

EECS4214

Digital Communications

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Week 6

Week 6 - Lecture 12

1 Review of the Last Lecture

2 Delta Modulation(contd.)

3 Multiplexing

Review of the Last Lecture

Review

Delta Modulator.

- Sending the difference b/w current sample & the sig. abx.
- More efficient - reduces $\frac{\text{No. of bits/sample}}{\text{lower BW}}$.
- Suitable for narrow-band CS, such as voice transmission where rapid changes are less frequent!
- Limitations.
 - Slope Overload.
 - Granular.

Will review in the lecture!

Delta Modulation(contd.)

Granular Noise:

- The granular noise error is uniformly distributed between $[-\Delta, \Delta]$

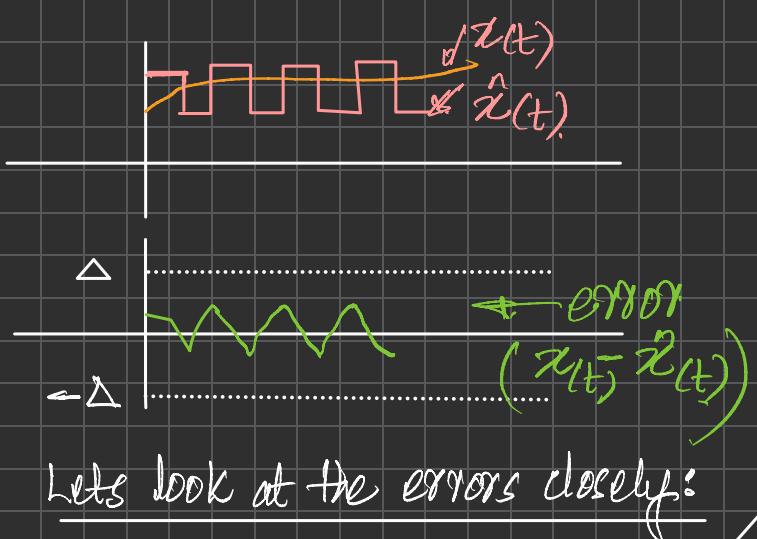


- Therefore, its probability distribution is given as:

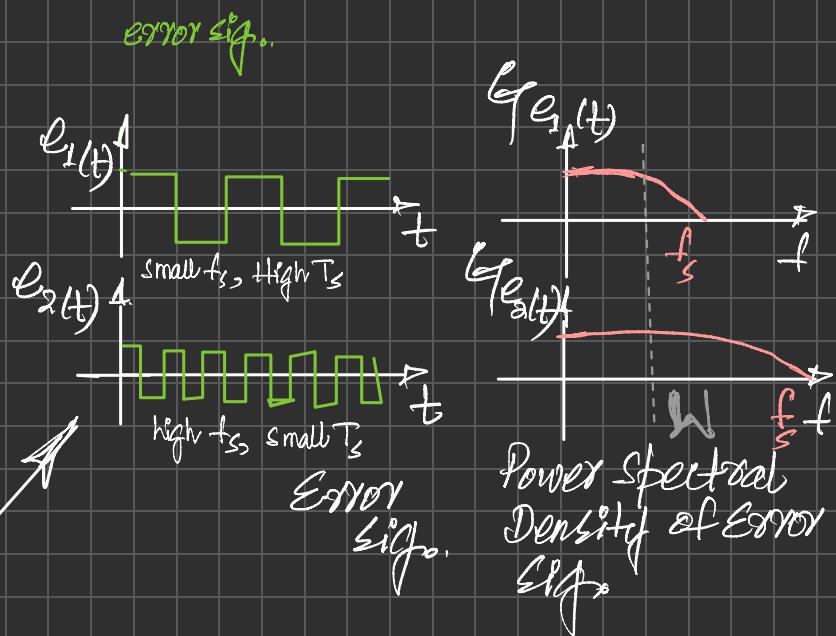
$$f_Q(q) = \begin{cases} \frac{1}{2\Delta} & -\Delta \leq q \leq \Delta \\ 0 & \text{otherwise} \end{cases}$$

$$E[q^2] = \int q^2 f_Q(q) dq = \frac{q^3}{3} \left(\frac{1}{2\Delta} \right) \Big|_{-\Delta}^{\Delta} = \frac{2\Delta^3}{3(2\Delta)}$$

$$= \frac{\Delta^2}{3}$$



Let's look at the errors closely:



Observation (1) The error decorrelates fast with high f_s .

Observation (2) The demodulator passes the error sig. through LPF,
Thus, Quantization Noise $\propto \frac{K}{f_s}$

Since the noise PSD is spreaded, LPF (K) \rightarrow less noise power!

Observation ③ Effective Noise Power $\leq \frac{W}{f_s} \frac{\Delta^2}{3}$

SQNR

$$SQNR = \frac{E[X^2(t)]}{N_0}$$

$$SQNR = 10 \log_{10} E[X^2(t)] - 10 \log_{10} \frac{W}{f_s} \frac{\Delta^2}{3}$$

Granular Noise

The granular noise is uniformly distributed between $-\Delta$ and Δ .

$$f_{n_g}(n_g) = \frac{1}{2\Delta}, -\Delta \leq n_g \leq \Delta$$

The average noise power can thus be given as: $\mathbb{E}[n_g^2] = \frac{\Delta^2}{3}$

After passing the noise power through a low pass filter of bandwidth W , the effective noise power can be given as

$$N_{\text{eff}} = \frac{W}{f_s} \mathbb{E}[n_g^2] = \frac{W}{f_s} \frac{\Delta^2}{3}$$

Problem

Consider a sinusoidal signal with maximum frequency of 3.4KHz and maximum amplitude of 1 volt. This speech signal is applied to a delta modulator whose bit rate is set at 60 kbit/sec. Explain the choice of an appropriate step size for the modulator.

Solution:

$$f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

$$f_m = 3.4 \text{ kHz}$$

$$A_m = 1V$$

$$R_b = 60 \text{ kbps}$$

$$\Delta = ?$$

We have derived.

$$A_m(2\bar{n}f_m) \leq f_s \Delta$$

$$f_s \geq \frac{A_m(2\bar{n}f_m)}{\Delta}$$

Problem

Consider a delta modulator system designed to operate at $W=4$ times the Nyquist rate for a signal. The step size of the quantizer is 400mV.

- Find the maximum amplitude of a 1KHz input sinusoid for which the delta modulator does not show slope overload.
- Find post-filtered output SNR when the filter bandwidth is W .

Solution:

$$E\{x^2\}$$

$$(a) f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

same as prev. as the I(p)
is sinusoidal here as well!

$$(b) \text{SNR}_o = \frac{\frac{A_m^2/2}{(\frac{W}{f_s})(\frac{\Delta^2}{3})}}{=} = \frac{3A_m^2 f_s}{2W\Delta^2} = \frac{3f_s (\frac{f_s \Delta}{2\pi f_m})^2}{2W\Delta^2} = \frac{3}{8\pi^2 W f_m^2 T_s^3}$$

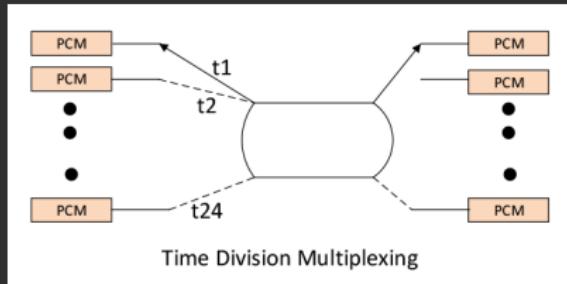
$$\begin{aligned} E\{x^2\} &= \int_0^T A_m^2 \sin^2 \omega_m t dt \\ &= A_m^2 / 2. \end{aligned}$$

Granular!

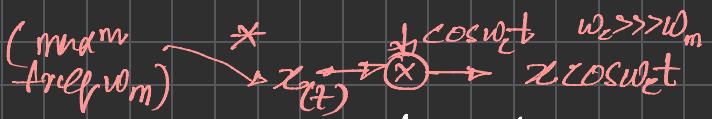
Multiplexing

What is Multiplexing?

- A method by which multiple analog or digital signals are combined to be sent over a shared communication medium
- Types of multiplexing
 - Frequency division multiplexing (~~Neat Page~~)
 - Time division multiplexing (~~Neat to Next Page~~)
 - Space division multiplexing or Spatial multiplexing

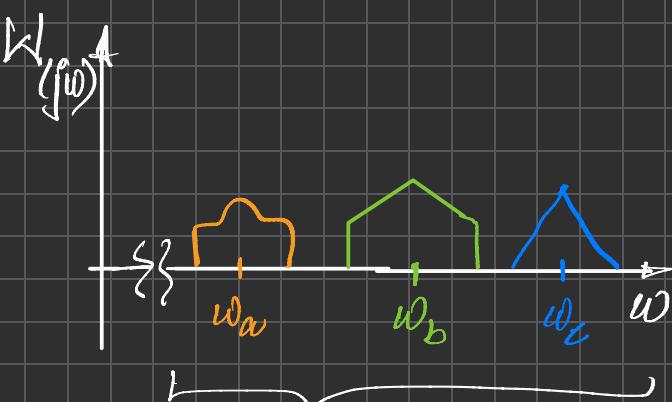
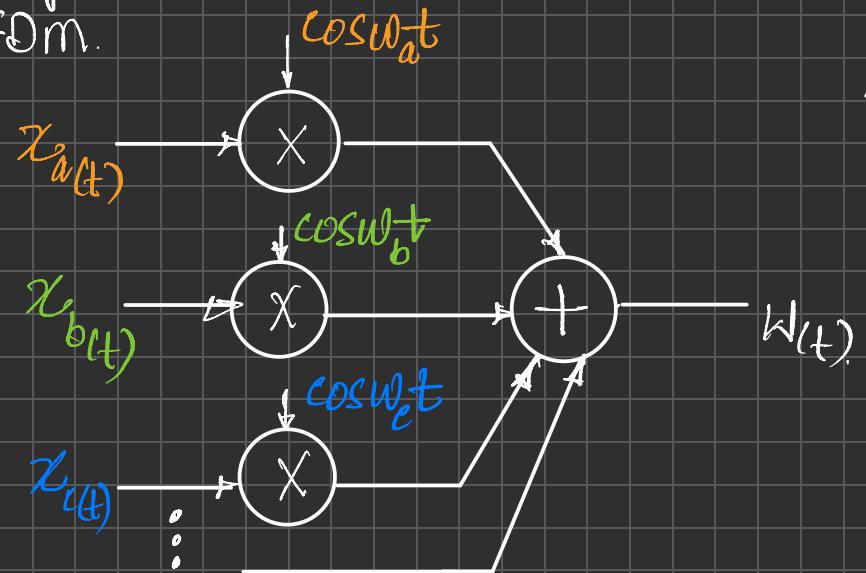


Freq. Division Multiplexing (FDM)



- To simultaneously transmit multiple signals over a single wideband channel
- if the individual voice signals, which are overlapping in freq., have their freq. content shifted by means of sinusoidal Am, for example - so that the spectra of the modulated signals no longer overlap.
- They can be transmitted simultaneously over the same channel.

- FDM.



FDM.
Example: Radio channels.

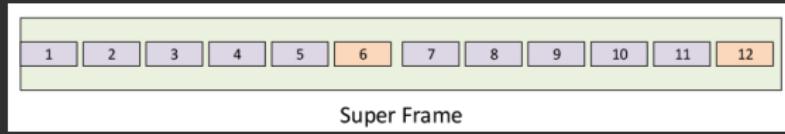
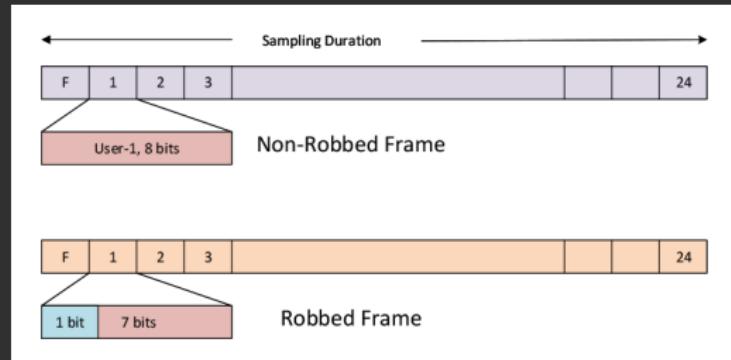
What is Multiplexing?

Time Division multiplexing

- T1-carrier system has been adopted in USA, Canada, and Japan
- Accommodate 24 voice signals
- The voice signals are filtered with low-pass filter (cutoff 3400 Hz)
- The filtered signals are sampled at 8KHz
- The μ -law companding technique is used with $\mu = 255$
- Sampling rate of 8KHz
- Each non-robed frame has twenty four 8-bit words from each user plus a single framing bit for synchronization
- Each frame consists of a total 193 bits. Each frame is of duration 125μ sec, correspondingly, the bit rate is 1.544 Mbps

Robbed/Non-Robbed Frames

- In a robbed frame, 1 bit per user is used for signaling purposes
- A super frame is composed of 12 frames and two frames are robbed (i.e., every 6th frame is a robbed frame)
- Total duration of a super frame is $12 \times 125\mu s$



Rate Calculations

■ Data rate per frame for all users

■ Data rate per frame for synchronization

■ Data rate per non-robbed frame for a user data

■ Data rate per robbed frame for a user data

■ Data rate per super frame of a user

■ Signaling data rate

$8000/6 \times 24 = 32 \text{ kbps}$.
Every 1st frame is
robbed for signalling.
= channels

$$24 \times 8 = 192 + 1 = 193; 193/125 \mu = 1.544 \text{ Mbps}$$

↑ framing bit in each frame is used for synchronization.

$$8000 \text{ frames/sec} \text{ as } f_s = 8 \text{ Ksamp./sec.}$$

$$1 \times 8000 = 8000 \text{ kbps.}$$

→ Nonrobbed: 8 bits/frame.

$$\Rightarrow 8 \times 8000 = 64 \text{ kbps.}$$

↑ for a user

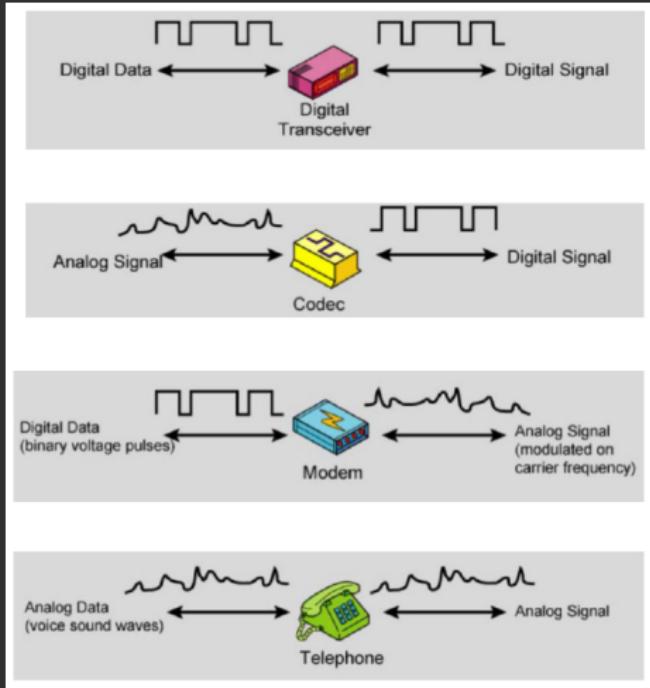
Robbed: 7 bits/frame.

$$\Rightarrow 7 \times 8000 = 56 \text{ kbps.}$$

$$\text{Data in n superframe} = \underbrace{10 \times 8}_{\text{Non Rob.}} + \underbrace{2 \times 7}_{\text{Robbed.}} = 94$$

$$8000/12 \times 94 = 62.67 \text{ kbps.}$$

Analog vs Digital (Data and Transmissions)



Thank You
Happy Learning

