LECTURE - 12 - Optimization Techniques

- Gradient Descent
- Gauss Newton
- Levenberg Marquardt
- Dog - leg.

Algebraic distance $\|Ah\|$ - linear least squares

(x, x') - homogeneous coordinates

$(\tilde{x}, \tilde{x}')$ - physical coordinates correspondence

Hx : $f_1(h_{11}, h_{12} \ldots h_{33})$ $\quad f_1(h_{11} \ldots h_{33}) = \dfrac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$

$f_2(h_{11}, h_{12} \ldots h_{33})$

$$f_2(h_{11} \ldots h_{33}) = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Hx - $\begin{pmatrix} f_1(p) \\ f_2(p) \end{pmatrix}$ $\quad p \to$ vector of 9 unknowns $(h_{11}, h_{12} \ldots h_{33})$

$$\|\tilde{x}' - f(p)\|^2.$$

for single correspondence : $\|\tilde{x}_i' - f^i(p)\|^2$

$\tilde{x}_i'$: 2-vec; $f^i(p)$: 2vec, fn of unknowns

For N correspondence : $\sum_{i=1}^{n} \|\tilde{x}'^{,i} - f^i(p)\|^2$

2N vector : $(f_1^1\ f_2^1\ f_1^2\ f_2^2 \ldots f_1^N\ f_2^N)^T$

Overall geometric error for N correspondence : $\|\bar{x} - \bar{f}(p)\|^2$

2N vector $(x_1'\ y_1' \ldots x_N'\ y_N')$

cost fn : $C(\bar{p}) = \|\bar{x} - \bar{f}(p)\|^2$

$$p = \arg\min_{\bar{p}} \|\bar{x} - f(p)\|^2$$

non linear eqn in p.

Non-linear least squares

- GRADIENT DESCENT

$$p_{k+1} = p_k - \gamma_k \nabla C\big|_{p=p_k}$$

∇C - gradient of C

$f: \mathbb{R}^n \to \mathbb{R}$

$$D_x(f) = \left(\frac{\partial f}{\partial x_1} \ldots \frac{\partial f}{\partial x_n}\right)^T \text{ - row vec}$$

$$\nabla_x(f) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

$f: \mathbb{R}^n \to \mathbb{R}^m$

$$D_x(f) = \text{Jacobian} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \\ \frac{\partial f_2}{\partial x_1} \\ \end{bmatrix}$$

$f : \mathbb{R}^N \to \mathbb{R}^M$.

$$D_x(f) = \text{Jacobian} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Hessian : $D_x(\nabla_x f)$

iterate till $\|p_{k+1} - p_k\| < \delta$ or $\|C_{p_{k+1}} - C_{p_k}\| < \epsilon$

$$C(\vec{p}) = \| x - f(p) \|^2 = (x - f(p))^T (x - f(p))$$

$$= \epsilon^T(\vec{p})\, \epsilon(\vec{p})$$

Chain Rule

$$D(g(f(x))) = D(g(f(x))) \cdot D f(x)$$

Product rule

$$D(f^T(x)\, g(x)) = f^T(x)\, D_x(g(x)) + g^T(x)\, D_x(f(x))$$

$$D_p(\epsilon^T(p)\, \epsilon(p)) = 2 \epsilon^T(p)\, D_\epsilon(\epsilon(p))$$

$$=$$

$$\nabla_p(\epsilon^T(p)\, \epsilon(p)) = 2\, D_\epsilon^T(\epsilon(p))\, \epsilon(p)$$

$$= 2\, J_\epsilon^T(\underset{p}{\cancel{\epsilon(p)}})\, \epsilon(p)$$

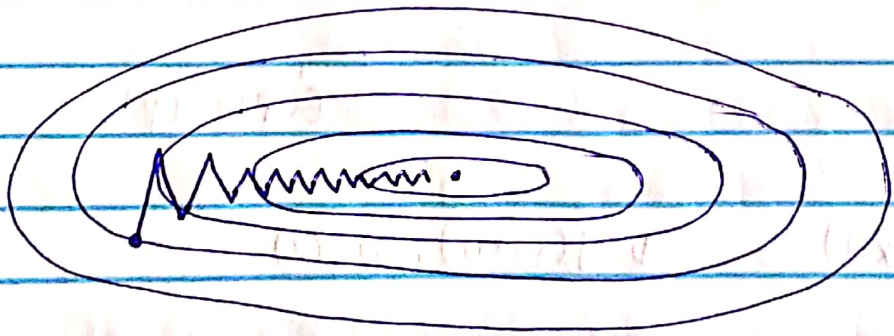
Jacobian

$\bar{E}(p) = \bar{X} - f(p)$ $\qquad J_E = -J_f(p)$

$J_E =$ $\qquad E = \begin{matrix} x_1^1 - f_1^1(p) \\ x_2^1 - f_2^1(p) \\ x_1^2 - f_1^2(p) \\ x_2^2 - f_2^2(p) \end{matrix}$

$J_E = \dfrac{\partial (x_1^1 - f_1^1(p))}{\partial p_1} \quad \ldots$

$\bar{P}_{k+1} = \bar{P}_k + 2\eta_k J_f(P_k) E(P_k)$



Gradient Descent

GAUSS - NEWTON METHOD

Assume at point p_k $\rightarrow$ next step will be to $p_k + \delta_p$

GN $\rightarrow$ at any position $\vec{p}$, find the best $\bar{\delta}_p$

Taylor Series : $f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots \frac{f^m(x_0)}{m!}(x - x_0)^m + \dots$

m^{th} derivative of $f(x)$ evaluated at x_0

~~$\bar{x}_{k+1} = f(p_k + \delta_p) \approx f(p_k) + J_f \bar{\delta}_{p_k}$~~

~~$J_f \rightarrow$ Jacobian of f wrt p.~~

~~$\bar{x} \approx f(p) + J_f \bar{\delta}_p \Rightarrow \bar{x} - f(p) = f(p) = J_f \delta_p$~~

~~$\bar{x} \approx f(p + \delta_p)$~~

assuming that current position $\bar{p}$ is already close to minimum, we can say that measurement vector $\bar{x}$ is close in value to $f(p)$, since p is supposed to minimize $\|x - f(p)\|^2$. In the same spirit, since the δ_p step is supposed to take us to an even better position vis-a-vis the minimum, ~~fp~~ $f(\bar{p} + \delta_p) =$ must be an even better approximation to $\bar{x}$

$$\bar{x} \approx f(p_k + \delta_p)$$

vector of knowns

$$f(p_k + \delta_p) \approx f(p_k) + J_f \delta_p$$ & taylor series

$J_f(p_k)$ partial differentiate f wrt $h_{11}, h_{12} \ldots h_{33}$

$$\bar{X} \approx f(p_k) + J_f \delta_p$$

$$\bar{x} - f(p_k) = J_f \delta_p$$

$$\delta_p : \epsilon(p_k) = J_f \delta_p$$

vector of KNOWN values.

solution to $Ax = b$ than minimises $\|Ax - b\|^2$ is $(A^T A)^{-1} A^T b$

$$\therefore \quad \delta_p = (J_f^T J_f)^{-1} J_f^T \epsilon(p_k)$$

* when $J_f^T J_f$ is diagonal, the GN & GD are identical with regard to the direction of step δ_p

The matrix is diagonal when cross derivatives $\frac{\partial f_i}{\partial p_j}$ $i=j$ are zero

LEVENBERG - MARQUARDT (LM)

→ combines best of GN and GD ; far from minima → GD
close to minima → GN

GN soln : $\delta_p = (J_f^T J_f)^{-1} J_f^T \epsilon(\bar{p})$

$(J_f^T J_f)\delta_p = J_F^T \epsilon(p)$

↓

modifying to incorporate both GD & GN.

$$(\underbrace{(J_F^T J_F)}_{n \times n} + \mu I)\delta_p = J_F^T \epsilon(p)$$

μ → damping coefficient; I → $n \times n$; δ_p → $n \times 1$; J_F^T → $n \times m$; $\epsilon(p)$ → $m \times 1$

n → no of unknow
m → no of rows

$f : \mathbb{R}^n \rightarrow \mathbb{R}^m$

μ large ⇒ GD $\mu = 0$ ⇒ GN

→ Starting with an initial guess p_0 → LM at each iteration k

↓

set value for damping coeff μ_k

↓

set δ_p to $(J_f^T J_F + \mu I)^{-1}$ $J_F^T \epsilon(p_k)$

↓

$p_{k+1} = p_k + \delta_p$

→ circular reasoning: best / μ_k depends on (p_k δ_p

best p_k

→ circular reasoning: best δ_p depends on μ_k

best μ_k depends on δ_p

→ when we are at p_k, → we already calculated μ_k

↓

use μ_k to compute

$$\delta_p = ((J_F^T J_F) + \mu I)^{-1} J_F^T \epsilon(p_k)$$

↓

$$p_{k+1} = p_k + \delta_p$$

↓

evaluate 'quality' of δ_p

↓

quality evaluated by sign of $C(p_k) - C(p_{k+1})$

if $C(p_k) - C(p_{k+1}) < 0$

⇒ quality must have passed over minima, so use higher μ_k

⇒ larger faith in GD.

Not pass → revert pt back to p_k i.e $p_{k+1} = p_k$

↓

set $\mu_{k+1} = 2\mu_k$

↓

recompute δ_p with $\mu_{k+1} = 2\mu_k$

More precisely

→ quality of computed δ_p → ratio of ACTUAL change in cost fn $C(p_k) - C(p_{k+1})$ to change in cost fn PREDICTED by a particular choice of μ_k

PREDICTED change is the change in cost fn assuming

$$X \approx f(p_k + \delta_p) \cong f(p_k) + J_F(p_k)\delta_p \quad \text{is accurate}$$

$$C(p) = \epsilon^T(\bar{p})\,\epsilon(p)$$

$$\therefore \quad C(p_k) - C(p_{k+1}) = \epsilon^T(p_k)\,\epsilon(p_k) - \epsilon^T(p_{k+1})\,\epsilon(p_{k+1})$$

actual change is the difference in cost fn based on the step. Predicted would be what it would have been if we actually went in the correct direction

$$X - f(p_k) = J_F(p_k)\,\delta_p$$

$$\cancel{\epsilon(p_k) = J_F(p_k)\,\delta_p}$$

$$\epsilon(p_{k+1}) = X - f(p_{k+1}) = X - f(p_k) - J_F(p_k)\,\delta_{p_k}$$

$$\epsilon(p_{k+1}) = \epsilon(p_k) + J_\epsilon(p_k)\,\delta_{p_k}$$

$$\epsilon(p_{k+1}) = \epsilon(p_k) - J_F \delta_{p_k}$$

$$c(p_k) - c(p_{k+1}) = -\left[\epsilon^T(p_{k+1})\,\epsilon(p_{k+1}) - \epsilon^T(p_k)\,\epsilon(p_k)\right]$$

$$= \cdot\left[\left(\epsilon(p_k) - J_F\delta_{p_k}\right)^T\left(\epsilon(p_k) - J_F\delta_{p_k}\right)\right) - \epsilon^T(p_k)\,\epsilon(p_k)\right]$$

$$= \cdot\left[\left(\epsilon^T(p_k) - \delta_{p_k}^T J_F^T\right)\left(\epsilon(p_k) - J_F\delta_{p_k}\right) - \epsilon^T(p_k)\,\epsilon(p_k)\right]$$

$$= \cdot\left[-\underbrace{\epsilon^T(p_k)\,J_F\delta_{p_k}}_{\text{scalar}} - \delta_{p_k}^T J_F^T \epsilon(p_k) + \delta_{p_k}^T J_F^T J_F \delta_{p_k}\right]$$

$$= 2\delta_{p_k}^T J_F^T \epsilon(p_k) - \delta_{p_k}^T J_F^T J_F \delta_{p_k}$$

Add & subtract $\delta_p^T \mu_k I \delta_p$

$$\therefore \epsilon(p_k) - \epsilon(p_{k+1}) = 2\delta_p^T J_F^T \epsilon(p_k) - \delta_p^T J_F^T J_F \delta_p + \delta_p^T \mu_k I \delta_p - \delta_p^T \mu_k I \delta_p$$

$$= 2\delta_p^T J_F^T \epsilon(p_k) - \delta_p^T \underbrace{\left(J_F^T J_F + \mu_k I\right)\delta_p}_{=\, J_F^T \epsilon(p_k)} + \delta_p^T \mu_k I \delta_p$$

$$= 2\delta_p^T J_F^T \epsilon(p_k) - \delta_p^T J_F^T \epsilon(p_k) + \delta_p^T \mu_k I \delta_p$$

$$= \delta_p^T J_F^T \epsilon(p_k) + \delta_p^T \mu_k I \delta_p$$

$$\rho_{k+1}^{LM} = \frac{C(p_k) - C(p_{k+1})}{\delta_p^T J_F^T E(p_k) + \delta_p^T \mu_k I \delta_p}$$

$\downarrow$

used in calculating value of damping coeff for next iteration

$$\mu_{k+1} = \mu_k \cdot \max\left\{\frac{1}{3},\ 1-(2\rho_{k+1}^{LM}-1)^3\right\}$$

→ $\rho_{k+1}^{LM} \leq 0 \Rightarrow (2\rho_{k+1}^{LM}-1)^3 < 0 \Rightarrow 2^{nd}$ term $\gg 1/3$

$\therefore\ \mu_{k+1} \gg 2\mu_k \Rightarrow$ strong steer in GD

→ $\rho_{k+1}^{LM} > 0 \Rightarrow \mu_{k+1} = \frac{\mu_k}{3} \Rightarrow$ steer towards GN

→ $0 \leq \rho_{k+1}^{LM} \leq 0.5 \Rightarrow 1-(2\rho_{k+1}^{LM}-1)^3 \geq 1 \Rightarrow \mu_{k+1} \geq \mu_k$

$\downarrow$

continue to bet on safety of GD

→ INITIALIZATION OF μ :

$$\mu_0 = \tau \max\{\text{diag}(J_F^T J_F)\} \qquad 0 \leq \tau \leq 1$$

largest value along diagonal is the direction along which cost fcn has steepest descent.

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LEVENBERG - MARQUARDT PSEUDO CODE

μ_k δ_p p_k p_{k+1}

$\tilde{X} = H_0 X$

Given H_0 and $(\tilde{X}, X)$

H_0 : initial estimate of H

X : set of observations - physical coordinates

$\tilde{X}$; set of corresponding pts - physical crds

$\tilde{X} = H_0 X$

$H_0 \to$ as col vector : p_0

$$f_1^i(p) = \frac{h_{11}x^i + h_{12}y^i + h_{13}}{h_{31}x^i + h_{32}y^i + h_{33}}$$

$$f_2^i(p) = \frac{h_{21}x^i + h_{22}y^i + h_{23}}{h_{31}x^i + h_{32}y^i + h_{33}}$$

Vector $\tilde{X}$: $[\tilde{x}_1 \ \tilde{y}_1 \ \tilde{x}_2 \ \tilde{y}_2 \ \ldots \ \tilde{x}_n \ \tilde{y}_n]^T$

vector $\bar{f}(p) = [f_1^1 \ f_2^1 \ f_1^2 \ f_2^2 \ \ldots \ f_1^n \ f_2^n]^T$

$\tilde{x}, x$ - known $\quad p = [h_{11} \ h_{12} \ h_{13} \ \ldots \ h_{33}]^T$ - unknown 'm'

- To be optimised ~~estimated~~ by LM

cost fcn $C(p) = \bar{\epsilon}^T(p)\, \bar{\epsilon}(p)$

$$\epsilon(p) = \tilde{X} - \bar{f}(p)$$

m - unknowns

n - observations

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J_F - Jacobian of $\bar{f}$ wrt p

$$J_F = \begin{matrix} \frac{\partial f_1^1}{\partial p_1} & \frac{\partial f_2^1}{\partial p_1} & \frac{\partial f_1^2}{\partial p_1} & \frac{\partial f_2^2}{\partial p_1} & \cdots & \frac{\partial f_1^n}{\partial p_1} & \frac{\partial f_2^n}{\partial p_1} \\ \frac{\partial f_1^1}{\partial p_2} & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & & & & & & \\ \frac{\partial f_1^1}{\partial p_m} & \frac{\partial f_2^1}{\partial p_m} & \frac{\partial f_1^2}{\partial p_m} & \frac{\partial f_2^2}{\partial p_m} & \cdots & \frac{\partial f_1^n}{\partial p_m} & \frac{\partial f_2^n}{\partial p_m} \end{matrix}$$

J_E - Jacobian of $E(\bar{p})$ wrt p. $J_E = -J_F$

~~write true~~

$$f = \begin{matrix} f_1^1 \\ f_2^1 \\ f_1^2 \\ f_2^2 \end{matrix} \qquad \begin{matrix} \frac{\partial f_1^1}{\partial p_1} & \frac{\partial f_1^1}{\partial p_2} & \cdots & \frac{\partial f_1^1}{\partial p_n} \\ \frac{\partial f_2^1}{\partial p_1} & \frac{\partial f_2^1}{\partial p_2} & \cdots & \frac{\partial f_2^1}{\partial p_n} \\ \frac{\partial f_1^2}{\partial p_1} & \frac{\partial f_1^2}{\partial p_2} & \cdots & \frac{\partial f_1^2}{\partial p_n} \\ \frac{\partial f_2^2}{\partial p_1} & \frac{\partial f_2^2}{\partial p_2} & \cdots & \frac{\partial f_2^2}{\partial p_n} \end{matrix}_{m \times n}$$

$f : \mathbb{R}$

while True

$$\mu_0 = \tau \max\{diag(J_F^T J_F)\} \quad 0 \le \tau \le 1$$

$$\delta_p = [(J_F^T J_F) + \mu I]^{-1} J_F$$

Set $\mu_0 = \tau \max\{diag(J_F^T J_F)\}$ $\quad 0 \le \tau \le 1$

p_0 = Initial estimate

while (True)

compute $\delta_p^k = [(J_F^T J_F) + \mu_k I]^{-1} J_F^T \epsilon(p_k)$

compute : $p_{k+1} = p_k + \delta_p^k$

If $c(p_{k+1}) < c(p_k)$:

then SET $p_{k+1} = p_k + \delta_p^k$

$$\rho_{LM} = \frac{c(p_k) - c(p_{k+1})}{\delta_p^{k^T} J_F^T \epsilon(p_k) + \delta_p^T \mu_k I \delta_p^k}$$

$$\mu_{k+1} = \mu_k \cdot \max\{\tfrac{1}{3}, 1-(2\rho^{LM}-1)^3\}$$

If $\delta_p^k <$ threshold

break

DOG-LEG

no hard & fast rules for initialization of r_0
set $r_0 = 1$

- combines GN & GD

→ Trust region of r_K around the point $\bar{p}_K$ - decides the step size

1. compute : $\delta_{p,GD} = \frac{\| J_F^T \epsilon(p_K) \|}{\| J_F J_F^T \epsilon(p_K) \|} J_F^T \epsilon(p_K)$

2. compute : $\delta_{p_{GN}} = \frac{1}{[J_F^T J_F + \mu_K I]} J_F^T \epsilon(p_K)$

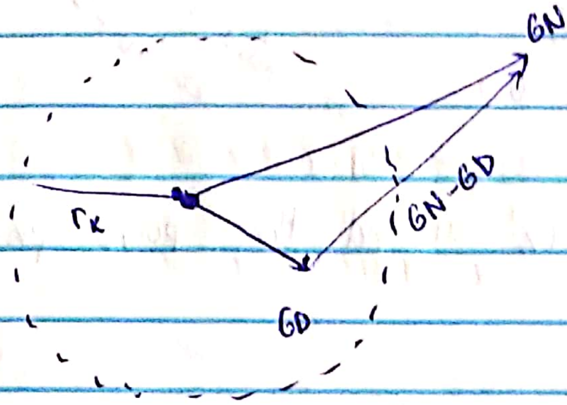
3. $$p_{K+1} = p_K + \begin{cases} \delta_{p,GN} & \text{if } \|\delta_{p,GN}\| < r_K \\ \delta_{p,GD} + \beta(\delta_{p,GN} - \delta_{p,GD}) & \text{if } \|\delta_{p,GD}\| < r_K < \|\delta_{p,GN}\| \\ \frac{r_K}{\|\delta_{p,GD}\|} \cdot \delta_{p,GD} & \text{otherwise} \end{cases}$$

4. β is obtained by solving $\| \delta_{p,GD} + \beta(\delta_{p,GN} - \delta_{p,GD}) \|^2 = r_K^2$

5. ~~(update r_K based on)~~ compute $\rho_{DL} = \frac{c(p_K) - c(p_{K+1})}{2\delta_p J_F^T \epsilon(p_K) - \delta_p^T J_F^T J_F \delta_p}$

6. $$r_{K+1} = \begin{cases} \frac{r_K}{4} & \text{if } \rho_{DL} < \frac{1}{4} \\ r_K & 1/4 < \rho_{DL} \leq 3/4 \\ 2r_K & \text{otherwise} \end{cases}$$

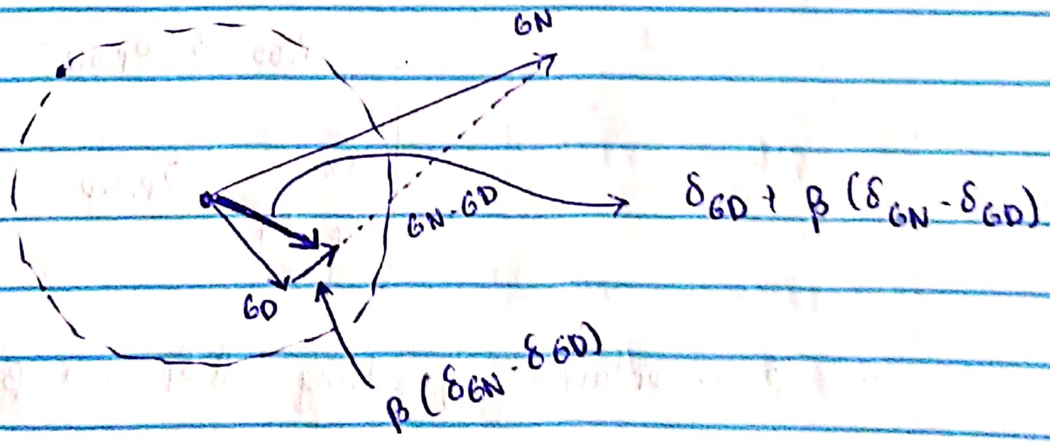
repeat until $\rho_{DL} \leq 0$



Choose β such that ~~δ_{GD}~~

$$\|\delta_{GD} + \beta(\delta_{GN} - \delta_{GD})\|^2 < r_k^2$$

falls within circle of radius r_k



Not quite in the direction of GD but making most of GD & GN directions

x_1 y_1

x_2 y_2

x_1^i y_1^i

x_2^i y_2^2

x_1

y_1

x_2

y_2

$\vdots$

$\tilde{x}_1$ $\tilde{y}_1$ $\tilde{z}$

$$f_1 = \frac{h_{11} x^i + h_{12} y^i + h_{13}}{h_{31} x^i + h_{32} y^i + h_{33}}$$

$$\frac{\partial f_1}{\partial h_{11}} = \frac{x^i}{(den)} \qquad \frac{\partial f_1}{\partial h_{21}} = 0$$

$$\frac{\partial f_1}{\partial h_{12}} = \frac{y^i}{den} \qquad \frac{\partial f_1}{\partial h_{22}} = 0$$

$$\frac{\partial f_1}{\partial h_{13}} = \frac{1}{den} \qquad \frac{\partial f_1}{\partial h_{23}} = 0$$

$$\frac{\partial f_1}{\partial h_{31}} = (num) \frac{-1}{(den)^2} \times x_i = \frac{-num \times (x^i)}{(den)^2}$$

$$\frac{\partial f}{\partial h_{32}} = -\frac{(num) \times y_i}{(den)^2}$$

$$\frac{\partial f}{\partial h_{33}} = \frac{-num}{(den)^2}$$

$$f_1 = \frac{h_{11}x_1 + h_{12}y_1 + h_{13}}{h_{31}x_1 + h_{32}y_1 + h_{33}}$$

$$f_2 = \frac{h_{21}x_1 + h_{22}y_1 + h_{23}}{h_{31}x_1 + h_{32}y_1 + h_{33}}$$

$num = h_{21}x_1 + h_{22}y_1 + h_{23}$

$den = h_{31}x_1 + h_{32}y_1 + h_{33}$

$num = h_{11}x_1 + h_{12}y_1 + h_{13}$

$den = h_{31}x_1 + h_{32}y_1 + h_{33}$

$$\frac{\partial f_1}{\partial h_{11}} = \frac{x_1}{den} \qquad \frac{\partial f_2}{\partial h_{11}} = 0$$

$$\frac{\partial f_1}{\partial h_{12}} = \frac{y_1}{den} \qquad \frac{\partial f_2}{\partial h_{12}} = 0$$

$$\frac{\partial f_1}{\partial h_{13}} = \frac{1}{den} \qquad \frac{\partial f_2}{\partial h_{13}} = 0$$

$$\frac{\partial f_1}{\partial h_{21}} = 0 \qquad \frac{\partial f_2}{\partial h_{21}} = \frac{x_1}{den}$$

$$\frac{\partial f_1}{\partial h_{22}} = 0 \qquad \frac{\partial f_2}{\partial h_{22}} = \frac{y_1}{den}$$

$$\frac{\partial f_1}{\partial h_{23}} = 0 \qquad \frac{\partial f_2}{\partial h_{23}} = \frac{1}{den}$$

$$\frac{\partial f_1}{\partial h_{31}} = \frac{-num * x_1}{(den)^2} \qquad \frac{\partial f_2}{\partial h_{31}} = \frac{-num \times x_1}{(den)^2}$$

$$\frac{\partial f_1}{\partial h_{32}} = \frac{-num * y_1}{(den)^2} \qquad \frac{\partial f_2}{\partial h_{32}} = \frac{-num \times y_1}{(den)^2}$$

$$\frac{\partial f_1}{\partial h_{33}} = \frac{-num}{(den)^2} \qquad \frac{\partial f_2}{\partial h_{33}} = \frac{-num}{(den)^2}$$

Jac : f_1^1

f_2^1

f_1^2

f_2^2

$$J_F = \begin{matrix} \frac{\partial f_1^1}{\partial p_1} & \frac{\partial f_1^1}{\partial p_2} & \cdots\cdots & \frac{\partial f_1^1}{\partial p_n} \\ \frac{\partial f_2^1}{\partial p_1} & \frac{\partial f_2^1}{\partial p_2} & \cdots\cdots & \frac{\partial f_2^1}{\partial p_n} \\ \frac{\partial f_1^2}{\partial p_1} & \frac{\partial f_1^2}{\partial p_2} & \cdots\cdots & \frac{\partial f_1^2}{\partial p_n} \\ \frac{\partial f_2^2}{\partial p_1} & \frac{\partial f_2^2}{\partial p_2} & \cdots\cdots & \frac{\partial f_2^2}{\partial p_n} \end{matrix}$$

556f $\frac{9}{2}$

293
2
586

Jacobian : $f : \mathbb{R}^n \to \mathbb{R}^m$

$f_1(x_1 \ldots x_n)$

J_F = $m \times n$ size

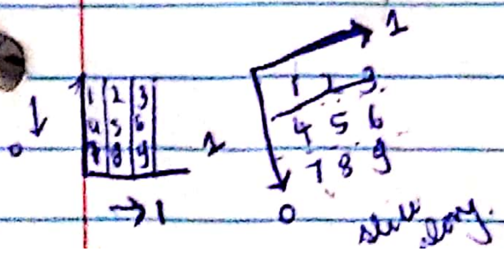
$f_2(x_1 \ldots x_n)$

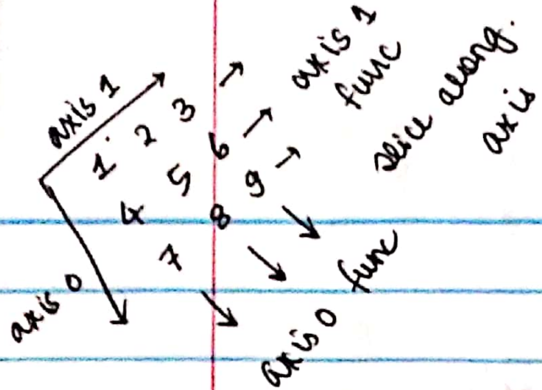
$\vdots$

n - cols : one for each variable.

$f_m(x_1 \ldots x_n)$

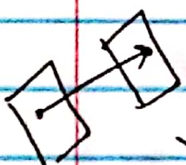
$n \times m$ $m \times n$





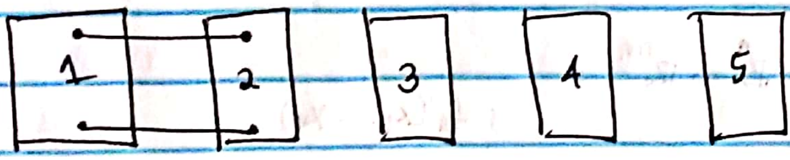
img1 img2

$x_1\ y_1$ $x_1'\ y_1'$



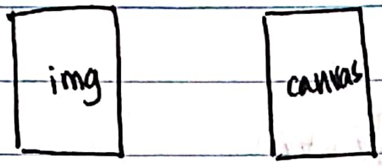
pts in 2 = H x pts in 1

HOMEWORK-5 - Image Mosaicing.



pts_in_2 = H * pts_in_1

blank canvas.



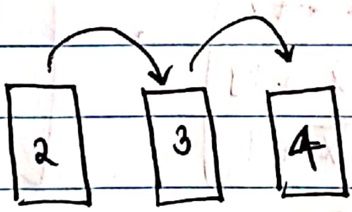
~~pts_in_canvas = H * pts_in_img.~~

pts_in_img = H * pts_canvas

bilinear txn

Project to central image. 3.

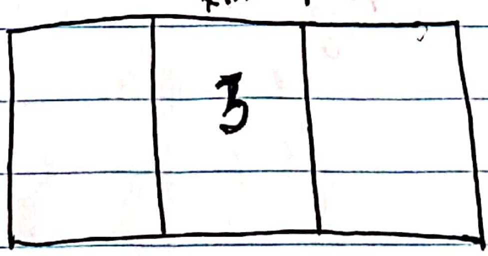
pts in 3 ref = H * pts in 1



H_{23} H_{34}

pts3 = H_{23} pts2   pts4 = H_{34} pts3

xmin ymin



H_{23}

$$\begin{pmatrix} A & t \\ v & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} A & \bar{t} \\ v & 1 \end{pmatrix} \begin{pmatrix} I & t \\ 0 & 1 \end{pmatrix}$$

$A \quad At + \bar{t}$

v

$$\begin{pmatrix} I & \bar{t} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & t \\ v & 1 \end{pmatrix} \quad I \quad t$$

$A \quad t + \bar{t}$

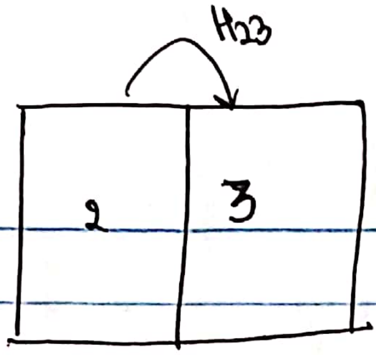
$v \quad 1$

$$\begin{bmatrix} I & \bar{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & t \\ v & 1 \end{bmatrix}$$

$$\begin{bmatrix} A + \bar{t}v & t + \bar{t} \\ v & 1 \end{bmatrix}$$

Scanned by CamScanner

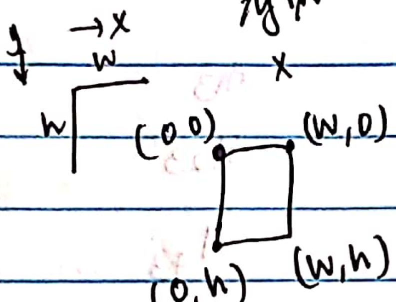
offset



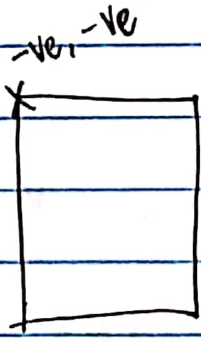
Offset

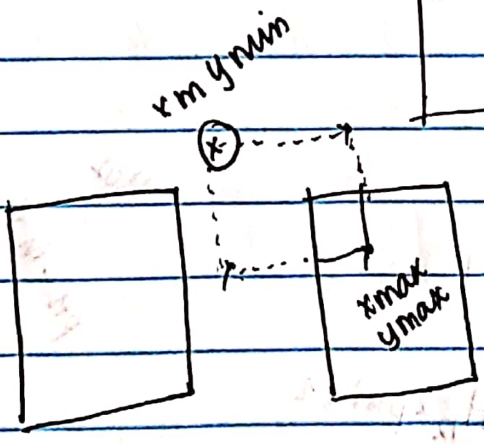
pts_in_3 = H_{23} x pts_2

~~x ymin~~ (0 0,) (0,w) (
~~y min~~



xmin, ymin, xmax, ymax





pts src = H x canvas

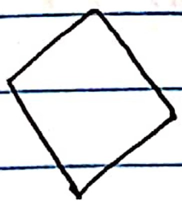
pts src (corners)

0 + xmin ymin

(0,0)

plot @ (0,0)

canvas, pts.



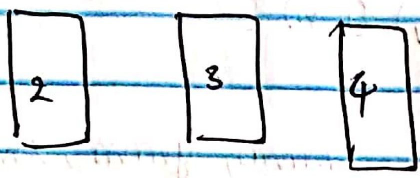
xmin ymin → (0,0)

(0 0) →

H23

pts in 3 = H23 x pts_in_3

H34 :



H_{13}

H_{23}

H_{43}

H_{53}

H_{33}

pts 3 = H23 x pts 2

pts ~~4~~ = H_{34} x pts ~~4~~ 3

pts 3 = H_{34}^{-1} pts 4

H_{43} pts 4 2

H_{13} =

H_{12}

pts in 2 = H_{12} x pts 1

pts in 3 = $H_{23} * H_{12}$ x pts in 1

= H_{13} = $H_{23} \times H_{12}$

H02

pts_in2 = H02 * ptsin0

~~H53~~ H_{45} =

pts in 5 = H_{45} pts in 4

pts in 4 = H_{34} x pts in 3

pts in 5 = $H_{45} \times H_{34}$ pts in 3

pts in 3 = $(H_{45} * H_{34})^{-1}$ pts in 5

Scanned by CamScanner

$$\text{pts in 3} = (H_{34})^{-1} \times (H_{45})^{-1} \text{ pts in 5}$$
23 94

$$= (H_{43} * H_{54})$$

$\underline{\underline{H_{53}}}$

$x_1 \quad y_1 \quad z_1$
$x_2 \quad y_2 \quad z_2$
$x_3 \quad y_3 \quad z_3$

pts in 3 = $H_{23} \times$ pts_in2

canvas : $H_{23} \times$ pts in 2
pts in 2 = $H_{23}^{-1} \times$ pts in canvas.

pts in canvas :

(0 0) (0 h) (w, 0) (0 h)
2

$\underline{\underline{\text{imgcrd}}}$ = H x word_crd
blank

pts in 4 = $H_{34} \times$ pts in 3.

pts in 3
canv = $(H_{34})^{-1}$

canvas = $H_{23} \times$ pts in 2

(0 0) (w h) (0 h) (w, 0)

img projecting to blank canvar.

$x_1 \quad x_2$
$y_1 \quad y_2$
$z_1 \quad z_2$

H34
pts in 4 = (H34) pts in3
pts in 3 = $(H_{34})^{-1}$ pts in 4
canvas =

offset

0 1 2 3 4 5 6 $\frac{5}{2}$
0 1 2 3 4 5 6
0 1 2 3
3, 2

0 1 2 3 4

H_{01} H_{12} H_{23} H_{34}

pts 1 = H_{01} * pts 0

pts 2 = H_{12} x pts 1

H_{*2}

pts 3 = H_{23} x pts 2 ⟹ pts 2 = $(H_{23})^{-1}$ pts 3

pts 4 = H_{34} x pts 3

pts 2 = $(H_{23})^{-1}$ pts 3

~~pts 2~~ =

pts in 2 = H_{12} * H_{01} x pts 0

pts 4 = H_{34} * H_{23} * pts 2

~~pts 0 = $(H_{12} * H_{01})^{-1}$ * pts 2~~

pts 2 = $(H_{23})^{-1}$ * $(H_{34})^{-1}$ pts 2

~~= $(H_{01})^{-1}$ * $(H_{12})^{-1}$ * pts 2~~

~~pt~~ H

pts in 2 = H_{12} * H_{01} * pts 0

= H_{02} = H.

$H_{32} = (H_{23})^{-1}$

$H_{42} = (H_{23})^{-1} (H_{34})^{-1}$

i = 4 j = 3

H_{42}

* $(H_{34})^{-1}$ * I ~~Hex~~

$H_{12} * H_{01}$

4

j = i-1 j = i-1

j = 2 $(2\ 3)^{-1}$ H_{34}

5

* $(34)^{-1}$ j = 3 j = 2

2 i = 1 3

3 4 32 23

1

i = 3 j = 2

$(H_{23})^{-1}$ H_2 * $(H_{j\ i-1,1})$

j = 2 j = 2

H_{01} H_{12} H_{23} H_{34} H_{45} H_{56}

0 1 2 3 4 5 6

$7/2 = 3$.

$H_{03} = H_{23} * H_{12} * H_{01}$

$H_{13} = H_{23} * H_{12}$

$H_{23} = H_{23}$

$H_{43} = (H_{34})^{-1}$

$H_{53} = (H_{34})^{-1} * (H_{45})^{-1}$

$H_{63} = (H_{34})^{-1} * (H_{45})^{-1} * (H_{56})^{-1}$

H02

pts_2 = H02 * pts_0

canvas = H02 * pts_0

0 1 2

$H_{01} * I$

3

j = 1

$H_{01} *$ temp

i = 4 5 6

j = 3

H02 :

j =

0 j = 0 2

1

43 34 $(34)^{-1} * I$

H_{01} * temp

H_{12}

53 i = 5 j = 4

j = 1

$(45)^{-1}$ I

1

j = 3

H_{12} * temp

H12

i = 1

$(H_{34})^{-1} * I$

j = 2

3

5

2

H13 =

j = 1

0 1 2

0 1 2 3 4

0 0 3

0

i = 6

$H_{23} * H_{12}$

4

j = 5

03 = 01

1

$(56)^{-1}$ I

H23 = 2

0 1 2 3

12

2

H23

0 1 2 3

23

j = 4

$(45)^{-1}$

02

12

22

32

42

j = 3

34^{-1}

2