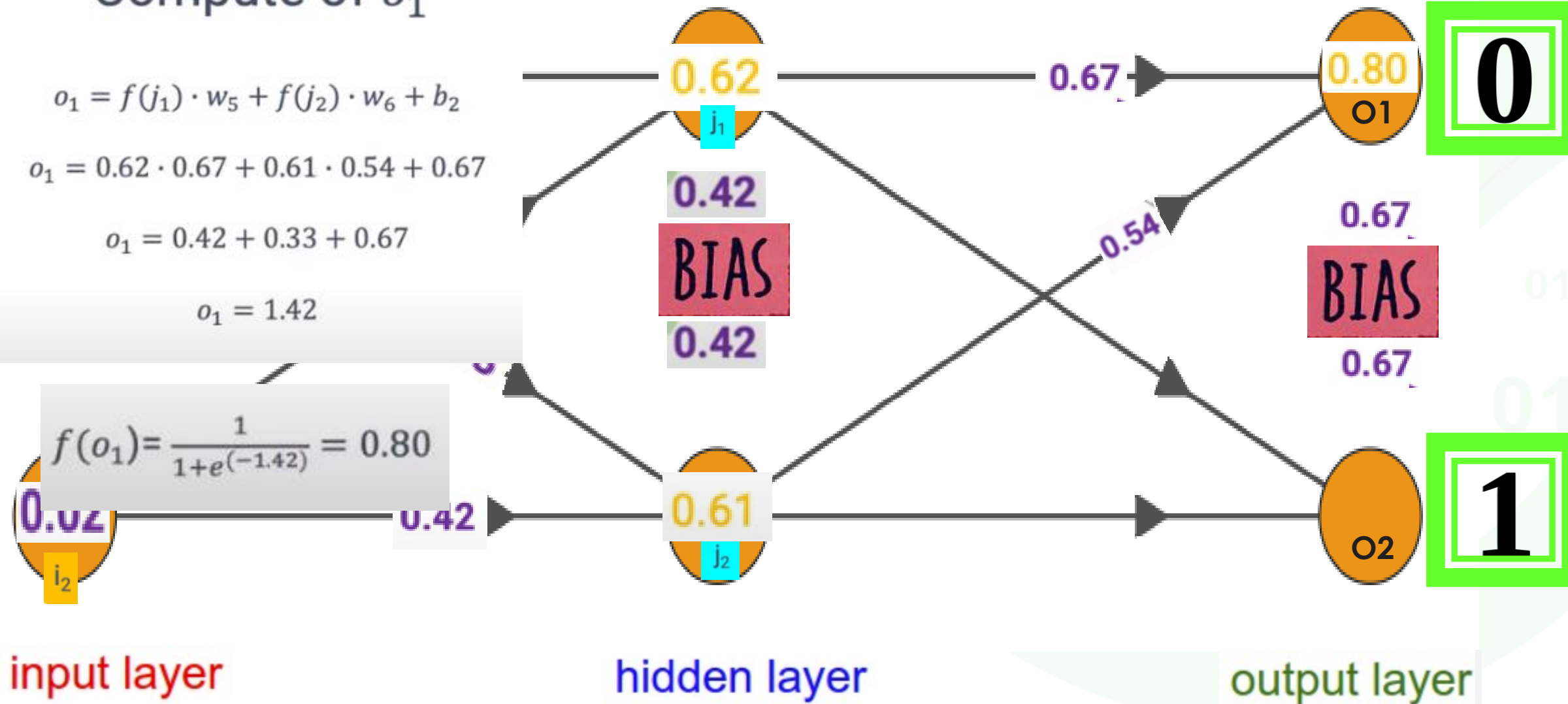
$$o_1 = f(j_1) \cdot w_5 + f(j_2) \cdot w_6 + b_2$$

$$o_1 = 0.62 \cdot 0.67 + 0.61 \cdot 0.54 + 0.67$$

$$o_1 = 0.42 + 0.33 + 0.67$$

$$o_1 = 1.42$$

$$f(o_1) = \frac{1}{1 + e^{(-1.42)}} = 0.80$$



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Compute of o_2

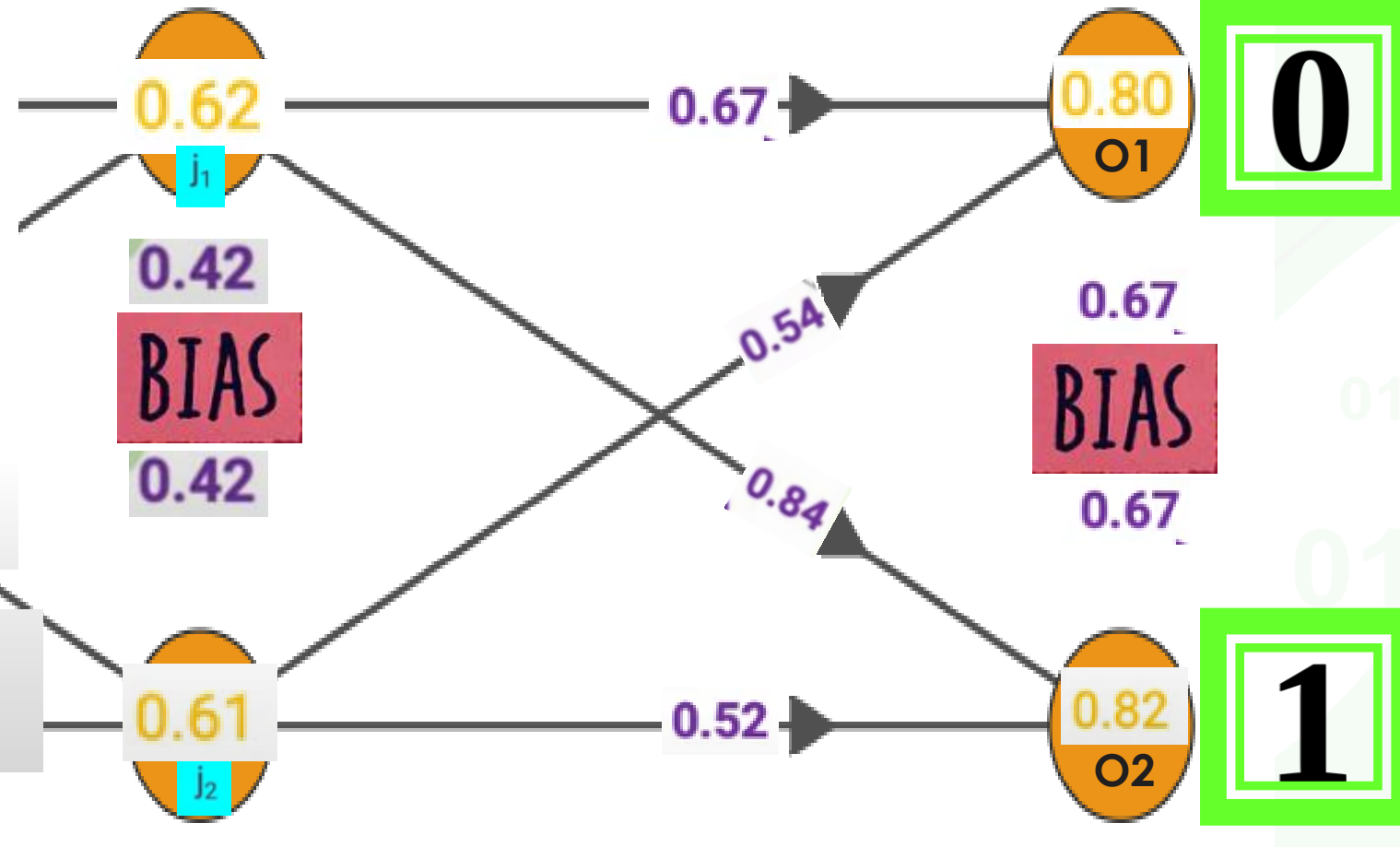
$$o_2 = f(j_1) \cdot w_5 + f(j_2) \cdot w_6 + b_2$$

$$o_2 = 0.62 \cdot 0.84 + 0.61 \cdot 0.52 + 0.67$$

$$o_2 = 0.52 + 0.32 + 0.67$$

$$o_2 = 1.51$$

$$f(o_2) = \frac{1}{1 + e^{(-1.51)}} = 0.82$$



input layer

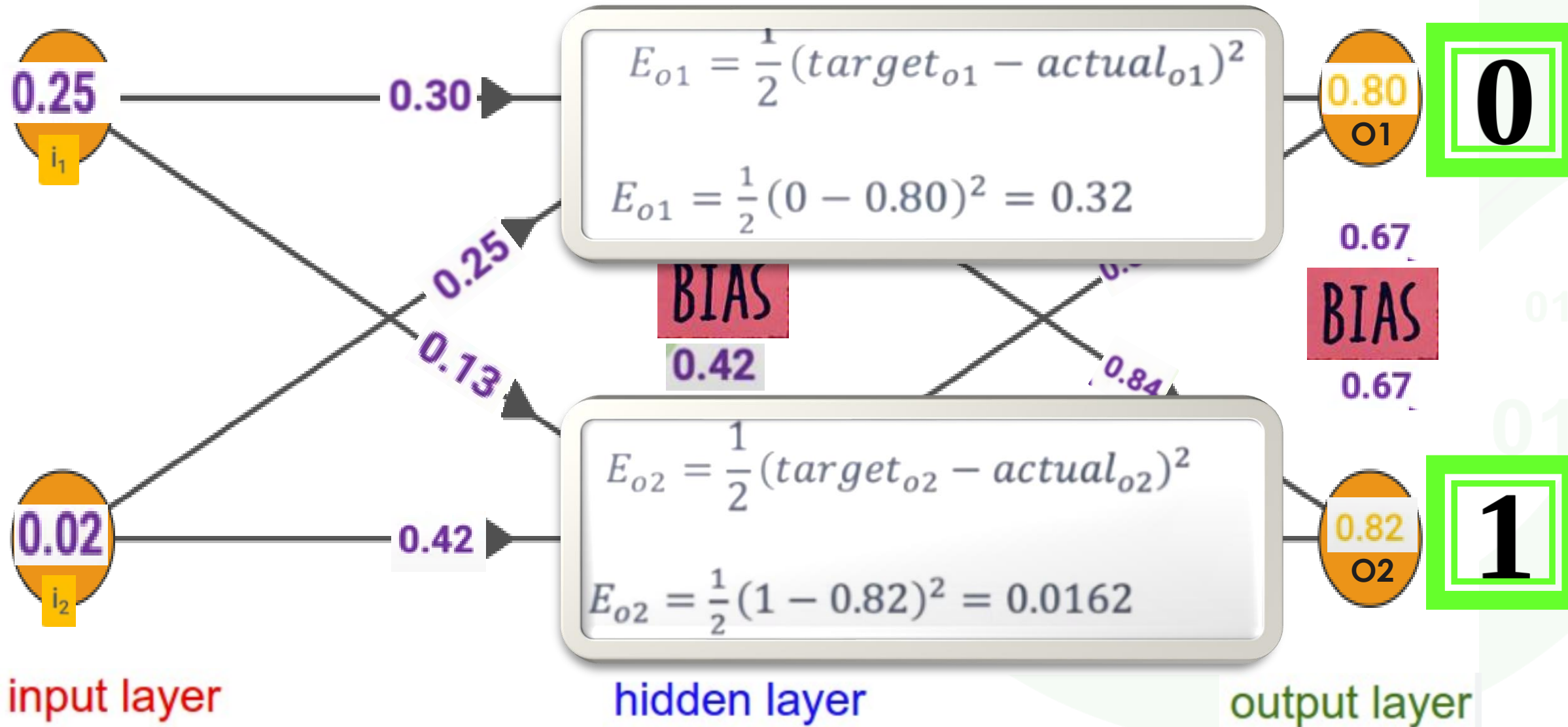
hidden layer

output layer

For each neuron:

$$E_{total} = \sum \frac{1}{2} (target - actual)^2$$

Calculate the error



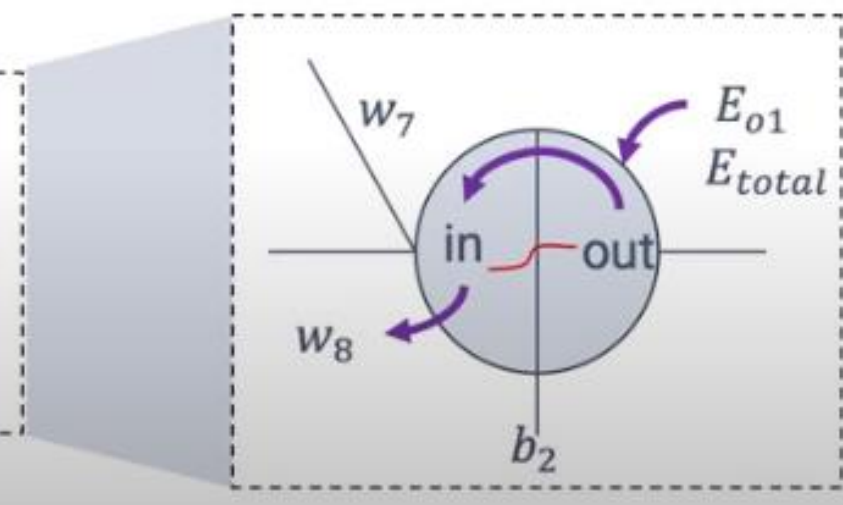


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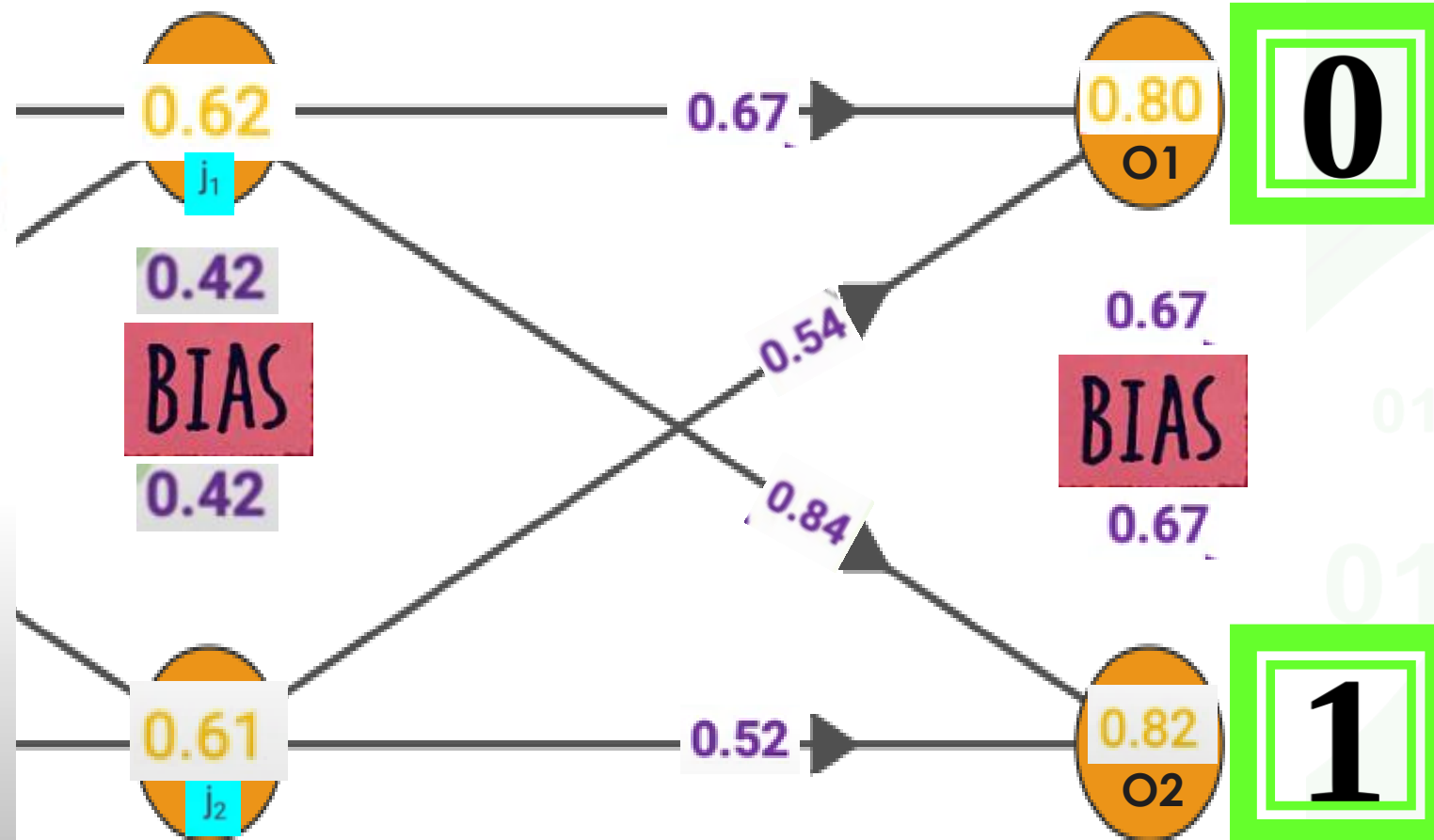
Backward propagation

Adjust weights for output layer

$$\frac{\partial E_{total}}{\partial w_8} = \frac{\partial E_{total}}{\partial out_{o1}} \cdot \frac{\partial out_{o1}}{\partial in_{o1}} \cdot \frac{\partial in_{o1}}{\partial w_8}$$



input layer



hidden layer

output layer



0.84 - (0.1) * (-0.027812) * (0.62) işlemini hesaplarsak:

0.84 + 0.00171787 = 0.84171787

$$\begin{aligned}\delta o_2 = & -(0.2 - 0.2) \text{ çık } 0.2 (1 - \text{çık } 0.2) \\ & -(1 - 0.82) 0.82 (1 - 0.82) \\ & -0.18 * 0.82 * 0.18 \\ & -0.027812\end{aligned}$$

ESKİ AĞIRLILIK (0.52)

Wyeni = Eski ağırlık - Learning rate

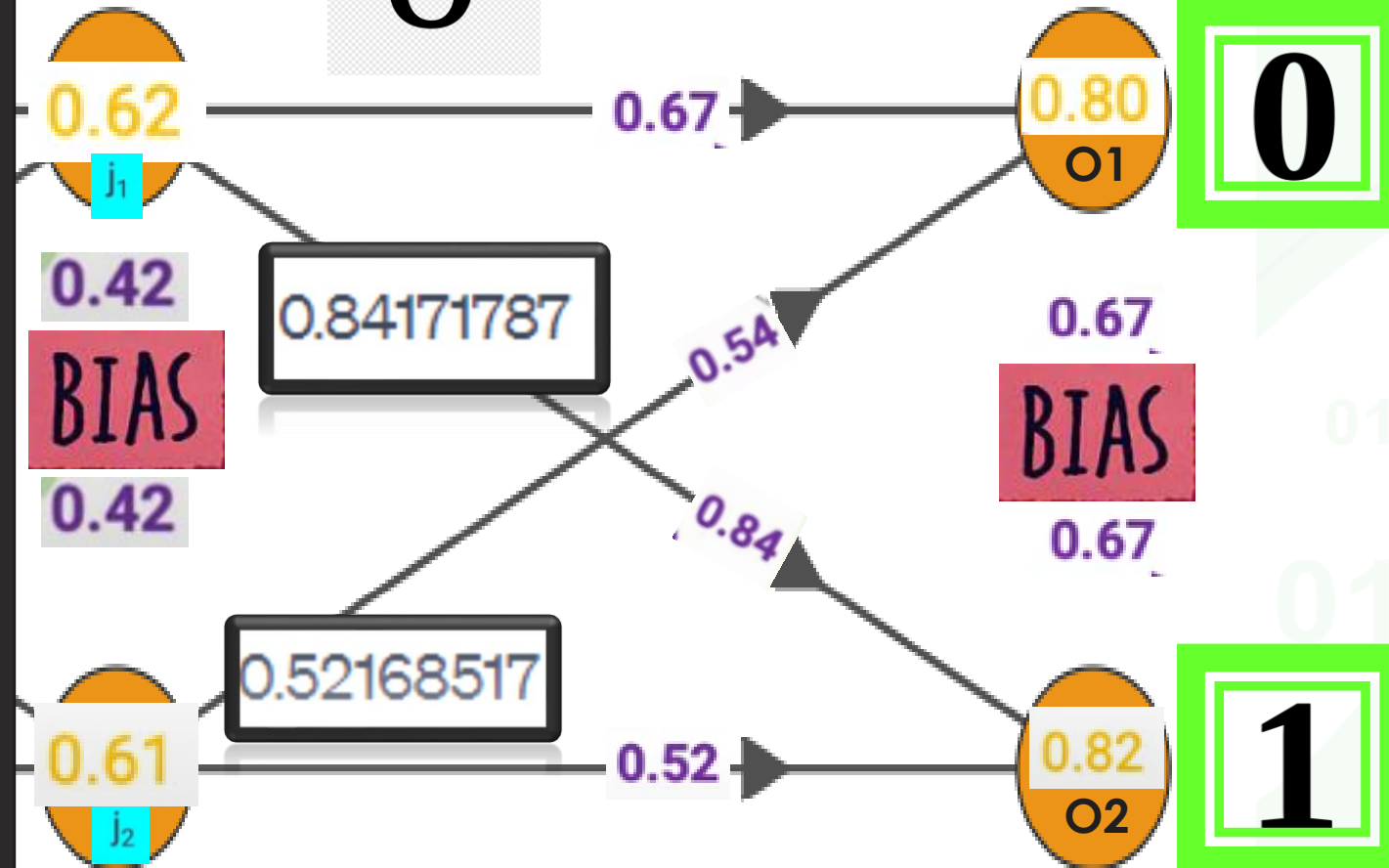
$\delta 0.2$ output J2

0.52 - (0.1) (-0.027812) (0.61)

0.52168517

Backward propagation

δ



input layer

hidden layer

output layer