Assignment 6: Distributed Memory representation and algorithm

1 Reduction

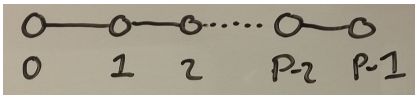
Consider the following three algorithms

```
reduce-star(p, P, val) {
  if (p == 0) {
    for (i=1; i<P;++i) {
      recv vald from i;
      val += vald;
    }
  }
  else {
    send val to 0;
  }
}
```

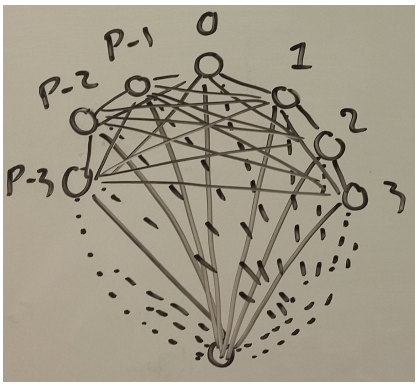
```
reduce-chain(p, P, val) {
  if ( p != P-1) {
    recv vald from p+1;
    val += vald;
  }
  if (p != 0) {
    send val to p-1;
  }
}
```

```
reduce-tree(p, P, val) {
  while (P>0) {
    if (p %2 == 1) {
      send val to p-1;
      return;
    }
    if (p %2 == 0){
      recv valp from p+1;
      val += valp;
    }
    p /= 2;
    P /= 2;
  }
}
```

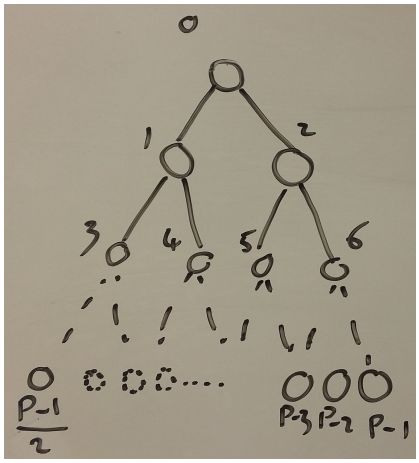
Consider the following three network structures



chain



clique



hierarchical

For each algorithm and each network structure, answer the following questions. (Run a small example if you have difficulty seeing how communication happens.) You might want to organise the results as a table.
Question: How much data transit on each link in total? What is the most loaded link?
Question: How much data is received/sends by each node? What is the node that receives/sends the most data?
Question: What is the longest chain of communication?
Question: What do you think is the best algorithm for each network structure? (One of the given algorithm or a different one.)

2 Heat Equation - 1D

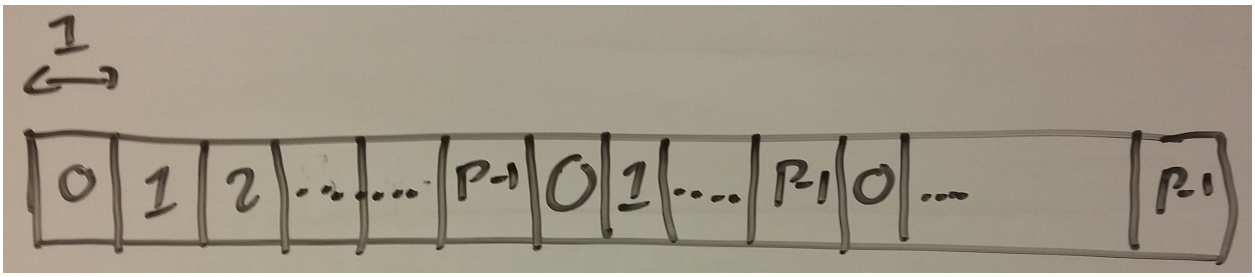
One dimensional heat equation is the simplest example of a stencil computation. It computes iteratively the following equation for a stencil of size N.

$$Heat^k[0] = \frac{2Heat^{k-1}[0] + Heat^{k-1}[i]}{3}$$

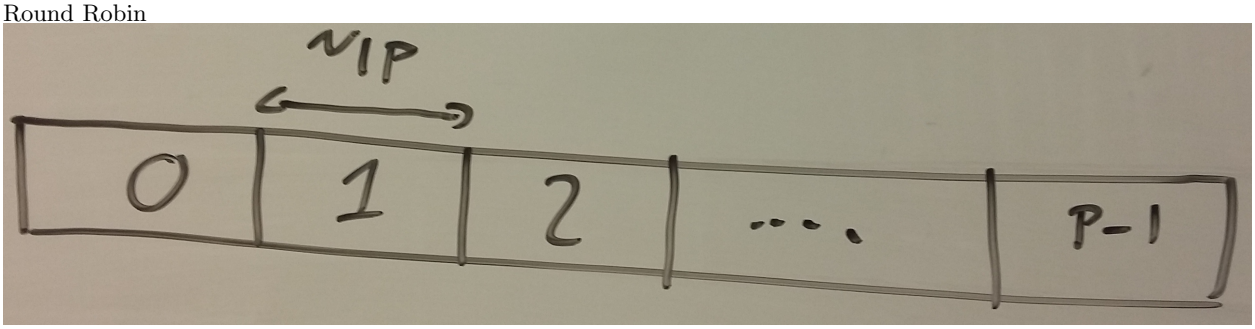
$$Heat^k[n-1] = \frac{2Heat^{k-1}[N-1] + Heat^{k-1}[i]}{N-2}$$

$$Heat^k[i] = \frac{Heat^{k-1}[i-1] + Heat^{k-1}[i] + Heat^{k-1}[i+1]}{3}, \forall 0 < i < N-1$$

Consider the following partitioning of the data



Round Robin



Block

(Assume network topology is a clique.)

Question: Write the algorithm that computes heat equation using this decomposition.

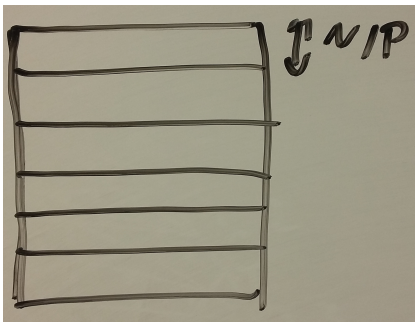
Question: How much communication happen per iteration of the heat equation?

Question: What data partitioning would you use? How much communication does it do per iteration of the heat equation (total, per link, per node)?

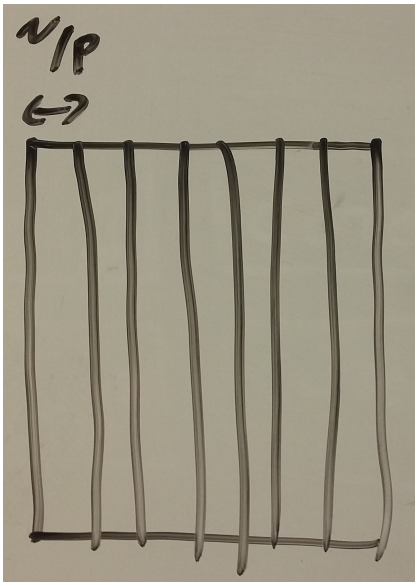
3 Dense Matrix Multiplication

Given a matrix A of size $N \times N$ and a vector x of size N, the value $y = Ax$ is given by $y[i] = \sum_j A[i][j]x[j]$. Or in other words, to compute $y[i]$ multiply element wise the ith row of the matrix by x and sum the values.

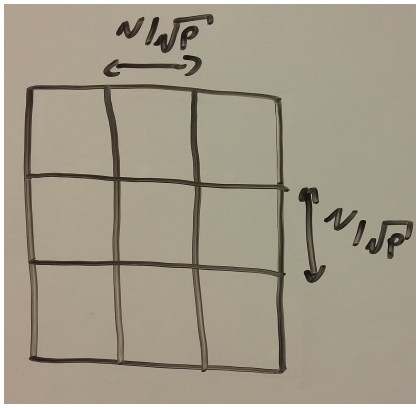
Consider the three data partitioning:



horizontal



vertical



blocks

(Assume the network topology is a clique.)

For each data partitioning:

Question: Write the algorithm that performs $y = Ax; x = y;$ 10 times in a loop.

Question: How much memory does each node need?

Question: How much communication does the algorithm do per iteration? (total, per link, per node)