Domain and Range in Set relation

All the values that can go into a relation are called domain (input)

All the values that come out of a relation are called the range (output)

The domain is the set of all first element of ordered pairs (x- coordinates)

The range is the set of all second elements of ordered pairs (y- coordinates)

Example

Domain

$$R = \{(1, 2), (3, 4), (5, 6)\}$$

~~Domain~~ Range

Example

State the domain and Range of the relation : $\{(1, 3), (-2, 7), (3, -3), (4, 5), (1, -3)\}$

Solution

domain : $\{1, -2, 3, 4\}$
Range : $\{-3, 3, 5, 7\}$

Ex 13

Let R be a relation on $\mathbb{N}$ defined by

$$R = \{(x, y) : x + 2y = 14\}$$

i, Write ~~down~~ R as a set of ordered pairs

ii, Find domain of R, range of R and inverse of R.

Composition of functions

Consider two functions f and g where the range of g is a subset of the domain of f.

The composite function fg is defined as

$$fg(x) = f[g(x)]$$

Note:

1, fg means g followed by f.

2, fg exists provided Range of $g \subseteq$ Domain of f

3, Domain of fg = Domain of g.

4, $ff^{-1}(x) = x$

5, $f^{-1}f(x) = x$

6, $(fg)^{-1} = g^{-1}f^{-1}$

Example

$$f(x) = 2x + 3 \qquad g(x) = x^2 - 3$$

find i, $fg(x)$ ii, $gf(x)$ iii, $f^2(x)$ iv $g^2(x)$

Solution

i, $fg(x) = f(g(x))$

$= f(x^2-3)$

$= 2(x^2-3)+3$

$= 2x^2-3$

ii, $gf(x) = (2x+3)^2 - 3$

$= 4x^2+12x+9-3$

$= 4x^2+12x+6$

iii, $f^2(x) = ff(x)$

$= 2(2x+3)+3$

$= 4x+6+3$

$= 4x+9$

iv, $g^2(x) = gg(x)$

$= (x^2-3)^2 - 3$

$= x^4 - 6x^2 + 9 - 3$

$= x^4 - 6x^2 + 6$

Inverse of a function

Steps : 1, Let $y = f(x)$

2, Make x subject of formula

3, Replace x by $f^{-1}(x)$ and y by x.

Example

Find an expression for ~~f(x)~~ $f^{-1}(x)$ where

i, $f(x) = 2x - 5$

ii, $f(x) = \frac{2x+1}{x-2}$, where $x \neq 2$

Solution

i, Let $y = f(x)$,

$y = 2x - 5$

$2x - 5 = y$

$x = \frac{y+5}{2}$

$\therefore f^{-1}(x) = \frac{x+5}{2}$

ii, Let $y = \frac{2x+1}{x-2}$

$y(x-2) = 2x+1$

$xy - 2y = 2x + 1$

$xy - 2x = 1 + 2y$

$x(y-2) = 1 + 2y$

$x = \frac{1+2y}{y-2}$

$\therefore f^{-1}(x) = \frac{1+2x}{x-2}$ $x \neq 2.$

Exercise 14

$f(x) = 2x - 1 \qquad g(x) = \frac{2x+5}{x-2} \quad x \neq 2$

a, find i, $f^{-1}(x)$

ii, $g^{-1}(x)$

b, find i, fg

ii, gf

iii, f^2

iv, g^2

v, $g^{-1}f^{-1}$