

# Permutations and combinations

↑  
Arrangement

↑  
selection.

## Permutations

The notation  $n!$

$n! = n$  factorial

$$3! = 3 \times 2 \times 1 = 6$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$9! = 9 \times 8 \times 7 \times \dots \times 2 \times 1 = 362880$$

Ex 1

Find the number of different arrangements of the letters of the word "BAG"

Nu bizin rod ton ban possibilities ki non ~~ky~~ karan mat sa 3 alphabet la

- BAC
- BCA
- GBA
- GAB
- AGB
- ABC

} 6 ways

→ Same as

$$3!$$

$$= 6 \text{ ways}$$

no of objects  
↓  
(letters)

∴ Number of permutations of  $n$  different objects =  $n!$

Ex 2

Find the number of different arrangements of the letters of the word "LOSED"

$$\text{No of different arrangements} = 6! = 720 \text{ ways}$$

Ex 3

In how many ways can 7 students stand in a line during the morning assembly?

$$\text{No of ways} = 7! = 5040 \text{ ways}$$

Ex 4

Find the number of different arrangements of the letters of the word "CONSIDER"

$$\text{No of ways} = 8! = 40320 \text{ ways}$$

Ex 5

In how many ways can 9 policemen stand in a row during a parade?

$$\text{No of ways} = 9! = 362880 \text{ ways}$$



Permutations of  $n$  objects, not all distinct

↓  
kan ena ban letter/  
object ki repeter

If in a set of  $n$  objects, there are  $n_1$  of the first kind and  $n_2$  of the second kind, then the permutation of these object is given by:

$$\frac{n!}{n_1! \times n_2!}$$

Ex(1)

Find the number of different arrangements of the letters of the word VIOLIN

1 2 3 4 5 6  
VIOLIN

I → two times

No. of object ⇒ 6

$$\text{No. of arrangements} = \frac{6!}{2!}$$

$$= 360 \text{ ways}$$

Ex 2

Find the number of different arrangements of the letters of the word "LANGUAGE"

1 2 3 4 5 6 7 8

A - 2

G - 2

No of object: 8

$$\text{No of arrangements} = \frac{8!}{2! \times 2!} = 10080 \text{ ways}$$

Ex 3

Find the number of different arrangements of the letters in the word "ENGLAND"

N → 2

No of objects = 7

$$\text{No of arrangements} = \frac{7!}{2!} = 2520 \text{ ways}$$

Ex (4)

Find the no of different arrangements of the letters of the word "BUSINESS"

S - 3

No of objects = 8

$$\text{No of arrangements} = \frac{8!}{3!} = 6720 \text{ ways}$$

Ex (5)

Find the no of different arrangements of the letters of the word "INDEPENDENT"

N - 3

D - 2

E - 3

No of objects = 11

$$\text{No of arrangements} = \frac{11!}{3! \times 2! \times 3!} = 554400 \text{ ways}$$



Permutations of  $n$  different objects, taking  $r$  at a time

↳ for every object  $k$  is even

The number of permutations of  $n$  objects taken  $r$  at a time is given by

total object  $\leftarrow n$   
 $P_r$   
 $\rightarrow$  number needed.

Ex 1

Eight different books are to be arranged in 5 empty spaces on a bookshelf.

In how many different ways can be done?

Total: 8

Needed: 5

$$\text{No of different ways} = {}^8P_5 = 6720 \text{ ways}$$

OR

$$\begin{array}{ccccccccc} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} & \textcircled{*} & & & \\ 8 & \times 7 & \times 6 & \times 5 & \times 4 & & & & \\ \hline & & & & & & & & \end{array} = 6720 \text{ ways}$$

↓  
5 object Needed

④ 1 → Pour ~~commencer~~ commencer nous avons 8 objets  
c'est à dire un kpu ena 8 possibilités pour  
premier place la.

2 → Nu in fin servi 1 nu rest 7. Alors sa  
pour 2<sup>em</sup> position la nu en 7 possibilités

3 → pareil nu in servi 1 2<sup>em</sup> pour 2<sup>em</sup> place la  
nu rest 6. Alors la pour 3<sup>em</sup> position nu  
ena 6 possibilités

↓  
pareil li continue

Ex 2

Ten athletes to run in 6 tracks in a running  
pitch. In how many different way can this be done?

• No of ways =  ${}^{10}P_6 = 151200$  ways

OR

$$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{1 \times 1 \times 1 \times 1 \times 1 \times 1} = 151200 \text{ ways}$$



Ex 3

How many 4 letter code words can be formed from the letters of the word: "INTEGRAL" 8 objects

$$8 \times 7 \times 6 \times 5 = 1680 \text{ ways}$$

OR

$$8P_4 = 1680 \text{ ways.}$$

Ex 4

Find the numbers of 3-letter code words that can be formed from the letters of the word

"SUNSET"

6 objects

a, using neither of the S

b, using both S

SUNSET

pas servi okan S

⇒ No res, 4 object

a,  $4 \times 3 \times 2 = 24 \text{ words}$

b, Bisin servi both S

$$\boxed{S} \times \boxed{S} \times 4 = 1 \times 1 \times 4 = 4 \text{ words}$$

Si mo in fini servi both S mo res 4 object

dan sa 4 la mo biza pren



Ex 6

Find the number of 4-letter code words that can be formed from the letters of the word "BOOKSHOP"

- a, using neither of the O's  
b, using all the O's

a, Permuti O  $\rightarrow$  No res 5 objects  
$$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 120 \text{ words}$$

b, Servi tou ban O

can 8 nu finiserv  
3 nu res 5

$$\underline{O} \times \underline{O} \times \underline{O} \times \underline{5}$$
$$\underline{1} \times \underline{1} \times \underline{1} \times \underline{5} = 5 \text{ words}$$

## Permutation with restriction - consecutive letters

Certain ban object  
biza 1 akoter 6

Ex 1

Find the different arrangements of the letters of the word "BOOK" in which the two letters 'O' are consecutive.

biza 1 akoter 6 [Combine li e compute li as 1 object]

∴ B O O K  
1 2 3 → 3 object

∴ No of arrangements =  $3! = 6$  ways

Ex 2

Find the different arrangements of the letters of the word "ELEPHANT" in which the letters 'E' are consecutive.

1 2 3 4 5 6 7  
E E L P H A N T

No of objects: 7

No of different arrangements =  $7! = 5040$  ways



Ex 3

Find the different arrangements of the letters of the word:

"EVERGREEN" in which the 'E' and 'R' are consecutive.

<sup>1</sup> <sup>2</sup> <sup>3</sup> <sup>4</sup> <sup>5</sup>  
(EEE) (RR) VGN

No of objects = 5

No of different arrangements =  $5! = 120$  ways

Ex 4

Find the different arrangements of the letters of the word: "PARLOUR" in which the two 'R' are not consecutive.

Tip: 1. Find no of ways without restriction  
2. Find no of ways they are consecutive

Then (1) - (2)

No of ways without restriction =  $\frac{7!}{(2!)} = 2520$  ways

<sup>1</sup> <sup>2</sup> <sup>3</sup> <sup>4</sup> <sup>5</sup> <sup>6</sup>  
(RR) PALOU → 6 objects

R → end 2

No of ways (R are consecutive) =  $6! = 720$  ways

$$\begin{aligned}
 \therefore \text{No. of ways (the O's are not consecutive)} \\
 &= 2520 - 720 \\
 &= 1800 \text{ ways}
 \end{aligned}$$

Ex 5

Find the different arrangement of the letters of the word: "ECONOMICS" in which the letters 'O' and 'C' are not consecutive.

$$\text{No. of ways without restriction} = \frac{9!}{2! \times 2!}$$

$$\begin{array}{ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
 & \text{O} & \text{O} & \text{C} & \text{C} & \text{E} & \text{N} & \text{M} & \text{I} & \text{S}
 \end{array}$$

= 90720 ways

$$\text{No. of ways (O and C are consecutive)} = 7! = 5040 \text{ ways}$$

$$\begin{aligned}
 \therefore \text{No. of ways (O and C are not consecutive)} &= 90720 - 5040 \\
 &= \underline{85680 \text{ ways}}
 \end{aligned}$$



Permutation with restriction - starting/ending with a letter.

Ex 1

Find the different arrangements of the letters of the word = "DESIGN" in which the first letter is "D"

D | E S I G N 5 remains  
 already used

$$5 \times 4 \times 3 \times 2 \times 1$$

must start by D

$$1 \times 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$

Ex 2

Find the different arrangements of the letters of the word "MEXICO" which

a, begin with the letter 'M'

b, start with the letter 'E' and end with the letter 'C'

$$M \quad 5 \times 4 \times 3 \times 2 \times 1$$

M | E X I C O 5 remains

a  $\underline{M} \times 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$

b  $\underline{E} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{C}$

E | M X I O | C  
 4 remains

$$1 \times 4 \times 3 \times 2 \times 1 \times 1 = 24 \text{ ways}$$

## Permutations with restrictions - digits

Ex(11)

Calculate the number of different 6-digits number which can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition and assuming that a number cannot begin with 0

Steps (1) Find <sup>total</sup> no of possibilities

(2) Find no of possibilities that can start by 0

Then (1) - (2)

$$\text{No of arrangements} = 6! \\ = 720 \text{ ways}$$

No of arrangements beginning with 0

(0) 1, 2, 3, 4, 5

$$0 \times 5 \times 4 \times 3 \times 2 \times 1$$

already used

we remain with 5 other digits

$$1 \times 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$



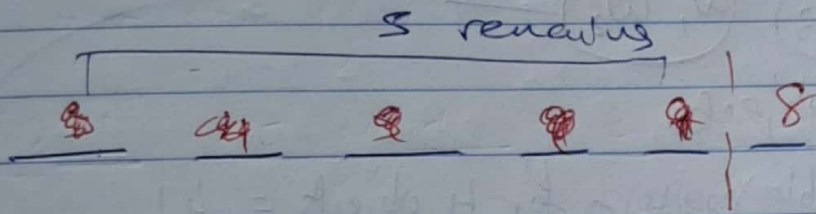
$$N^2 \text{ of arrangements (cannot begin with '0')} = 720 - 120 = 600 \text{ ways}$$

Ex(2)

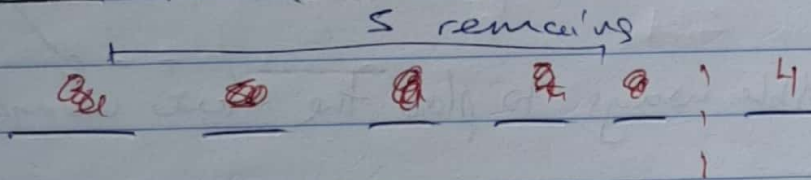
The digits of the number 345987 are arranged so that the resulting number is even.

Find the number of ways in which this can be done.

For even  $\rightarrow$  must end by 8 or 4



$$5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120 \text{ ways}$$

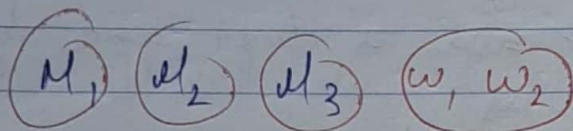


$$5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120 \text{ ways}$$

## Permutations with restrictions - human beings

Ex(1)

In how many ways can three men and two women stand in a queue if two women must stand next to each other.



4 objects

No. of possible position for 4 objects =  $4!$

No. of possible ways to place the two women =  $2!$

$$\begin{aligned}\text{Total No. of ways} &= 4! \times 2! \\ &= 48 \text{ ways}\end{aligned}$$

Ex(2)

In how many ways can 4 boys and 3 girls stand in a queue if the 4 boys must stand next to each other.



4 objects.

$B_1, B_2, B_3, B_4$   $G_1, G_2, G_3$

No. of ways for 4 objects =  $4!$

No. of ways for the 4 boys =  $4!$

Total no. of ways =  $4! \times 4! = 576$  ways

## Combinations

- It is the selection of a specified number of things from a larger group.
- From a given set of  $n$  objects, if a subset of  $r$  objects is to be formed without regard to the order it is called a combination of  $n$  objects, taken  $r$  at a time.
- The total number of such combinations is denoted by

$${}^n C_r$$

total                      combination

Ex 1

In how many ways can 11 football players be selected from a team of 15 to play a match?

No. of ways to select 11 players from 15

$$= {}^{15} C_{11} = 1365 \text{ ways}$$

choisir 11 de 15



Ex 2  
In how many ways can 5 students be selected from a class of 12 to participate in a debate?

$$\text{No of ways} = {}^{12}C_5 = 792 \text{ ways}$$

Ex 3

In how many ways can 2 boys and 4 girls be selected from a group of 4 boys and 7 girls to sing a song.

- No of ways to select 2 boys from 4

$$= {}^4C_2$$

- No of ways to select 4 girls from 7

$$= {}^7C_4$$

$$\text{Total} = {}^4C_2 \times {}^7C_4 = 210 \text{ ways}$$

Ex 4

In how many ways can 3 boys and 5 girls be selected from a group of 5 boys and 10 girls to dance on a stage?

No. of ways to select 3 boys from 5 =  ${}^5C_3$

No. of ways to select 5 girls from 10 =  ${}^{10}C_5$

Total no. of ways =  ${}^5C_3 \times {}^{10}C_5 = 2520$  ways

Ex 5

A committee of 4 people is to be chosen from 4 women and 5 men. The committee must contain at least 1 woman. Calculate the number of different committees that can be formed.

Minimum 1 woman

1 woman and 3 men =  ${}^4C_1 \times {}^5C_3 = 20$

OR

2 women and 2 men =  ${}^4C_2 \times {}^5C_2 = 60$

OR

3 women and 1 man =  ${}^4C_3 \times {}^5C_1 = 20$

OR

4 women and 0 man =  ${}^4C_4 \times {}^5C_0 = 1$

Total =  $20 + 60 + 20 + 1 = 121$  ways



Ex 6

A committee of 5 people is to be chosen from 4 boys and 6 girls. The committee must contain at most 3 girls. Calculate the number of different committees that can be formed?

Maximum 3 girls

or

$$3 \text{ girls and } 2 \text{ boys} = {}^6C_3 \times {}^4C_2 = 120 \text{ ways}$$

OR

$$2 \text{ girls and } 3 \text{ boys} = {}^6C_2 \times {}^4C_3 = 60 \text{ ways}$$

OR

$$1 \text{ girl and } 4 \text{ boys} = {}^6C_1 \times {}^4C_4 = 6 \text{ ways}$$

$$\text{Total} = 120 + 60 + 6 \\ = 186 \text{ ways.}$$

Ex 7

① ② ③ ④ ⑤ ⑥ ⑦  
Allan, Ben, Charles, David, Emily, Frederic, Gaëlle

4 persons are to be chosen.

In how many ways can the selection be made if

- i, there is no restriction.
- ii, Allan must be chosen.
- iii, Both Ben and Emily must be chosen.
- iv, Frederic must not be chosen.
- v, Either Allan or Ben but not both must be chosen.

Solution

i, No. of ways =  ${}^7C_4 = 35$  ways

ii, No. of ways =  $1 \times {}^6C_3$

= 20 ways

from new calculation 6 choose 3



iii) No of ways =  $1 \times 1 \times {}^5C_2$

Ben      Emily

↓      ↓

← From remaining five choose 2.

$= 10 \text{ ways}$

iv) No of way =  ${}^6C_4$

from 7 take out 1 person  
we remain with six  
from the six choose 4.

$= 15 \text{ ways}$

v) No of ways =  ${}^2C_1 \times {}^5C_3$

from Allan and  
Ben choose  
any 1

↓  
from remaining 5  
choose 3

$= 20 \text{ ways}$