SYNTAX ANALYSIS

4.1 ROLE OF THE PARSER :

Parser for any grammar is program that takes as input string w (obtain set of strings tokens from the lexical analyzer) and produces as output either a parse tree for w , if w is a valid sentences of grammar or error message indicating that w is not a valid sentences of given grammar. The goal of the parser is to determine the syntactic validity of a source string is valid, a tree is built for use by the subsequent phases of the computer. The tree reflects the sequence of derivations or reduction used during the parser. Hence, it is called parse tree. If string is invalid, the parse has to issue diagnostic message identifying the nature and cause of the errors in string. Every elementary subtree in the parse tree corresponds to a production of the grammar.

There are two ways of identifying an elementry sutree:

1. By deriving a string from a non-terminal or
2. By reducing a string of symbol to a non-terminal.

The two types of parsers employed are:

a. Top down parser: which build parse trees from top(root) to bottom(leaves)
b. Bottom up parser: which build parse trees from leaves and work up the root.

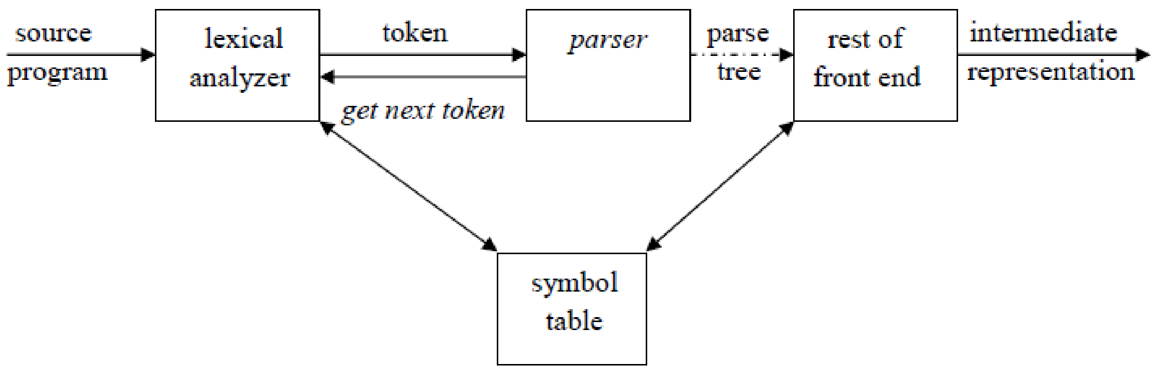


Fig . 4.1: position of parser in compiler model.

4.2 CONTEXT FREE GRAMMARS

Inherently recursive structures of a programming language are defined by a context-free Grammar. In a context-free grammar, we have four triples G(V,T,P,S).

Here , V is finite set of terminals (in our case, this will be the set of tokens)

T is a finite set of non-terminals (syntactic-variables)

P is a finite set of productions rules in the following form

$A \rightarrow \alpha$ where A is a non-terminal and α is a string of terminals and non-terminals (including the empty string)

S is a start symbol (one of the non-terminal symbol)

L(G) is the language of G (the language generated by G) which is a set of sentences.

A sentence of L(G) is a string of terminal symbols of G. If S is the start symbol of G then ω is a sentence of L(G) iff $S \Rightarrow \omega$ where ω is a string of terminals of G. If G is a context-free grammar, L(G) is a context-free language. Two grammar G_1 and G_2 are equivalent, if they produce same grammar.

Consider the production of the form $S \Rightarrow \alpha$, If α contains non-terminals, it is called as a sentential form of G. If α does not contain non-terminals, it is called as a sentence of G.

4.2.1 Derivations

In general a derivation step is

$\alpha A\beta \Rightarrow \alpha\gamma\beta$ is sentential form and if there is a production rule $A\rightarrow\gamma$ in our grammar. where α and β are arbitrary strings of terminal and non-terminal symbols $\alpha 1 \Rightarrow \alpha 2 \Rightarrow \ldots \Rightarrow \alpha n$ (αn derives from $\alpha 1$ or $\alpha 1$ derives αn). There are two types of derivaion

1 At each derivation step, we can choose any of the non-terminal in the sentential form of G for the replacement.

2 If we always choose the left-most non-terminal in each derivation step, this derivation is called as left-most derivation.

Example:
E → E + E | E – E | E * E | E / E | - E
E → (E)
E → id

Leftmost derivation :

E → E + E

→ E * E+E →id* E+E→id*id+E→id*id+id

The string is derive from the grammar w= id*id+id, which is consists of all terminal symbols

Rightmost derivation

E → E + E

→ E+E * E→E+ E*id→E+id*id→id+id*id

Given grammar G : E → E+E | E*E | (E) | - E | id

Sentence to be derived : – (id+id)

LEFTMOST DERIVATION	RIGHTMOST DERIVATION
E → - E	E → - E
E → - (E)	E → - (E)
E → - (E+E)	E → - (E+E)
E → - (id+E)	E → - (E+id)
E → - (id+id)	E → - (id+id)

- String that appear in leftmost derivation are called **left sentinel forms.**
- String that appear in rightmost derivation are called **right sentinel forms.**

Sentinels:

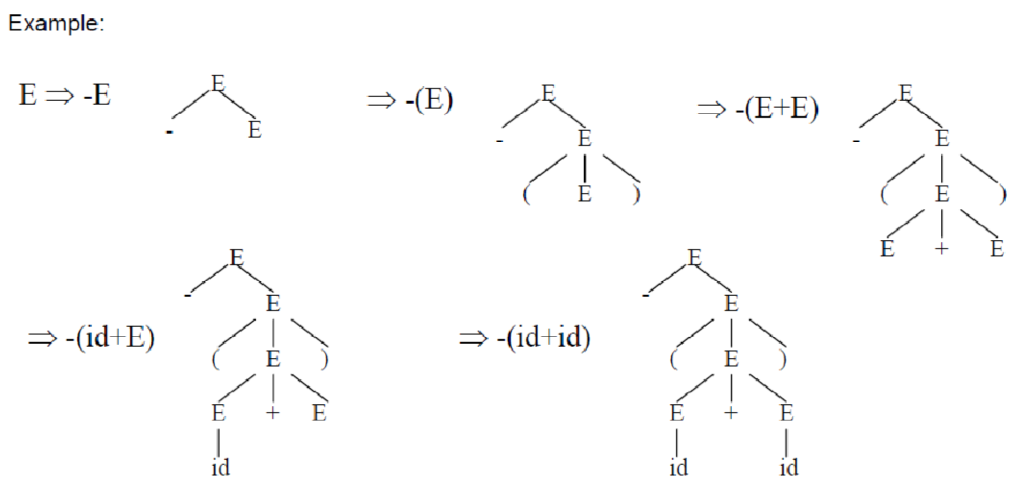
- Given a grammar G with start symbol S, if S → α , where α may contain non-terminals or terminals, then α is called the sentinel form of G.

Yield or frontier of tree:

- Each interior node of a parse tree is a non-terminal. The children of node can be a terminal or non-terminal of the sentinel forms that are read from left to right. The sentinel form in the parse tree is called **yield** or **frontier** of the tree.

4.2.2 PARSE TREE

- Inner nodes of a parse tree are non-terminal symbols.
- The leaves of a parse tree are terminal symbols.
- A parse tree can be seen as a graphical representation of a derivation.



Ambiguity:

A grammar that produces more than one parse for some sentence is said to be **ambiguous grammar**.

Example : Given grammar G : E → E+E | E*E | (E) | - E | id

The sentence id+id*id has the following two distinct leftmost derivations:

E → E+ E	E → E* E
E → id + E	E → E + E * E
E → id + E * E	E → id + E * E
E → id + id * E	E → id + id * E
E → id + id * id	E → id + id * id

The two corresponding parse trees are :

Example:

To disambiguate the grammar E → E+E | E*E | E^E | id | (E), we can use precedence of operators as follows:

^ (right to left)

/,* (left to right)

-,+ (left to right)

We get the following unambiguous grammar:

E → E+T | T

T → T*F | F

F → G^F | G

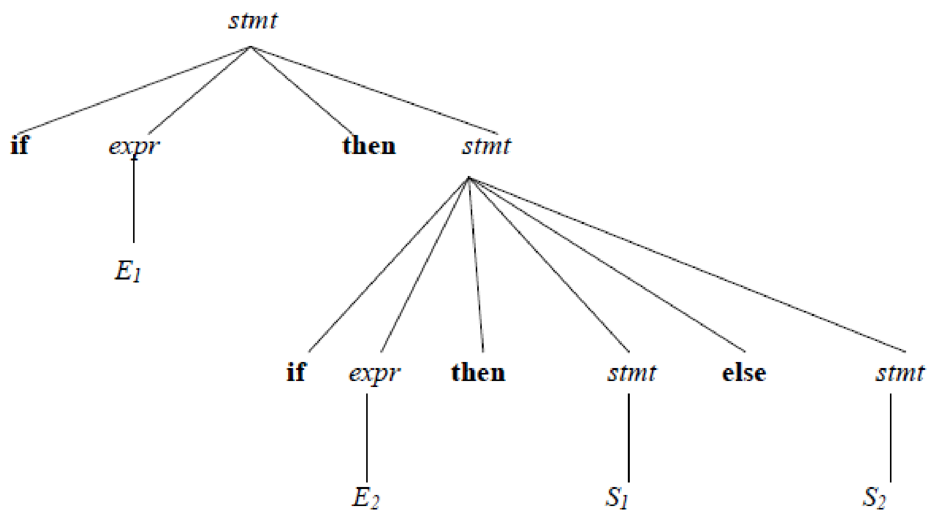
G → id | (E)

Consider this example, G: *stmt* → **if** *expr* **then** *stmt* | **if** *expr* **then** *stmt* **else** *stmt* | **other**

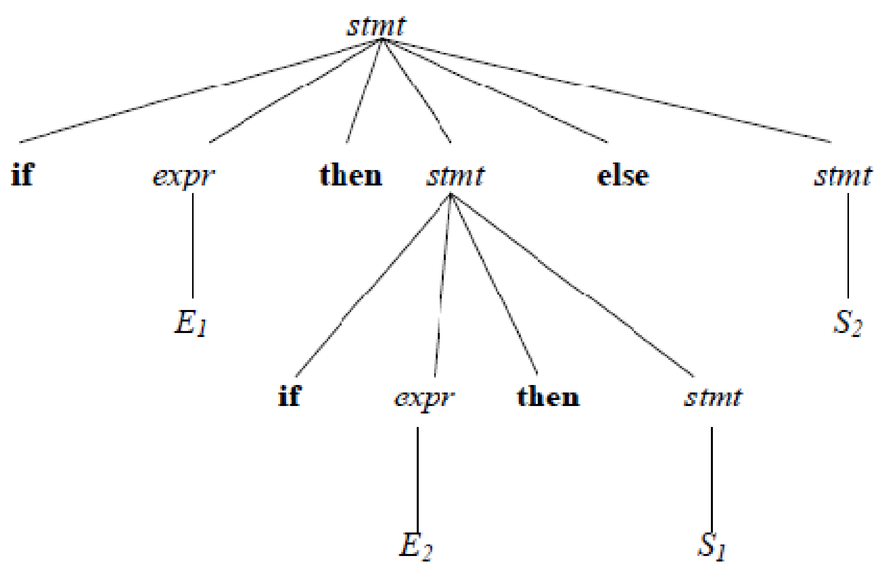
This grammar is ambiguous since the string **if E1 then if E2 then S1 else S2** has the following

Two parse trees for leftmost derivation :

1.



2.



To eliminate ambiguity, the following grammar may be used:

stmt → *matched_stmt* | *unmatched_stmt*

matched_stmt → **if** *expr* **then** *matched_stmt* **else** *matched_stmt* | **other**

unmatched_stmt → **if** *expr* **then** *stmt* | **if** *expr* **then** *matched_stmt* **else** *unmatched_stmt*

Eliminating Left Recursion:

A grammar is said to be *left recursive* if it has a non-terminal *A* such that there is a derivation A=>Aα for some string α. Top-down parsing methods cannot handle left-recursive grammars. Hence, left recursion can be eliminated as follows:

If there is a production $A \rightarrow A\alpha \mid \beta$ it can be replaced with a sequence of two productions

$$A \rightarrow \beta A'$$
$$A' \rightarrow \alpha A' \mid \varepsilon$$

Without changing the set of strings derivable from A.

Example : Consider the following grammar for arithmetic expressions:

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

First eliminate the left recursion for E as

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

Then eliminate for T as

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

Thus the obtained grammar after eliminating left recursion is

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

$F \rightarrow (E) \mid id$

Algorithm to eliminate left recursion:

1. Arrange the non-terminals in some order A1, A2 . . . An.
2. **for** *i* := 1 **to** *n* **do begin**

    for *j* := 1 **to** *i*-1 **do begin**

        replace each production of the form $Ai \rightarrow Aj\ \gamma$

        by the productions $Ai \rightarrow \delta1\ \gamma \mid \delta2\gamma \mid \ldots \mid \delta k\ \gamma$

        where $Aj \rightarrow \delta1 \mid \delta2 \mid \ldots \mid \delta k$ are all the current Aj-productions;

    end

        eliminate the immediate left recursion among the Ai-productions

  end

Left factoring:

Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive parsing. When it is not clear which of two alternative productions to use to expand a non-terminal A, we can rewrite the A-productions to defer the decision until we have seen enough of the input to make the right choice.

If there is any production $A \rightarrow \alpha\beta1 \mid \alpha\beta2$, it can be rewritten as

$A \rightarrow \alpha A'$

$A' \rightarrow \beta1 \mid \beta2$

Consider the grammar , G : $S \rightarrow iEtS \mid iEtSeS \mid a$

$E \rightarrow b$

Left factored, this grammar becomes

$S \rightarrow iEtSS' \mid a$

$S' \rightarrow eS \mid \varepsilon$

$E \rightarrow b$

TOP-DOWN PARSING

It can be viewed as an attempt to find a left-most derivation for an input string or an attempt to construct a parse tree for the input starting from the root to the leaves.

Types of top-down parsing :

1. Recursive descent parsing
2. Predictive parsing

1. RECURSIVE DESCENT PARSING

- Recursive descent parsing is one of the top-down parsing techniques that uses a set of recursive procedures to scan its input**.**
- This parsing method may involve **backtracking**, that is, making repeated scans of the input.

Example for backtracking :

Consider the grammar G : $S \rightarrow cAd$

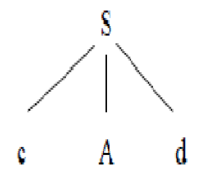
$A \rightarrow ab \mid a$

and the input string w=cad.

The parse tree can be constructed using the following top-down approach :

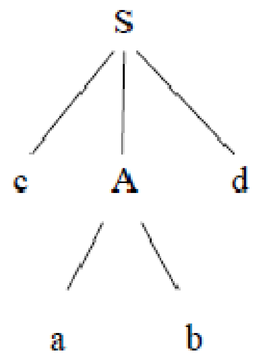
Step1:

Initially create a tree with single node labeled S. An input pointer points to ‘c’, the first symbol of w. Expand the tree with the production of S.



Step2:

The leftmost leaf ‘c’ matches the first symbol of w, so advance the input pointer to the second symbol of w ‘a’ and consider the next leaf ‘A’. Expand A using the first alternative.



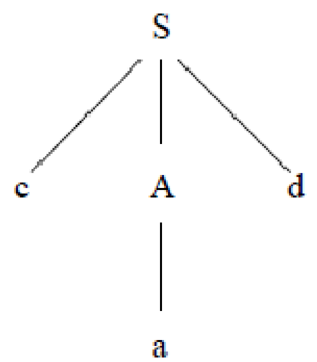
Step3:

The second symbol ‘a’ of w also matches with second leaf of tree. So advance the input pointer to third symbol of w ‘d’**.** But the third leaf of tree is b which does not match with the input symbol **d.**

Hence discard the chosen production and reset the pointer to second position. This is called **backtracking.**

Step4:

Now try the second alternative for A.



Now we can halt and announce the successful completion of parsing.

Example for recursive decent parsing:

A left-recursive grammar can cause a recursive-descent parser to go into an infinite loop. Hence, **elimination of left-recursion** must be done before parsing.

Consider the grammar for arithmetic expressions

E → E+T | T

T → T*F | F

F → (E) | id

After eliminating the left-recursion the grammar becomes,

E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | id

Now we can write the procedure for grammar as follows:

Recursive procedure:

Procedure E()

begin

    T( );

    EPRIME( );

End

Procedure EPRIME( )

    begin

        If input_symbol=’+’ then

        ADVANCE( );

        T( );

        EPRIME( );

    end

Procedure T( )

    begin

        F( );

        TPRIME( );

    End

Procedure TPRIME( )

begin

If input_symbol='*' then

ADVANCE( );

F( );

TPRIME( );

end

Procedure F( )

begin

If input-symbol='id' then

ADVANCE( );

else if input-symbol='(' then

ADVANCE( );

E( );

else if input-symbol=')' then

ADVANCE( );

end

else ERROR( );

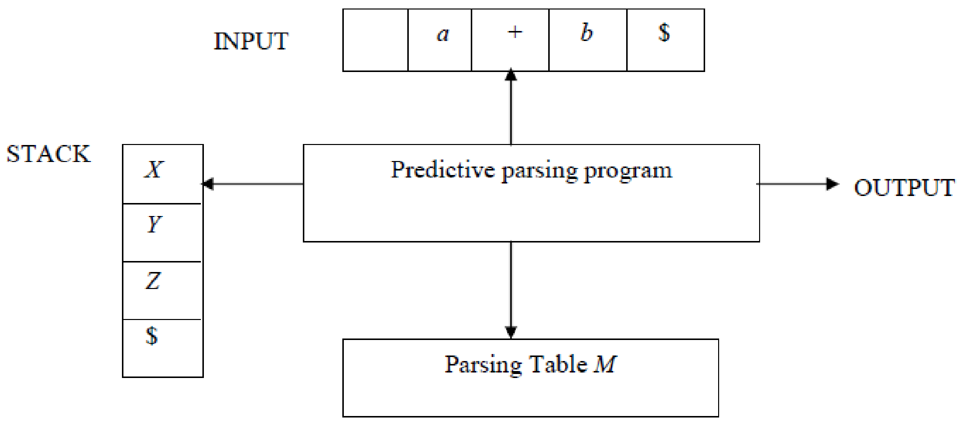
Stack implementation:

PROCEDURE	**INPUT STRING**
E( )	**id**+id*id
T( )	**id**+id*id
F( )	**id**+id*id
ADVANCE( )	id**+**id*id
TPRIME( )	id**+**id*id
EPRIME( )	id**+**id*id
ADVANCE( )	id+**id***id
T( )	id+**id***id
F( )	id+**id***id
ADVANCE( )	id+id*id
TPRIME( )	id+id*****id
ADVANCE( )	id+id*****id
F( )	id+id*****id
ADVANCE( )	id+id***id**
TPRIME( )	id+id***id**

2. PREDICTIVE PARSING

- ✓ Predictive parsing is a special case of recursive descent parsing where no backtracking is required.
- ✓ The key problem of predictive parsing is to determine the production to be applied for a non-terminal in case of alternatives.

Non-recursive predictive parser



The table-driven predictive parser has an input buffer, stack, a parsing table and an output stream.

Input buffer:

It consists of strings to be parsed, followed by $ to indicate the end of the input string.

Stack:

It contains a sequence of grammar symbols preceded by $ to indicate the bottom of the stack. Initially, the stack contains the start symbol on top of $.

Parsing table:

It is a two-dimensional array $M[A, a]$, where **'*A*'** is a non-terminal and **'*a*'** is a terminal.

Predictive parsing program:

The parser is controlled by a program that considers X, the symbol on top of stack, and a, the current input symbol. These two symbols determine the parser action. There are three possibilities:

1. If $X = a = \$$, the parser halts and announces successful completion of parsing.
2. If $X = a \neq \$$, the parser pops X off the stack and advances the input pointer to the next input symbol.
3. If X is a non-terminal , the program consults entry $M[X, a]$ of the parsing table M. This entry will either be an X-production of the grammar or an error entry.

If $M[X, a] = \{X \rightarrow UVW\}$,the parser replaces X on top of the stack by UVW

If $M[X, a]$ = **error**, the parser calls an error recovery routine.

Algorithm for nonrecursive predictive parsing:

Input : A string *w* and a parsing table *M* for grammar *G*.

Output : If *w* is in *L*(*G*), a leftmost derivation of *w*; otherwise, an error indication.

Method : Initially, the parser has $*S* on the stack with *S*, the start symbol of *G* on top, and *w*$ in the input buffer. The program that utilizes the predictive parsing table *M* to produce a parse for the input is as follows:

set *ip* to point to the first symbol of *w*$;

repeat

let *X* be the top stack symbol and *a* the symbol pointed to by *ip*;

if *X* is a terminal or $ **then**

if *X* = *a* **then**

pop *X* from the stack and advance *ip*

else e*rror*()

else /* *X* is a non-terminal */

if $M[X, a] = X \rightarrow Y1Y2 \ldots Yk$ **then begin**

pop *X* from the stack;

push *Yk*, *Yk-1*, … ,*Y1* onto the stack, with *Y1* on top;

output the production $X \rightarrow Y1\ Y2 \ldots Yk$

end

else *error*()

until *X* = $

Predictive parsing table construction:

The construction of a predictive parser is aided by two functions associated with a grammar G :

1. FIRST
2. FOLLOW

Rules for first( ):

1. If *X* is terminal, then FIRST(*X*) is {X}.
2. If $X \rightarrow \varepsilon$ is a production, then add ε to FIRST(*X*).
3. If *X* is non-terminal and $X \rightarrow a\alpha$ is a production then add *a* to FIRST(X).

4. If X is non-terminal and $X \rightarrow Y_1\ Y_2 \ldots Y_k$ is a production, then place a in FIRST(X) if for some i, a is in FIRST(Yi), and ε is in all of FIRST($Y1$),…,FIRST(Yi-1); that is, $Y1$,….Yi-1 => ε. If ε is in FIRST(Y_j) for all j=1,2,..,k, then add ε to FIRST(X).

Rules for follow( ):

1. If S is a start symbol, then FOLLOW(S) contains $.

2. If there is a production $A \rightarrow \alpha B\beta$, then everything in FIRST(β) except ε is placed in follow(B).
3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B\beta$ where FIRST(β) contains ε, then everything in FOLLOW(A) is in FOLLOW(B).

Algorithm for construction of predictive parsing table:

Input : Grammar G

Output : Parsing table M

Method :

1. For each production $A \rightarrow \alpha$ of the grammar, do steps 2 and 3.
2. For each terminal a in FIRST(α), add $A \rightarrow \alpha$ to $M[A, a]$.
3. If ε is in FIRST(α), add A → α to $M[A, b]$ for each terminal b in FOLLOW(A). If ε is in FIRST(α) and $ is in FOLLOW(A) , add $A \rightarrow \alpha$ to $M[A, \$]$.
4. Make each undefined entry of M be **error**.

Example:

Consider the following grammar :

E → E+T | T

T → T*F | F

F → (E) | id

After eliminating left-recursion the grammar is

E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | id

First( ) :

FIRST(E) = { (, id}

FIRST(E’) ={+ , ε }

FIRST(T) = { (, id}

FIRST(T’) = {*, ε }

FIRST(F) = { (, id }

Follow( ):

FOLLOW(E) = { $,) }

FOLLOW(E’) = { $,) }

FOLLOW(T) = { +, $,) }

FOLLOW(T') = { +, $,) }

FOLLOW(F) = {+, * , $,) }

Predictive parsing table :

NON-TERMINAL	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E'→ ε
T	T → FT'			T → FT'		
T'		T'→ ε	T'→ *FT'		T' → ε	T' → ε
F	F → id			F → (E)		

Stack implementation:

stack	Input	Output
$E	id+id*id $	
$E'T	id+id*id $	E → TE'
$E'T'F	id+id*id $	T → FT'
$E'T'id	id+id*id $	F → id
$E'T'	+id*id $	
$E'	+id*id $	T' → ε
$E'T+	+id*id $	E' → +TE'
$E'T	id*id $	
$E'T'F	id*id $	T → FT'
$E'T'id	id*id $	F → id
$E'T'	*id $	
$E'T'F*	*id $	T' → *FT'
$E'T'F	id $	
$E'T'id	id $	F → id
$E'T'	$	
$E'	$	T' → ε
$	$	E' → ε

LL(1) grammar:

The parsing table entries are single entries. So each location has not more than one entry. This type of grammar is called LL(1) grammar.

Consider this following grammar:

S → iEtS | iEtSeS | a

E → b

After eliminating left factoring, we have

S → iEtSS' | a

S'→ eS | ε

E → b

To construct a parsing table, we need FIRST() and FOLLOW() for all the non-terminals.

FIRST(S) = { i, a }

FIRST(S') = {e, ε }

FIRST(E) = { b}

FOLLOW(S) = { $,e }

FOLLOW(S') = { $,e }

FOLLOW(E) = {t}

Parsing table:

NON-TERMINAL	a	b	e	i	t	$
S	S → a			S → iEtSS'		
S'			S' → eS S' → ε			S' → ε
E		E → b				

Since there are more than one production, the grammar is not LL(1) grammar.

Actions performed in predictive parsing:

1. Shift
2. Reduce
3. Accept
4. Error

Implementation of predictive parser:

1. Elimination of left recursion, left factoring and ambiguous grammar.
2. Construct FIRST() and FOLLOW() for all non-terminals.
3. Construct predictive parsing table.
4. Parse the given input string using stack and parsing table.

BOTTOM-UP PARSING

Constructing a parse tree for an input string beginning at the leaves and going towards the root is called bottom-up parsing.

A general type of bottom-up parser is a **shift-reduce parser**.

SHIFT-REDUCE PARSING

Shift-reduce parsing is a type of bottom-up parsing that attempts to construct a parse tree for an input string beginning at the leaves (the bottom) and working up towards the root (the top).

Example:

Consider the grammar:

S → aABe

A → Abc | b

B → d

The sentence to be recognized is **abbcde.**

REDUCTION (LEFTMOST)		**RIGHTMOST DERIVATION**
a**b**bcde	(A → b)	**S** → aA**B**e
a**Abc**de	(A → Abc)	→ a**A**de
aA**d**e	(B → d)	→ a**A**bcde
aABe	(S → aABe)	→ abbcde
S		

The reductions trace out the right-most derivation in reverse.

Handles:

A handle of a string is a substring that matches the right side of a production, and whose reduction to the non-terminal on the left side of the production represents one step along the reverse of a rightmost derivation.

Example:

Consider the grammar:

E → E+E
E → E*E
E → (E)
E → id

And the input string $id_1+id_2*id_3$

The rightmost derivation is :

$E \rightarrow \underline{\mathbf{E+E}}$
$\rightarrow E+\underline{\mathbf{E*E}}$
$\rightarrow E+E*\underline{\mathbf{id_3}}$
$\rightarrow E+\underline{\mathbf{id_2}}*id_3$
$\rightarrow \underline{\mathbf{id_1}}+id_2*id_3$

In the above derivation the underlined substrings are called **handles.**

Handle pruning:

A rightmost derivation in reverse can be obtained by "**handle pruning**".
(i.e.) if w is a sentence or string of the grammar at hand, then $w = \gamma_n$, where γ_n is the n^{th} right-sentinel form of some rightmost derivation.

Stack implementation of shift-reduce parsing :

Stack	Input	Action
\$	$id_1+id_2*id_3$ \$	shift
\$ id_1	$+id_2*id_3$ \$	reduce by E→id
\$ E	$+id_2*id_3$ \$	shift
\$ E+	id_2*id_3 \$	shift
\$ $E+id_2$	$*id_3$ \$	reduce by E→id
\$ E+E	$*id_3$ \$	shift
\$ E+E*	id3 \$	shift
\$ E+E*id3	\$	reduce by E→id
\$ E+E*E	\$	reduce by E→ E *E
\$ E+E	\$	reduce by E→ E+E
\$ E	\$	accept

Actions in shift-reduce parser:

- shift – The next input symbol is shifted onto the top of the stack.
- reduce – The parser replaces the handle within a stack with a non-terminal.
- accept – The parser announces successful completion of parsing.
- error – The parser discovers that a syntax error has occurred and calls an error recovery routine.

Conflicts in shift-reduce parsing:

There are two conflicts that occur in shift shift-reduce parsing:

1. **Shift-reduce conflict**: The parser cannot decide whether to shift or to reduce.

2. **Reduce-reduce conflict**: The parser cannot decide which of several reductions to make.

1. Shift-reduce conflict:

Example:

Consider the grammar:

E→E+E | E*E | id and input id+id*id

Stack	Input	Action	Stack	Input	Action
$ E+E	*id $	Reduce by E→E+E	$E+E	*id $	Shift
$ E	*id $	Shift	$E+E*	id $	Shift
$ E*	id $	Shift	$E+E*id	$	Reduce by E→id
$ E*id	$	Reduce by E→id	$E+E*E	$	Reduce by E→E*E
$ E*E	$	Reduce by E→E*E	$E+E	$	Reduce by E→E*E
$ E			$E		

2. Reduce-reduce conflict:

Consider the grammar:

M → R+R | R+c | R
R → c
and input c+c

Stack	Input	Action	Stack	Input	Action
$	c+c $	Shift	$	c+c $	Shift
$ c	+c $	Reduce by R→c	$ c	+c $	Reduce by R→c
$ R	+c $	Shift	$ R	+c $	Shift
$ R+	c $	Shift	$ R+	c $	Shift
$ R+c	$	Reduce by R→c	$ R+c	$	Reduce by M→R+c
$ R+R	$	Reduce by M→R+R	$ M	$	
$ M	$				