Permutations and Combinations

Arrangement — Permutations

selection — Combinations

Permutations

The notation $n!$

$n! = n$ factorial

$3! = 3 \times 2 \times 1 = 6$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$9! = 9 \times 8 \times 7 \times --- \times 2 \times 1 = 362\,880$

Ex 1

Find the number of different arrangements of the letters of the word "BAG"

Nu bizin rod tou ban possibilité ki nou ~~kg~~ kapav met sa 3 alphabet la

- BAG
- BGA
- GBA
- GAB
- AGB
- ABG

$\Big\}$ 6 ways → Same as $3!$ = 6 ways

no of objects (letters)

∴ Number of permutations of n different objects = $n!$

Ex 2

Find the number of different arrangement of the letters of the word "CLOSED"

No of different arrangements = 6! = 720 ways

Ex 3

In how many ways can 7 students stand in a line during the morning assembly?

No of ways = 7! = 5040 ways

Ex 4

Find the number of different ~~ways~~ arrangements of the letters of the word "CONSIDER"

No of ways = 8! = 40320 ways

Ex 5

In how many ways can 9 policemen stand in a row during a parade?

No of ways = 9! = 362880 ways.

Permutations of n objects, not all distinct

$\underbrace{\text{not all distinct}}$
↓
Kan ena ban lettre/ objet ki repeter

If in a set of n objects there are n_1 of the first kind and n_2 of the second kind, then the permutation of these object is given by:

$$\frac{n!}{n_1! \times n_2!}$$

Ex(1)

Find the number of different arrangements of the letters of the word violin

1 2 3 4 5 6
VIOLIN   I → two times

No of object ⇒ 6

$$\text{No of arrangements} = \frac{6!}{2!}$$

$= 360$ ways

Ex 2

Find the number of different arrangements of the letters of the word "LANGUAGE"
1 2 3 4 5 6 7 8

A - 2
G - 2

No of object : 8

No of arrangements = $\frac{8!}{2! \times 2!}$ = 10080 ways

Ex 3

Find the number of different arrangements of the letters in the word "ENGLAND"

N → 2
No of objects = 7

No of arrangements = $\frac{7!}{2!}$ = 2520 ways

Ex (4)

Find the no of different arrangement of the letters of the word "BUSINESS"

S – 3

No of objects : 8

No of arrangement $= \frac{8!}{3!} = 6720$ ways

Ex (5)

Find the no of different arrangements of the letters of the word "INDEPENDENT"

N – 3
D – 2
E – 3

No of objects = 11

No of arrangements $= \frac{11!}{3! \times 2! \times 3!} = 554400$ ways

Permutations of n different objects, taking r at a time

↳ pa servi tou object ki ena

The number of permutation of n objects taken r at a time is given by

total object ← ^nP_r → number needed.

Ex 1

Eight different books are to be arranged in 5 empty spaces on a bookshelf.
In how many different ways can be done?

Total: 8
Needed: 5

No of different ways = $^8P_5$ = 6720 ways

OR

$$\underbrace{\underset{①}{8} \times \underset{②}{7} \times \underset{③}{6} \times \underset{④}{5} \times \underset{⑤}{4}}_{\text{5 object Needed}} = 6720 \text{ ways}$$

(*) 1 → Pou ~~commencer~~ commencer nou ena 8 object c a dir nu kpv ena 8 possibilité pou premier place la.

2 → Nu in fin servi 1 nu ress 7 Alors sa pou 2em position la nu en 7 possibilite

3 → parey nu in servi 1 2em pou 2em place la nu ress 6. Ler la pou 3em position nu ena 6 possibilite

↓

parey li continie

Ex 2

Ten athletes to run in 6 tracks in a running pitch. In how many different way can this be done?

- No of ways = $^{10}P_6$ = 151 200 ways

OR

$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} = 151200$ ways

Ex3

How many 4 letters code words can be formed from the letters of the word: "INTEGRAL"

$\underbrace{\text{INTEGRAL}}_{8 \text{ objects}}$

$\underline{8} \times \underline{7} \times \underline{6} \times \underline{5} = 1680$ ways

OR

$^8P_4 = 1680$ ways.

Ex4

Find the number of 3-letter code words that can be formed from the letters of the word "SUNSET"

$\overbrace{\text{SUNSET}}^{6 \text{ objects}}$

a, using neither of the S

b, using both S

pas servi okens

⇒ Nu ress 4 object

a, $\underline{4} \times \underline{3} \times \underline{2} = 24$ words

b, Bisin servi both S

$\underline{\boxed{S}} \times \underline{\boxed{S}} \times \underline{4} =$

$\underline{1} \times \underline{1} \times \underline{4} = 30$ words

Si mo in fini servi both s mo ress 4 object

dan sa 4 la mo bizin pren 1

Ex 6

Find the number of 4- letter code words that can be formed from the letters of the word "BOOKSHOP"

a, using neither of the O's

b, using all the O's

a, Pa servi O → Nu ress 5 objects

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 120$ words

b, Servi tou bam O

dan 8 nu fini servi
3 nu ress 5

$\underline{O} \times \underline{O} \times \underline{O} \times \underline{5}$

$\underline{1} \times \underline{1} \times \underline{1} \times \underline{5} = 5$ words

Permutation with restriction - consecutive letters

Certain ban object bizin 1 akoter lot

Ex1

Find the different arrangements of the letters of the word: "BOOK" in which the two letters 'O' are consecutive.

bizin 1 akoter lot [Combine li e compte li as 1 object]

$\therefore$ B OO K

1 2 3 → 3 object

$\therefore$ No of arrangements = $3! = 6$ ways.

Ex2

Find the different arrangements of the letters of the word "ELEPHANT" in which the letters 'E' are consecutive

1 2 3 4 5 6 7

EE L P H A N T   No of objects: 7

No of different arrangement = $7! = 5040$ ways

Ex 3

Find the different arrangements of the letters word:
"EVERGREEN" in which the 'E' and 'R' are consecutive.

$\overset{1}{(EEE)}$ $\overset{2}{(RR)}$ $\overset{3}{V}\overset{4}{G}\overset{5}{N}$ No of objects = 5

No of different arrangements = 5! = 120 ways

Ex 4

Find the different arrangement of the letters of the word: "PARLOUR" in which the two 'R' are not consecutive.

Tip: 1) Find no of ways without restriction
2) Find no of ways they are consecutive
- Then (1) - (2)

No of ways without restriction = $\frac{7!}{2!}$ = 2520 ways
R → ena 2

$\overset{1}{(RR)}$ $\overset{2}{P}\overset{3}{A}\overset{4}{L}\overset{5}{O}\overset{6}{U}$ → 6 objects

No of ways (R are consecutive) = 6! = 720 ways

$\therefore$ No. of ways (the R's are not consecutive)

$= 2520 - 720$

$= 1800$ ways

Ex 5

Find the different arrangement of the letters of the word: "ECONOMICS" in which the letters 'O' and 'C' are not consecutive

No. of ways without restriction $= \dfrac{9!}{2! \times 2!}$

$= 90720$ ways

1 2 3 4 5 6 7

OO CC ENMIS

No. of ways (O and C are consecutive) $= 7! = 5040$ ways

$\therefore$ No. of ways (O and C are not consecutive) $= 90720 - 5040$

$= 85680$ ways

Permutation with restriction - starting / ending with a letter.

Ex1

Find the different arrangements of the letters of the word: "DESIGN" in which the first letter is "D"

already used D | ESIGN — 5 remains

D → $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

must start by D

$\underline{1} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$ ways

Ex2

Find the different arrangements of the letters of the word "MEXICO" which

a, begin with the letter 'M'

b, start with the letter 'E' and end with the letter 'C'

M | E, X, I, C, O — 5 - remains

M $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

a, $\underline{1} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$ ways

b, $\underline{E}$ $\underline{4}$ $\underline{3}$ $\underline{2}$ $\underline{1}$ $\underline{C}$ E | M X I O | C — 4 remains

$\underline{1} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{1} = 24$ ways

Permutations with restrictions – digits

Ex(1)

Calculate the number of different 6-digits number which can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition and assuming that a number cannot begin with 0

- Steps (1) Find total no of possibilities

(2) find no of possibilities that can start by 0

Then (1) - (2)

No of arrangements $= 6!$

$= 720$ ways

No of arrangements beginning with 0

0, 1, 2, 3, 4, 5

$\underline{0} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

already used

we remain with 5 other digits

$\underline{1} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$ ways

No. of arrangements (cannot begin with '0') = 720 − 120
= 600 ways

Ex(2)

The digits of the number 345987 are arranged so that the resulting number is even. Find the number of ways in which this can be done.

For even → Must end by 8 or 4

$\underbrace{\underline{3}\ \ \underline{4}\ \ \underline{5}\ \ \underline{9}\ \ \underline{7}}_{5\ \text{remains}}\ |\ \underline{8}$

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{1} = 120$ ways

$\underbrace{\underline{3}\ \ \underline{8}\ \ \underline{5}\ \ \underline{9}\ \ \underline{7}}_{5\ \text{remains}}\ |\ \underline{4}$

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{1} = 120$ ways

Permutations with restrictions - human beings

Ex(1)

In how many ways can three men and two women stand in a queue if two women must stand next to each other.

M_1 M_2 M_3 (W_1, W_2)

4 objects

No of possible position for 4 objects = 4!

No of possible ways to place the two women = 2!

Total No of ways = $4! \times 2!$
= 48 ways

Ex(2)

In how many ways can 4 boys and 3 girls stand in a queue if the 4 boys must stand next to each other.

4 objects.

$B_1 \quad B_2 \quad B_3 \quad B_4 \quad G_1 \quad G_2 \quad G_3$

No of ways for 4 objects = $4!$

No of ways for the 4 boys = $4!$

Total no of ways = $4! \times 4!$ = 576 ways

Combinations

- It is the selection of a specified number of things from a larger group.
- From a given set of n objects, if a subset of r objects is to be formed without regard to the order it is called a combination of n objects taken r at a time
- The total number of such combinations is denoted by

$$^{n}C_{r}$$

total

comien nu bizin

Ex 1

In how many ways can 11 football players be selected from a team of 15 to play a match?

No. of ways to select 11 players from 15

$$= {}^{15}C_{11} = 1365 \text{ ways}$$

choisir 11 depi 15

Ex2
In how many ways can 5 students be selected from a class of 12 to participate in a debate?

$$\text{No of ways} = {}^{12}C_5 = 792 \text{ ways}$$

Ex3
In how many ways can 2 boys and 4 girls be selected from a group of 4 boys and 7 girls to sing a song.

- No of ways to select 2 boys from 4
$$= {}^{4}C_2$$
- No of ways to select 4 girls from 7
$$= {}^{7}C_4$$

$$\text{Total} = {}^{4}C_2 \times {}^{7}C_4 = 210 \text{ ways}$$

Ex4

In how many ways can 3 boys and 5 girls be selected from a group of 5 boys and 10 girls to dance on a stage?

No. of ways to select 3 boys from 5 = $^5C_3$

No. of ways to select 5 girls from 10 = $^{10}C_5$

Total no. of ways = $^5C_3 \times {}^{10}C_5 = 2520$ ways

Ex5

A committee of 4 people is to be chosen from 4 women and 5 men. The committee must contain at least 1 woman. Calculate the number of different committees that can be formed.

Minimum 1 women

1 woman and 3 men = $^4C_1 \times {}^5C_3 = 20$

OR

2 women and 2 men = $^4C_2 \times {}^5C_2 = 60$

OR

3 women and 1 man = $^4C_3 \times {}^5C_1 = 20$

OR

4 women and 0 man = $^4C_4 \times {}^5C_0 = 1$

Total = 20 + 60 + 20 + 1 = 121 ways

Ex 6

A committee of 5 people is to be chosen from 4 boys and 6 girls. The committee must contain at most 3 girls. Calculate the number of different committees that can be formed?

Maximum 3 girls

~~No of~~

3 girls and 2 boys $= {}^6C_3 \times {}^4C_2 = 120$ ways

OR

2 girls and 3 boys $= {}^6C_2 \times {}^4C_3 = 60$ ways

OR

1 girl and 4 boys $= {}^6C_1 \times {}^4C_4 = 6$ ways

Total $=$ ~~[illegible]~~ $120 + 60 + 6$

$= 186$ ways.

Ex 7

1 Allan, 2 Ben, 3 Charles, 4 David, 5 Emily, 6 Frederic, 7 Gaëlle

4 persons are to be chosen.

In how many ways can the selection be made if

i) there is no restriction.
ii) Allan must be chosen.
iii) Both Ben and Emily must be chosen.
iv) Federic must not be chosen
v) Either Allan or Ben but not both must be chosen

Solution

i) No. of ways $= {}^7C_4 = \underline{35 \text{ ways}}$

ii) No. of ways $= \underset{\text{Allan}}{1} \times {}^6C_3$ from remaining 6 choose 3

$= \underline{20 \text{ ways}}$

Ben — Emily — From remaining five choose 2

iii) No of ways = $1 \times 1 \times {}^5C_2$

= 10 ways

from 7 take out Federick we remain with six from the six choose 4

iv) No of ways = ${}^6C_4$

= 15 ways

v) No of ways = ${}^2C_1 \times {}^5C_3$

from Allan and Ben choose any 1

from remaining 5 choose 3

= 20 ways