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Difference Operator :-

For a given function $y(x)$ we define an operator Δ called the difference operator by $\Delta f(x) = f(x+h) - f(x)$

$$\Delta^2 f(x) = \Delta[\Delta f(x)]$$
$$= \Delta[f(x+h) - f(x)]$$
$$= \Delta f(x+h) - \Delta f(x)$$
$$= f(x+h+h) - f(x+h) - [f(x+h) - f(x)]$$
$$= f(x+2h) - 2f(x+h) + f(x)$$

$$\Delta^3 f(x) = \Delta^2[\Delta f(x)]$$
$$= \Delta^2[f(x+h) - f(x)]$$
$$= \Delta^2 f(x+h) - \Delta^2 f(x)$$
$$= f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x).$$

$\Delta^4 f(x)$
$$= f(x+4h) - 4f(x+3h) + 6f(x+2h) - 4f(x+h) + f(x)$$

(Translating) Translation Operator :- (shifting)

We define the translation (or) shifting Operator E by $Ef(x) = f(x+h)$

$$E^2 f(x) = E(E(f(x)))$$
$$= E[f(x+h)$$
$$= f(x+2h)$$
$$\vdots$$
$$E^n f(x) = f(x+nh)$$

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Relationship b/w E and Δ :-

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Delta f(x) = [E f(x)] - f(x)$$
$$\Delta f(x) = [E-1] f(x)$$
$$\therefore \boxed{\Delta = E-1}$$

(or)

$$\boxed{E = \Delta + 1}$$

1) $\Delta [f(x) + g(x)] = \Delta f(x) + \Delta g(x)$

Sol'n:- Suppose that

$$A(x) = f(x) + g(x)$$
$$\Delta [A(x)] = A(x+h) - A(x)$$
$$= f(x+h) + g(x+h) - [f(x) + g(x)]$$
$$= [f(x+h) - f(x)] + [g(x+h) - g(x)]$$
$$= \Delta f(x) + \Delta g(x).$$

2) $\Delta [\alpha f(x)] = \alpha \Delta f(x)$

Sol'n:- $\Delta [\alpha f(x)] = \alpha f(x+h) - \alpha f(x)$

$$= \alpha [f(x+h) - f(x)]$$
$$= \alpha \Delta f(x).$$

3) $\Delta [f(x) \cdot g(x)] = f(x) \cdot \Delta g(x) + g(x+h) \Delta f(x)$

$$\Delta [f(x) \cdot g(x)] = f(x+h) \cdot g(x+h) - f(x) \cdot g(x)$$
$$= f(x+h) g(x+h) - f(x) g(x+h) +$$
$$f(x) g(x+h) - f(x) g(x)$$

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$$= g(x+h)[f(x+h)-f(x)] + f(x)[g(x+h)-g(x)]$$

$$\Delta[f(x)\cdot g(x)] = g(x+h)\,\Delta f(x) + f(x)\,\Delta g(x).$$

4) $$\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,\Delta f(x) - f(x)\,\Delta g(x)}{g(x)\,g(x+h)}.$$

Q1) $\Delta[2x^2+3x]$

Here $f(x) = 2x^2+3x$

$$\Delta f(x) = f(x+h) - f(x)$$

$$= 2[x+h]^2 + 3[x+h] - [2x^2+3x]$$

$$= 2[x^2+h^2+2hx] + 3x+3h - 2x^2 - 3x$$

$$= \cancel{2x^2} + 2h^2 + 4hx + \cancel{3x} + 3h - \cancel{2x^2} - \cancel{3x}$$

$$= 2h^2 + 4hx + 3h$$

Q2) $E[4x-x^2]$

Sol:- $f(x) = 4x - x^2$

$$E[f(x)] = f(x+h)$$

$$= 4(x+h) - (x+h)^2$$

$$= 4x + 4h - x^2 - h^2 - 2xh$$

Q3) $\Delta^2[x^3-x^2]$

Soln:- Here $f(x) = x^3 - x^2$.

$$\Delta^2[f(x)] = f(x+2h) - 2f(x+h) + f(x).$$

$$= [x+2h]^3 - [x+2h]^2 - 2[(x+h)^3 - (x+h)^2]$$

$$+ [x^3 - x^2].$$

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$= x^3+8h^3+6x^2h+6h^2x - [x^2+4h^2+4xh) -$
$2[x^3+h^3+3x^2h+3xh^2-x^2-h^2-2xh] + x^3-x^2.$

$= \cancel{x^3}+8h^3+\cancel{6x^2h}+\cancel{6h^2}x-x^2-4h^2-4hx-2\cancel{x^3}-2h^3-$
$\cancel{6x^2h}-\cancel{6xh^2}+2\cancel{x^2}+2h^2+4\cancel{x}h+\cancel{x^3}-\cancel{x^2}$

$= 6h^3-2h^2.$

4) $\boxed{\begin{aligned}\Delta(b^x) &= b^{x+h}-b^x \\ &= b^x[b^h-1]\end{aligned}}$ → Formula.

Q) $\Delta[e^{rx}] = e^{r(x+h)}-e^{rx}$
$= e^{rx}[e^{rh}-1],$

Q) $\Delta[\sin rx]$
$= \sin r(x+h)-\sin rx$
$= \sin(rx+rh)-\sin rx$

$\because \boxed{\sin A-\sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}.$

$= 2\cos\left(\frac{rx+rh+rx}{2}\right]\sin\left[\frac{\cancel{rx}+rh-\cancel{rx}}{2}\right]$
$= 2\cos\left[\frac{2rx+rh}{2}\right]\sin\left[\frac{rh}{2}\right]$
$= 2\cos[rx+rh/2]\sin[rh/2].$

Q) $\Delta[xe^x]$

doing Here

$\Delta[f(x)\cdot g(x)] = g(x+h)\Delta f(x) + f(x)^{x+h}\cdot\Delta g(x)$
$= x\Delta e^x + e^{x+h}\Delta x$
$= x[e^{x+h}-e^x]+e^{x+h}[\cancel{x}+h-\cancel{x}]$
$= x[e^{x+h}-e^x]+he^{x+h}$
$= (x+h)e^{x+h}-xe^x.$

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Q) ΔC

$$\Delta f(x) = f(x+h) - f(x)$$
$$= C - C = 0.$$

* $\Delta(10!) = 0$

$\Delta(2005) = 0.$

Q) prove that $\Delta E = E\Delta$

Sol'n:- L.H.S

$$\Delta E\, f(x) = \Delta[f(x+h)]$$
$$= f(x+2h) - f(x+h) \rightarrow (1)$$

R.H.S

$$E\,\Delta[f(x)] = E\,[f(x+h) - f(x)]$$
$$= f(x+2h) - f(x+h) \rightarrow (2)$$

from (1) & (2), we get

$$\Delta E\,[f(x)] = E\Delta[f(x)]$$
$$\Delta E = E\Delta$$

* Factorial polynomial to General polynomial:-

$x^{(1)} = x$

$x^{(2)} = x(x-h)$

$x^{(2)} = x^2 - hx$

$x^{(3)} = x(x-h)(x-2h)$

$= (x^2 - hx)(x-2h)$

$x^{(3)} = x^3 - 2hx^2 - hx^2 + 2h^2x$

$= x^3 - 3hx^2 + 2h^2x.$

Eg:- $x^{(2)} + 3x^{(1)}$

$= x(x-h) + 3x$

$= x^2 - hx + 3x$

$= x^2 + (3-h)x$

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General polynomial to Factorial polynomial :-

$x = x^{(1)}$

$x(x-h) = x^{(2)}$

$x^2 = x^{(2)} - hx^{(1)}$

$x^3 = x^{(3)} + 3hx^{(2)} - 2h^2x^{(1)}$

eg :-

$x^2 + x$

$= x^{(2)} - hx^{(1)} + x^{(1)}$

$= x^{(2)} + (1-h)x^{(1)}$.

Q) Express $2x^2+3x+1$ in factorial polynomial.

Sol'n:- $2x^2+3x+1 = ax^{(2)} + bx^{(1)} - c \rightarrow ①$

$2x^2+3x+1 = a\,x(x-1) + bx + c$

Comparing coefficients $x^2 \rightarrow 2 = a$

$x \rightarrow 3 = -a+b$

$3 = -2+b \Rightarrow \boxed{b=5}$

Constant term $\rightarrow c = 1$

$2x^2+3x+1 = 2x^{(2)} + 5x^{(1)} + 1$.

h=1

* $\Delta x = (x+1) - x$

$\Delta x = 1$.

* $\Delta x^{(2)} = \Delta x(x-1)$

$= \Delta[x^2 - x]$

$= (x+1)^2 - (x+1) - x^2 + x$.

$= \cancel{x^2} + \cancel{1} + 2x - \cancel{x} - \cancel{1} - \cancel{x^2} + \cancel{x}$

$= 2x^{(1)}$

* $\Delta x^2 = \Delta(\Delta x)$

$= \Delta(1)$

$= 0$.

* $\Delta x^{(1)} = \Delta x = 1$

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* $\Delta^5 [x^4 + x) = 0.$
* $\Delta^5 (x^{(3)} + x^{(2)}) = 0.$
* $\Delta^2 (x^{(1)}) = 0.$

Note :- IF $[x^{(1)}]$ power is less than $\Delta^{(2)}$ power. Then it is 0.

Difference Equation:-

A difference eq'n is defined as a relationship of the form.

$$F\left[x, y, \frac{\Delta y}{\Delta x}, \frac{\Delta^2 y}{\Delta x^2}, \ldots\ldots \frac{\Delta^n y}{\Delta x^n}\right] = 0 \rightarrow (1)$$

$\Delta x = h$

$\Delta x^P = (\Delta x)^P = h^P.$

Now eq(1) implies a relationship Connecting.

$x, f(x), f(x+h), \ldots\ldots f(x+nh)$

$G[x, f(x), f(x+h), \ldots f(x+nh)] = 0$ where $y = f(x).$

$$F\left(x, y, \frac{\Delta y}{\Delta x}, \frac{\Delta^2 y}{\Delta x^2}\right) = 0.$$

Q) $\dfrac{\Delta^2 y}{\Delta x^2} + \dfrac{\Delta y}{\Delta x} + y = x.$

$= \dfrac{1}{h^2} \Delta^2 [f(x)] + \dfrac{1}{h} \Delta f(x) + f(x) = x.$

$= \dfrac{f(x+2h) - 2f(x+h) + f(x)}{h^2} + \dfrac{f(x+h) - f(x)}{h} + f(x) = x$

$= f(x+2h) - 2f(x+h) + f(x) + hf(x+h) - hf(x) + h^2 f(x) = h^2 x$

$= f(x+2h) + (h-2) f(x+h) + [1-h+h^2] f(x) = h^2 x$

$\Rightarrow G[x, f(x), f(x+h), f(x+2h)] = 0.$

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Order of Difference Equation :-

Order is the difference b/w the largest and Smallest arguments for the function f divided by h.

$$\text{Order} = \frac{\overset{L\cdot A}{(x+2h)} - \overset{S\cdot A}{x}}{h} = 2$$

Q) $G[x, f(x+h), f(x+2h) \ldots\ldots f(x+nh)] = 0.$

$$\text{Order} = \frac{(x+nh)-(x+h)}{h}$$

$$= \frac{\cancel{x}+nh-\cancel{x}-h}{h} = \frac{h(n-1)}{h} = n-1.$$

Q) $\frac{\Delta^2 y}{\Delta x^2} - 3\frac{\Delta y}{\Delta x} + 2y = 4x^{(2)}.$

$$= \frac{1}{h^2}\Delta^2[f(x)] - \frac{3}{h}\Delta f(x) + 2f(x) = 4x^{(2)}$$

$$= \frac{f(x+2h)-2f(x+h)+f(x)}{h^2} - 3\frac{[f(x+h)-f(x)]}{h} + 2f(x) = 4x^{(2)}.$$

$$= f(x+2h)-2f(x+h)+f(x) - 3hf(x+h)+3hf(x) + 2h^2 f(x) = 4x^{(2)}h^2.$$

$$= G[x, f(x), f(x+h), f(x+2h)]$$

$$\text{Order} = \frac{\cancel{x}+2h-\cancel{x}}{h} = \frac{2\cancel{h}}{\cancel{h}}$$

$$= 2$$

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Subscript form of Difference Equation :-

$f(x+2h) + 3f(x+h) + f(x) = \sin x.$

put $x = a+kh$ put $a=0$ then $\boxed{x=kh}$.

$f(x) = f(a+kh) = y_k$

$f(x+h) = f(a+kh+k) = f[a+(k+1)h]$

$= y_{k+1}.$

$f(x+2h) = y_{k+2}$

$y_{k+2} + 3y_{k+1} + y_k = \sin kh.$

$f(x+2h) + 3f(x+h) + 2f(x) = x$

$y_{k+2} + 3y_{k+1} + 2y_k = hk.$

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sub form is $\boxed{y_{k+2} = y_{k+1} + y_k}$; $y_0 = 0,\ y_1 = 1$

$k=0 \quad y_2 = y_1 + y_0 = 1+0 = 1$

$k=1 \quad y_3 = y_2 + y_1 = 1+1 = 2.$

* $y_{n+k} + y_{n+(k-1)} + \dots + y_n = 0.$

$E^k y_n + E^{k-1} y_n + \dots + y_n = 0.$

$[E^k + E^{k-1} + \dots 1] y_n = 0. \rightarrow (1)$

$\phi(E) y_n = 0.$

Let $y_n = x^n.$

$x^{n+k} + x^{n+(k-1)} + \dots + x^n = 0$

$[x^k + x^{k-1} + \dots 1] x^n = 0$

$\phi[x] x^n = 0$

$\rightarrow (2)$

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* If α, β, γ are distinct roots, then
$$c_1(\alpha)^n + c_2(\beta)^n + c_3(\gamma)^n.$$

* If α, α, α are repeated roots then
sol'n is $[c_1 + c_2 n + c_3 n^2](\alpha)^n$

* If $\alpha, \alpha, \beta, \beta$ are roots of any eq'n then
$$[c_1 + c_2 n](\alpha)^n + [c_3 + c_4 n](\beta)^n.$$

* Demorvies theorem $\rightarrow (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$

* Solve the eq'n $y_{k+2} - 3y_{k+1} + 2y_k = 0.$

Sol^n:- Order $= \cancel{k} + 2 - \cancel{k} = 2.$

characteristic eq'n :- $\alpha^2 - 3\alpha + 2 = 0$
$$\alpha^2 - 2\alpha - \alpha + 2 = 0$$
$$\alpha(\alpha - 2) - 1(\alpha - 2) = 0$$
$$(\alpha - 1)(\alpha - 2) = 0$$

then roots are $\alpha = 1, 2.$
$$y_k = c_1(1)^k + c_2(2)^k.$$
$$y_{k+1} = c_1(1)^{k+1} + c_2(2)^{k+1}$$
$$y_{k+2} = c_1(1)^{k+2} + c_2(2)^{k+2}$$

then ① $\Rightarrow c_1 + c_2(2)^{k+2} - 3[c_1 + c_2(2)^{k+1}] + 2[c_1 + c_2(2)^k]$
$= \cancel{c_1} + 4c_2 2^k - 3\cancel{c_1} - 6c_2(2)^k + 2\cancel{c_1} + 2(2)^k c_2$
$= 0.$

* Solve $y_{k+2} - 7y_{k+1} + 10y_k = 0$

Order $= 2$

characteristic eq'n :- $\alpha^2 - 7\alpha + 10 = 0$
$$\alpha^2 - 5\alpha - 2\alpha + 10 = 0$$

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$\alpha(\alpha-5)-2(\alpha-5)=0$

$(\alpha-2)(\alpha-5)=0$

$\alpha=2,5.$

$y_k = c_1(2)^k + c_2(5)^k.$

Q) Solve $y_{k+3}-6y_{k+2}+11y_{k+1}-6y_k=0.$

order = 3

$C.E.T = \alpha^3-6\alpha^2+11\alpha-6=0.$

$\Rightarrow \alpha^2-5\alpha+6=0$

$\alpha^2-3\alpha-2\alpha+6=0$

$(\alpha-3)(\alpha-2)=0$

$\alpha=3,2$

1	1	-6	11	-6
	0	1	-5	6
	1	-5	6	0

$y_k=c_1(1)^k+c_2(2)^k+c_3(3)^k=0$

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Q) $8y_{k+3}-12y_{k+2}+6y_{k+1}-y_k=0.$

Order $= k+3-k = 3.$

characteristic Eq'n $= 8\alpha^3-12\alpha^2+6\alpha-1=0.$

$p \rightarrow$ divisors of 1.

$q \rightarrow$ division of 8

$\frac{\pm1}{\pm1}, \frac{\pm1}{\pm2}, \frac{\pm1}{\pm4}, \frac{\pm1}{\pm8}$

1/2	8	-12	6	-1
		4	-4	1
	8	-8	2	0

$(\alpha-1/2)[8\alpha^2-8\alpha+2]=0$

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$8\alpha^2 - 8\alpha + 2 = 0$

$4\alpha^2 - 4\alpha + 1 = 0$

$(2\alpha - 1)^2 = 0 \Rightarrow \alpha = 1/2, 1/2.$

$y_k = [c_1 + c_2 k + c_3 k^2] (1/2)^k.$

Q) $y_{k+2} - 4y_{k+1} + 4y_k = 0$; $y_0 = 1, y_1 = 3.$

Order = 2

$C.E = \alpha^2 - 4\alpha + 4 = 0$

$= \alpha^2 - 2\alpha - 2\alpha + 4 = 0$

$= \alpha(\alpha - 2) - 2(\alpha - 2) = 0$

$= (\alpha - 2)(\alpha - 2) = 0$

$\Rightarrow \alpha = 2, 2$

$y_k = (c_1 + c_2 k)(2)^k. \rightarrow (1)$

put $k = 0$ then

$y_0 = c_1$

$\therefore \boxed{c_1 = 1}$

put $k = 1$ then

$y_1 = (c_1 + c_2)(2)$

$3 = (1 + c_2)(2)$

$\frac{3}{2} = 1 + c_2 \Rightarrow c_2 = (1 - 3/2) \Rightarrow c_2 = 1/2.$

$\therefore \; y_k = [1 + \frac{1}{2} k](2^k).$

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* Q) $y_{k+2} - 6y_{k+1} + 9y_k = 0$; $y_0 = 0$; $y_1 = 4.$

$k + 2 - k = 2$

$C.E = \alpha^2 - 6\alpha + 9 = 0$

$\alpha = 3, 3$

$y_k = (c_1 + c_2 k)(3)^k \rightarrow (1)$

put $k=0$

$y_0 = c_1$

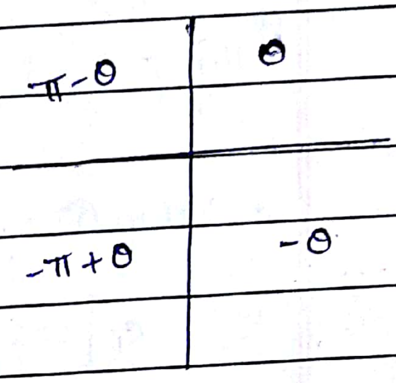
$\boxed{c_1 = 0}$

put $k=1$ then

$y_1 = (c_1+c_2)(3)$

$4 = 3c_2$

$\boxed{c_2 = 4/3}$



$\therefore \; y_k = \left[0 + \frac{4}{3}k\right](3)^k$ //

Q) $y_{k+2} + y_{k+1} + y_k = 0.$

$\alpha^2 + \alpha + 1 = 0$

$$\alpha = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2-4}}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2}$$

$$y_k = c_1\left[\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right]^k + c_2\left[\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right]^k \rightarrow (1)$$

$$\frac{-1}{2} + i\frac{\sqrt{3}}{2} = r[\cos\theta + i\sin\theta]$$

comparing real & imaginary roots.

$r\cos\theta = -1/2 \rightarrow (2)$

$r\sin\theta = \sqrt{3}/2 \rightarrow (3)$

Squaring & adding (2) & (3) we get

$r^2 = 1/4 + \frac{3}{4} \Rightarrow r^2 = 1 \Rightarrow r = 1$ (-1 is not ~~taken bcz~~ r is a distance)

$\cos\theta = -1/2$; $\sin\theta = \sqrt{3}/2$

$\theta = \pi - \pi/3$

$\boxed{\theta = \frac{2\pi}{3}}$

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$$-\frac{1}{2}+\frac{i\sqrt{3}}{2}=\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right]$$

$$\Rightarrow\left[\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right]^k=\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right]^k$$

$$=\cos\frac{2\pi k}{3}+i\sin\frac{2\pi}{3}k.$$

$\Rightarrow$

$$\text{lly}\left[\frac{-1}{2}-\frac{i\sqrt{3}}{2}\right]^k=\cos\frac{2\pi}{3}k-i\sin\frac{2\pi}{3}k.$$

$\therefore$ From (1) we get

$$y_k=c_1\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}k\right]+c_2\left[\cos\frac{2\pi}{3}k-i\sin\frac{2\pi}{3}k\right]$$

$$y_k=(c_1+c_2)\cos\frac{2\pi}{3}k+[ic_1-ic_2]\sin\frac{2\pi}{3}k.$$

$$=A\cos\frac{2\pi}{3}k+B\sin\frac{2\pi}{3}k.$$

V.IMP

2) $y_{k+2}-2y_{k+1}+5y_k=0$

C.E $\Rightarrow$ $\alpha^2-2\alpha+5=0$

$$\alpha=\frac{2\pm\sqrt{(-2)^2-4(1)(5)}}{2}$$

$$=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm4i}{2}=1\pm2i.$$

$$y_k=c_1(1+2i)^k+c_2(1-2i)^k.\quad -(1)$$

$$1+2i=r[\cos\theta+i\sin\theta]$$

Comparing:

$$r\cos\theta=1\rightarrow(2)$$

$$r\sin\theta=2\rightarrow(3)$$

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Squaring & adding ② & ③

$$r^2 = (1)^2 + (2)^2$$

$$= 5$$

$$r = \sqrt{5}$$

③ ÷ ②

$$\frac{\cancel{r} \sin\theta}{\cancel{r} \cos\theta} = \frac{2}{1} \Rightarrow \tan\theta = 2 \Rightarrow \theta = \tan^{-1}(2).$$

$$(1+2i) = \sqrt{5}\,(\cos\theta + i\sin\theta)$$

$$(1+2i)^k = (\sqrt{5})^k [\cos\theta + i\sin\theta]^k$$

$$= (\sqrt{5})^k [\cos k\theta + i\sin k\theta].$$

Ily

$$(1-2i) = (\sqrt{5})^k [\cos k\theta - i\sin k\theta].$$

∴ From ①, we get.

$$y_k = c_1(\sqrt{5})^k [\cos k\theta + i\sin k\theta] + c_2(\sqrt{5})^k [\cos k\theta - i\sin k\theta]$$

$$y_k = (\sqrt{5})^k [(c_1+c_2)\cos k\theta + (ic_1 - ic_2)\sin k\theta]$$

$$= (\sqrt{5})^k [A\cos k\theta + B\sin k\theta)$$

$$\theta = \tan^{-1}(2)$$

Q) egn $c_1(\frac{1}{2} + i\frac{\sqrt{3}}{2})^k + c_2(\frac{1}{2} - i\frac{\sqrt{3}}{2}k).$

$$y_k \Rightarrow r = \frac{1}{2} + \frac{3}{2} = 1$$

$$\text{and } \theta = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan\frac{\pi}{3}$$

$$\boxed{\theta = \frac{\pi}{3}}$$

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Q) Write the Sol'n of difference equation if roots of its characteristic eq'n is as follows:
$1, 2, 1, 1, 2, 3, 3, 1 \pm i\sqrt{3}$

Sol'n ① Corresponding to the roots 1, 1, 1
$(c_1 + c_2 k + c_3 k^2)(1)^k$

Corresponding to the roots 2, 2.
$(c_4 + c_5 k)(2)^k$

Corresponding to the roots 3, 3.
$(c_6 + c_7 k)(3)^k$

$r\cos\theta = \sqrt{3}$
$r\sin\theta = 1$
$\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$.

Corresponding to the roots $1 \pm i\sqrt{3}$
$(2)^k \left[c_8 \cos\frac{k\pi}{3} + c_9 \sin\frac{k\pi}{3}\right]$

General Sol'n is

$(c_1 + c_2 k + c_3 k^2)(1)^k + (c_4 + c_5 k)(2)^k + (c_6 + c_7 k)(3)^k +$
$(2)^k \left[c_8 \cos\frac{k\pi}{3} + c_9 \sin\frac{k\pi}{3}\right]$.

order of this eq'n is 9. (depending on no. of roots).

* Casorati Wronskian

$f_1(k), f_2(k), f_3(k)$

$$w = \begin{vmatrix} f_1(0) & f_2(0) & f_3(0) \\ f_1'(0) & f_2'(0) & f_3'(0) \\ f_1''(0) & f_2''(0) & f_3''(0) \end{vmatrix} \neq 0$$

Two f'ns are called L.I and L.D otherwise.

Q) Check whether the following functions are L.I (or) L.D

① k^2+k+1, $2k+1$, k^3.

sub $k=0$

$$\begin{vmatrix} k^2+k+1 & 2k+1 & k^3 \\ 2k+1 & 2 & 3k^2 \\ 2 & 0 & 6k \end{vmatrix} \neq 0 \qquad \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 0 & 0 \end{vmatrix}$$

$(k^2+k+1)(12k) - (2k+1)$

$= 1(0) - 1(0) + 0$

$= 0$

$\therefore$ These are linearly dependent.

② k^2+k+1, $3k^2+4k+5$, 6^k ; put $k=0$.

$$\begin{vmatrix} 1 & 5 & 1 \\ 1 & 4 & \log 6 \\ 2 & 6 & (\log 6)^2 \end{vmatrix}$$

$= 1(4(\log 6)^2 - 6\log 6) - 5((\log 6)^2 - 2\log 6) + [6-8]$

$= 4(\log 6)^2 - 6\log 6 - 5(\log 6)^2 + 10\log 6 - 2$

$= -(\log 6)^2 + 4\log 6 - 2 \neq 0$

$\therefore$ These are linearly independent.

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NON-HOMOGENUS D.E.

* Solution of non-homogenous difference Eq'n.

Q) Solve D.E $y_{k+2} - 7y_{k+1} + 10y_k = 1$.

$$y_{k+2} - 7y_{k+1} + 10y_k = 1$$

Sol'n:- It's characteristic eq'n is $\alpha^2 - 7\alpha + 10 = 0$.

$\alpha = 2, 5$

homogenous $\leftarrow$ $y_k^{(h)} = c_1(2)^k + c_2(5)^k$.
$\rightarrow$ ①

[* particular Sol'n:- A Sol'n which does not contain any constants in the solution is called particular Sol'n].

$y_k^{(p)} = d$, $^{||}y$. $y_{k+1}^{(p)} = d$; $y_{k+2}^{(p)} = d$.

$d - 7d + 10d = 1$

$4d = 1 \Rightarrow \boxed{d = 1/4}$

$\therefore y_k = y_k^{(h)} + y_k^{(p)}$

$= c_1(2)^k + c_2(5)^k + 1/4$

Q) $y_{k+2} - 5y_{k+1} + 6y_k = 2$

C.E $= \alpha^2 - 5\alpha + 6 = 0$

$\alpha = 3, 2$.

$y_k^{(h)} = c_1(2)^k + c_2(3)^k \rightarrow$ ①

$y_k^{(p)} = d$; $y_{k+1}^{(p)} = d$, $y_{k+2}^{(p)} = d$

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$d - 5d + 6d = 2$

$2d = 2$

$d = 1$

$y_k = y_k^{(h)} + y_k^{(p)}$

$= c_1 (2)^k + c_2 (3)^k + 1$

Q) $y_{k+2} - 3y_{k+1} + 2y_k = 5.$

$C \cdot E = \alpha^2 - 3\alpha + 2$

$\Rightarrow \alpha = 1, 2.$

$y_k^{(h)} = c_1 (1)^k + c_2 (2)^k \rightarrow ①$

$y_k^{(p)} = d, \quad y_{k+1}^{(p)} = d, \quad y_{k+2}^{(p)} = d.$

$d - 3d + \cancel{2d} = 5$

$\cancel{-2d = 3}$ $\cancel{d}$ ①

$d = -3/2$

$y_k^{(p)} = d, \quad y_{k+1}^{(p)} = d, \quad y_{k+2}^{(p)} = d$

$d - 3d + 2d = 5 \Rightarrow 0 = 5$

$\therefore$ which is not possible.

$y_k^{(p)} = dk, \quad y_{k+1}^{(p)} = d(k+1), \quad y_{k+2} = d(k+2)$

$d(k+2) - 3d(k+1) + 2dk = 5$

$\cancel{dk} + 2d - 3\cancel{d}k - 3d + 2\cancel{d}k = 5$

$-d = 5$ (or) $\boxed{d = -5}$

$y_k = y_k^{(p)} + y_k^{(h)}$

$= c_1 (1)^k + c_2 (2)^k - 5k.$

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~~[Homogenous Sol'n of Eq'n :-]~~ x

Q) $y_{k+2} + 3y_{k+1} + 2y_k = 7k+5$

$\alpha = -1, -2$

$y_k^{(h)} = c_1(-1)^k + c_2(-2)^k \rightarrow (1)$

$y_k^{(p)} = P_0 k + P_1 \; ; \; y_{k+1}^{(p)} = P_0(k+1) + P_1 \; , \; y_{k+2}^{(p)} = P_0(k+2) + P_1$

$P_1 + P_0(k+2) + 3\{P_0(k+1) + P_1\} + 2(P_0 k + P_1) = 7k+5$

$P_0 + 3P_0 + 2P_0 = 7$ (Comparing k coefficients).

$6P_0 = 7 \Rightarrow \boxed{P_0 = 7/6}$

Comparing Constants:

$2P_0 + P_1 + 3P_0 + 3P_1 + 2P_1 = 5$

$5P_0 + 6P_1 = 5$

$5(7/6) + 6P_1 = 5 \Rightarrow 6P_1 = 5 - \frac{35}{6}$

$\boxed{P_1 = \frac{-5}{36}}$

$\therefore y_k^{(p)} = \frac{7}{6}k - \frac{5}{36}$

$y_k = y_k^{(h)} + y_k^{(p)}$

$= c_1(-1)^k + c_2(-2)^k + \frac{7}{6}k - \frac{5}{36}$ //

Q) $y_{k+2} + 5y_{k+1} + 6y_k = k+1$

$\alpha^2 + 5\alpha + 6 = 0$

$\alpha^2 + 3\alpha + 2\alpha + 6 = 0$

$\alpha(\alpha+3) + 2(\alpha+3) = 0$

$(\alpha+3)(\alpha+2) \Rightarrow \alpha = -3, -2$

$y_k = c_1(-2)^k + c_2(-3)^k$.

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$y_k^{(P)} = P_0 k + P_1$

$y_{k+1}^{(P)} = P_0(k+1) + P_1$

$y_{k+2}^{(P)} = P_0(k+2) + P_1$

$P_0(k+2) + P_1 + 5\{P_0(k+1) + P_1\} + 6\{P_0 k + P_1\} = k+1$

$P_0 k + 2P_0 + P_1 + 5P_0 k + 5P_0 + 5P_1 + 6P_0 k + 6P_1 = k+1$

k coeff

$P_0 + 5P_0 + 6P_0 = 1$

$12P_0 = 1$

$P_0 = 1/12.$

$\cancel{y_k = \frac{1}{12}.1}$

Constansts

$2P_0 + P_1 + 5P_0 + 5P_1 + 6P_1 = 1$

$7P_0 + 12P_1 = 1$

$7\left(\frac{1}{12}\right) + 12P_1 = 1$

$12P_1 = 1 - \frac{7}{12}$

$P_1 = \frac{5}{12 \times 12}$

$P_1 = \frac{5}{144}$

$y_k^{(P)} = \frac{1}{12}k + \frac{5}{144}$

$y_k = y_k^{(h)} + y_k^{(P)}$

$= c_1(-2)^k + c_2(-3)^k + \frac{1}{12}k + \frac{5}{144}.$

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Q) $y_{k+2} + 3y_{k+1} + 2y_k = 1\cdot 2^k$

$y_k^{(h)} = c_1(-1)^k + c_2(-2)^k$

$y_k^{(p)} = d\cdot 2^k, \; y_{k+1}^{(p)} = d\cdot 2^{k+1}, \; y_{k+2}^{(p)} = d\cdot 2^{k+2}.$

$\Rightarrow d\cdot 2^{k+2} + 3[d\, 2^{k+1}] + 2[d\cdot 2^k] = 2^k.$

$2^k[4d + 6d + 2d] = 2^k$

$12d = 1 \Rightarrow \boxed{d = \frac{1}{12}}.$

$y_k^{(p)} = \frac{1}{12}\cdot 2^k$

$y_k = y_k^{(h)} + y_k^{(p)}$

$= c_1(-1)^k + c_2(-2)^k + \frac{1}{12}\cdot 2^k.$ //

Q) $y_{k+2} - 7y_{k+1} + 12y_k = 4^k$

$y_k^{(h)} = c_1(4)^k + c_2(3)^k$

$y_k^{(p)} = d\, 4^k, \; y_{k+1}^{p} = d\cdot 4^{k+1}, \; y_{k+2} = d\cdot 4^{k+2}.$

$\Rightarrow d\cdot 4^{k+2} - 7\{d\cdot 4^{k+1}\} + 12\{d\cdot 4^k\} = 4^k$

$4^k\{16d - 28d + 12d\} = 4^k$

$\cancel{4^k}\{28d - 28d\} = \cancel{4^k}$

$\boxed{d = 0}$

$\therefore$ This is not possible

$y_k^{(p)} = dk\, 4^k$

$y_{k+1}^{(p)} = d(k+1)\cdot 4^{k+1}$

$y_{k+2}^{(p)} = d(k+2)\cdot 4^{k+2}$

$d(k+2)\cdot 4^{k+2} - 7\{d(k+1)\cdot 4^{k+1}\} + 12\{dk\cdot 4^k\} = 4^k$

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$\Rightarrow d(k+2)\cdot 4^{k+2} - 7d(k+1)\cdot 4^{k+1} + 12dk4^k = \cancel{2}4^k$

$\Rightarrow \cancel{4^k}[16d(k+2) - 7d(k+1)\cdot 4 + 12dk] = \cancel{4^k}$

$\Rightarrow 16\cancel{dk} + 32d - \cancel{28dk} - 28d + 12\cancel{dk} = 1$

$\Rightarrow \quad 4d = 1 \quad \Rightarrow \boxed{d = \frac{1}{4}}$

$y_k^{(p)} = \frac{1}{4}k\cdot 4^k \Rightarrow 4^{k-1}$

$y_k = y_k^{(h)} + y_k^{(p)}$

$= c_1(4)^k + c_2(3)^k + 4^{k-1}$ //

Q) $y_{k+2} - 3y_{k+1} + 2y_k = 3k+2.$

$\alpha = 1, 2$

$y_{(k)}^{(h)} = c_1(1)^k + c_2(2)^k \rightarrow (1)$

$y_k^{(p)} = P_0 k + P_1$

$y_{k+1}^{(p)} = P_0(k+1) + P_1$

$y_{k+2}^{(p)} = P_0(k+2) + P_1$

$\Rightarrow P_0(k+2) + P_1 - 3\{P_0(k+1) + P_1\} + 2\{P_0 k + P_1\} = 3k+2.$

Comparing Coefficients of k

$P_0 - 3P_0 + 2P_0 = 3$

$0 = 3.$

then $y_k =$

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* If i) $f(K) = K^2$

then $y_K^{(P)} = P_0 + P_1 K + P_2 K^2$.

ii) $f(K) = 5 \cdot 2^K + 3 \cdot 7^K$

$y_K^{(P)} = d_0 \cdot 2^K + d_1 \cdot 7^K$.

iii) $f(K) = K + 2^K$

$y_K^{(P)} = P_0 + P_1 K + P_2 \cdot 2^K$

iv] $f(K) = K \cdot 2^K$

$y_K^{(P)} = (P_0 + P_1 K)\, P_2 \cdot 2^K$

$= (P_0 P_2 + P_1 P_2 K) \cdot 2^K$

$= \cancel{(A+KB) \cdot 2^K} = (A + BK)\, 2^K$

* Generating function:- Let $a_0, a_1, a_2 \ldots, a_n$ be the elements of sequence then the generating f'n is denoted by $G(a, z)$ and given by

$$G[a, z] = \sum_{n=0}^{\infty} a_n z^n.$$

Theorem 1:-

If $a_n = b$, $n \geq 0$. find Generating f'n.

Sol'n:-

$$G(a, x) = \sum_{n=0}^{\infty} a_n z^n$$

$$= a_0 z^0 + a_1 z^1 + a_2 z^2 + \cdots \infty$$

$$= b + bz + bz^2 + \cdots \infty$$

$$G[a, x] = \frac{b}{1-z}$$

$S_\infty = \frac{a}{1-r}$

$r = \frac{T_2}{T_1}$

$r = \frac{bz}{\cancel{b}} = z$.

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Theorem 2 :- If $a_n = b^n$ $n \geq 0$ find G.F

Sol'n :- $G[a, x] = \sum_{n=0}^{\infty} a_n z^n$.

$= a_0 z^0 + a_1 z^1 + a_2 z^2 + \dots \infty$

$= 1 + bz + bz^2 + \dots \infty$

$r = \frac{bz}{1} = bz$

$= \frac{1}{1-bz}$.

* Eg :- $a^n = 5^n$

then directly $= \frac{1}{1-5z}$.

* Eg :- $\frac{1}{1-3z} + \frac{1}{1-5z}$

Sol'n :- $(3)^n + (5)^n$.

Q) Solve the difference eq'n $a_{n+2} - 3a_{n+1} + 2a_n = 0$.
$a_0 = 1$, $a_1 = 2$ using generation function Method.

Sol'n :-

The given equation is
$a_{n+2} - 3a_{n+1} + 2a_n = 0$.

$a_{n+2} z^n - 3a_{n+1} z^n + 2a_n z^n = 0$.

$\frac{z^2}{z^2}[a_2 + a_3 z + \dots] - \frac{3z}{z}[a_1 + a_2 z + a_3 z^2 + \dots] + 2G(a,z) = 0$.

$\frac{1}{z^2}[a_2 z^2 + a_3 z^3 + a_4 z^4 + \dots] - \frac{3}{z}[a_1 z + a_2 z^2 + \dots] + 2G(a,z) = 0$

$\frac{1}{z^2}[G(a,z) - a_0 - a_1 z] - \frac{3}{z}[G(a,z) - a_0] + 2G(a,z) = 0$.

$\rightarrow [G(a,z) - a_0 - a_1 z] - 3z[G(a,z) - a_0] + 2z^2 G(a,z)] = 0$.

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$[1-3z+2z^2]\,G(a,z) - a_0 - a_1 z + 3z a_0 = 0.$

$[2z^2-3z+1]\,G[a,z] - 1 - 2z + 3z = 0.$

$[2z^2-3z+1]\,G(a,z) - 1 + z = 0.$

$(2z^2-3z+1)\,G(a,z) = (1-z)$

$$G(a,z) = \frac{1-z}{2z^2-3z+1}$$

$$G(a,z) = \frac{1-z}{2z^2-2z-z+1}$$

$$= \frac{1-z}{2z(z-1)-1(z-1)}$$

$$= \frac{1-z}{(2z-1)(z-1)}$$

$$G(a,z) = \frac{1}{1-2z}$$

$$a_n = (2)^n.$$

31/Aug

Operator Method:-

$$y_{k+2} + 3y_{k+1} + 2y_k = f(k)$$

$$E^2 y_k + E y_k + 2y_k = f(k)$$

$$y_k(E^2 + E + 2] = f(k).$$

$$\cancel{y_k(}\phi\, 2(y_k) = f(k)$$

For homogenous Sol'n:

$$\phi E = 0.$$

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$$E^2+3E+2=0$$

$$E^2+\cancel{8}E+2E+2=0$$

$$E(E+1)+2(E+1)=0$$

$$(E+1)(E+2)=0$$

$$E=-1,-2$$

$$y_k^{(h)} = C_1(-1)^k + C_2(-2)^k$$

$$y_k = \frac{1}{\phi E}\cdot f(k)$$

Case (i) when $f(k)=a$.

Consider

$$\phi E(a) = (E^2+3E+2)a$$

$$= E^2(a)+3E(a)+2(a)$$

$$= [a+3a+2a]$$

$$= [1+3+2]a$$

$$\phi E(a) = \phi(1)a$$

$$\therefore \frac{1}{\phi(1)}(a) = \frac{1}{\phi(E)}\cdot a.$$

$$\rightarrow \frac{1}{E^2+3E+10}\cdot a$$

$$= \frac{1}{(1)^2+3(1)+10}\cdot a$$

$$= \frac{1}{14}a.$$ //

Case (2) :-

When $f(k)=2^k$.

Consida

$$\phi E = 2^k = (E^2+3E+2]2^k.$$

$$= [E^2(2^k)+3E(2^k)+2(2^k).]$$

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$= (2^{k+2}) + 3(2^{k+1}) + 2(2^k)$

$= 2^k(4+3+2)$

$\phi E(2^k) = 2^k(12)$

$\phi E(2^k) = \phi(2)(2^k)$

$\frac{1}{\phi(2)} \cdot 2^k = \frac{1}{\phi E} \cdot 2^k$

Eg:- $\frac{1}{E^2+3E+5}(3^k)$

$= \frac{1}{(3)^2+3(3)+5}(3^k)$

$= \frac{1}{23} \cdot 3^k$

$\frac{1}{E^2+3E+10} \cdot 5^k$

$= \frac{1}{(5)^2+3(5)+10} \cdot 5^k$

$= \frac{5^k}{50}$

Case (3):- When $f(k)$ is a polynomial of degree n

$y_k = \frac{1}{\phi E} \cdot f(k)$ $\left(\because \Delta = E-1, \ E = \Delta+1 \right)$

$\frac{1}{\phi(\Delta+1)}[G(k')]$.

Write denominator in ascending powers of Δ & apply binomial theorem.

$(1+\Delta)^{-1} = 1-\Delta+\Delta^2+\dots\infty$

$(1-\Delta)^{-1} = 1+\Delta+\Delta^2+\dots\infty$

$(1+\Delta)^{-2} = 1-2\Delta+3\Delta^2+\dots\infty$

$(1-\Delta)^{-2} = 1+2\Delta+3\Delta^2+\dots\infty$

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Q] $\frac{1}{E+2} \cdot k$

$\Rightarrow \frac{1}{(\Delta+1)+2} \cdot k$

$\Rightarrow \frac{1}{\Delta+3} \cdot k^{(1)}$

$\Rightarrow \frac{1}{3+\Delta} k^{(1)}$

$\Rightarrow \frac{1}{3(\Delta+\frac{1}{3})} k^{(1)} \Rightarrow \frac{1}{3}\left[1-\frac{\Delta}{3}+\left(\frac{\Delta}{3}\right)^2+\cdots\infty\right] k^{(1)}$

$\Rightarrow \frac{1}{3}\left[1-\frac{\Delta}{3}\right] \cdot k^{(1)}$

$\Rightarrow \frac{1}{3}\left[k^{(1)}-\frac{1}{3}\right] \cdot = \frac{1}{3}\left[k-\frac{1}{3}\right]$

Q) $\frac{1}{E+5}(k+2)$.

$\Rightarrow \frac{1}{\Delta+1+5}(k)$

$\Rightarrow \frac{1}{\Delta+6}(k)$

$\Rightarrow \frac{1}{6}\left[\frac{1}{1+\frac{\Delta}{6}}\right) k^{(1)}$

$\Rightarrow \frac{1}{6}\left(1+\frac{\Delta}{6}\right)^{-1} \cdot k^{(1)}$

$\Rightarrow \frac{1}{6}\left(1-\frac{\Delta}{6}\right) \cdot k^{(1)}$

$\Rightarrow \frac{1}{6}\left(k^{(1)}-\frac{1}{6}\right] = \frac{1}{6}\left[k-\frac{1}{6}\right]$.

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Q) $\frac{1}{E-2} \cdot k^2$

$\frac{1}{(E-1-1)} \cdot k^2$

$\Rightarrow k^{(2)} = k(k-1)$

$k^{(2)} = k^2 - k^{(1)}$

$k^2 = k^{(2)} + k^{(1)}$

$\Rightarrow \frac{1}{(\Delta-1)} \cdot k^2$

$\Rightarrow \frac{1}{(\Delta-1)} \cdot \left[k^{(2)} + k^{(1)}\right]$

$\Rightarrow \frac{1}{-1+\Delta}\left[k^{(2)} + k^{(1)}\right]$

$\Rightarrow \frac{-1}{1-\Delta}\left[k^{(2)} + k^{(1)}\right]$

$\Rightarrow -\left[1-\Delta\right]^{-1} \cdot \left[k^{(2)} + k^{(1)}\right]$

$\Rightarrow -1\left[1+\Delta+\Delta^2 + \dots \infty\right]\left[k^{(2)} + k^{(1)}\right]$

$\Rightarrow -\left[k^{(2)} + k^{(1)} + 2k^{(1)} + 1 + 2\right]$

$\Rightarrow -\left[k^{(2)} + 3k^{(1)} + 3\right]$

$\Rightarrow -[\cdot$

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Reduction of Order Method

1) $a_{n+2} - 3a_{n+1} + 2a_n = 0.$

$[E^2 - 3E + 2]a_n = 0.$

$[E-1][E-2]a_n = 0.$

put. $(E-2)a_n = y_n$

$(E-1)y_n = 0$

$y_n = c_1(1)^n$

$(E-2)a_n = c_1(1)^n$

$a_n = a_n^{(h)} + a_n^{(P)}$

$= c_2(2)^n + \frac{1}{E-2} c_1(1)^n.$

$= c_2(2)^n + \frac{1}{1-2} c_1(1)^n$

$= c_2(2)^n - c_1(1)^n$

$= A(2)^n + B(1)^n.$

2) $a_{n+2} - 5a_{n+1} + 6a_n = 5^n.$

$E^2 - 5E + 6 = 0$

$E^2 + E - 6E + 6 = 0$

$= \frac{5 \pm \sqrt{25 - 4(1)(6)}}{2(1)}$

$(E-2)(E-3)a_n = 5^n$

$= \frac{5 \pm 1}{2} = 2, 3.$

put $(E-3)a_n = y_n$

$(E-2)y_n = 5^n$

$y_n = y_n^{(h)} + y_n^{(P)}$

$\Rightarrow c_1(2)^n + \frac{1}{E-2}(5)^n$

$$y_n = c_1 (2)^n + \frac{1}{3}(5)^n$$

$$(E-3)\, a_n = c_1 (2)^n + \frac{1}{3}(5)^n$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$= c_2 (3)^n + \frac{1}{E-3}\left[c_1 (2)^n + \frac{1}{3} 5^n \right]$$

$$= c_2 (3)^n + \frac{1}{E-3} c_1 (2)^n + \frac{1}{E-3}\left(\frac{1}{3} 5^n \right)$$

$$= c_2 (3)^n + \frac{1}{2-3} c_1 (2)^n + \frac{1}{5-3}\left(\frac{1}{3} \cdot 5^n \right)$$

$$= c_2 (3)^n - c_1 (2)^n + \frac{1}{6} 5^n .$$

12th Sep

partial Order Relation

A Relation R

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{Relation}

→ Consider a non-empty set A. then Relation R is subset of A×A.

$$\boxed{R \subseteq A \times A}$$

Eg:- $A = \{1,2\}$

$A \times A = \{1,2\} \times \{1,2\}$

$= \{(1,1),(1,2),(2,1),(2,2)\}$

$R_1 = \{(1,1),(1,2)\}$; $R_2 = \{(1,2),(2,2)\}$

$R_3 = \{(1,2)(2,1)\}$ (sum).

* Counting of Relations:-

$2 \times 2 \times 2 \times 2$

$= 2^4 = 2^{2\times 2}$

$= 2^{n(A) \times n(A)}$

$= 2^{[n(A)]^2}$

if there are 2 sets A, B then $2^{n(A) \times n(B)}$ $n(A)=5, n(B)=2$

then $2^{5\times 2} = 2^{10} = 1024$.

* Reflexive Relation:-

let A be any non-empty set. Then a relation R is called reflexive relation.

if $(a,a) \in R$ $\forall a \in A$

$A = \{1,2,3\}$

$R_1 = \{(1,1),(2,2),(1,2),(2,1)\}$ → Not reflexive

$R_2 = \{(1,1),(2,2),(3,3)\}$ → reflexive.

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Counting of reflexive relation:-

→ $A = \{1, 2, 3\}$

$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$R = \{(1,1), (2,2), (3,3)\}$

Symmetric Relation:- let n be a non empty set define a relation on R as

$(a, b) \in R \Rightarrow (b, a) \in R$

$A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (3,3), (1,2)\}$

This relation is reflecxive but not symmetric.

$R_2 = \{(1,1), (1,2), (2,1), (1,3), (3,1)\}$

$R_3 = \{(1,1), (2,2), (3,3)\}$

Reflexive & Symmetric.

Counting of no. of symmetric relation.

$A = \{1, 2, 3\}$

	1	2	3
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

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Transitive relation :- Let A be any non empty set. Then a relation R defined on A is called transitive relation.

if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

If N is a set of Natural number defined a relation on N $R = \{(a,b) : a \le b\}$

prove that this relation is reflexive & Transition. but not Symmetric.

i) Reflexive :-

$a \le a \quad \forall a \in \mathbb{N}$

$(a,a) \in R \quad \forall a \in \mathbb{N}$

$\Rightarrow$ Relation is reflexive.

ii) Transitive :-

$(a,b) \in R$ & $(b,c) \in R$

$a \le b$ & $b \le c$ & $a \le c$

$(a,c) \in R$

so relation is transitive.

iii) Symmetric :-

Let $(a,b) \in R$

$a \le b \not\Rightarrow b \le a$

$(b,a) \in R$

$\rightarrow$ relation is not Symmetric

Let L be the set of all lines in a plane Define a relation R on L

$R = \{(L_1, L_2) : L_1 \perp L_2\}$

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Show that this relation is Symmetric but its is neither reflexive nor transitive.

I Reflexive :-

No line can be $\perp$ to itself

$L_1 \perp l_1 \quad \forall L_1 \in L$

$(l_1, l_1) \in R \quad \forall L_1 \in L$

$\rightarrow$ Relation is not reflexive.

II Symmetric :-

Let $(l_1, l_2) \in R$

$l_1 \perp l_2$

$l_2 \perp l_1$

$\Rightarrow (l_2, l_1) \in R$

Relation is symmetric.

III Transitive :-

Let $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$

$L_1 \perp l_2$ & $L_2 \perp l_3$

$l_1 \perp l_3$

$\Rightarrow (l_1, l_3) \in R$.

Equivalence Relation :- if any relation R is Symmetric, reflexive & transitive then this relation is called equalence ~~relation~~.

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Ex: If R is the set of real numbers define R.

$R = \{ (x,y) : x - y \in I \}$

prove R is equalence.

$x - x = 0 \in I \quad \forall \; x \in R$

reflexive.

2) Symmetric

let $(x,y) \in R$

$x - y \in I$

$-(x-y) \in I$

$y - x \in I$

$(y,x) \in R.$

$\therefore$ Relation is reflexive.

12th sep

Partial Order Relation

A relation R defined on set A is called as partial Order relation if it is

1. Reflexive
2. Anti-Symmetric

if $(a,b) \in R$ and $(b,a) \in R$

then $\boxed{a=b}$

3. Transitive.

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Q) If N is the set of Natural Numbers defined a relation R on N as $R = \{(a,b) : \text{if } a/b\}$ prove that this Relation is POR.

① Reflexive :

clearly $a/a \ \forall a \in N$

$(a,a) \in R \ \forall a \in N$

∴ This relation is reflexive.

② Anti-Symmetric :

$(a,b) \in R$ and $(b,a) \in R$

a/b and b/a

This is possible when $a=b$.

∴ This relation is Anti-Symmetric.

③ Transitive :-

Let $(a,b) \in R$ and $(b,c) \in R$

a/b and b/c

$\Rightarrow a/c \Rightarrow (a,c) \in R$

∴ This relation is transitive.

Hence this is called partial Order Relation.

Q) Let S be a non-empty set and P(S) denotes the power set of S. Define a Relation ~~R on P(S)~~

$R = \{(A,B) : A \leq B\}$

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1) Reflexive

We know that every set is a subset of itself

i.e., $A \subseteq A \quad \forall \quad A \in P(S)$

$(A, A) \in R$

$\therefore$ It is reflexive.

(2) Anti-Symmetric:

Let $(A,B) \in R$ and $(B,A) \in R$

$A \subseteq B$ and $B \subseteq A$

$\boxed{A = B}$

(3) Transitive property:-

Let $(A,B) \in R$ and $(B,C) \in R$

$A \subseteq B$ and $B \subseteq C$

$\Rightarrow A \subseteq C \Rightarrow (A, C) \in R$

So, this Relation is Transitive.

Hence this relation is partial Order Relation.

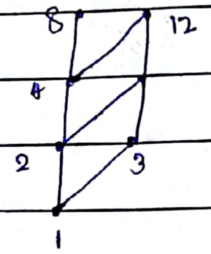
Hasse Diagram

Q) Draw the Hasse diagram representing the partial Ordering.

$R = \{(a,b) : a \text{ divides } b\}$ on $\{1,2,3,4,6,8,12\}$

Soln:-

$$R = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12) \\ (2,2), (2,4), (2,6), (2,8), (2,12), (3,3), (3,6) \\ (3,12), (4,4), (4,8), (4,12), (6,6), (6,12), (8,8) \\ (12,12) \end{array} \right\}$$



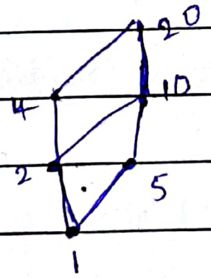
14th sep — Draw the hasse diagram for this set $(D_{20}, |)$ under divisibility.

Sol'n:-

$D_{20} \to$ divisors of 20

$D_{20} = \{1, 2, 4, 5, 10, 20\}$

$R = \{(1,1), (1,2), (1,4), (1,5), (1,10), (1,20)$
$(2,1), (2,4), (2,10), (2,20), (4,20), (4,4)$
$(5,5), (5,10), (5,20), (10,10), (10,20), (20,20)\}$

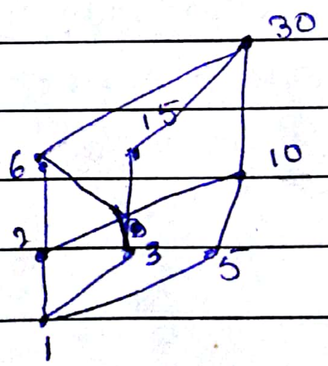


Maximal element = 20

Minimal element = 1

& $(D_{30}, |)$

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$



Maximal elt = 30

Minimal = 1

Greatest → all elements of set should be connected with that element (lies at top)

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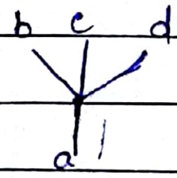
Least → (lies at bottom)

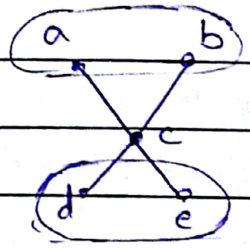


Minimal = a

Maximal elt = b, c, d

Greatest elt = Nil

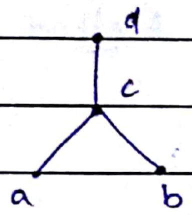
least elt = a.

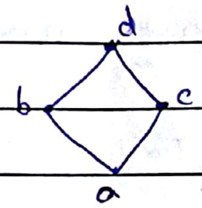


Minimal elt = d, e | least elt = Nil

Maximal elt = a, b

Greatest elt = Nil





Min elt = a, b

Max elt = d

Greatest elt = d

least = Nil

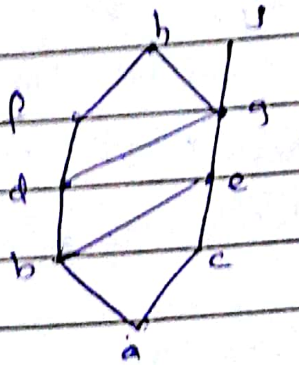
Min elt = a

Max elt = d

Greatest = d

least elt = a

l·b (lower bound)	u·b (upper bond)	g·l·b (greatest lower bond)	l·u·b (lowest upper bond)	Subset
a	e, f, j, h	a	e	{a, b, c}
a, c, b, e, d, f	–	f	–	{i, h}
a, b	g, h	b	g.	{b, d, g}



$A = \{a, b, c, d, e, f, g, h, i\}$

Given the hasse diagram. Find lower and upper bor
of subsets 1) $\{a, b, c\}$ 2) $\{i, h\}$ ③ $\{b, d, e\}$

17/sep

Variation of parameters Method

Q) $y_{k+2} - 3y_{k+1} + 2y_k = 5^k.$

$\cancel{y_1}$

$y_k^{(h)} = c_1(1)^k + c_2(2)^k.$

$y_k^{(p)} = k_1(1)^k + k_2(2)^k.$

$$K_1 = -\Delta^{-1}\left[\frac{(5)^k}{(1)^{k+1}}\right] \qquad K_2 = \Delta^{-1}\left[\frac{5^k}{2^{k+1}}\right]$$

$$= -\frac{1}{\Delta}\left[5^k\right] \qquad = \frac{1}{E-1}\left[\left(\frac{5}{2}\right)^k \cdot \frac{1}{2}\right]$$

$$= -\frac{1}{E-1}\left[5^k\right] \qquad = \frac{1}{2} \cdot \frac{1}{E-1}\left(\frac{5}{2}\right)^k$$

$$= -\frac{1}{5-1}\left[5^k\right] \qquad = \frac{1}{2} \cdot \frac{1}{\frac{5}{2}-1}\left(\frac{5}{2}\right)^k$$

$$= -\frac{1}{4}\left[5^k\right] \qquad = \frac{1}{\cancel{2}} \cdot \left[\frac{\cancel{2}}{3}\right] \cdot \left(\frac{5}{2}\right)^k$$

$$= \frac{1}{3}\left(\frac{5}{2}\right)^k$$

$\therefore y_k^{(p)} =$

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$$\therefore\ y_k^{(p)} = -\frac{1}{4}[5^k)(1)^k + \frac{1}{3}\left(\frac{5}{2}\right)^k(2)^k.$$ //

Q) $y_{k+2} - 5y_{k+1} + 4y_k = k^2.$

$y_k = c_1(1)^k + c_2(4)^k$

$y_k^{(p)} = k_1(1)^k + k_2(4)^k.$

$$\Delta k_1 = \frac{-k^2}{(1)^{k+1}}$$

$= -\Delta^{-1} \cdot k^2$

$$= -\frac{1}{E-1}\left(k^{(2)} + k^{(1)}\right)$$

$= -\Delta^{-1}\left(k^{(2)} + k^{(1)}\right)$

(Δ = differentiation
$1/\Delta$ = integration)

$$\therefore = -\left[\frac{k^{(3)}}{3} + \frac{k^{(2)}}{2}\right].$$ //

Q) $2^{k+1} \cdot \Delta k_1 + 3^{k+1}\ \Delta k_2 = 0 \rightarrow 1$

$2^{k+1}\ \Delta k_1 + 2 \cdot 3^{k+1} \Delta k_2 = k^2 \rightarrow 2$

Subtract 2 from 1

$2^{k+1} \cdot \Delta k_1 + 2 \cdot 3^{k+1} \Delta k_2 = k^2$

$\cancel{2^{k+1}} \cdot \Delta k_1 + 1 \cdot 3^{k+1} \Delta k_2 = 0$

$3^{k+1} \cdot \Delta k_2 = k^2$

$$\Delta k_2 = \frac{k^2}{3^{k+1}}$$

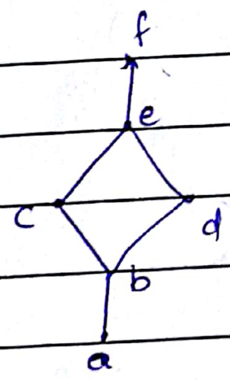
$\therefore$ substitute value of Δk_2 in eq (1)

$$(1) \Rightarrow 2^{k+1}\Delta k_1 + \cancel{1\cdot 3^{k+1}}\left[\frac{k^2}{\cancel{3^{k+1}}}\right] = 0$$

$$\Rightarrow 2^{k+1}\cdot\Delta k_1 = -k^2$$

$$\Rightarrow \Delta k_1 = \frac{-k^2}{2^{k+1}}$$

Lattice :- A partially Ordered Set in which every pair of elements has a least upper bond and the greatest lower bond is called a lattice.



Subset	u.b	l.b	g.l.b	l.u.b
{c, e}	e, f	c, b, a	c	e
{a, f}	f	a	a	f
{b, e}	e, f	b, a	b	e

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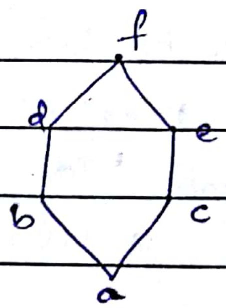

Subset	u.b	l.b	g.l.b	l.u.b
b, c	d, e, f	a	a	—

26th Sep
Wednesday

UNIT-6

Number Theory

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Divisibility:- Let a & b be two integers, where $a \neq 0$ then a we say that a divides b if there exist integer c such that $b/a = c$.
$\Rightarrow b = ac$

Q) Check that $3/7$ and $3/12$.

$\frac{7}{3} = 2.333\ldots$

$3 \nmid 7$

$\frac{12}{3} = 4$, where 4 is an integer.

$\therefore \Rightarrow 3/12.$ //

properties of divisibility:-

1. if $a|b$ and $a|c \Rightarrow a|(b+c)$.
2. if $a|b$ then $a|bc$.
3. if $a|b$ and $b|c$ then $a|c$
4. if $a|b$ and $b|c$ then $a|bm+cn$.

as $a|b$ ~~an~~ then $a|bm$
also $a|c$ then $a|cn$
$\Rightarrow a|bm+cn$.

→ Let a be any integer, and b be any +ve integer then there exist integers q and r such that
$a = bq + r$. Where $q \to$ quotient, $r \to$ remainder
$0 \leq r \leq b$

$42 - 11$

$-11 + 9$

Q) what are q, r when 101 is divisible by 11.

$101 = 11(9) + 2$

$11 \overline{)\,101\,}(9$
$\;\;\underline{99}$
$\;\;\;\;2$

Quotient = 9

Remainder = 2

Q) what are q, r when -11 is divisible by 3.

$-11 = 3(-4) + 1$

$3 \overline{)\,-11\,}(-4$
$\;\;\underline{+12}$
$\;\;\;\;\boxed{1}$ — 1 in posi

* Modular Arithmetic :- If a and b are integers and n is +ve Integer then a is Concurrent to b modulo n if n divides a-b

$a \equiv b \pmod{n}$ $\quad (\because \equiv \to$ concurrent$)$

if $n \mid (a-b)$

Eg:- $15 \equiv 3 \pmod{4}$

Q) determine whether 17 is concurrent to 5 modulo 6 & whether 24 and 14 are Concurrent to module 6.

Sol:- if $17 \equiv 5 \pmod{6}$

$6 \mid (17-5)$

$6 \mid 12$

concurrent

if $24 \equiv 14 \pmod{6}$

$6 \mid (24-14)$

$6 \mid 10$

non-current

Theorem:

→ let m be a +ve integer then the integer a & b are concurrent modulo m if there exist, an integer k Such that $a = b + mk$.

Pf'n: $a \equiv b \pmod{m}$

$m \mid (a-b)$

There must exist $\exists$ integer k such that

$$\frac{a-b}{m} = k$$

$$a - b = mk$$

$$a = b + mk$$

28th Sep **Theorem:-**

If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then

$$a + c \equiv (b+d) \pmod{m}$$

$$ac \equiv bd \pmod{m}$$

Pf'n: Given that

$a \equiv b \pmod{m}$ & $c \equiv d \pmod{m}$

$\Rightarrow m \mid (a-b)$ & $m \mid c-d$

$\Rightarrow m \mid (a-b) + (c-d)$

$\Rightarrow m \mid (a+c) - (b+d)$

$\therefore a + c \equiv (b+d) \pmod{m}$

$\rightarrow m \mid (a-b)$ & $m \mid (c-d)$

$\frac{a-b}{m} = k_1$, $\frac{c-d}{m} = k_2$

$a = b + mk_1$, $c = d + mk_2$

$ac = (b+mk_1)(d+mk_2)$

$ac = bd + bmk_2 + dmk_1 + m^2 k_1 k_2$

$ac - bd = m[bk_2 + k_1 d + mk_1 k_2]$

$\frac{ac-bd}{m} = Q \Rightarrow m \mid (ac-bd) \Rightarrow ac \equiv bd \pmod{m}$

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* **Prime number**:- A number n is called prime no. if it has exactly 2 divisors 1 and number itself.

Q) check that 101 is prime number (or) not.

$10 < \sqrt{101} < 11$

prime no's less than 11 are

2, 3, 5, 7

$2 \nmid 101$, $3 \nmid 101$, $5 \nmid 101$, $7 \nmid 101$

⇒ if all (2,3,5,7) are not divisible then 101 is a prime number.

* **Greatest Common divisior or H.C.F**:- Let a & b be two numbers then d is called G.c.d. of a & b if $d|a$ and $d|b$. also if ∃ some other number c such that $c|a$ and $c|b$ then $c|d$.

eg:- $g.c.d(20, 24) = 4$ //.

2, 4
c d

⇒ $20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 3^0 \cdot 5^1$

$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3^1 \cdot 5^0$

$l.cm(20, 24) = 2^3 \cdot 3^1 \cdot 5^1 = 8 \times 15 = 120$

⇒ $a = P_1^{\alpha_1} \cdot P_2^{\alpha_2}$

$b = P_1^{\beta_1} \cdot P_2^{\beta_2} \cdots P_n$

$G.C.D(a,b) = P_1^{\min(\alpha_1, \beta_1)} \cdot P_2^{\min(\alpha_2, \beta_2)} \cdots P_n^{\min(\alpha_n, \beta_n)}$

$l.c.m(a,b) = P_1^{\max(\alpha_1, \beta_1)} \cdot P_2^{\max(\alpha_2, \beta_2)} \cdots P_n^{\max(\alpha_n, \beta_n)}$

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Euclidean Algorithm

→ This is used to find H.C.F of 2 numbers.

Q) Find g.c.d (414, 662) also write g.c.d as a linear Combination of given numbers.

$$
\begin{array}{l}
414)\ 662\ (1 \\
\quad\ \ 414 \\
\quad\ \ 248)\ 414\ (1 \\
\qquad\qquad 248 \\
\qquad\qquad 166)\ 248\ (1 \\
\qquad\qquad\qquad\ \ 166 \\
\qquad\qquad\qquad\ \ 82)\ 166\ (2 \\
\qquad\qquad\qquad\qquad\ \ 164 \\
\qquad\qquad\qquad\qquad\ \ \boxed{2})\ 82\ (41 \\
\qquad\qquad\qquad\qquad\qquad\ 82 \\
\qquad\qquad\qquad\qquad\qquad\ 0
\end{array}
$$

∴ HCF(414, 662) = 2.

2nd Oct

* Divident = Divisor · Quotient + remainder.

$662 = 414 \cdot 1 + 248$

$414 = 248 \cdot 1 + 166$

$248 = 166 \cdot 1 + 82$

$166 = 82 \cdot 2 + 2$

$82 = 2 \cdot 41$ ✗ (bcz of no remainder)

$2 = 166 - \underbrace{82}_{} \cdot 2$

$= 166 - [248 - 166 \cdot 1] \cdot 2$

$= 1 \cdot 166 - 2 \cdot 248 - 166 \cdot 2$

$= 3 \cdot 166 - 2 \cdot 248$

$= 3 \cdot [414 - 248 \cdot 1] - 2 \cdot 248$

$= 3 \cdot 414 - 3 \cdot 248 - 2 \cdot 248$

$= 3 \cdot 414 - 5 \cdot 248$

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$= 3\cdot414 - 5\,[662 - 414\cdot1]$

$= 3\cdot414 - 5\cdot662 + 414\cdot5$

$2 = 8\cdot414 + (-5)\cdot662$

$2 = a\cdot414 + b\cdot662.$

* Find H.C.F and write linear Combination.

1) (135, 225)

$135\,)\,225\,(\,1$
135
$90\,)\,135\,(\,1$
90
$45\,)\,90\,(\,2$
90
$\times$

HCF = 45

$\Rightarrow 225 = 135\cdot1 + 90$

$135 = 90\cdot1 + 45$

$90 = 45\cdot2 + \times$

$45 = 135 - 90\cdot1$

$= 135 - [225 - 135\cdot1]\cdot1$

$= 135 - 225\cdot1 + 135\cdot1$

$= 2\cdot135 - 225\cdot1 \Rightarrow 2\cdot135 + (-1)\cdot225 = 45$

~~2~~ $\{\ 45 = a\,135 + b\,225.$ //

* **Linear Congruences:**- A Congruence of the form $ax \equiv b \pmod m$ where $m \to$ +ve integer and a, b are integers and $x \to$ variable. is called a linear congruence.

→ $x+5=7$, 2 satisfies this eq'n.

* $ax \equiv b \pmod m$

when $(a, m = 1)$ ~~wh~~ then sol'n exist.
g.c.d of

→ what are solutions of linear congruence
$3x \equiv 4 \pmod 7$.

→ $7 / 3x-4$

$x = 1$ ✗ sub 7 : add 7

$x = 2$ ✗ $\{-8, -1, 6, 13, 20 \ldots\}$

$x = 3$ ✗

$x = 4$ ✗

$x = 5$ ✗

$x = 6$ ✓

* $7x \equiv 2 \pmod 3$

$3 / 7x - 2$

$x = 1$ ✗

$x = 2$ ✓

⇒ Sol'n set: $\{\ldots -4, -1, 2, 5, 8 \ldots\}$

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* If $a \cdot \bar{a} \equiv 1 \mod m$ then $\bar{a} \to$ inverse of a.

→ Find the inverse of 3 (mod 7)

Sol'n:- $3\bar{a} \equiv 1 \mod 7$

$7 / 3\bar{a} - 1$

$\bar{a} = 1$ ×

$\bar{a} = 2$ ×

$\bar{a} = 3$ ×

$\bar{a} = 4$ ×

$\boxed{\bar{a} = 5}$.

∴ inverse of 3 is 5.

→ inverse of 6 (mod 11)

Sol'n:- $6\bar{a} = 1 \mod 11$

$11 / 6\bar{a} - 1$

$\bar{a} = 1$ ×

$\bar{a} = 2$ ✓

$\boxed{\bar{a} = 2}$

Imp Chinese - Remainder Theorem

$x \equiv a_1 (\mod m_1)$

$x \equiv a_2 (\mod m_2)$

$x \equiv a_3 (\mod m_3)$

g.c.d $(m_1, m_2) = (m_2, m_3) = (m_3, m_1) = 1$

$M = m_1 \cdot m_2 \cdot m_3$

$M_1 = \frac{M}{m_1}$, $M_3 = \frac{M}{m_3}$

$M_2 = \frac{M}{m_2}$

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$M_1 y_1 \equiv 1 \pmod{m_1}$

$M_2 y_2 \equiv 1 \pmod{m_2}$

$M_3 y_3 \equiv 1 \pmod{m_3}$.

$x \equiv [M_1 a_1 y_1 + M_2 a_2 y_2 + M_3 a_3 y_3] \bmod M$

Q) Use Chinese Remainder Theorem to find a solution to the system of concurrence.

1) $x \equiv 2 \pmod 3$

2) $x \equiv 1 \pmod 4$

3) $x \equiv 3 \pmod 5$

Sol'n:- $\therefore a_1 = 2;\ a_2 = 1,\ a_3 = 3.;\ m_1 = 3,\ m_2 = 4,\ m_3 = 5$

G.c.d of $(3,4) = 1,\ (4,5) = 1,\ (5,3) = 1$.

$M = 3 \cdot 4 \cdot 5$

$= 60$

$M_1 = \frac{M}{m_1} = \frac{60}{3} = 20$

$M_2 = \frac{M}{m_2} = \frac{60}{4} = 15$

$M_3 = \frac{M}{m_3} = \frac{60}{5} = 12.$

$M_1 y_1 \equiv 1 \pmod{m_1}$

$20 y_1 \equiv 1 \pmod 3$

$\therefore 3 / (20y_1 - 1)$

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$y_1 = 1x$
$y_1 = 2$
$y_1 = 2$ is soln of above (eq'n) congruence.

→ $M_2 y_2 \equiv 1 \pmod{m_2}$
$15 y_2 = 1 \pmod 4$
→ $4 / (15 y_2 - 1)$
$y_2 = 3$ sol'n of above Congruence

→ $M_3 y_3 = 1 \pmod{m_3}$
$12 y_3 = 1 \pmod 5$
$5 / (12 y_3 - 1)$
$\therefore y_3 = 3$ is soln of above Congruence.

$x \equiv [M_1 a_1 y_1 + M_2 a_2 y_2 + M_3 a_3 y_3] \pmod M$
$\equiv [20 \cdot 2 \cdot 2 + 15 \cdot 1 \cdot 3 + 12 \cdot 3 \cdot 3] \pmod{60}$
$\equiv [233] \pmod{60}$
$x \equiv 53 \pmod{60}$

$60) 233 (3$
180
53

[→ Find the System if any] x

Q) Find all the solutions of any System of Concurrency.

$x \equiv 5 \pmod 6$
$x \equiv 3 \pmod{10}$
$x \equiv 8 \pmod{15}$

$x \equiv 5 \pmod 3$ $\quad$ $x \equiv 5 \pmod 2$
$x \equiv 2 \pmod 3$ $\quad$ $x \equiv 1 \pmod 2$

$x \equiv 3 \pmod 2$, $\quad$ $x \equiv 3 \pmod 5$
$x \equiv 1 \pmod 2$, $\quad$ $x \equiv 3 \pmod 5$

$x \equiv 8 \pmod 3$, $\quad$ $x \equiv 8 \pmod 5$
$x \equiv 2 \pmod 3$, $\quad$ $x \equiv 3 \pmod 5$

$\therefore$ $x \equiv 2 \pmod 3$
$x \equiv 1 \pmod 2$
$x \equiv 3 \pmod 5$

$M = 3 \cdot 2 \cdot 5 = 30$

$\Rightarrow M_1 = \frac{M}{m_1} = \frac{30}{3} = 10$

$M_2 = \frac{M}{m_2} = \frac{30}{2} = 15$

$M_3 = \frac{M}{m_3} = \frac{30}{5} = 6$

$M_1 y_1 \equiv 1 \pmod 3$
$10 y_1 = 1 \pmod 3$
$3 / 10y_1 - 1 \Rightarrow \boxed{y_1 = 1}$

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$M_2 y_2 \equiv 1 \pmod 2$

$15 y_2 \equiv 1 \pmod 2$

$2 / 15y_2 - 1$

$\boxed{y_2 = 1}$

$M_3 y_3 \equiv 1 \pmod 5$

$6 y_3 = 1 \pmod 5$

$5 / 6y_3 - 1$

$y_3 = 1$

$x \equiv [M_1 a_1 y_1 + M_2 a_2 y_3 + M_3 a_3 y_3] \bmod M$

$\equiv 53 \pmod{30}$

$\equiv 23 \pmod{30}$ //

8 Oct

- Q)

$x \equiv 1 \pmod 2$ $\quad m_1 = 2,\ m_2 = 3,\ m_3 = 5,\ m_4 = 11$

$x \equiv 2 \pmod 3$ $\quad a_1 = 1,\ a_2 = 2,\ a_3 = 3,\ a_4 = 4$

$x \equiv 3 \pmod 5$

$x \equiv 4 \pmod{11}$

G.c.d $\Rightarrow (m_1, m_2) = (m_2, m_3) = (m_3, m_4) = (m_4, m_1) = 1$

$M = m_1 \cdot m_2 \cdot m_3 \cdot m_4$

$= 2 \times 3 \times 5 \times 11$

$= 330$

$M_1 = \frac{M}{m_1} = \frac{330}{2} = 165$

$M_2 = \frac{M}{m_2} = \frac{330}{3} = 110$

$M_3 = \frac{M}{m_3} = \frac{330}{5} = 66$

$M_4 = \frac{M}{m_4} = \frac{330}{11} = 30.$

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$\frac{73}{2+9}$ $\cancel{8}$

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$M_1 y_1 \equiv 1 \pmod{m_1}$

$165 y_1 = 1 \pmod 2$

$\Rightarrow y_1 = 2 / (165 y_1 - 1) \Rightarrow \boxed{y_1 = 1}$

$M_2 y_2 = 1 \pmod{m_2}$

$110 y_2 = 1 \pmod 3$

$y_2 = 3 / (110 y_2 - 1)$

$\boxed{y_2 = 2}$

$M_3 y_3 = 1 \pmod{m_3}$

$66 y_3 = 1 \pmod 5$

$y_3 = 5 / (66 y_3 - 1)$

$\boxed{y_3 = 1}$

$M_4 y_4 = 1 \pmod{14}$

$30 y_4 = 1 \pmod{14}$

$y_4 = 11 / (30 y_4 - 1) \Rightarrow \boxed{y_4 = 7}$

$x \equiv [M_1 a_1 y_1 + M_2 a_2 y_2 + M_3 a_3 y_3 + M_4 a_4 y_4] \pmod M$

$\equiv [165 \cdot 1 \cdot 1 + 110 \cdot 2 \cdot 1 + 66 \cdot 3 \cdot 1 + 30 \cdot 4 \cdot 7] \pmod{330}$

$\Rightarrow [165 + 220 + 198 + 840] \pmod{330}$

$= 1643 = 323$

$\frac{12}{7}$ $\frac{7}{84}$

165, 220, 198, 840, 1423, 220, + 1643

1643 / 330

Q) $x \equiv 7 \pmod 9$ 3×3

$x \equiv 4 \pmod{12}$ 3×4

$x \equiv 16 \pmod{21}$ 7×3

Sol'n:-

$\rightarrow x \equiv 7 \pmod 3$, $x \equiv 7 \pmod 3$

$x \equiv 1 \pmod 3$, $x \equiv 1 \pmod 3$

$\rightarrow x \equiv 4 \pmod 3$, $x \equiv 4 \pmod 4$

$x \equiv 1 \pmod 3$, $x \equiv 0 \pmod 4$

$\rightarrow x \equiv 16 \pmod 3$, $x \equiv 16 \pmod 7$

$x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 7$

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$x \equiv 1 \pmod 3$

$x \equiv 0 \pmod 4$ $\therefore a_1 = 1, a_2 = 0, a_3 = 2.$

$x \equiv 2 \pmod 7$ $m_1 = 3, m_2 = 4, m_3 = 7$

$M = m_1 \cdot m_2 \cdot m_3 = 3 \cdot 4 \cdot 7 = 84$

$M_1 = \frac{M}{m_1} = 28, M_2 = 21, M_3 = 12.$

$\rightarrow M_1 y_1 \equiv 1 \pmod{m_1}$

$28 y_1 = 1 \pmod 3$

$3 / (28y_1 - 1)$

$\therefore \boxed{y_1 = 1}$

$M_2 y_2 \equiv 1 \pmod{m_2}$

$21 y_2 \equiv 1 \pmod 4$

$4 / (21y_2 - 1)$

$\boxed{y_2 = 1}$

$\rightarrow M_3 y_3 = 1 \pmod{m_3}$

$12 y_3 = 1 \pmod 7$

$7 / (12y_3 - 1) \Rightarrow \boxed{y_3 = 3}$

$x \equiv [M_1 a_1 y_1 + M_2 a_2 y_2 + M_3 a_3 y_3] \pmod{84}$

$\equiv [28 \cdot 1 \cdot 1 + 21 \cdot 0 \cdot 1 + 12 \cdot 2 \cdot 3] \pmod{84}$

$\equiv 100 \pmod{84}$

$\equiv 16 \pmod{84}.$ //

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Cryptography

→ Caesar Cipher :-

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	1	2	3	4	5	6	7	8	9	10	11	12	13

O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25

$$f(p) = p+3 \pmod{26}$$

→ Encrypt this Msg 'AB' using Caesar Cipher Method

Sol'n:- Given text message is 'AB'.

write in numericals.

0 1

increase each numerical by 3.

3 4

Again write these numericals in alphabets.

DE.

* if the numericals after increasing by 3 is greater than 26 then we write the remainder there.

10/oct What is the Secret message ~~reduced~~ Produce from the message "MEET YOU IN THE PARK". using Caesar Cipher Method.

Sol'n:- "MEET YOU IN THE PARK"

"12 4 4 19 24 14 20 8 13, 19 7 4 15 0 17 10.

Applying Ceasar Cipher Method.

$f(p) = p+3 \pmod{26}$

" $\overline{15}\ \overline{7}\ \overline{7}\ \overline{22}$, $\overline{1}\ \overline{17}\ \overline{23}$ $\overline{11}\ \overline{16}$ $\overline{22}\ \overline{10}\ \overline{7}$ $\overline{18}\ \overline{3}\ \overline{20}\ \overline{13}$ "

write these no's in alphabets.

" PHHW BRX LQ WKH SOUN"

Q) Decrypt "DQG" using Caesar Cipher method.

"D Q G"

3 16 6

" 0 13 3 "

A N D

* Shift Cipher Method

$f(p) = p+k \pmod{26}$

where k is any +ve integer.

Encrypt this message "AN" using $k=23$.

Soln: Given message is "AN"

write this message in numbers " 0 13"

Apply Shift Cipher method.

$f(p) = p+23 \pmod{26}$

" 23 10 "

" X K "

Q) Encrypt 'STOP GLOBAL WARMING' using $k=11$

Soln: Given message is

" STOP GLOBAL WARMING"

" $\underbrace{\overline{18}\ \overline{19}\ \overline{14}\ \overline{15}}$ $\underbrace{\overline{6}\ \overline{11}\ \overline{14}\ \overline{1}\ \overline{0}\ \overline{11}}$, $\underbrace{\overline{22}\ \overline{0}\ \overline{17}\ \overline{12}\ \overline{8}\ \overline{13}\ \overline{6}}$, "

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" 3 4 25 0   17 22 25 12 11 22   7 11 2 23 19 24 17 "
" DEZA   RWZMLW   HLCXTYR "

Q) Decrypt the Message using Shift Cipher Method.

"LEWLYPLUTL PZ H NYLHA ALHJOLY"
with $k=7$

" 11 4 22 11 24 15 11 20 19 11   15 25   7   13 24 11 7 0   0 11 7 9 14 11 "

" ~~8 1 19 8 21 12 8 17 16 8   12 22   4   10 21 8 4~~ "

" 4 23 15 4 17 8 4 13 12 4,   8 18   0   6 17 4 0 19,   19 4 0 2 7 4 "

→ " EXPERIENCE   IS A GREAT   TEACHER "

Affine Method

$$f(p) = ap+b \pmod{26}$$

Q) Use the mapping $f(p) = 3p+4 \pmod{26}$ and encrypt this word "AT"

Sol^n) "A T"

" 0   19 "

" 4   9 "

" E   J "

$$26 \overline{)\,61\,}(2$$
$$\underline{52}$$
$$9$$

$ap+b=t$

$ap=t-b$

$p = \left(\frac{t-b}{a}\right)$

UNIT- IV

GRAPH THEORY

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14th Oct

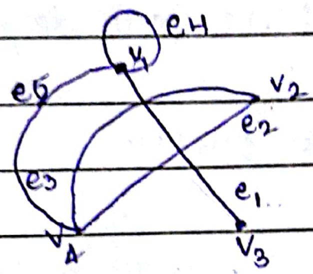
* Application of graph theory in Computer science:

→ In computer science graphs are used to represent Network of Communication and flow of computation

* Graph:- It is denoted by 'G'. Graph 'G' is a triple (V, E, G)

→ Draw a graph 'G' with the vertex $V = \{v_1, v_2, v_3, v_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$, $g(e_1) = \{v_1, v_3\}$, $g(e_2) = \{v_2, v_4\}$, $g(e_3) = \{v_2, v_4\}$, $g(e_4) = \{v_1, v_1\}$, $g(e_5) = \{v_1, v_4\}$

Sol:



$deg(v_1) = 4$
$deg(v_2) = 2$
$deg(v_3) = 1$
$deg(v_4) = 3$.

* Self loop:- An edge whose end vertices are same.

* parallel edges:- Two (or) more edges are called parallel edges when they have same initial and end edges

* Incidence:- Let e be an edge with end Vertices u, v, then the (unit) v is called Incident with e, and e is called incident with v.

$u \xrightarrow{e} v$

* Adjacent Vertices:- Two Vertices are said to adjacent if there is an edge b/w those two Vertices.

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* **Degree of a Vertex:-** Let G be a graph and v be a vertex in G then degree of v is the no. of edges passes through that vertex.
→ where we ~~can't~~ count degree of self loop twice.

* **Degree Sequence of the Graph:-** Let 'G' be a graph the degree of all vertices arranged in increasing order (or) ascending order is called "Degree Sequence".

* **Isolated Vertex:-** A vertex in a graph 'G' is called as isolated vertex iff its ~~(vert)~~ degree is "zero(0)".

* **pendent Vertex:-**
→ Iff its degree is One(1).

* **Null Graph:-**
→ If its has no edges then it is called Null Graph.

* **Simple Graph:-** A Graph having no self loop and no parallel edges.

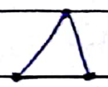
* **Multi Graph:-** A Graph 'G' having no Self loop but containing parallel edges.

* **Psuedo Graph:-** A graph having both self loop and parallel edges.

* **Complete Graph:-** In a Simple Graph 'G' if there exists edge between every distinct vertices.

15/oct

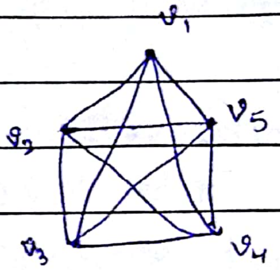
Q) Draw the Complete Graph for 5 and 6 points.



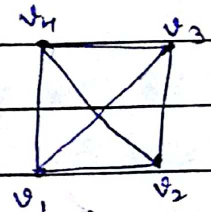
(3) points

K_3 Graph

[bcz degree of each vertex is 2 so K_2]



(5) points K_4



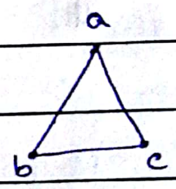
K_3.

* Regular Graph :- A Graph 'G' is said to be regular if degree of each Vertex is Same.

→ Every Complete Graph is called Regular graph. but every regular graph is not a Complete graph

* Handshaking Theorem :-

It states that Sume of the degree of all the Vertices in a graph 'G' is equal to twice the number of edges.



Soln:- $deg(a) = 2$; $deg(b) = 2$; $deg(c) = 2$

therefore,

$deg(a) + deg(b) + deg(c) = 6$

$= 2 \times 3$

$= 2e$ [$e \rightarrow$ no. of edges].

$$\sum_{i=1}^{n} deg(V_i) = 2e$$

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Theorem 2:- In any graph 'G' no. of odd degree Vertices are always even.

→ Let 'G' be the graph, $u_1, u_2 \ldots u_m$ be the odd degree Vertices and $v_1, v_2, \ldots v_k$ be the even degree Vertices. In totality no. of vertices are n.

Sol'n:- $u_1, u_2, \ldots u_m$ $[m+k=n]$
$v_1, v_2 \ldots v_k$

Applying Handshaking Theorem

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

$$\sum \deg(u_i) + \sum \deg(v_i) = 2e$$

Odd degree Vertex   even degree Vertex.

$$\underset{O}{\sum} \deg(u_i) = 2e - \underset{e}{\sum} \deg(v_i)$$

So, this is possible when no. of odd Vertices are even.

Q) Is the following Graph with the following degree of Vertices is possible or not.

(i) (2,2,2,2) ii) (1,1,1,1) iii) (1,1,2,2) iv) (1,1,1,2)

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Theorem :- In a Simple Graph 'G', the max no. of edges is equal to $n(n-1)/2$.

Sol'n:- Let 'G' be a graph with vertices $v_1, v_2, \ldots v_n$. Now vertex 'v_1' can be joined with the remaining 'n-1' vertices. Ily the max degree is again 'n-1'. proceding in this. we can say that max degree of v_n vertex is ~~...~~ 'n-1'.

Apply Handshaking theorem.

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

$$\deg(v_1) + \deg(v_2) + \cdots + \deg(v_n) = 2e.$$

$$\text{Max} \deg(v_1) + \text{Max} \deg(v_2) + \cdots + \text{Max} \deg(v_n) = 2e$$

$$(n-1) + (n-1) + \cdots + (n-1) = 2e$$

$$n(n-1) = 2e$$

$$\boxed{e = n(n-1)/2.}$$

Q) Is the following suitable possible in a Simple Graph 'G' with five vertices where the degree of each vertex is 1, 2, 3, 6, 2.

A No. because in this we have six so not possible bcz the max no of vertex ~~degree~~ is 4 but for 6 it is 5. So it is false.

Q) In Graph 'G' with 5 Vertices having 1, 2, 3, 2, 2. find ~~the~~ no. of edges.

$\Rightarrow$ $1+2+3+2+2 = 10$

$= 2\times 5 = 2e$

$\therefore e = 5$

~~if possible~~

Q) Find the no. of edges in Graph with 10 vertices & degree of each vertex is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Sol^n:- Applying Handshaking theorem.

$$\sum_{i=1}^{10} deg(v_i) = 2e$$

$$deg(v_1) + deg(v_2) + \dots + deg(v_{10}) = 2e$$

$$1+2+\dots+10 = 2e \qquad \left(\frac{n(n+1)}{2}\right)$$

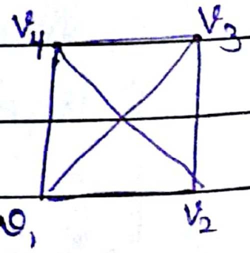
$$\frac{10(11)}{2} = 2e \Rightarrow 55 = 2e$$

$$\boxed{e = 55/2}$$

Theorem:-

Q) In a Complete Graph no. of edges is $n(n-1)/2$.

Sol^n:- Apply handshaking theorem



$$\sum_{i=1}^{10} deg(v_i) = 2e$$

$$deg(v_1) + deg(v_2) + \dots + deg(v_{10}) = 2e$$

$$(n-1) + (n-1) + \dots + (n-1) = 2e$$

$$n(n-1) = 2e$$

$$\boxed{e = \frac{n(n-1)}{2}}$$

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* degree in both graphs should be equal.
* No. of vertices & edges should be equal.

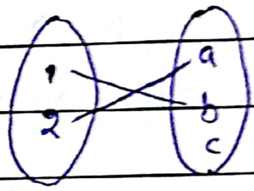
Isomorphoic graphs

1) **One-One Mapping:-** A mapping $f: A \to B$ is called 1-1 mapping $\forall\ x_1, x_2 \in A$.

if $x_1 \neq x_2$

$\Rightarrow f(x_1) \neq f(x_2)$.

Eg:-



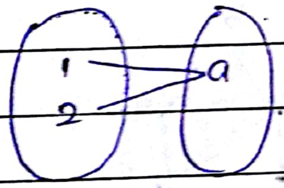
2) **On-to Mapping:-**

- A mapping $f: A \to B$ is called Onto if $\forall\ y \in B$.

$\exists$ some $x \in A$.

s.t $f(x) = y$.

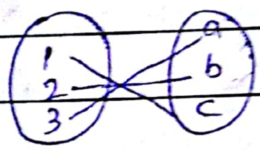
Eg:-

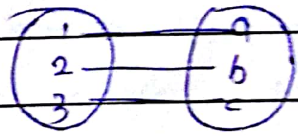


3) **Bijective Mapping:-**

- A mapping $f: A \to B$ is called Bijective if it is 1-1 and Onto.

Eg:-





* How to Check the Graph is Isomorphok (or) not.

→ Two Graphs 'G_1' and 'G_2' are called isomorphic if there exist ($\exists$) if $\exists\ f: G_1 \to G_2$. such that

1) f is 1-1 and onto

2) f preserves adjancies

· $(x, y) \in E$, then $(f(x), f(y)) \in E^2$

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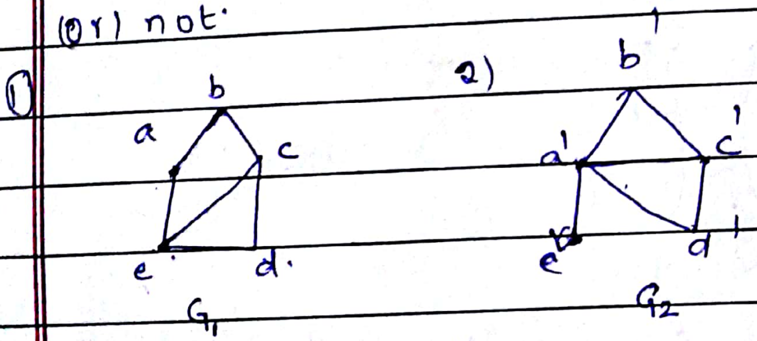
iii) f preserves non adjiancies.
$(x,y) \notin E_1$ then $(f(x), f(y)) \notin E_2$.

→ Steps to check Isomorphic (or) not.
→ First of all count no. of vertices in each graph and they must be same in counting.
→ Count the no. of edges in each graph and they must be equal in counting.
→ Count the degree of each vertex in both the graphs

* Check that whether the following graphs are isomorphic (or) not.



→ no. of vertices is same
→ no. of edges in both are graph are 6.
→ In the graph 'G_2' degree of vertex a' is 4, but there does not any vertex of degree '4' in Grap 'G_1'.
∴ It implies that these graphs are not isomorphic.

29/oct

Dijkastra's Algorithm

* What is the length of shortest path b/w A and z in the below graph.

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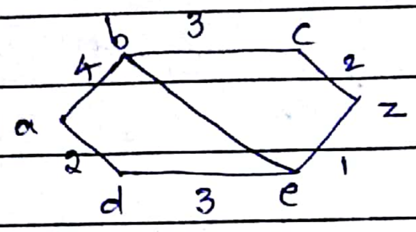
Eg:-



a — b = 4
a — d = 2 ✓

(min of b,d is d so we take d].

d — a = 2 ✗
d — e = 3

* if already ~~rea~~ written Once on left we cant ~~right~~ write it again.

d $\xrightarrow{2+3=5}$ e = 5 ✓

b — e = 7 ✓ (3)
b — e = 7 ✗ (3)

(min of b,e = b)

* [if one is repeated again then we should ✗ it)

e — z = 6
5 1

z

∴ Once the given one is given we no need to take for it. ~~and~~

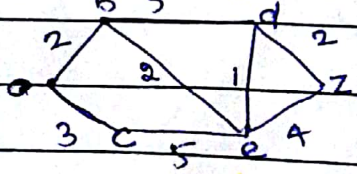
a → d → e → z

2 + 3 + 1 = 6 = Min distance.

∴ Here we dont take d-b bcz then the right ones are not ticked.

Eg:-

a to z



a — b = 2 ✓
a — c = 8 ✓

e — d = 4+1=5 ✓
e — z = 4+4=8.

b — d ⇒ 2+5=7 ✗
b — e = 2+2=4 ✓

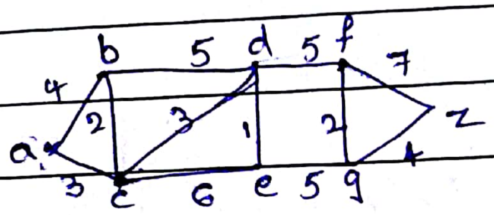
d — z = 5+2=7 ✓

z

c — e = 3+5=8 ✗

a → b → e → d – z = 2+2+1+2=7

Q)



g — z = 12+4=16 ✓

a — b = 4 ✓
a — c = 3 ✓

z

c — b = 3+2=5 ✗
c — e = 3+6=9 ✗
c — d = 3+3=6 ✓

a → c – d – e – g – z
3+3+1+5+4 = 16

b — d = 4+5=9 ✗

d — e = 6+1=7 ✓
d — f = 6+5=11 ✓

e — g = 7+5=12 ✓

f — z = 11+7=18 ✗
f — g = 11+2=13 ✗

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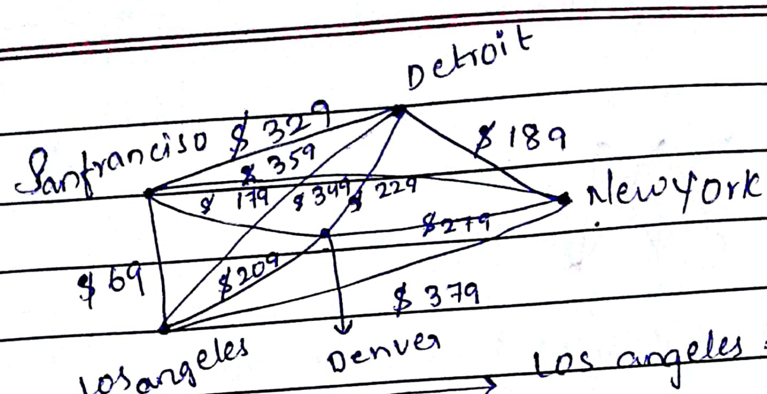
Q)



Los angeles = 69 ✓

San — Detroit = 329 $
San — Newyor = $ 359
San — Denver = $ 179

Denver — Detroit = 179 + 329
Denver — ~~LOS an = $ 209 + 179~~
Denver — New york = $ 279 + 179

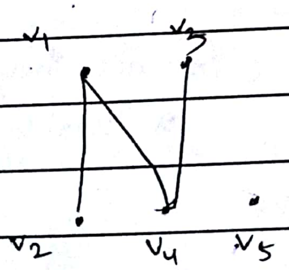
✗

31/Oct

* **Bi-partite graph:-**

$V = \{v_1, v_2, v_3, v_4, v_5\}$

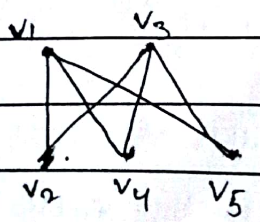
$x_1 = \{v_1, v_3\}$ $x_2 = \{v_2, v_4, v_5\}$



* **Complete Bi-partite graph:-**

$V = \{v_1, v_2, v_3, v_4, v_5\}$

$x_1 = \{v_1, v_3\}$, $X_2 = \{v_2, v_4, v_5\}$



$K_{mn} = K_{23}$

$m \rightarrow$ no. of vertices in (x_1)

$n \rightarrow (x_2)$

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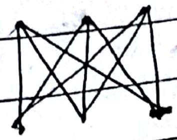
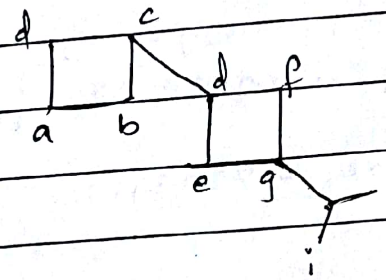
* $K_{3,3}$



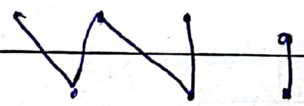
→ No. of edges in $K_{m,n}$ graph = $m \cdot n$

i) m ii) m/n iii) $m+n$ iv) $m \cdot n$ ✓

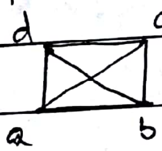
* <u>Connected</u> <u>Graph</u>:-

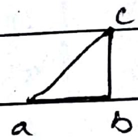


* <u>dis Connected</u>



* <u>Subgraph</u>:-





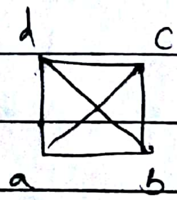
$V' \subseteq V$, $E' \subseteq E$.

Graph $G(V, E)$ → vertices, Edge.

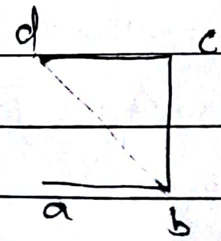
$G'(V', E')$

* In this they are taking as parts of graph.

* <u>Spanning</u> <u>Subgraph</u>:-



$G(V, E)$



$G'(V', E')$

$V' = V$, $E' \neq E$

* In this also we take parts of graph but every vertex should be present.

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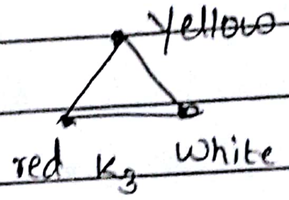
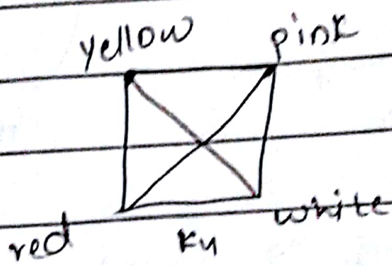
Cycle :-

* Chromatic no :-





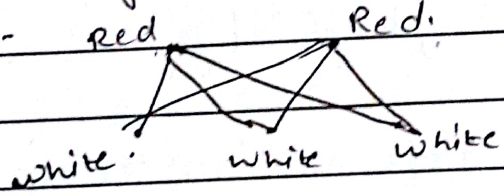
$\chi(K_3) = 3$ $\chi(K_4) = 4$

→ same colours should not be used in adj edges.

lly $\chi(K_n) = n$.

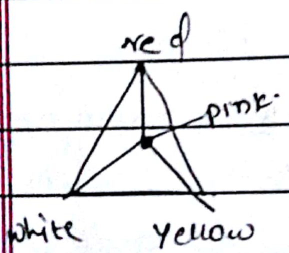
* for bi-partite chromatic no. does not depend on no. of edges

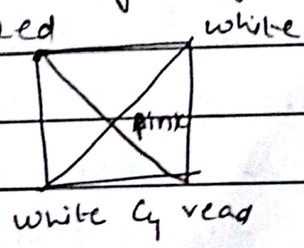
Eg :-



$\chi(K_{m,n} = 2)$.

* Chromatic no for Cycle graph.





$$\chi(W_n) = \begin{cases} 4 & \text{if } n \text{ is odd} \\ 3 & \text{if } n \text{ is even.} \end{cases}$$

$\therefore \chi(W_{11})$: 11 → odd so 4.

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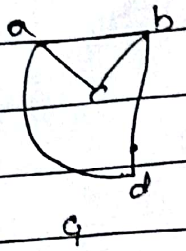
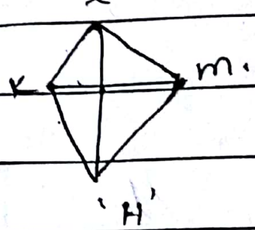
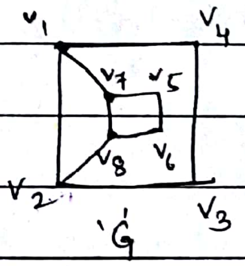
Q) Show that Graph shown in figure are isomorphic Or not.

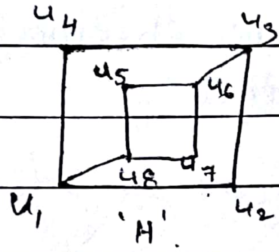




Sol'n:-
→ In both the graphs no. of vertices are equal.
→ In graph 'G' there are 5 edges & in Graph 'G' there are '6' edges
→ Hence these graphs are not isomorphic

** Imp
Q)

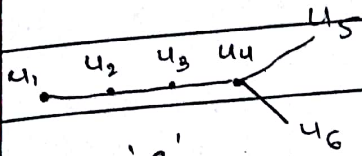




Soln:- In both the graps no. of vertices are equal.
→ No. of edges in both the graphs are equal.
→ In both the graph there are four vertices of degree 3 and four vertices of degree 2.
→ Now in the graph H vertex u_2 is a vertex of degree 2 and this Vertex Connected to two vertices u_1 & u_3 which are of degree 3. but in the graph 'G' there does not exist any vertex of degree '2' which is Connected to 2 vertices of degree 3. It implies that the structure of these two graphs are different.
→ Hence, these graphs are not isomorphic.

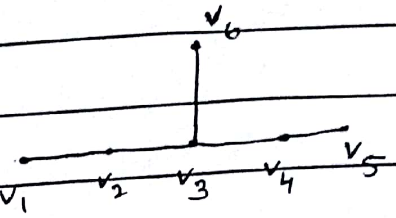
Q) Check the following graphs are isomorphic (or) not.

(i)



'G'.

(ii)



Sol'n:- No. of vertices & edges in both the graphs are same.

→ No. of vertices of degree '1' in both the graphs are '3'.

→ No. of vertices of degree '2' in both the graphs → 2.

degree '3' → '1'.

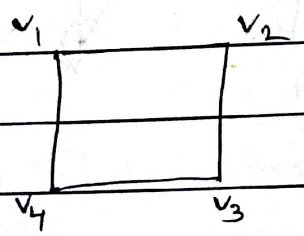
→ Despite satisfy these three properties these graphs are not isomorphic because in Graph 'G' u_4 vertex is connected to 2 pendent vertices.

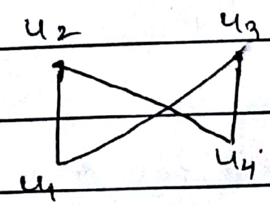
→ In Graph 'H' vertex 'v_3' is connected to only one (1) pendent vertices.

→ but here the structure is different.

→ Hence these are not isomorphic.

Q) Check that they are isomorphic (or) not.





→ In both graphs no. of vertices are same.

→ No. of edges in both graphs are same.

→ In both graphs we have four (4) vertices of degree '2'.

→ define a mapping.

$f(v_1) = u_1$
$f(v_2) = u_3$
$f(v_3) = u_2$
$f(v_4) = u_4$

→ $f(v_1)$ - we have to check the degree of u_1'
→ $f(v_2)$ - as v_2 is connected v_1 so, the value (u) should also connects to u_1'

Adjacency matrix of Graph 'G'

	v_1	v_2	v_3	v_4
v_1	0	1	0	1
v_2	1	0	1	0
v_3	0	1	0	1
v_4	1	0	1	0

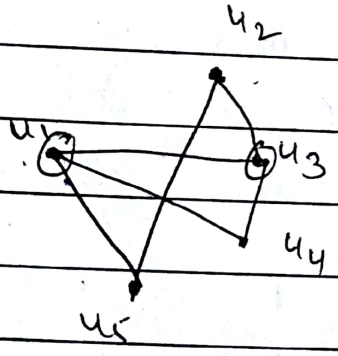
Here we have to write the value $f(v)$....

Graph H'.

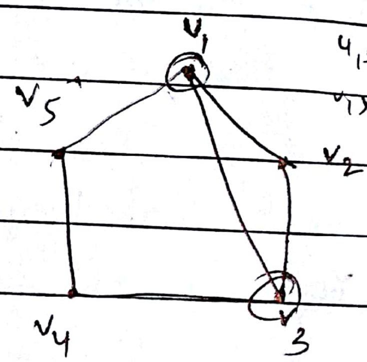
	u_1	u_3	u_4	u_2
u_1	0	1	0	1
u_3	1	0	1	0
u_4	0	1	0	1
u_2	1	0	1	0

* Check that the following Graphs are isomorphic (or) not.

Q)



ii)



u_1, u_3 = degree 3
v_1, v_3 = degree 3.

→ No. of vertices are same.
→ No. of edges are same.
→ In this there are 2 vertices of degree 3. and 3 vertices of degree 2.
→ Define the Mapping.

$f(u_1) = v_3$
$f(u_3) = v_1$
$f(u_4) = v_2$.

we take u_4 bcz it is connected to two sides of degree 3.

$f(u_5) = v_4$

$f(u_2) = v_5$

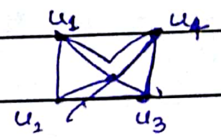
Adjacency ~~graph~~ matrix Graph 'G'

Adjacency ~~graph~~ matrix of Graph 'H'

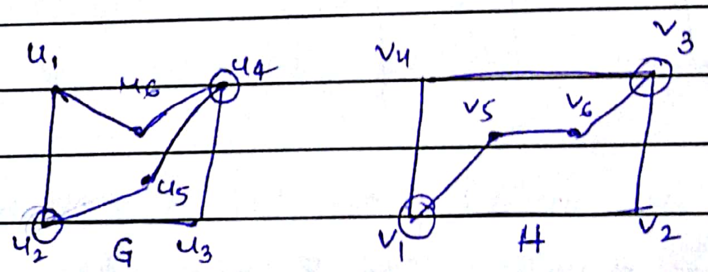
	u_1	u_2	u_3	u_4	u_5
u_1	0	0	1	1	1
u_2	0	0	1	0	1
u_3	1	1	0	1	0
u_4	1	0	1	0	0
u_5	1	1	0	0	0

	v_3	v_5	v_1	v_2	v_4
v_3	0	0	1	1	1
v_5	0	0	1	0	1
v_1	1	1	0	1	0
v_2	1	0	1	0	0
v_4	1	1	0	0	0

9/NOV. Check whether the graphs are isomorphic (or) not.



1) Imp



Sol: → In (the) both the graphs no. of vertices are same.

→ No. of edges are same.

→ In both the graphs we have 2 vertices of degree 3 and 4 vertices of degree 2.

→ Mapping.

$f(u_2) = v_1$, $f(u_3) = v_4$

$f(u_4) = v_3$, $f(u_6) = v_6$

$f(u_5) = v_2$, $f(u_1) = v_5$

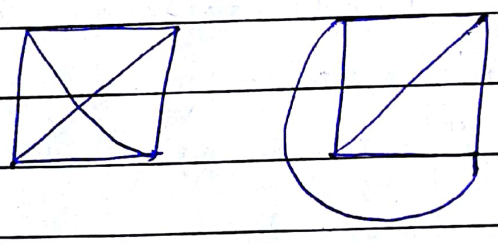
Adjancy matrix of Graph 'G'

~~Adjancy~~ matrix of Graph 'H'.

	u_1	u_2	u_3	u_4	u_5	u_6
u_1	0	1	0	0	0	1
u_2	1	0	1	0	1	0
u_3	0	1	0	1	0	0
u_4	0	0	1	0	1	1
u_5	0	1	0	1	0	0
u_6	1	0	0	1	0	0

	v_5	v_1	v_4	v_3	v_2	v_6
v_5	0	1	0	0	0	1
v_1	1	0	1	0	1	0
v_4	0	1	0	1	0	0
v_3	0	0	1	0	1	1
v_2	0	1	0	1	0	0
v_6	1	0	0	1	0	0.

* **planar Graph :-** A graph is said to be planar if it can be drawn in a plane. So that no edges cross.
→ for example the Graph 'K_4' is a planar Graph.



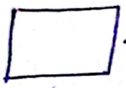
* **Euler's Theorem :-** prove that in a planar Graph with 'V' no. of vertices and 'E' no. of edges and 'R' no. of regions. ($V-E+R=2.$)

Sol :- We know prove this $V-E+R=2$ with the help of the mathematical Induction.

Step 1 :- $V=2,\ E=1,\ R=1$

$V-E+R = 2-1+1 = 2.$

Result is true, in this case.

 $\rightarrow G_K$ graph.

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Step 2:- let us Suppose that the result is true for this G_K graph, with $V_K \rightarrow$ no. of vertices, E_K no. of edges and $R_K \rightarrow$ no. of regions.

$$\boxed{V_K - E_K + R_K = 2}$$

Step 3:-

Subpart 1:- In this we increase one edge with the already existing two vertices.

G_{K+1}

$V_{K+1} = V_K$, $E_{K+1} = E_K + 1$, $R_{K+1} = R_K + 1$

$\therefore \rightarrow V_{K+1} - E_{K+1} + R_{K+1}$

$= V_K - (E_K + 1) + (R_K + 1)$

$= V_K - E_K - \cancel{1} + R_K + \cancel{1}$

$= V_K - E_K + R_K = 2.$

G_{K+1}

Subpart 2:-

We ~~know~~ increase One Vertex and now join this vertex with the One already existing Vertex.

$\rightarrow V_{K+1} = V_K + 1$, $E_{K+1} = E_K + 1$, $R_{K+1} = R_K$

$V_{K+1} - E_{K+1} + R_{K+1} = (V_K + 1) - (E_K + 1) + R_K$

$= V_K - E_K + R_K$

$= 2.$

$\rightarrow \therefore$ Result is true for G_{K+1} Graph.

$\rightarrow$ It implies that result is true for every planar graph.

* **Strongly Connected Graph:-** A directed graph is Called Strongly Connected if there is a directed path from any node (vertex) U to V. and vice-Versa.

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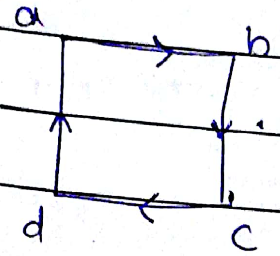
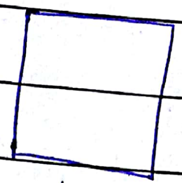


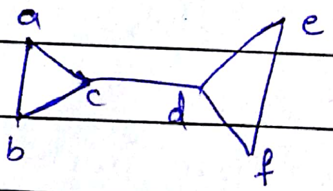
* In this we can go from a to b as well as b to a with direction.

* Weakly Connected graph :-
A directed graph is called a weakly connected graph if its undirected graph is connected. i.e., the graph obtained after the neglecting directions.



* Cut points/cut Vertices :-
Consider a graph 'G' a cut point for graph 'G' is a Vertex 'v' such that 'G-{v}' has more connected components than 'G'.

→ The subgraph G-{v} is obtained by deleting the vertex v from graph 'G' and also deleting all the edges connected to this vertex v.



* Cut Set :-
Consider a connected graph 'G'. A cut set for 'G' is a smallest set of edges such that removal of the set disconnects the graph, whereas the removal of any proper subset of this set left a connected subgraph.

→ for example determine the cut set for the Graph given below.

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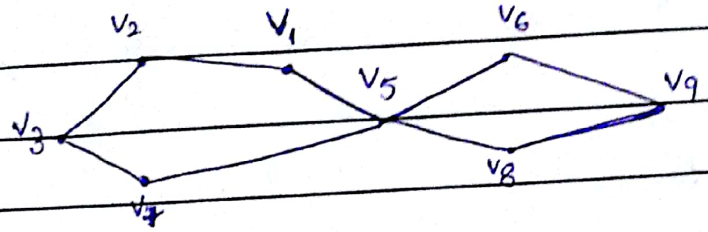

$\{(v_1, v_5), (v_7, v_5)\}$

$\{(v_5, v_6), (v_5, v_8)\}$

→ for this graph the edge set $\{(v_1, v_5), (v_7, v_5)\}$ is a cut set after the removal of this set, we left with a disconnected sub graph while after the removal of any of its proper subset. we left with a connected sub graph.

* Euler's path, Euler circuit, Graph (euler's)
* Hamilitonian path, circuit, Graph.

UNIT-5

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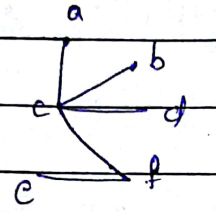
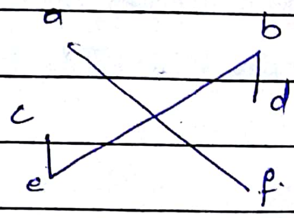
TREE

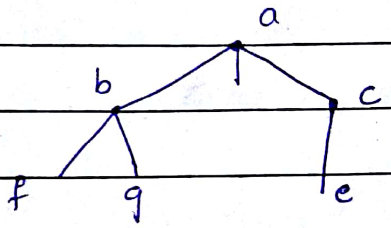


→ It is Connected graph so it is a tree.

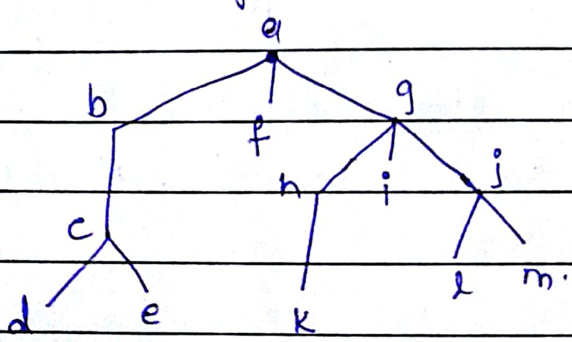


→ not a tree (bcz of disconnected graph).

Rooted tree :-



→ In the rooted tree T. find the parents of c, the children of g, the Sibilings of H all ~~and the sisters~~ ancestors of e, descendants of b all internal Vertices & leaf.



'T'.

1) parent of c is 'b'
2) children of g is h, i, j.
3) sibilings of H is i, j.

→ leaf :- Which don't have further Children.

→ internal vertices means the one which are further divided into children.

4) All ~~and sisters~~ ancestors of e are c, b, a.

5) Descandands of b are c, d, e.

6) All internal Vertices c, b, a, g, h, j.

7) leafs are d, e, f, k, i, l, m

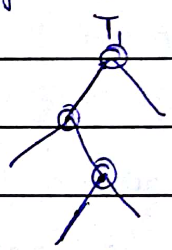
* M-ARY tree :- A rooted tree is m-ary tree if every internal vertex has no more than m childre

→ An m-ary tree with m = 2 is Called a binary tree.

* Full M-ary tree :-

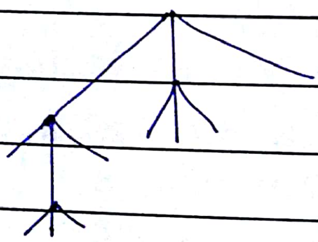
→ If internal vertices has exactly m = 2.

→ Are the rooted is Full m-ary tree for some +ve integer 'm'.



→ This tree is full binary tree.

Here $\boxed{m=2}$



Here $\boxed{m=3}$

→ Tree is full 3-ary tree.

* Ordered tree :-

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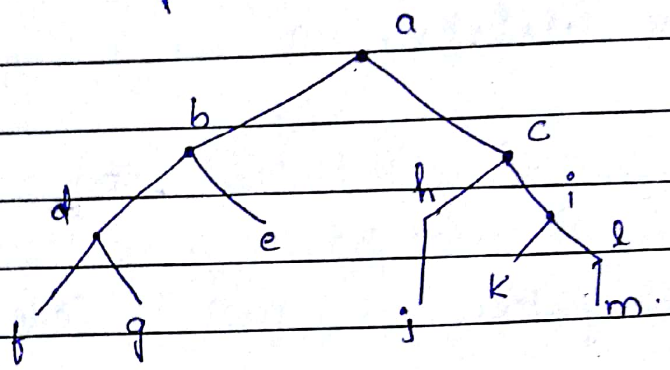
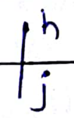
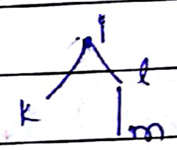
Q) what are the left and the right children of 'D' in the binary tree. what are the left and right subtree of 'c'.



1) left child of d if f. 2) right child of d is g.

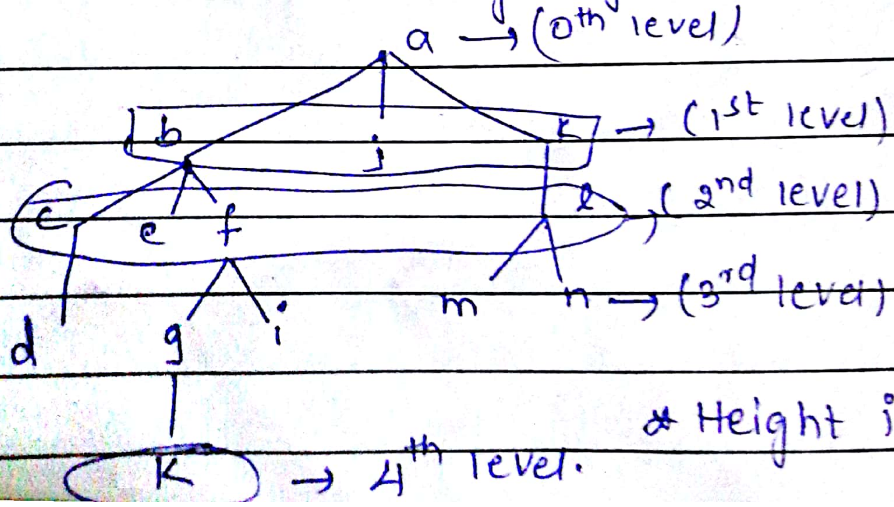
3) left subtree of c 4) right subtree of c





* The level of vertex 'v' in a rooted tree is the leng of unique path from the root to this vertex, the level of the root is defined to be zero. The heigth of the rooted tree is the longest path from the root to the vertex

Q) Find the level of each Vertex in the rooted tree and what is the height of this tree.



* Height is '4'.

Mon
12/Nov

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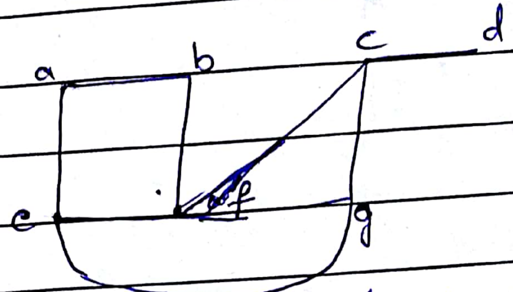
Spanning Tree :- Let 'G' be a graph a Spanning tree of 'G' is that tree Containing every vertex of 'G'.

eg: Find the Spanning Tree of the graph.



* This graph is Connected but it Contains Circuit
* Remove {a,e} i.e., we remove One circuit from the graph but still this graph Contains Circuits
* Now we remove {c,f} from the graph we get a Connected graph but there is again a circuit in this graph.
* Next we remove {e,g} edge from the graph, now as this graph is Connected and it doesn't Contain any Circuit so. It is a tree. but as this tree Contain all the Vertices of the graph 'G'. Hence it is a Spanning tree.

Spanning Tree.

*10M

Minimum Spanning Tree :- A minium S.T in a Connected Weighted graph is a Spanning Tree that has a Smallest possible sum of weights of its edges.

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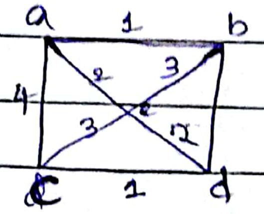
* Krushkal's Algorithm:-

→ It is a way for finding Min Spanning Tree.

Q) Use krushkal's Algorithm to find a Min Spanning Tree for the given weighted graph.



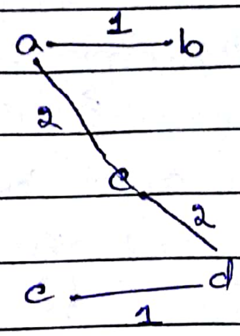
Soln:-

1 < ab ✓, cd ✓

2 < ae ✓, de ✓

3 < ce ✗, eb ✗, ~~ed~~, bd ✗

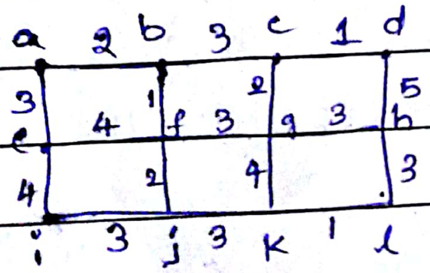
4 – ac ✗



Weight of Spanning Tree = 2+2+1+1 = 6.

* We shouldn't use the One (side) which forms a cycle bcz in Spanning there shouldn't be a Tree.

Q) Find the Min Spanning Tree for the given graph



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* if No. of vertices are 'n' then spannings should be 'n-1'.

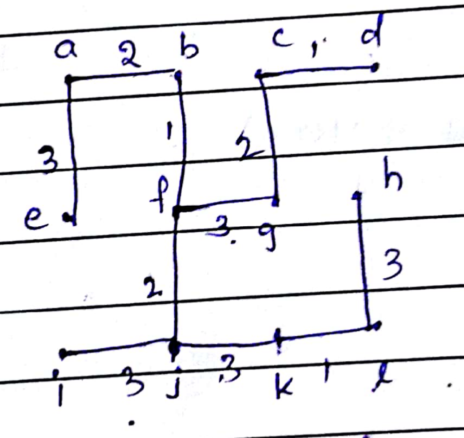
1
- cd ✓
- bf ✓
- kl ✓

4
- gk ✗
- ef ✗
- ei ✗

2
- ab ✓
- eg ✓
- fj ✓

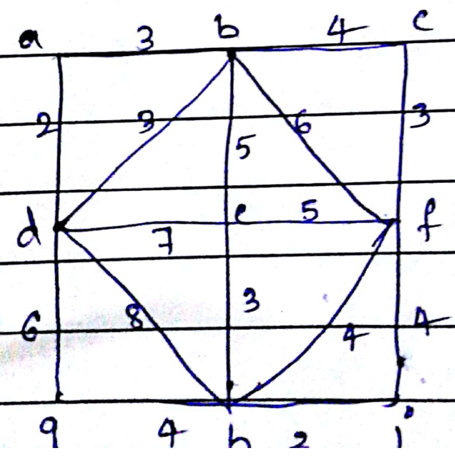
5
- dh ✗

3
- bc ✗
- ae ✓
- fg ✓
- gh ✗
- hl ✓
- jk ✓
- ij ✓



Weight of Min Spanning Tree (MST) = 24.

Q)



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3 — ab ✓, bd ✗, cf ✓, ch ✓

5 — be ✗, ef ✗

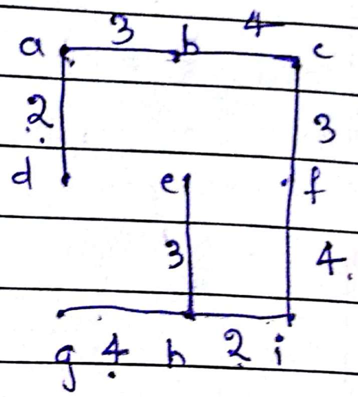
2 — ad ✓, hi ✓

6 — bf ✗, dg .

7 — de

4 — bc ✓, fi ✓, fh ✗, gh ✓

8 — dh



Weight of MST =