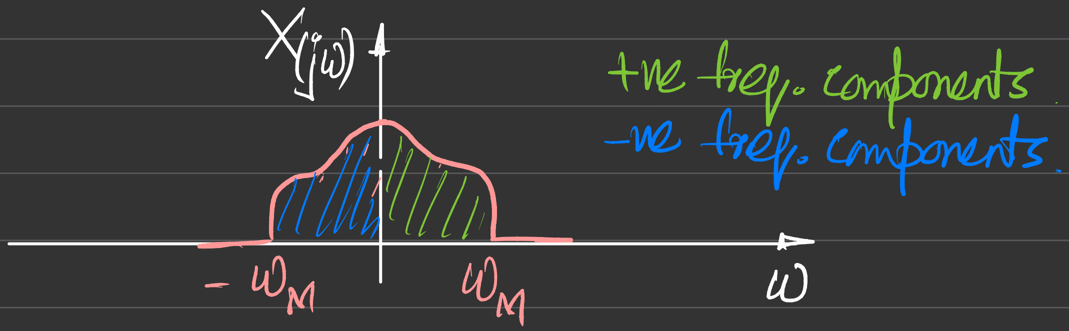
Signal Analysis & Communication ECE355

Ch. 8.4: Single-sideband Amplitude Modulation (AM)
Ch. 8.5: AM with Pulse-Train Carrier.

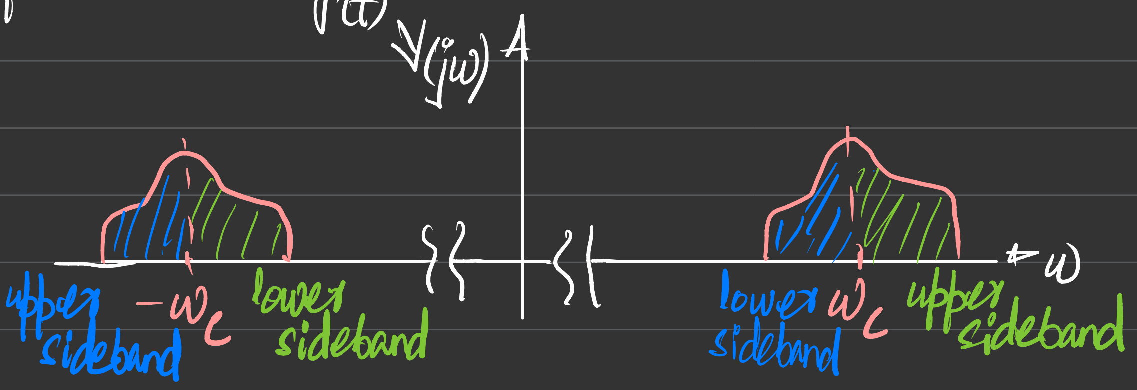
Lecture 34
04-12-2023

Ch. 8.4. Single-Sideband (SSB) Sinusoidal AM

- For the AM sys. discussed earlier, the total BW of the original sig. $x_{(t)}$ is $2\omega_M$, including both positive & negative frequencies, where ω_M is the highest freq. present in $x_{(t)}$
- With the use of complex exponential carrier ($e^{j\omega_c t}$), the spectrum is "$X(j(\omega-\omega_c))$", & the total width of the freq. band over which there is energy from the sig. is still $2\omega_M$.
- With a sinusoidal carrier, on the other hand, the spectrum of the sig. is shifted to $+\omega_c$ & $-\omega_c$, $\frac{1}{2}[X_{(j(\omega-\omega_c))} + X_{(j(\omega+\omega_c))}]$ thus twice the BW is required.
- The spectrum of $x_{(t)}$ is:



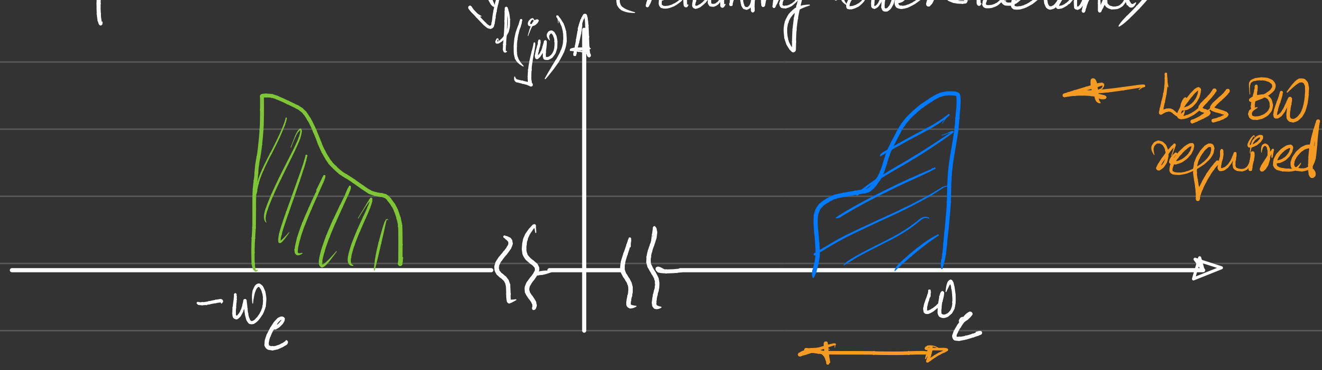
- The spectrum of $y_{(t)}$ is:



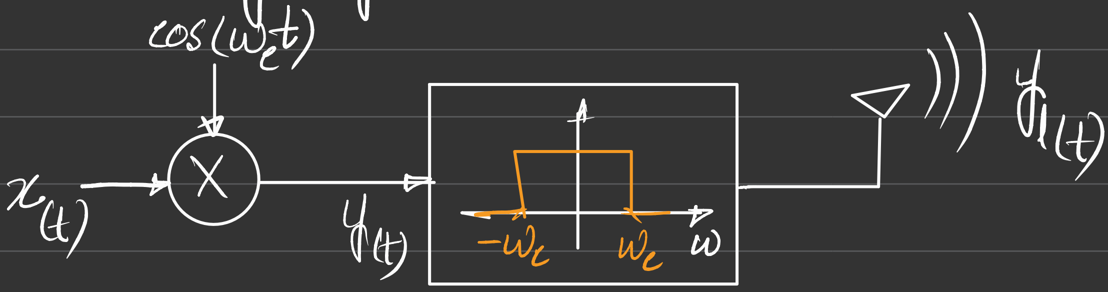
- Upper sideband contains the entire spectrum of $x_{(t)}$.
- Lower sideband contains the entire spectrum of $x_{(t)}$.
- $X_{(j\omega)}$ can be recovered if only the upper sidebands or the lower

sidebands can be retained.

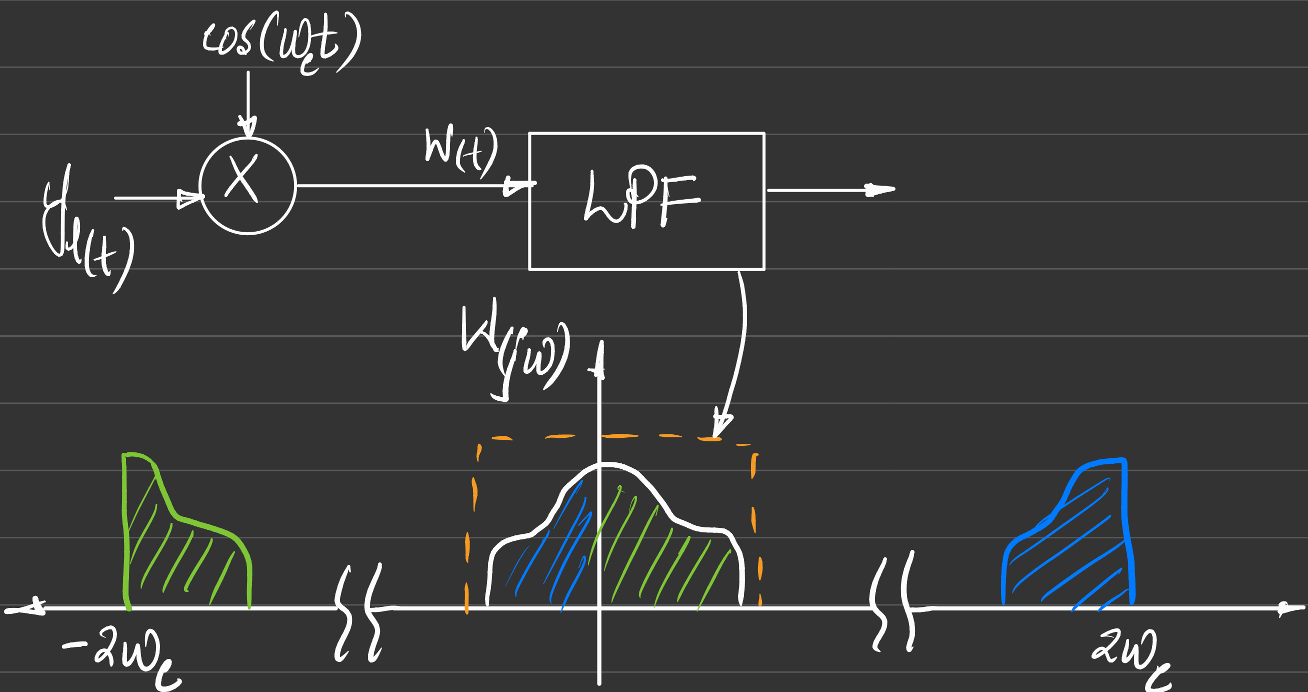
- For example: (retaining lower sideband)



$\Rightarrow$ transmitting only the lower sideband.

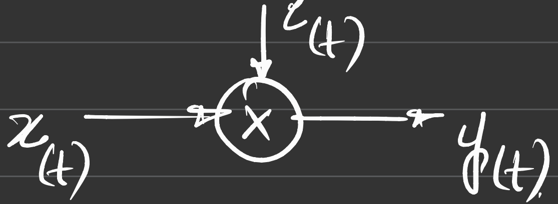


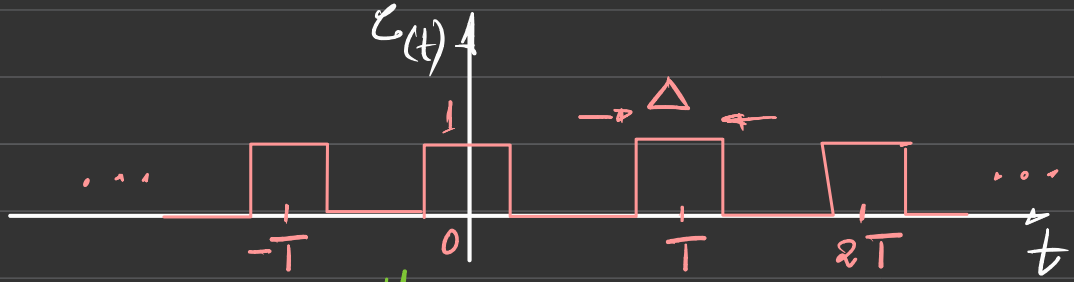
- At the receiver:

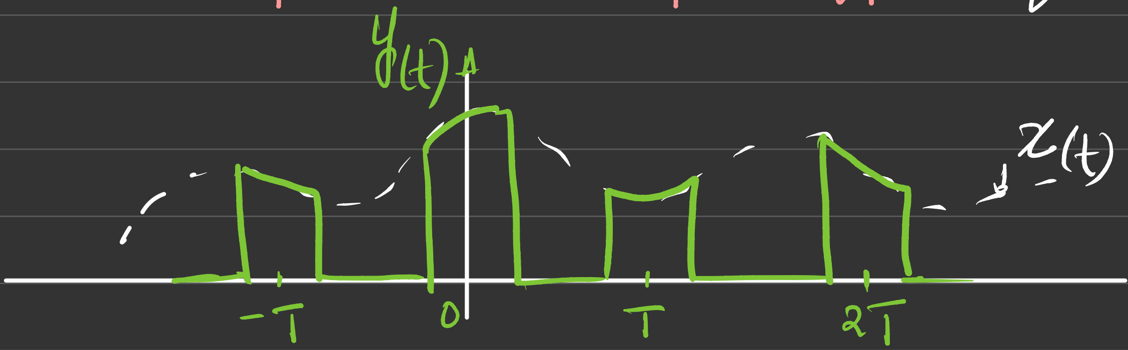


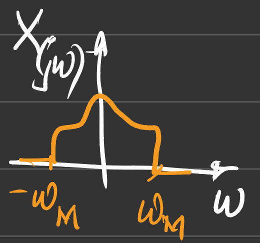
Ch. 8.5. AM With Pulse Train Carrier

- Use of a carrier that is a pulse train.









$$y(t) = x(t)\, c(t)$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)\, C(j(\omega-\theta))\, d\theta$$

where

$$C(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k\, \delta(\omega - k\omega_c)$$

FT of a periodic pulse train

$\omega_c = \frac{2\pi}{T} \Rightarrow \omega_c T = 2\pi$

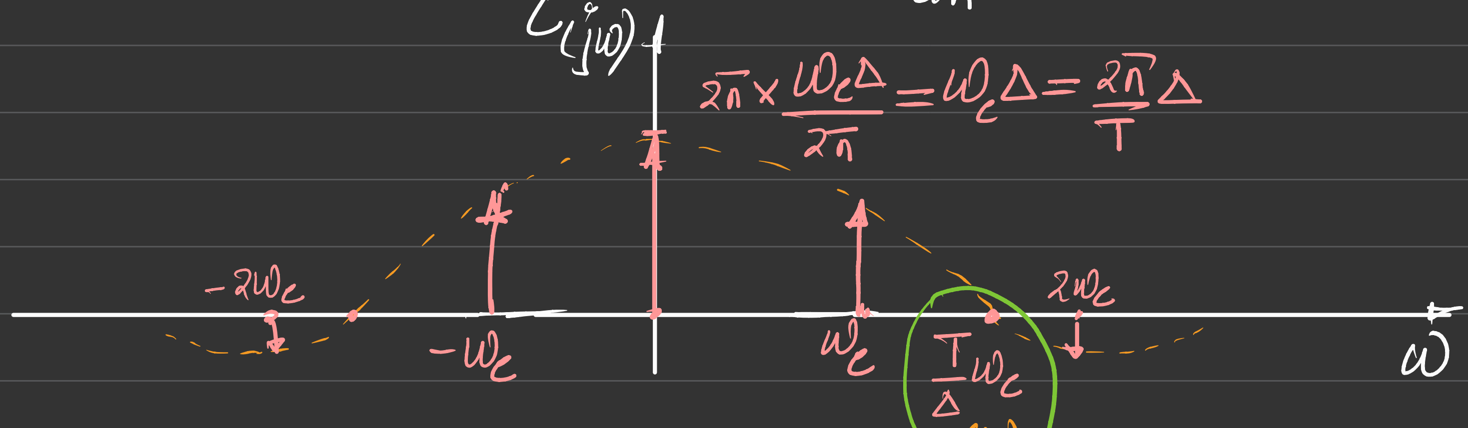
where

$$a_k = \frac{1}{T} \int_T c(t)\, e^{-jk\omega_c t}\, dt$$

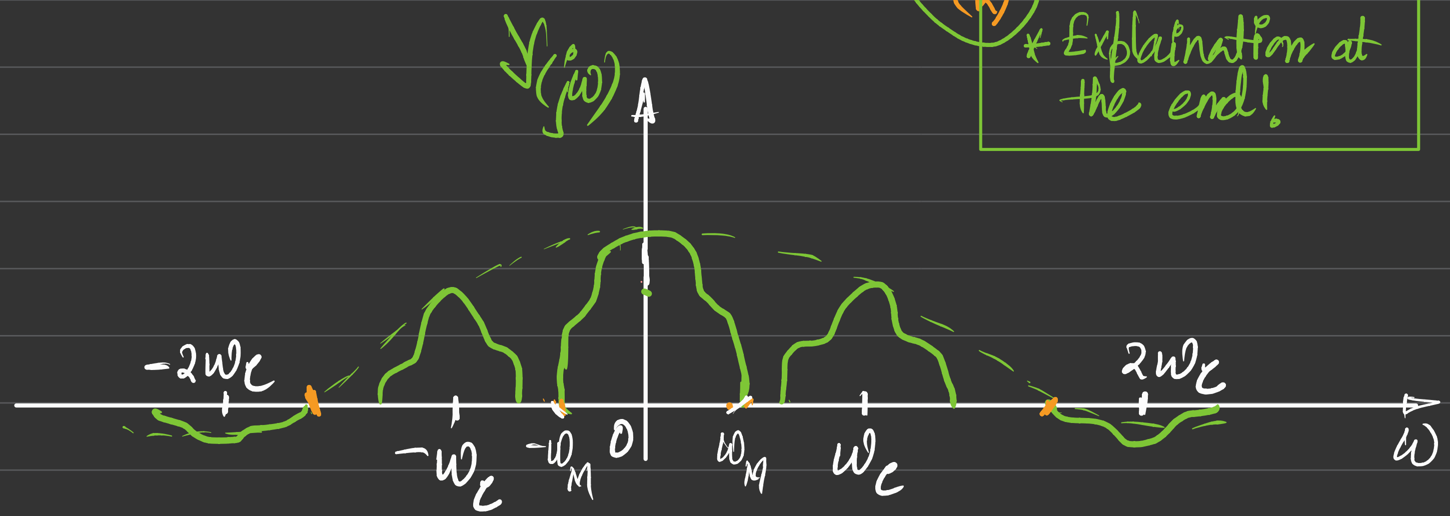
$$a_k = \frac{1}{T}\int_{-\Delta/2}^{\Delta/2} e^{-jk\omega_c t}\,dt$$

$$= \frac{1}{T} \times \frac{1}{-jk\omega_c}\, e^{-jk\omega_c t}\Big|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{\sin(k\omega_c \Delta/2)}{k\pi} = \frac{\sin\left(\frac{k\omega_c\Delta}{2\pi}\times\pi\right)}{\frac{k\omega_c\Delta}{2\pi}\times\pi} \times \frac{\omega_c\Delta}{2\pi}$$



* Explaination at the end!



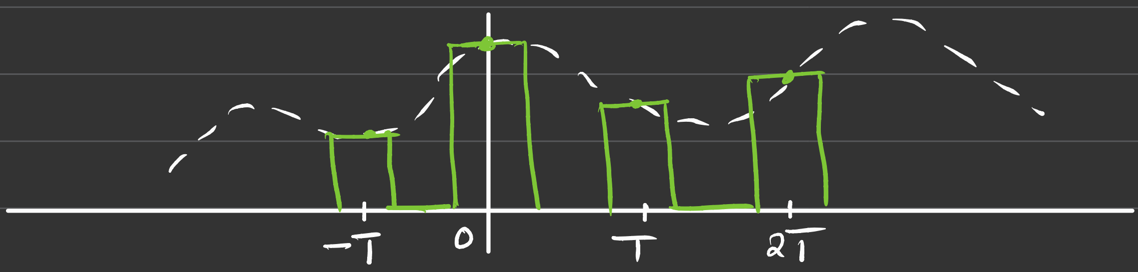
Receiver: Can recover $z(t)$ by LPF or by bandpass filter. (in case if $a_0 = 0$ or close to zero - BPF the one with larger a_k then demodulate)

Benefit: Low Complexity to implement.

if $\omega_c > 2\omega_M$ as per Nyquist sampling theorem.

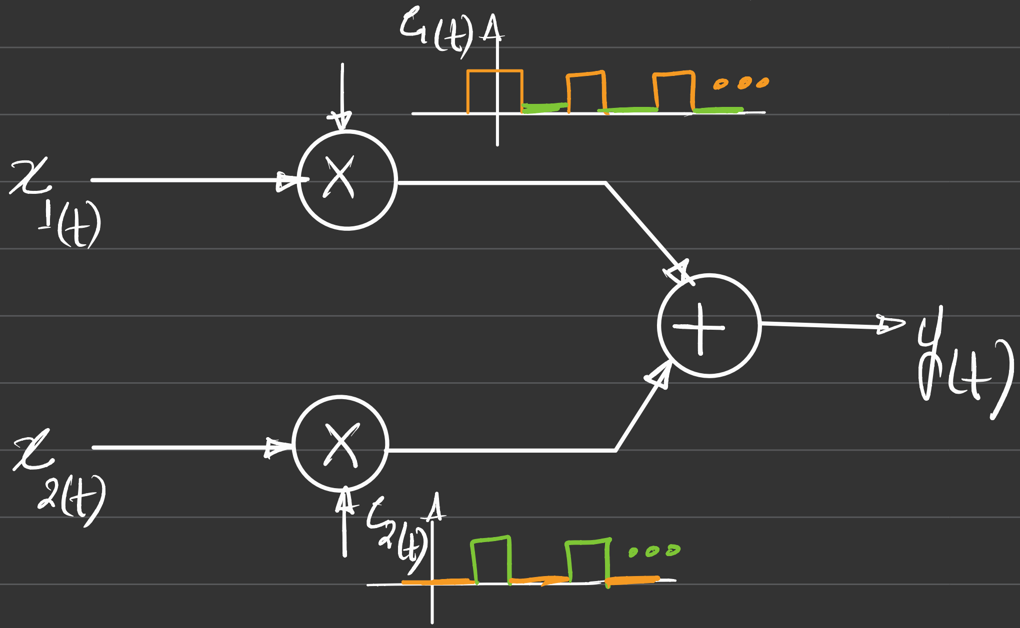
NOTE:

- Δ is not important.
- ($\Delta \to 0 \Rightarrow$ transmit samples $x(nT)$ only)
- Modern systems simply send pulse trains.



Example: Ethernet over twisted pair.

TIME DIVISION MULTIPLEXING



- Using time-shifted pulse trains to transmit several signals over a single channel.
- Each sig. in effect assigned a set of time slots of duration Δ that repeats every 'T' sec. & that does not overlap with the slots assigned to the other signals.
- The smaller the ratio Δ/T, the larger the number of signals that can be transmitted over a channel.

* Explaination.

- We Know: $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$

$= 0 \quad \text{for} \quad x = \pm 1, \pm 2, \ldots$

- Lets look at the case of $x = 1$. $\text{sinc}(1) = \frac{\sin \pi(1)}{\pi(1)} = 0$

- Let's apply it to the function we have in hand.

$$\text{sinc}\left(\frac{k\omega_c \Delta}{2\pi}\right) = \frac{\sin\left(\pi \times \frac{k\omega_c \Delta}{2\pi}\right)}{\pi \frac{k\omega_c \Delta}{2\pi}}$$

here $x = \frac{k\omega_c \Delta}{2\pi}$

$\text{sinc}(x) = 0$

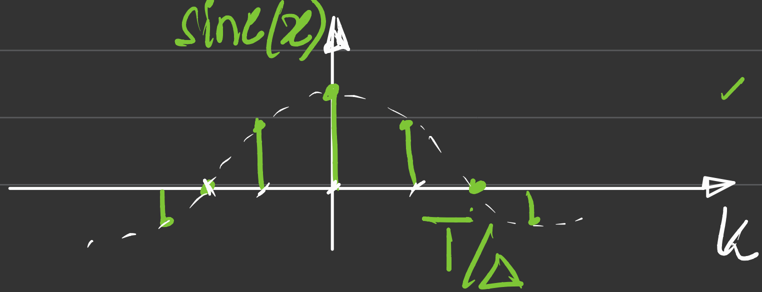
Let's look at the case when $x = 1$.

$$\Rightarrow x = \frac{k\omega_c \Delta}{2\pi} = 1.$$

$$\Rightarrow k = \frac{2\pi/\omega_c}{\Delta} = \frac{T}{\Delta}$$

Now we talk of in terms of k or $k\omega_0$

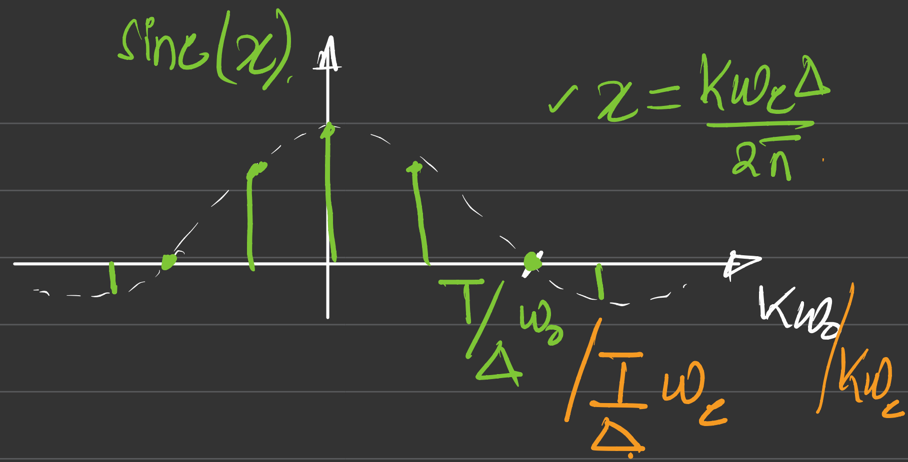
k :



$, x = \frac{k\omega_c \Delta}{2\pi}$

$k\omega_0 / k\omega_c$

here



i.e., $\boxed{\text{if } \operatorname{sinc}\left(\frac{k\omega_c\Delta}{2\pi}\right) = 0 \text{ at } k = \frac{T}{\Delta}}$

$\Rightarrow \boxed{\operatorname{sinc}\left(\frac{k\omega_c\Delta}{2\pi}\right) = 0 \text{ at } k = \frac{T}{\Delta}\omega_c}$

The rest is just the insertion of a_k in $C(j\omega)$

& then in $Y(j\omega) = \frac{1}{2\pi} X(j\omega) * \left[2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c)\right]$

and k of course here take integer values.

Remember

[We may have plot in CTFS as a_k vs. k.

or a_k vs. $k\omega_0$

& we have plot of CTFT as $C(j\omega)$ vs. ω

Check Lecture Notes-15 please!