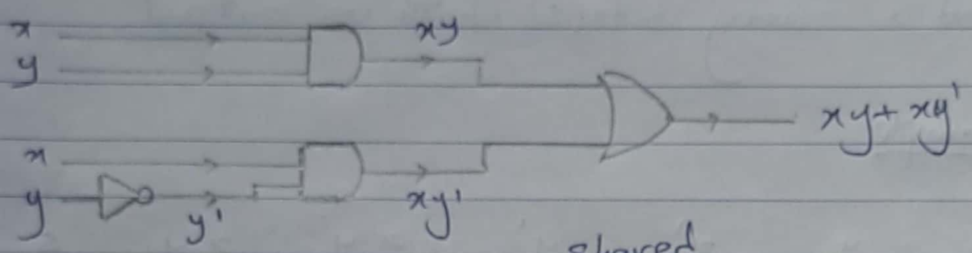
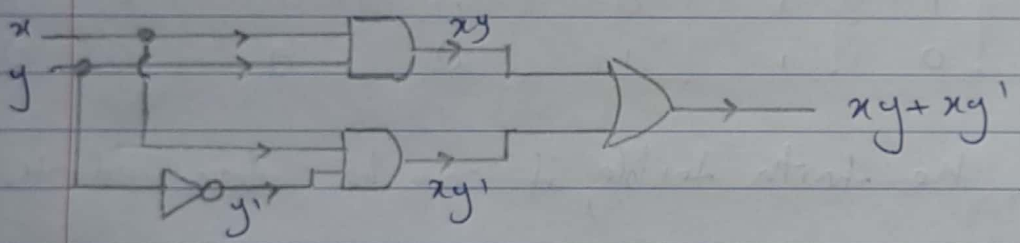
Combination of gates

When basic gates are interconnected, combinational circuits are formed.

i) Separate inputs for each gate.

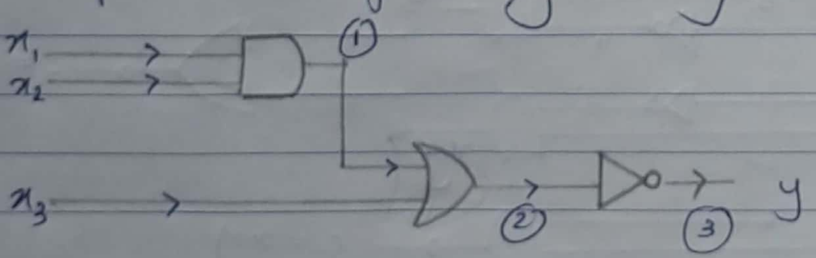


ii) Branching (inputs are ~~stored~~ shared)



Example

Compute the output by tracing the following circuit.



1 : ~~x_1+x_2~~ $x_1 x_2$

2 : $x_1 x_2 + x_3$

3 : $(x_1 x_2 + x_3)'$ $\therefore\ y = (x_1 x_2 + x_3)'$

Adders

Half adder

Half adder is a combinational arithmetic that adds two numbers and produces a sum bit (s) and a carry bits (s) both as output.

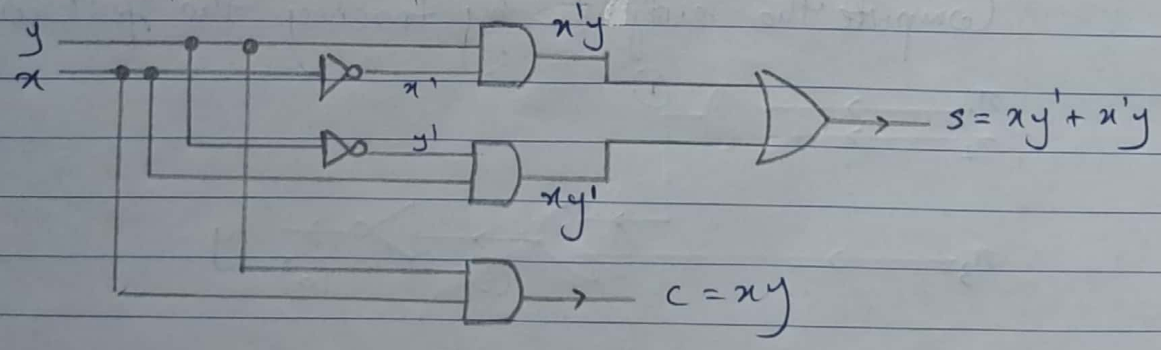
Example

x	y	x'	y'	xy'	$x'y$	s	c
1	1	0	0	0	0	0	1
1	0	0	1	1	0	1	0
0	1	1	0	0	1	1	0
0	0	1	1	0	0	0	0

from the truth table, it can be observed that,

$$s = xy' + x'y \quad \text{and} \quad c = xy.$$

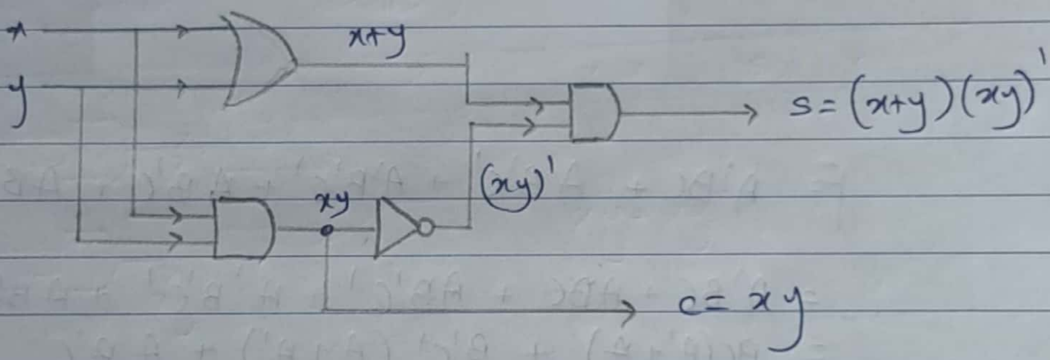
Circuit for the half adder.



We can observe that

$$
\begin{aligned}
xy' + x'y &= 1 + xy' + x'y + 1 \\
&= xx' + xy' + x'y + yy' \\
&= x(x'+y') + y(x'+y') \\
&= (x+y)(x'+y') \\
&= (x+y)(xy)'
\end{aligned}
$$

Hence the half adder can be simplified as



Example

Use Boolean algebra to simplify the following expressions and also draw the circuit of the simplified expression.

a, $F = A'BC + AB'C' + A'B'C' + AB'C + ABC$

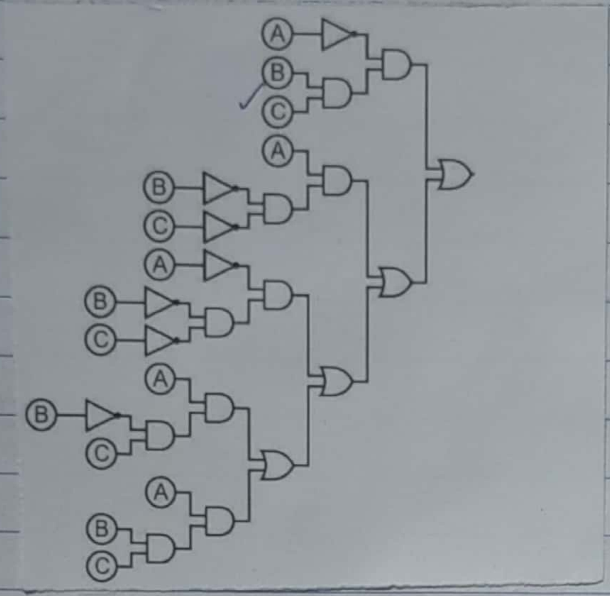
b, $F = A[B + C(AB + AC)]$
c, $F = A'B'C + (A+B+C)' + A'B'C'D$
d, $F = AB + ABC + \bar{A}B + A\bar{B}C$
e, $F = A + \bar{A}B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$

Ex16

Solution

a)

$F = A'BC + AB'C' + A'B'C' + AB'C + ABC$



$F = A'BC + AB'C' + A'B'C' + AB'C + ABC$

$= A'BC + ABC + AB'C' + A'B'C' + AB'C$

$= BC(A'+A) + B'C'(A+A') + AB'C$

$= BC + B'C' + AB'C$

$= B'C' + BC + AB'C$

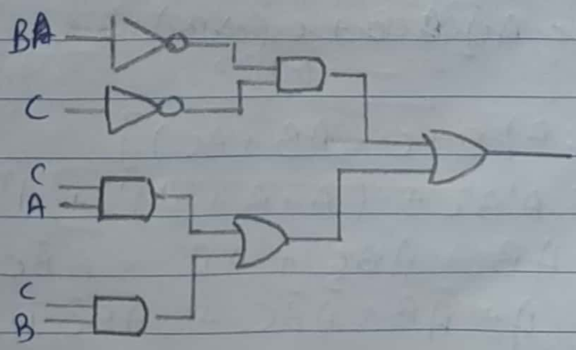
$= B'C' + C(B + AB')$ $\quad (b+ab') = b+a$

$= B'C' + C(B+A)$

$= B'C' + C(A+B)$

$= B'C' + CA + CB$

Simplified circuit :



b) $F = A[B + C(AB + AC)]$

$= A[B + CAB + CAC]$

$= A[B + CAB + ACC]$

$= A[B + CAB + AC]$

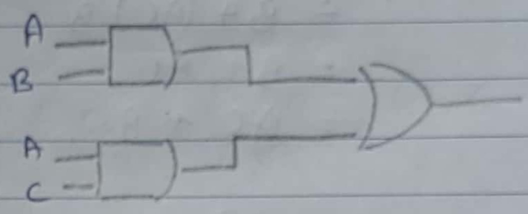
$= AB + AACB + AAC$

$= AB + ACB + AC$

$= AB + AC(B+1)$

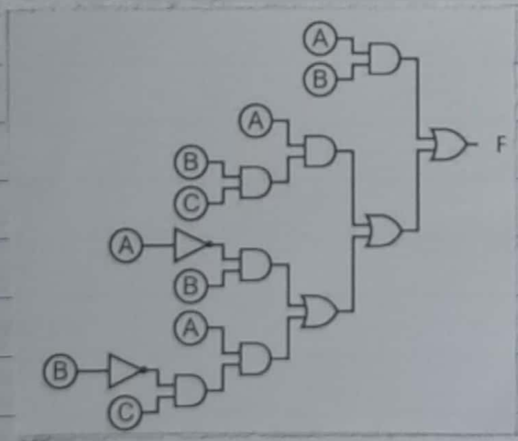
$= AB + AC(1)$

$F = AB + AC$

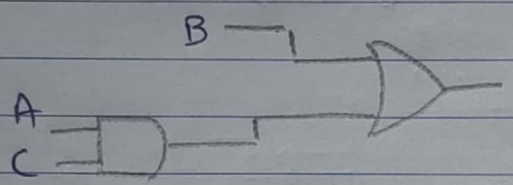


Example

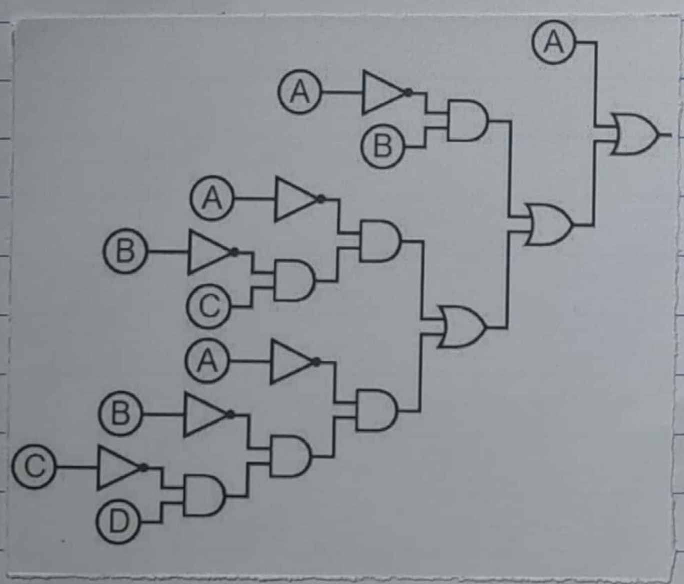
For the logical circuit below write down the output expression and simplify it. Include the simplified logic gates



$F = AB + ABC + \bar{A}B + A\bar{B}C$
$= AB(1+C) + \bar{A}B + A\bar{B}C$
$= AB(1) + \bar{A}B + A\bar{B}C$
$= AB + \bar{A}B + A\bar{B}C$
$= B(A+\bar{A}) + A\bar{B}C$
$= B(1) + A\bar{B}C$
$= B + \bar{B}(AC)$
$= B + (AC)\bar{B}$ $\qquad b + ab' = b + a$
$= B + AC$
$= AC + B$



Exercise 18
i) Write the output expression
ii) Simplify it
iii) Draw the simplified logic gates



Karnaugh map method.

Karnaugh map method is a graphical method for simplifying Boolean expressions involving six or fewer variables that are expressed in the sum of products form and that represent combinational circuits. Simplification requires identification of terms in the Boolean expression which can be combined. The terms which can be combined can be easily found out from Karnaugh maps.

A Karnaugh map (k-map) is a diagram consisting of squares. If the Boolean expression contains n variables, the corresponding k-map will have 2^n squares, each of which represents a minterm. A '1' is placed in the square representing a minterm if it is present in the given expression. A '0' is placed in the square that corresponds to the minterm not present in the expression. The simplified Boolean expression that represents the output is then obtained by combining or grouping adjacent squares that contain 1. Adjacent squares are those that represent minterms differing by only one litreral

To identify adjacent cells (squares) in the k-map for grouping, the following points may be borne in mind.

1, The number of cells in a group must be a power of 2, i.e, 1, 2, 4, 8, 16, ---.

2, A cell containing 1 may be included in any number of groups.

3. To minimize the expression to the maximum possible extent, largest possible groups must be preferred, viz. a group of two cells should not be considered if these cells can be included in a group of four cells and so on.

4. Adjacent cells exists not only within the interior of the K-map, but also at the extremes of each column and each row viz. the top cell in any column is adjacent to the bottom cell in the same column. The left most cell in any row is adjacent to the rightmost cell in that row.

Example

Consider the following truth table using K-map, write an expression of F.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

3 variables, A, B, C.

$\Rightarrow 2^3 \Rightarrow$ 8 squares

C \ AB	00	01	11	10
0	0	1	1	1
1	0	0	0	1

Red part:

A : 0 1 → varies
B : 1 1 → stays same ⇒ B
C : 0 0 → Reverse ⇒ $\bar{C}$

⇒ $B\bar{C}$

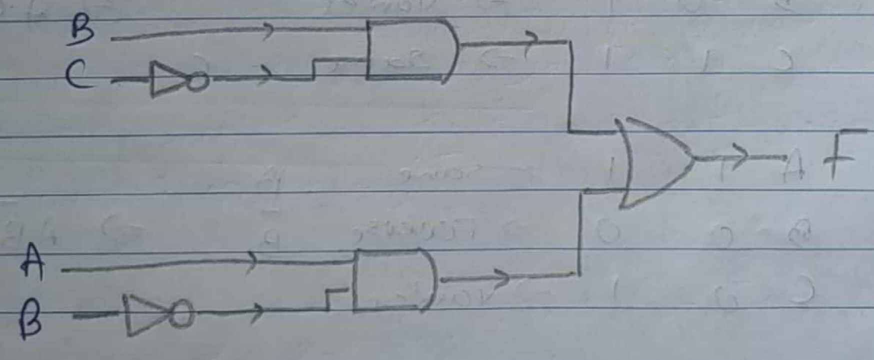
Green part.

A : 1 1 → stays same ⇒ A
B : 0 0 → reverse ⇒ $\bar{B}$
C : 0 1 → varies

⇒ $A\bar{B}$

Thus $f = B\bar{C} + A\bar{B}$

Circuit



Example

Write down the expression function using k-map method for the ~~truth~~ truth table below.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

3 variables

2^3 squares $\Rightarrow$ 8 squares

C \ AB	00	01	11	10
0	0	0	1	1
1	1	1	0	1

Green

A 0 0 → Reverse $\bar{A}$

B 0 1 → Varies $\Rightarrow \bar{A}C$

C 1 1 → same C

Black

A 1 1 – same A

B 0 0 → reverse $\bar{B}$ $\Rightarrow A\bar{B}$

C 0 1 → varies

Red.

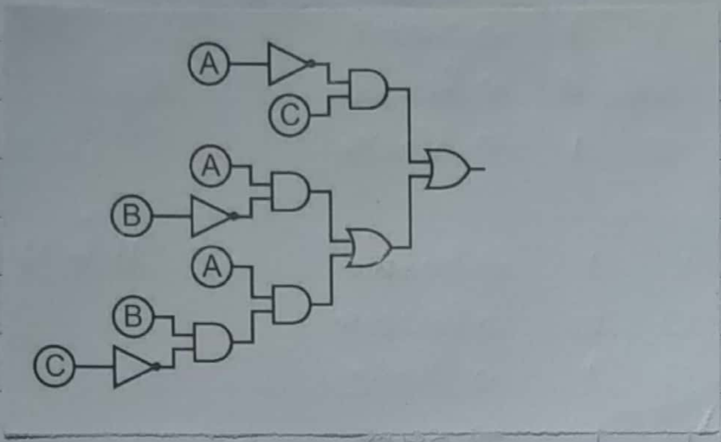
A 1 → same A

B 1 → same B $\Rightarrow AB\bar{C}$

C 0 → reverse $\bar{C}$

$\therefore F = \bar{A}C + A\bar{B} + AB\bar{C}$

The circuit is :



Example

For the following k-map,
i) Write the function
ii) Draw the truth table
iii) Draw the circuit. ~~[illegible]~~

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	0	0	0	1
10	1	1	0	0

Green: ~~A+B~~ A 0 0 → Reverse ⟹ $\bar{A}C\bar{D}$
B 0 1 → Varies
C 1 1 → Same
D 0 0 → Reverse

Black A 1 1 → same ⇒ $A B \bar{C}$

B 1 1 → same

C 0 0 → Reverse

D 0 1 → Varies

Red A 1 1 → same ⇒ $A \bar{B} D$

B 0 0 → Reverse

C 0 1 → Varies

D 1 1 → same

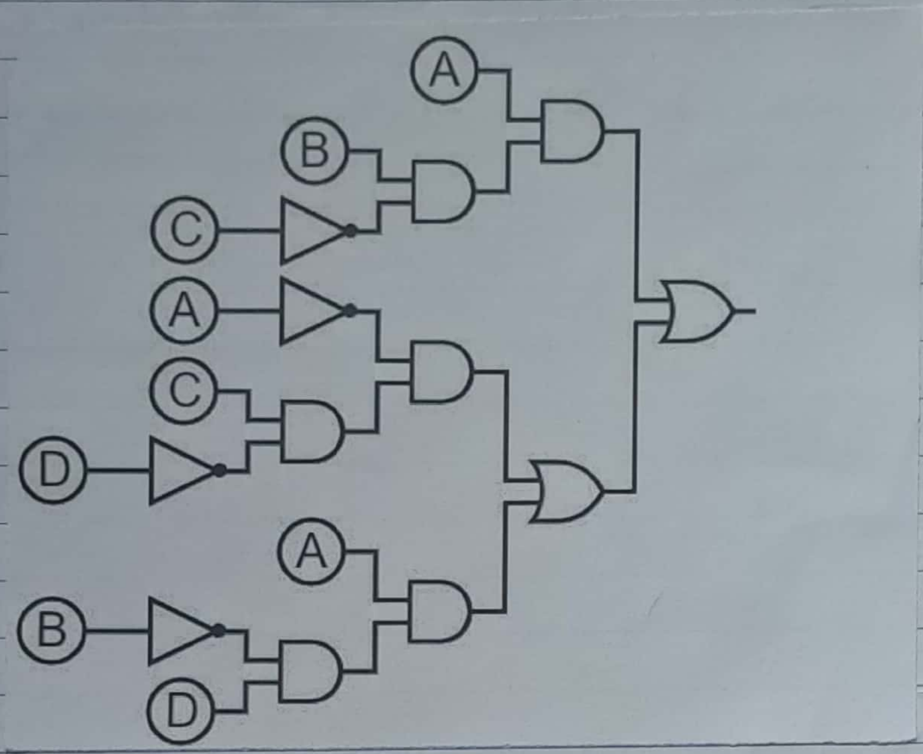
$$\therefore F = \bar{A} C \bar{D} + A B \bar{C} + \cancel{BD}\ A \bar{B} D$$

$$F = A \bar{B} D + \bar{A} C \bar{D} + A \bar{B} D$$

The truth table is

A	B	C	D	F.
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

The circuit is :



Exercise 19

for the following K-map,

i) write the function

ii) Draw the truth table

CD \ AB	00	01	11	10
00	1	0	0	0
01	1	0	1	1
11	1	0	1	1
10	1	0	0	0

Example

For the following Boolean Expression, write the k-map.

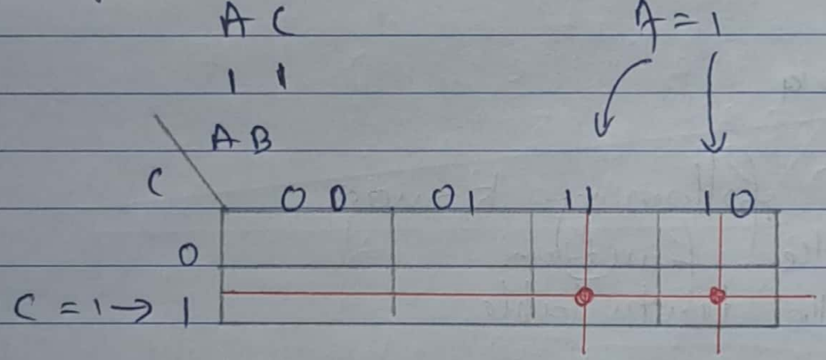
$F = AC + A\bar{B}$

3 variables $\Rightarrow$ 8 squares

$F = \underbrace{AC}_{1^{st}\ part} + \underbrace{A\bar{B}}_{2^{nd}\ part}$

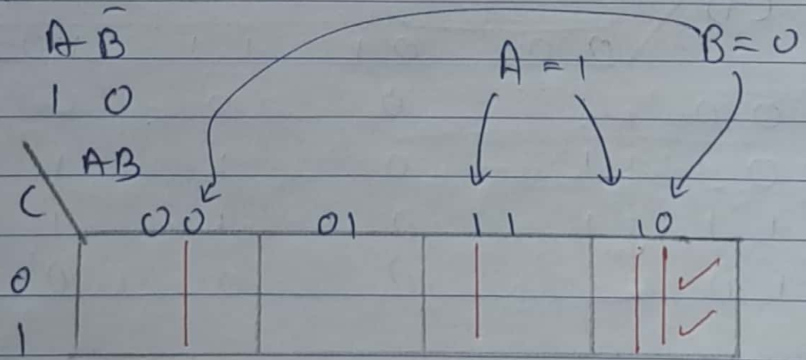
first part.

AC
$1\ 1$



2nd part.

$A\bar{B}$
$1\ 0$



Hence.

C \ AB	00	01	11	10
0	0	0	0	1
1	0	0	1	1

~~Exercise~~

Exercise 20

For the following Boolean Expressions, write their respective k-map

a) $F = A\bar{B} + \cancel{\bar{A}CB}\ \bar{A}CD + AB\bar{C}$

b) $F = A\bar{B} + AC + \bar{A}B\bar{C}$

c) $F = C + A\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C$