Indefinite Integrals

A function $F(x)$ is said to be an ***anti-derivative*** of a function $f(x)$ if $\boldsymbol{F'(x) = f(x)}$ for all x in the domain of f.

The set of all anti-derivatives of f is the ***indefinite integral*** of f with respect to x, denoted by

$$\int f(x)dx$$

The symbol ∫ is an ***integral sign***. The function f is the ***integrand*** of the integral and x is the ***variable of integration***.

Once we have found one anti-derivative F of a function f, the other anti-derivatives of f differ from F by a constant.

We indicate this in the integral notation in the following way:

$$\int f(x)dx = F\left(x\right) + C \text{ ....(1)}$$

The constant *C* is the ***constant of integration*** or ***arbitrary constant***. Equation (1) is read, "The indefinite integral of f with respect to x is $F(x)+C$." When we find $F(x)+C$, we say that we have ***integrated*** f and ***evaluated*** the integral.

The standard arithmetic rules for indefinite integration are

1. $\int k\, f(x)\, dx = k\int f(x)\, dx$ **Constant Multiple Rule**
2. $\int -f(x)\, dx = -\int f(x)\, dx$ **Rule for Negatives**
3. $\int [f(x) \pm g(x)]dx = \int f(x)\, dx \pm \int g(x)\, dx$

   Sum and Difference Rule
4. $\frac{d}{dx}\left(\int f(x)\, dx\right) = f(x)$

Integration Formulas: Let a, b, c & d are any constants.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

2. $\int dx = x + C$ & $\int k\, dx = kx + C$; $\boldsymbol{k}$ is any constant.

3. $\int (ax+b)^n dx = \frac{1}{a} \times \frac{(ax+b)^{n+1}}{n+1} + C, n \neq -1$

4. $\int \frac{1}{x} dx = \log|x| + C$

5. $\int \frac{1}{ax+b} dx = \frac{1}{a}\left(\log|ax+b|\right) + C$

6. $\int e^x dx = e^x + C$

7. $\int e^{ax} dx = \frac{1}{a}\left(e^{ax}\right) + C$

8. $\int e^{ax+b} dx = \frac{1}{a}\left(e^{ax+b}\right) + C$

9. $\int a^x dx = \frac{a^x}{\log_e a} + C\ , a > 0, a \neq 1$

10. $\int a^{mx} dx = \frac{1}{m}\left(\frac{a^{mx}}{\log_e a}\right) + C\ , a > 0, a \neq 1$

11. $\int a^{mx+n} dx = \frac{1}{m}\left(\frac{a^{mx+n}}{\log_e a}\right) + C\ , a > 0, a \neq 1$

Integrals of Trigonometric Functions:

12. $\int \sin x\,dx = -\cos x + C$

13. $\int \sin(ax)\,dx = -\frac{1}{a}\left(\cos(ax)\right) + C$

14. $\int \sin(ax+b)\,dx = -\frac{1}{a}\left(\cos(ax+b)\right) + C$

15. $\int \cos x\,dx = \sin x + C$

16. $\int \cos(ax)\,dx = \frac{1}{a}\left(\sin(ax)\right) + C$

17. $\int \cos(ax+b)\,dx = \frac{1}{a}\left(\sin(ax+b)\right) + C$

18. $\int \tan x\,dx = -\log\left|\cos x\right| + C$ (or)

$\int \tan x\,dx = \log\left|\sec x\right| + C$

19. $\int \tan(ax)\,dx = -\frac{1}{a}\log\left|\cos(ax)\right| + C$ (Or)

$\int \tan(ax)\,dx = \frac{1}{a}\log\left|\sec(ax)\right| + C$

20. $\int \tan(ax+b)\,dx = -\frac{1}{a}\log\left|\cos(ax+b)\right| + C$ (Or)

$\int \tan(ax+b)\,dx = \frac{1}{a}\log\left|\sec(ax+b)\right| + C$

21. $\int \cot x\,dx = \log\left|\sin x\right| + C$ (Or)

$\int \cot x\,dx = -\log\left|\csc x\right| + C$

22. $\int \cot(ax)\,dx = \frac{1}{a}\log\left|\sin(ax)\right| + C$ (Or)

$\int \cot(ax)\,dx = -\frac{1}{a}\log\left|\csc(ax)\right| + C$

23. $\int \cot(ax+b)\,dx = \frac{1}{a}\log\left|\sin(ax+b)\right| + C$ (Or)

$\int \cot(ax+b)\,dx = -\frac{1}{a}\log\left|\csc(ax+b)\right| + C$

24. $\int \sec x\,dx = \log\left|\sec x + \tan x\right| + C$ (Or)

$\int \sec x\,dx = -\log\left|\sec x - \tan x\right| + C$ (Or)

$\int \sec x\,dx = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

25. $\int \sec(ax)\,dx = \frac{1}{a}\log\left|\sec(ax) + \tan(ax)\right| + C$ (Or)

$\int \sec(ax)\,dx = -\frac{1}{a}\log\left|\sec(ax) - \tan(ax)\right| + C$ (Or)

$\int \sec(ax)\,dx = \frac{1}{a}\log\left|\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right)\right| + C$

26. $\int \sec(ax+b)\,dx = \frac{1}{a}\log\left|\sec(ax+b) + \tan(ax+b)\right| + C$

(Or)

$\int \sec(ax+b)\,dx = -\frac{1}{a}\log\left|\sec(ax+b) - \tan(ax+b)\right| + C$

(Or)

$\int \sec(ax+b)\,dx = \frac{1}{a}\log\left|\tan\left(\frac{\pi}{4} + \frac{ax+b}{2}\right)\right| + C$

27. $\int \csc x\,dx = \log\left|\csc x - \cot x\right| + C$ (Or)

$\int \csc x\,dx = -\log\left|\csc x + \cot x\right| + C$ (Or)

$\int \csc x\,dx = \log\left|\tan\left(\dfrac{x}{2}\right)\right| + C$

28. $\int \csc(ax)\,dx = \dfrac{1}{a}\log\left|\csc(ax) - \cot(ax)\right| + C$ (Or)

$\int \csc(ax)\,dx = -\dfrac{1}{a}\log\left|\csc(ax) + \cot(ax)\right| + C$ (Or)

$\int \csc(ax)\,dx = \dfrac{1}{a}\log\left|\tan\left(\dfrac{ax}{2}\right)\right| + C$

29. $\int \csc(ax+b)\,dx = \dfrac{1}{a}\log\left|\csc(ax+b) - \cot(ax+b)\right| + C$

(Or)

$\int \csc(ax+b)\,dx = -\dfrac{1}{a}\log\left|\csc(ax+b) + \cot(ax+b)\right| + C$

(Or)

$\int \csc(ax+b)\,dx = \dfrac{1}{a}\log\left|\tan\left(\dfrac{ax+b}{2}\right)\right| + C$

30. $\int \sec^2 x\,dx = \tan x + C$

$\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + C$

$\int \sec^2(ax+b)\,dx = \dfrac{1}{a}\tan(ax+b) + C$

31. $\int \csc^2 x\,dx = -\cot x + C$

$$\int \csc^2 (ax)\,dx = -\frac{1}{a}\cot(ax) + C$$

$$\int \csc^2 (ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$$

32. $\int \sec x \tan x\,dx = \sec x + C$

$$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + C$$

$$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$$

33. $\int \csc x \cot x\,dx = -\csc x + C$

$$\int \csc(ax)\cot(ax)\,dx = -\frac{1}{a}\csc(ax) + C$$

$$\int \csc(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\csc(ax+b) + C$$

Integrals of hyperbolic functions:

34. $\int \sinh x\,dx = \cosh x + C$

$$\int \sinh(ax)\,dx = \frac{1}{a}\cosh(ax) + C$$

$$\int \sinh(ax+b)\,dx = \frac{1}{a}\cosh(ax+b) + C$$

35. $\int \cosh x\,dx = \sinh x + C$

$$\int \cosh(ax)\,dx = \frac{1}{a}\sinh(ax) + C$$

$$\int \cosh(ax+b)\,dx = \frac{1}{a}\sinh(ax+b) + C$$

36. $$\int \tanh x\,dx = \log_e(\cosh x) + C = \ln(\cosh x) + C$$

$$\int \tanh(ax)\,dx = \frac{1}{a}\log_e(\cosh(ax)) + C$$

$$= \frac{1}{a}\ln(\cosh(ax)) + C$$

$$\int \tanh(ax+b)\,dx = \frac{1}{a}\log_e(\cosh(ax+b)) + C$$

$$= \frac{1}{a}\ln(\cosh(ax+b)) + C$$

37. $$\int \coth x\,dx = \log_e(\sinh x) + C = \ln(\sinh x) + C$$

$$\int \coth(ax)\,dx = \frac{1}{a}\log_e(\sinh(ax)) + C$$

$$= \frac{1}{a}\ln(\sinh(ax)) + C$$

$$\int \coth(ax+b)\,dx = \frac{1}{a}\log_e(\sinh(ax+b)) + C$$

$$= \frac{1}{a}\ln(\sinh(ax+b)) + C$$

38. $$\int \operatorname{sech} x\,dx = \operatorname{Tan}^{-1}(\sinh x) + C$$

$$\int \operatorname{sech}(ax)\,dx = \frac{1}{a}\operatorname{Tan}^{-1}(\sinh(ax)) + C$$

$$\int \operatorname{sech}(ax+b)\,dx = \frac{1}{a}\operatorname{Tan}^{-1}(\sinh(ax+b)) + C$$

39. $\int \operatorname{csch} x\,dx = \log_e\left(\tanh\left(\frac{x}{2}\right)\right) + C = \ln\left(\tanh\left(\frac{x}{2}\right)\right) + C$

$$\int \operatorname{csch}(ax)\,dx = \frac{1}{a}\log_e\left(\tanh\left(\frac{ax}{2}\right)\right) + C$$

$$= \frac{1}{a}\ln\left(\tanh\left(\frac{ax}{2}\right)\right) + C$$

$$\int \operatorname{csch}(ax+b)\,dx = \frac{1}{a}\log_e\left(\tanh\left(\frac{ax+b}{2}\right)\right) + C$$

$$= \frac{1}{a}\ln\left(\tanh\left(\frac{ax+b}{2}\right)\right) + C$$

40. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$

$$\int \operatorname{sech}^2(ax)\,dx = \frac{1}{a}\tanh(ax) + C$$

$$\int \operatorname{sech}^2(ax+b)\,dx = \frac{1}{a}\tanh(ax+b) + C$$

41. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

$$\int \operatorname{csch}^2(ax)\,dx = -\frac{1}{a}\coth(ax) + C$$

$$\int \operatorname{csch}^2(ax+b)\,dx = -\frac{1}{a}\coth(ax+b) + C$$

42. $\int \operatorname{sech} x \tanh x\ dx = -\operatorname{sech} x + C$

$$\int \operatorname{sech}(ax)\tanh(ax)\ dx = -\frac{1}{a}\operatorname{sech}(ax) + C$$

$$\int \operatorname{sech}(ax+b)\tanh(ax+b)\,dx = -\frac{1}{a}\operatorname{sech}(ax+b)+C$$

43. $\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$

$$\int \operatorname{csch}(ax)\coth(ax)\,dx = -\frac{1}{a}\operatorname{csch}(ax)+C$$

$$\int \operatorname{csch}(ax+b)\coth(ax+b)\,dx = -\frac{1}{a}\operatorname{csch}(ax+b)+C$$

Some special integrals:

44. $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\mathrm{Tan}^{-1}\left(\frac{x}{a}\right)+C$ (Or)

$$\int \frac{1}{x^2+a^2}dx = -\frac{1}{a}Cot^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{x^2+1}dx = \mathrm{Tan}^{-1}(x)+C = -Cot^{-1}(x)+C$$

45. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+C$

$$\int \frac{1}{x^2-1}dx = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$$

46. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+C$ (Or)

$$\int \frac{1}{a^2-x^2}dx = \tanh^{-1}\left(\frac{x}{a}\right)+C \quad for\ |x|<a.$$ (Or)

$$\int \frac{1}{a^2-x^2}dx = \coth^{-1}\left(\frac{x}{a}\right)+C \quad for\ |x|>a.$$

$$\int \frac{1}{1-x^2}dx = \frac{1}{2}\log\left|\frac{1+x}{1-x}\right|+C$$ (Or)

$$\int \frac{1}{1-x^2}\,dx = \tanh^{-1}(x) + C \quad for \ |x| < 1.$$ (Or)

$$\int \frac{1}{1-x^2}\,dx = \coth^{-1}(x) + C \quad for \ |x| > 1.$$

47. $$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$ (Or)

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log_e\left(\frac{x+\sqrt{x^2+a^2}}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1}(x) + C = \log_e\left(x+\sqrt{x^2+1}\right) + C$$

48. $$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad on \ (a,\infty)$$ (Or)

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log_e\left(\frac{x+\sqrt{x^2-a^2}}{a}\right) + C \quad on \ (a,\infty)$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = -\cosh^{-1}\left(-\frac{x}{a}\right) + C \quad on \ (-\infty,-a)$$ (Or)

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = -\log_e\left(\frac{-x+\sqrt{x^2-a^2}}{a}\right) + C \quad on \ (-\infty,-a)$$

For example

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1}(x) + C \quad on \ (1,\infty)$$ (Or)

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \log_e\left(x+\sqrt{x^2-1}\right) + C \quad on \ (1,\infty)$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = -\cosh^{-1}(-x) + C \quad on \;\; (-\infty,-1) \quad \text{(Or)}$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = -\log_e\left(-x+\sqrt{x^2-1}\right) + C \quad on \;\; (-\infty,-1)$$

49. $$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \operatorname{Sin}^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = -\operatorname{Cos}^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{Sin}^{-1}(x) + C = -\operatorname{Cos}^{-1}(x) + C$$

50. $$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + C$$

51. $$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + C$$

52. $$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\operatorname{Sin}^{-1}\left(\frac{x}{a}\right) + C$$

53. $$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{Sec}^{-1}(x) + C = -\operatorname{Csc}^{-1}(x) + C$$

54. $$\int \frac{1}{x\sqrt{x^2+1}}\,dx = -\operatorname{csch}^{-1}|x| + C$$

55. $$\int \frac{1}{x\sqrt{1-x^2}}\,dx = -\operatorname{sech}^{-1}|x| + C$$

56. $$\int \frac{1}{x\sqrt{x^2+a^2}}\,dx = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + C$$

57. $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right)+C$

58. $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$

59. $\int \frac{1}{|x|\sqrt{x^2+a^2}}\,dx = \operatorname{csch}^{-1}\left(\frac{x}{a}\right)+C$

60. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)|+C$

61. $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)}+C$

62. $\int (f(x))^n\, f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1}+C$

Integration by parts:

If $\boldsymbol{u}$ and $\boldsymbol{v}$ are two functions of $\boldsymbol{x}$, then

$$\int uv\,dx = u\left(\int v\,dx\right)-\int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$\boldsymbol{i.e.\ the\ integral\ of\ the\ product\ of\ two\ \ functions}$$
$$= (\boldsymbol{First\ function})\times(\boldsymbol{integral\ of\ second\ function})$$
$$-\boldsymbol{integral\ of}\ \begin{Bmatrix}(\boldsymbol{Derivative\ of\ first\ function})\times\\ (\boldsymbol{integral\ of\ second\ function})\end{Bmatrix}$$

Note:

1. Proper choice of first and second function.

In the above rule there are two terms on RHS and in both the terms the integral of the second function is involved. Therefore in the product of two functions if one of the two functions is not directly integrable $\left(e.g., \log x, \sin^{-1} x, \cos^{-1} x \ etc.\right)$ we take it as the first function and the remaining function is taken as the second function.

If there is no other function, then unity (1) is taken as the second function.

If in the integral both the functions are easily integrable, then the first function is chosen in such a way that the derivative of the function is a simple function and the function thus obtained under the integral sign is easily integrable than the original function.

2. We can also choose the first function as the function which comes first in the word ***ILATE,*** where

I – stands for the inverse trigonometric function
L – Stands for the logarithmic functions
A – Stands for the algebraic functions
T – Stands for the trigonometric functions
E – Stands for the exponential functions

1. $\int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + C$
2. $\int e^{kx} \left[kf(x) + f'(x)\right] dx = e^{kx} f(x) + C$; ***k*** is any constant
3. $\int xe^x \, dx = (x-1)e^x + C$
4. $\int \log x \, dx = x \log x - x + C$
5. $\int x \log x \, dx = \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C$

Integrals of inverse trigonometric functions:

1. $\int \text{Sin}^{-1} x\,dx = x\,\text{Sin}^{-1} x + \sqrt{1-x^2} + C$

2. $\int \text{Cos}^{-1} x\,dx = x\,\text{Cos}^{-1} x - \sqrt{1-x^2} + C$

3. $\int \text{Tan}^{-1} x\,dx = x\,\text{Tan}^{-1} x - \log\left(\sqrt{1+x^2}\right) + C$

4. $\int Cot^{-1}x\,dx = xCot^{-1}x + \log\left(\sqrt{1+x^2}\right) + C$

5. $\int \text{Sec}^{-1} x\,dx = x\,\text{Sec}^{-1} x - \cosh^{-1} x + C$; for $x \in (1, \infty)$ (Or)

$$\int \text{Sec}^{-1} x\,dx = x\,\text{Sec}^{-1} x - \log_e\left(x + \sqrt{x^2-1}\right) + C$$

$\int \text{Sec}^{-1} x\,dx = x\,\text{Sec}^{-1} x - \cosh^{-1}(-x) + C$; For $x \in (-\infty, -1)$

(Or) $\int \text{Sec}^{-1} x\,dx = x\,\text{Sec}^{-1} x - \log_e\left(-x + \sqrt{x^2-1}\right) + C$

6. $\int \text{Csc}^{-1} x\,dx = x\,\text{Csc}^{-1} x + \cosh^{-1} x + C$; for $x \in (1, \infty)$ (Or)

$$\int \text{Csc}^{-1} x\,dx = x\,\text{Csc}^{-1} x + \log_e\left(x + \sqrt{x^2-1}\right) + C$$

$\int \text{Csc}^{-1} x\,dx = x\,\text{Csc}^{-1} x + \cosh^{-1}(-x) + C$; For $x \in (-\infty, -1)$

(Or) $\int \text{Csc}^{-1} x\,dx = x\,\text{Csc}^{-1} x + \log_e\left(-x + \sqrt{x^2-1}\right) + C$

Integrals of inverse hyperbolic functions:

1. $\int \sinh^{-1} x\,dx = x\sinh^{-1} x - \sqrt{x^2+1} + C$

$$\int \sinh^{-1}\left(\frac{x}{k}\right)dx = x\sinh^{-1}\left(\frac{x}{k}\right) - \sqrt{x^2+k^2} + C$$

2. $\int \cosh^{-1} x\,dx = x\cosh^{-1} x - \sqrt{x^2-1} + C$

$$\int \cosh^{-1}\left(\frac{x}{k}\right)dx = x\cosh^{-1}\left(\frac{x}{k}\right) - \sqrt{x^2-k^2} + C$$

3. $\int \tanh^{-1} x\,dx = x\tanh^{-1} x + \frac{1}{2}\log_e\left(1-x^2\right) + C\ \left(for\,|x|<1\right)$

$$\int \tanh^{-1}\left(\frac{x}{k}\right)dx = x\tanh^{-1}\left(\frac{x}{k}\right) + \frac{k}{2}\log_e\left(k^2-x^2\right) + C\ \left(for\,|x|<k\right)$$

4. $\int \coth^{-1} x\,dx = x\coth^{-1} x + \frac{1}{2}\log_e\left(x^2-1\right) + C\ \ \left(for\ |x|>1\right)$

$$\int \coth^{-1}\left(\frac{x}{k}\right)dx = x\coth^{-1}\left(\frac{x}{k}\right) + \frac{k}{2}\log_e\left(x^2-k^2\right) + C\ \ \left(for\ |x|>k\right)$$

5. $\int \operatorname{sech}^{-1} x\,dx = x\operatorname{sech}^{-1} x - \tan^{-1}\left(\frac{x\sqrt{\frac{1-x}{1+x}}}{x-1}\right) + C;\ for\ x\in(0,1)$

$$\int \operatorname{sech}^{-1}\left(\frac{x}{k}\right)dx = x\operatorname{sech}^{-1}\left(\frac{x}{k}\right) - \tan^{-1}\left(\frac{x\sqrt{\frac{k-x}{k+x}}}{x-k}\right) + C;\ for\ x\in(0,k)$$

6. $\int \operatorname{csch}^{-1} x\,dx = x\operatorname{csch}^{-1} x + \log_e\left(x+\sqrt{x^2+1}\right) + C;\ for\ x\in(0,k)$

$$\int \operatorname{csch}^{-1}\left(\frac{x}{k}\right)dx = x\operatorname{csch}^{-1}\left(\frac{x}{k}\right) + k\log_e\left(\frac{x+\sqrt{x^2+k^2}}{k}\right) + C;\ for\ x\in(0,k)$$

Useful formulas:

1. $\int e^{ax}\sin(bx+c)\,dx=\frac{e^{ax}}{a^2+b^2}\Big(a\sin(bx+c)-b\cos(bx+c)\Big)+C$

2. $\int e^{ax}\sin(bx)\,dx=\frac{e^{ax}}{a^2+b^2}\Big(a\sin(bx)-b\cos(bx)\Big)+C$

3. $\int e^{x}\sin x\,dx=\frac{e^{x}}{2}(\sin x-\cos x)+C$

4. $\int e^{ax}\cos(bx+c)\,dx=\frac{e^{ax}}{a^2+b^2}\Big(a\cos(bx+c)+b\sin(bx+c)\Big)+C$

5. $\int e^{ax}\cos(bx)\,dx=\frac{e^{ax}}{a^2+b^2}\Big(a\cos(bx)+b\sin(bx)\Big)+C$

6. $\int e^{x}\cos x\,dx=\frac{e^{x}}{2}(\cos x+\sin x)+C$

7. $\int \sin^2 x\,dx=\frac{x}{2}-\frac{1}{4}\sin(2x)+C$

8. $\int \cos^2 x\,dx=\frac{x}{2}+\frac{1}{4}\sin(2x)+C$

9. $\int \tan^2 x\,dx=\tan x-x+C$

10. $\int \cot^2 x\,dx=-\cot x-x+C$