TUTORIAL 1

1. Differentiate the following functions w.r.t. *x*:

(i) $\ln[\sqrt{3x-1}+\sqrt{3x+1}]$;

(ii) $\dfrac{\sqrt{x-5}\,(x+2)^3}{\sqrt{x+5}(x-2)^2}$;

(iii) $\sqrt{x}\sin^{-1}(\sqrt{x})$;

(iv) $\tan^{-1}(2\sin\sqrt{x})$;

(v) $5^{-3x}e^{\cos 5x}$.

2. If $y = 3x\sqrt{1-x^2}\cos^{-1}x + 2x^3 + (\cos^{-1}x)^2$, show that

$$\frac{dy}{dx} = 3x(2x-1) + \frac{(1-6x^2)\cos^{-1}x}{\sqrt{1-x^2}}.$$

3. Find $\dfrac{d^2y}{dx^2}$ if $x = a[\sin t - t\cos t]$ and $y = b[\cos t + t\sin t]$.

4. If $y = \tan^{-1} x$, prove that $\left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$. Differentiate three more times, and find the values of the first five derivatives of $\tan^{-1} x$ at $x = 0$. Hence, or otherwise, find the first three non-zero terms in the Maclaurin's expansion of $\tan^{-1} x$.

5. Expand $x\cos x$ about $x = \frac{\pi}{2}$ as far as the term involving x^4 .

6. Let $y = e^{-2x}\sin 3x$.

a. Show that $\frac{dy}{dx} = 3e^{-2x}\cos 3x - 2y$, and hence show that $\frac{d^2y}{dx^2} = -13 - 4\frac{dy}{dx}$.

b. Find Maclaurin's series for y, up to and including the term in x^4.

7. If $y = (\sin^{-1} x)^2$**,** prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2 .$$

Hence using Maclaurin's expansion, or otherwise, prove that the first three non-zero terms in the expansion of $(\sin^{-1} x)^2$ are

$$x^2 + \frac{1}{3}x^4 + \frac{8}{45}x^6 .$$

ANSWERS

1. **(i)** $\dfrac{3}{2\sqrt{9x^2-1}}$;

(ii) $\dfrac{(2+x)^2\,(230-25x-5x^2+x^3)}{\sqrt{-5+x}\;(-2+x)^3\,(5+x)^{3/2}}$

(iii) $\dfrac{1}{2\sqrt{1-x}}+\dfrac{\mathrm{Sin}^{-1}[\sqrt{x}]}{2\sqrt{x}}$

(iv) $\dfrac{\mathrm{Cos}[\sqrt{x}]}{\sqrt{x}\,\left(1+4\,\mathrm{Sin}^2[\sqrt{x}]\right)}$

(v) $-5^{-3x}\,e^{\mathrm{Cos}[5x]}\,(3\,\mathrm{Ln}[5]+5\,\mathrm{Sin}[5x])$

3. $-\dfrac{b\,\mathrm{Cosec}^3[t]}{a^2\,t}$

4. $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}$

5. $-\dfrac{1}{2}\pi\left(x-\dfrac{\pi}{2}\right)-\left(x-\dfrac{\pi}{2}\right)^2+\dfrac{1}{12}\pi\left(x-\dfrac{\pi}{2}\right)^3+\dfrac{1}{6}\left(x-\dfrac{\pi}{2}\right)^4$

6. $3x-6x^2+\dfrac{3}{2}x^3+5x^4$

TUTORIAL 2

1. Find the following integrals:

(i) $\int \tan^2 ax + \cot^2 bx \, dx$;

(ii) $\int \frac{x^6 + 2}{\sqrt{x^7 + 14x - 1}} \, dx$;

(iii) $\int 3x^2 \sin^{-1} 2x \, dx$;

(iv) $\int \frac{dx}{\sqrt{-5 + 18x - 9x^2}}$;

(v) $\int \frac{dx}{4 + 3\cos x}$;

(vi) $\int \frac{x + 7}{10 - 7x + x^2} \, dx$;

(vii) $\int 2x \cdot 5^x \, dx$;

(viii) $\int \frac{dx}{13 + 6x + x^2}$.

2. Evaluate the following definite integrals:

(a) $\int_{-2}^{-1} \frac{dx}{6 - 13x - 5x^2}$;

(b) $\int_0^1 \frac{1}{8 + 6\sin x} \, dy$;

(c) $\int_0^1 \frac{1}{\sqrt{7 + 6x - x^2}} \, dx$;

3. Sketch on separate diagrams each of the following curves whose equations in polar coordinates are given by

(i) $r = a(2 + \cos\theta)$

(ii) $r = a(3 + 2\sin\theta)$

for $0 \le \theta \le 2\pi$, where a is a positive constant.

Find the area of the region enclosed by the curves in each case.

4. **(i)** Sketch on the same diagram the polar curves $r = 2\cos\theta$ and $r = 2 - 2\cos\theta$. Find the area outside $r = 2\cos\theta$ and inside $r = 2 - 2\cos\theta$.

(ii) Sketch the curve with polar equation

$$r = a(1 + \cos 3\theta), \qquad 0 \le \theta \le 2\pi.$$

Find the area of the region enclosed by one loop of the curve.

(iii) Figure 1 shows the polar curves of $r = 1 + \cos\theta$ and $r = \sin 2\theta$. Show that the area of the shaded region is given by $2 + \dfrac{\pi}{2}$.

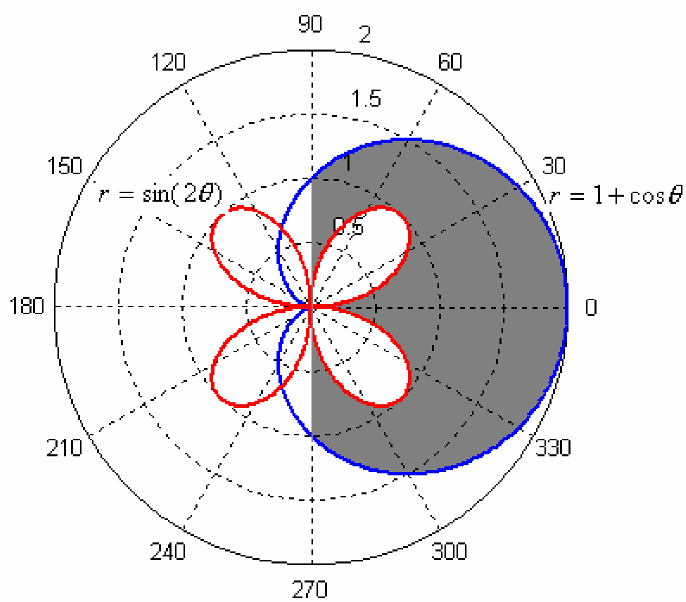


Figure 1

ANSWERS

1. **(i)** $\frac{\text{Cot}[b\,x]}{b} + \frac{\text{Tan}[a\,x]}{a} + c$;

(ii) $\frac{2}{7}\sqrt{-1+14\,x+x^7} + c$;

(iii) $3\left(\sqrt{1-4\,x^2}\left(\frac{1}{36}+\frac{x^2}{18}\right)+\frac{1}{3}\,x^3\,\text{Sin}^{-1}[2\,x]\right)+c$;

(iv) $\frac{1}{3}\,\text{Sin}^{-1}\left[\frac{3}{2}\,(-1+x)\right]+c$;

(v) $\frac{2\,\text{Tan}^{-1}\left[\frac{\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{7}}\right]}{\sqrt{7}}+c$;

(vi) 4 ln[x-5]-3 ln[x-2]+c;

(vii) $\left\{\frac{2\,5^x\,(-1+x\,\ln[5])}{(\ln[5])^2}\right\}+c$;

(viii) $\frac{1}{2}\,\text{Tan}^{-1}\left[\frac{3+x}{2}\right]+c$.

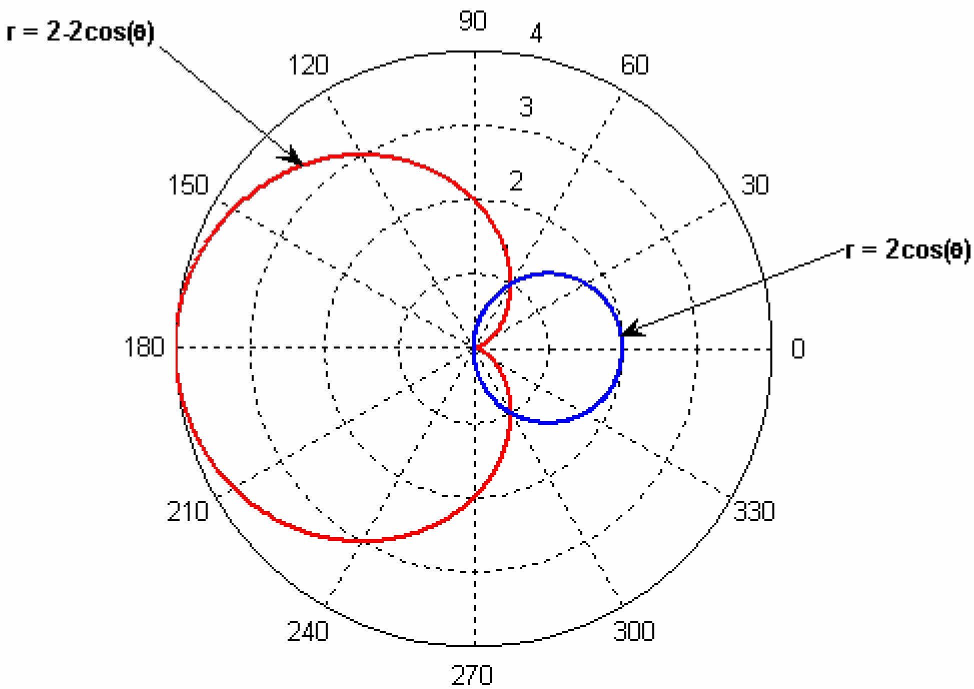
2. **(a)** $\frac{1}{17}\ln\left[\frac{24}{7}\right]$;

(b) $\dfrac{-\mathrm{Tan}^{-1}\left[\frac{3}{\sqrt{7}}\right]+\mathrm{Tan}^{-1}\left[\frac{3+4\,\mathrm{Tan}\left[\frac{1}{2}\right]}{\sqrt{7}}\right]}{\sqrt{7}}$ = 0.0948391;

(c) $-\frac{\pi}{6}+\mathrm{Sin}^{-1}\left[\frac{3}{4}\right]=0.32446$;

3. **(i)** $\frac{9}{2}\pi a^2$ **(ii)** $11\pi a^2$

4. **(i)** $4\sqrt{3}-11\pi/3$;



(ii) $\frac{1}{2}\pi a^2$