EECS4214

Digital Communications

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Week 6

Week 6 - Lectures 11

1 Review of the Last Lecture

2 Delta Modulation

Review of the Last Lecture

Review

Formatting

Sampling

↓

Quantization

↓

PCM Coding

OR both together!

Each Sample → bits (depending how many levels)

Is there an alternative - MORE EFFICIENT WAY?

Will review in the lecture!

Delta Modulation

Delta Modulation

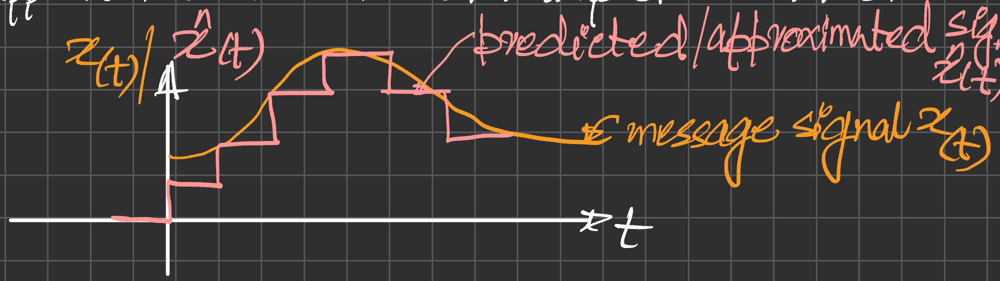
- Sample the base-band signal at a rate much higher than the Nyquist rate to increase the correlation between adjacent samples of the signal, so as to permit the use of a single quantization level strategy.
- The difference between the input and the approximation is quantized into only two levels, corresponding to positive and negative differences, respectively,
- In PCM, we encode every sample; whereas, in delta modulation we encode the difference between two sample values obtained at sampling instants as Δ.

Delta Modulation.

- The design of a PCM sys. involves many operations, which tend to make its practical implementation rather costly.
- To simplify the sys. design, we may use another digital pulse modulation technique - Delta Modulation (DM)

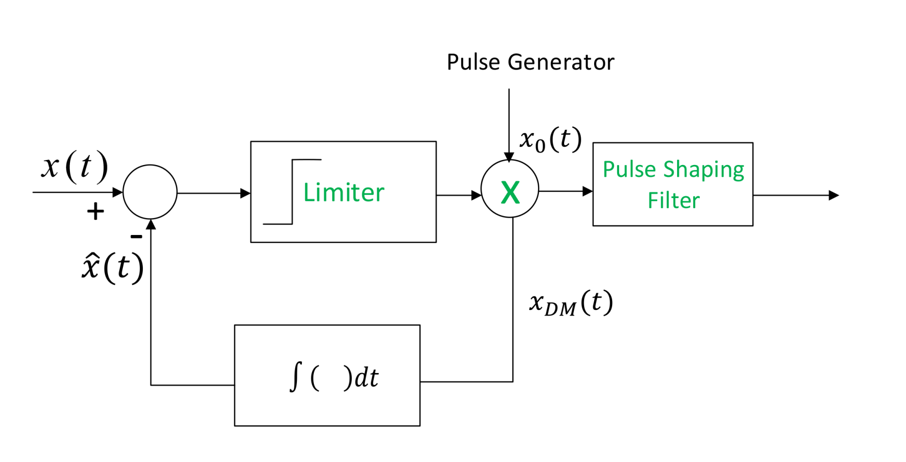
Basic Considerations

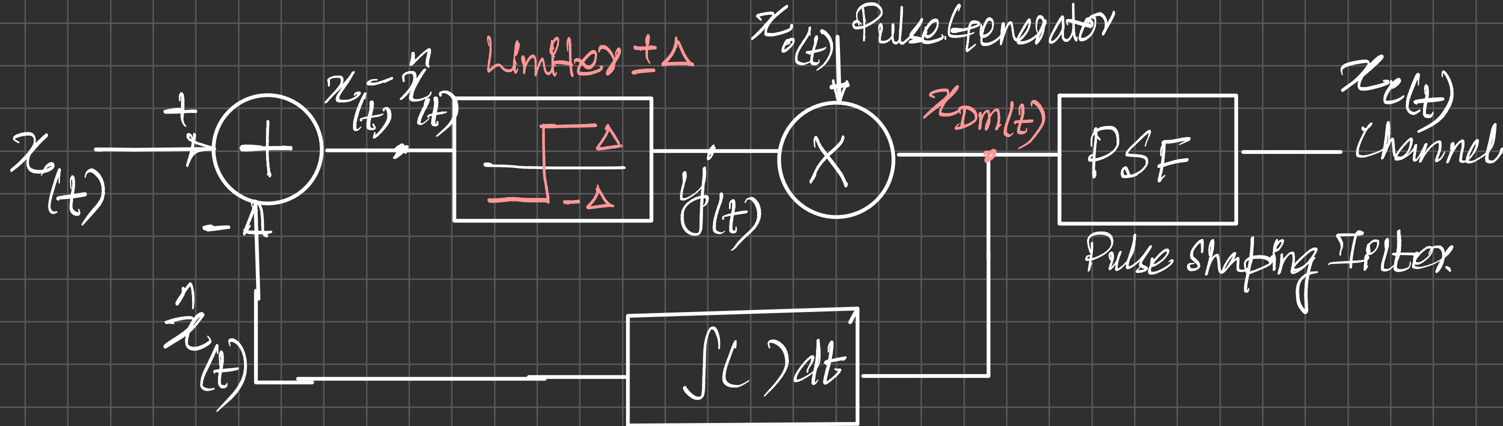
- An incoming sig. is oversampled - to purposely increase the correlation between the adjacent samples of the sig. (we'll see later!)
- DM provides a staircase approximation to the oversampled version of the message sig.



- Unlike PCM, the difference between the I/p sig. & its approximation is quantized into only two levels - namely, $\pm\Delta$, corresponding to positive & negative differences.
  - If $x_{(t)} - \hat{x}_{(t)} \geq 0$ we encode $+\Delta$
  - if $x_{(t)} - \hat{x}_{(t)} < 0$ we encode $-\Delta$
- Note: If we sample fast enough, the change between two samples is not big enough (we'll see later)
- Schematic Representation - Next page!

Schematic Representation





- Pulse Train $\quad x_{o(t)} = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$ $\quad T_s$ - Sampling Duration.

- Limiter

$$y_{(t)} = \begin{cases} +\Delta & (x_{(t)} - \hat{x}_{(t)}) \geq 0 \\ -\Delta & (x_{(t)} - \hat{x}_{(t)}) < 0 \end{cases}$$

- Delta-modulated signal $\quad x_{DM(t)} = \sum_{k=-\infty}^{\infty} d_k \delta(t-kT_s)$

where $d_k \in \{-\Delta, \Delta\}$

- Quantized Output Sig.

$$\hat{x}(t) = \int_{-\infty}^{t} x_{DM}(t)\,dt$$

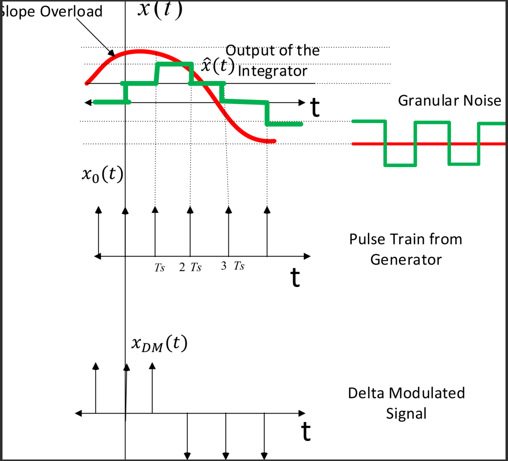
$$= \sum_{k=-\infty}^{\infty} d_k \int_{-\infty}^{t} \delta(t-kT_s)\,dt$$

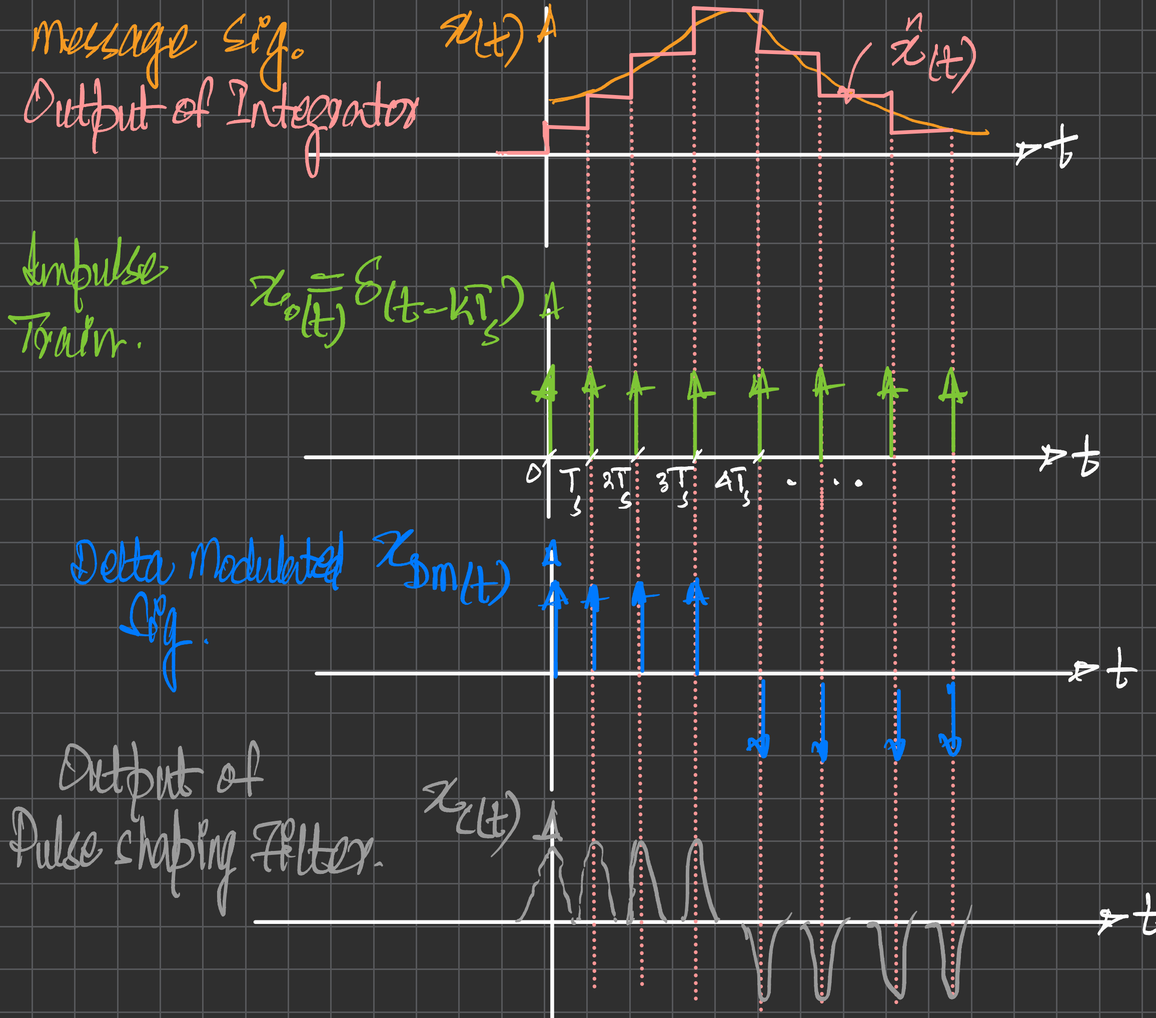
$$= \sum_{k=-\infty}^{\infty} \underbrace{d_k}_{\downarrow} \, u(t-kT_s)$$

$$= \sum_{k=-\infty}^{\infty} \overbrace{\Delta\,\mathrm{sgn}\big(x(t)-\hat{x}(t)\big)}^{*}\, u(t-kT_s)$$

$*\; d_k = \begin{cases} +\Delta \\ -\Delta \end{cases}$ depending on sgn argument

Graphical Illustration




message sig.
Output of Integrator
$x(t)$
$\hat{x}(t)$
t
Impulse Train.
$x_0(t) = \delta(t - kT_s)$
0
T_s
$2T_s$
$3T_s$
$4T_s$
$\cdots$
t
Delta Modulated Sig.
$x_{DM}(t)$
t
Output of Pulse shaping Filter.
$x_c(t)$
t

Schematic Representation of Receiver.

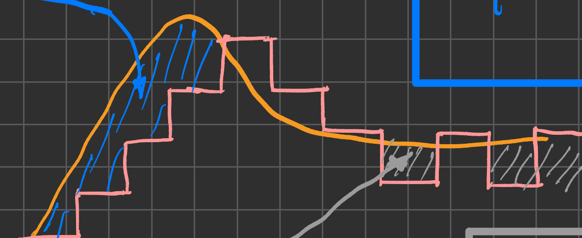
- The Integrator receives & recovers $\hat{x}(t)$, which is an approximation of the original sig.

Asynchronous Demodulation

$$x(t) \longrightarrow \boxed{\int(\,)\,dt} \longrightarrow \boxed{\text{LPF}} \longrightarrow \hat{x}(t)$$

- **Two Types of Noise:**

1. Slope Overload.



$\boxed{\text{Slope Overload} \uparrow \text{ if } \Delta\uparrow}$

2. Granular

$\boxed{\text{Granular} \uparrow \text{ if } \Delta\downarrow.}$

Trade-off between Slope Overload & Granular Noise

- To reduce slope overload, larger 'Δ' is needed.
- To reduce granular, smaller 'Δ' is needed.

(I) Slope Overload

- To avoid slope overload:

$$|x_{(t+T_s)} - x_{(t)}| \leq \Delta$$

Change in the sig. should be less than Δ

$$\frac{|x_{(t+T_s)} - x_{(t)}|}{T_s} \leq \frac{\Delta}{T_s} \quad \text{— (A)}$$

- We can write that the slope of the input sig. (or the rate of change of the I/P sig.) at any point is always at any point is always less than its max^m slope.

$$\frac{|x_{(t+T_s)} - x_{(t)}|}{T_s} \leq \left|\frac{dx_{(t)}}{dt}\right|_{max} \quad - \quad B$$

- Using A & B, we can write a condition that needs to be satisfied to prevent overload.

$$\left|\frac{dx(t)}{dt}\right|_{max} \leq f_s \Delta$$

- From here, we can also see that the slope overload can be prevented by either increasing f_s or Δ.

- Example:

See the slide!

Slope Overload

To avoid slope overload, the necessary condition is:

$$\left|\frac{dx(t)}{dt}\right|_{\max} \leq f_s \Delta$$

Clearly, slope overload can be prevented either by increasing quantization step Δ or sampling frequency f_s.

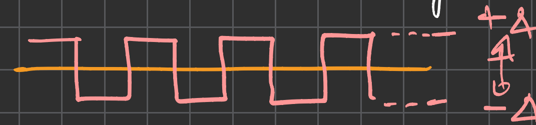
Example: Assume $x(t) = A_m \cos(2\pi f_m t)$, where $A_m = 1\text{V}$ and $f_m = 800\text{Hz}$. If $\Delta = 0.1\text{V}$, determine f_s.

Solution:

$$f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

Granular Noise:

- The granular noise error is uniformly distributed between $[-\Delta \;\; \Delta]$

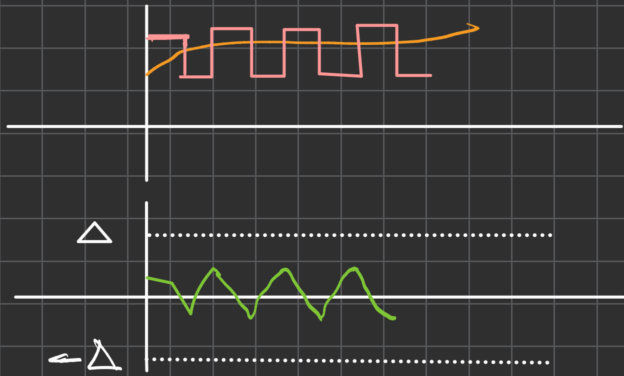


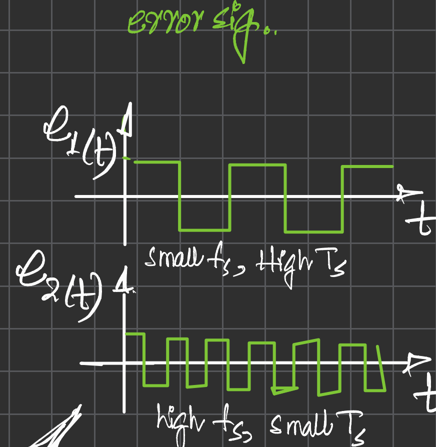
- Therefore, its probability distribution is given as:

$$f_Q(q) = \begin{cases} \dfrac{1}{2\Delta} & -\Delta \leq q \leq \Delta \\ 0 & \text{otherwise} \end{cases}$$

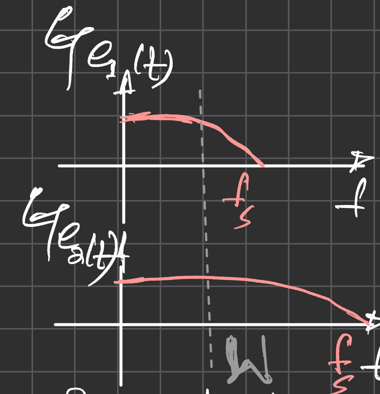
$$E[q^2] = \int q^2 f_Q(q)\,dq = \frac{q^3}{3}\left(\frac{1}{2\Delta}\right)\Bigg|_{-\Delta}^{\Delta} = \frac{2\Delta^3}{3(2\Delta)}$$

$$= \frac{\Delta^2}{3}$$





Error sig.



Power Spectral Density of Error sig.

Lets look at the errors closely:

Observation ① The error decorrelates fast with high f_s.

Observation ② The demodulator passes the error sig. through LPF,

Thus, Quantization Noise $\downarrow \propto W / f_s \uparrow$

Since the noise PSD is spreaded, LPF (W) ← less Noise Power!

∴ Observation ③ Effective Noise Power ≃ $\frac{W}{f_s} \frac{\Delta^2}{3}$

SQNR

$$SQNR = \frac{E[x^2(t)]}{N_0}$$

$$\boxed{SQNR = 10\log_{10} E[x^2(t)] - 10\log_{10} \frac{W}{f_s}\frac{\Delta^2}{3}}$$

Granular Noise

The granular noise is uniformly distributed between $-\Delta$ and Δ.

$$f_{n_g}(n_g) = \frac{1}{2\Delta}, -\Delta \leq n_g \leq \Delta$$

The average noise power can thus be given as: $\mathbb{E}[n_g^2] = \frac{\Delta^2}{3}$

After passing the noise power through a low pass filter of bandwidth W, the effective noise power can be given as

$$N_{\text{eff}} = \frac{W}{f_s}\mathbb{E}[n_g^2] = \frac{W}{f_s}\frac{\Delta^2}{3}$$

Problem

Consider a sinusoidal signal with maximum frequency of 3.4KHz and maximum amplitude of 1 volt. This speech signal is applied to a delta modulator whose bit rate is set at 60 kbit/sec. Explain the choice of an appropriate step size for the modulator.

Solution:

$$f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

Problem

Consider a delta modulator system designed to operate at W=4 times the Nyquist rate for a signal. The step size of the quantizer is 400mV.

a) Find the maximum amplitude of a 1KHz input sinusoid for which the delta modulator does not show slope overload.

b) Find post-filtered output SNR when the filter bandwidth is W.

Solution:

$$\text{(a) } f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

$$\text{(b) SNR}_o = \frac{A_m^2/2}{(\frac{W}{f_s})(\frac{\Delta^2}{3})} = \frac{3A_m^2 f_s}{2W\Delta^2} = \frac{3f_s(\frac{f_s\Delta}{2\pi f_m})^2}{2W\Delta^2} = \frac{3}{8\pi^2 W f_m^2 T_s^3}$$

Thank You
Happy Learning
☺