EECS4214

Digital Communications

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Week 6

Week 6 - Lecture 12

1 Review of the Last Lecture

2 Delta Modulation(contd.)

3 Multiplexing

Review of the Last Lecture

Review

Delta Modulator.

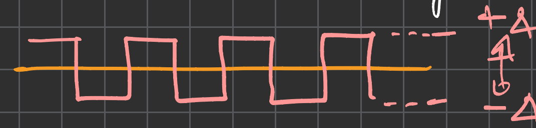
- Sending the difference b/w current sample & the sig. apx.
- More efficient – reduces No. of bits/sample, lower BW.
- Suitable for Narrow-band CS, such as voice transmission where rapid changes are less frequent!
- Limitations.
  - Slope Overload.
  - Granular.

Will review in the lecture!

Delta Modulation(contd.)

Granular Noise:

- The granular noise error is uniformly distributed between $[-\Delta \; \Delta]$

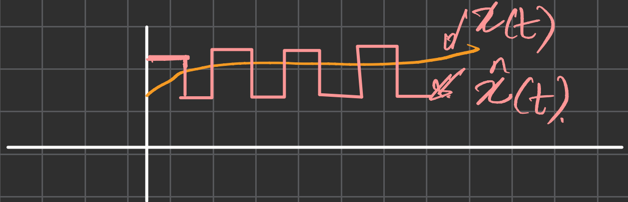


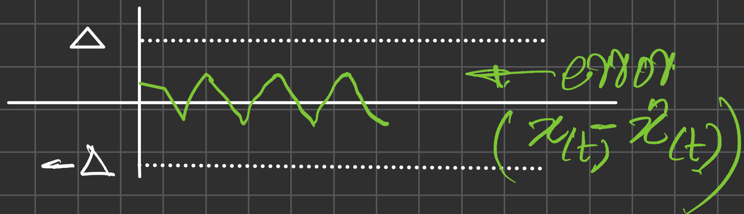
- Therefore, its probability distribution is given as:

$$f_Q(q) = \begin{cases} \dfrac{1}{2\Delta} & -\Delta \leq q \leq \Delta \\ 0 & \text{otherwise} \end{cases}$$

$$E[q^2] = \int q^2 f_Q(q)\,dq = \frac{q^3}{3}\left(\frac{1}{2\Delta}\right)\Bigg|_{-\Delta}^{\Delta} = \frac{2\Delta^3}{3(2\Delta)}$$

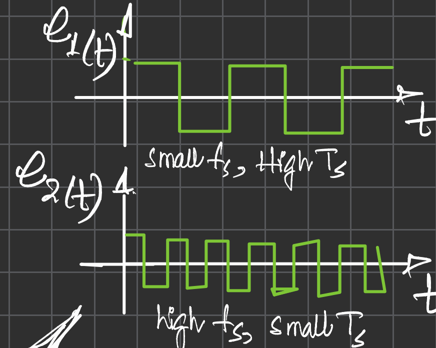
$$= \frac{\Delta^2}{3}$$



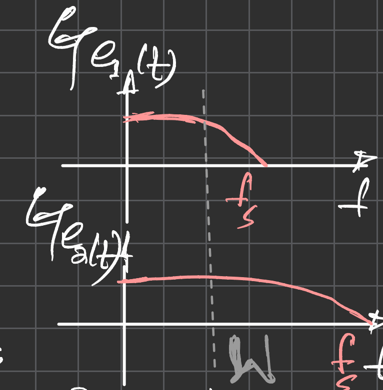


Lets look at the errors closely:

error sig.



Error sig.



Power Spectral Density of Error sig.

<u>Observation ①</u> The error decorrelates fast with high f_s.

<u>Observation ②</u> The demodulator passes the error sig. through LPF,

Thus, Quantization Noise $\downarrow \propto W / f_s \uparrow$

Since the noise PSD is spreaded, LPF (W) ⟵ less Noise Power!

∴ Observation ③ Effective Noise Power $\simeq \frac{W}{f_s} \frac{\Delta^2}{3}$

SQNR

$$SQNR = \frac{E[x^2(t)]}{N_0}$$

$$\boxed{SQNR = 10 \log_{10} E[x^2(t)] - 10 \log_{10} \frac{W}{f_s} \frac{\Delta^2}{3}}$$

Granular Noise

The granular noise is uniformly distributed between $-\Delta$ and Δ.

$$f_{n_g}(n_g) = \frac{1}{2\Delta}, -\Delta \leq n_g \leq \Delta$$

The average noise power can thus be given as: $\mathbb{E}[n_g^2] = \frac{\Delta^2}{3}$

After passing the noise power through a low pass filter of bandwidth W, the effective noise power can be given as

$$N_{\text{eff}} = \frac{W}{f_s}\mathbb{E}[n_g^2] = \frac{W}{f_s}\frac{\Delta^2}{3}$$

Problem

Consider a sinusoidal signal with maximum frequency of 3.4KHz and maximum amplitude of 1 volt. This speech signal is applied to a delta modulator whose bit rate is set at 60 kbit/sec. Explain the choice of an appropriate step size for the modulator.

Solution:

$$f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

$f_m = 3.4$ kHz.
$A_m = 1V$
$R_b = 60$ kbps.
$\Delta = ?$

We have derived.

$A_m(2\pi f_m) \leq f_s \Delta$.

$$\boxed{f_s \geq \frac{A_m(2\pi f_m)}{\Delta}}$$

Problem

Consider a delta modulator system designed to operate at W=4 times the Nyquist rate for a signal. The step size of the quantizer is 400mV.

a) Find the maximum amplitude of a 1KHz input sinusoid for which the delta modulator does not show slope overload.

b) Find post-filtered output SNR when the filter bandwidth is W.

Solution:

same as prev. as the i/p is sinusoid here as well!

$E[x^2]$

(a) $f_s \geq \frac{2\pi f_m A_m}{\Delta}$

(b) $\text{SNR}_o = \frac{A_m^2/2}{\underbrace{(\frac{W}{f_s})(\frac{\Delta^2}{3})}} = \frac{3A_m^2 f_s}{2W\Delta^2} = \frac{3f_s(\frac{f_s\Delta}{2\pi f_m})^2}{2W\Delta^2} = \frac{3}{8\pi^2 W f_m^2 T_s^3}$

$E[x^2] = \int_0^T A_m^2 \sin^2 \omega_m t \, dt$

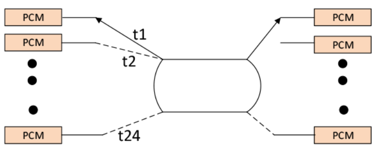
$= A_m^2/2$.

Granular!

Multiplexing

What is Multiplexing?

- A method by which multiple analog or digital signals are combined to be sent over a shared communication medium
- Types of multiplexing
  - Frequency division multiplexing (Next Page).
  - Time division multiplexing (Next to Next Page)
  - Space division multiplexing or Spatial multiplexing

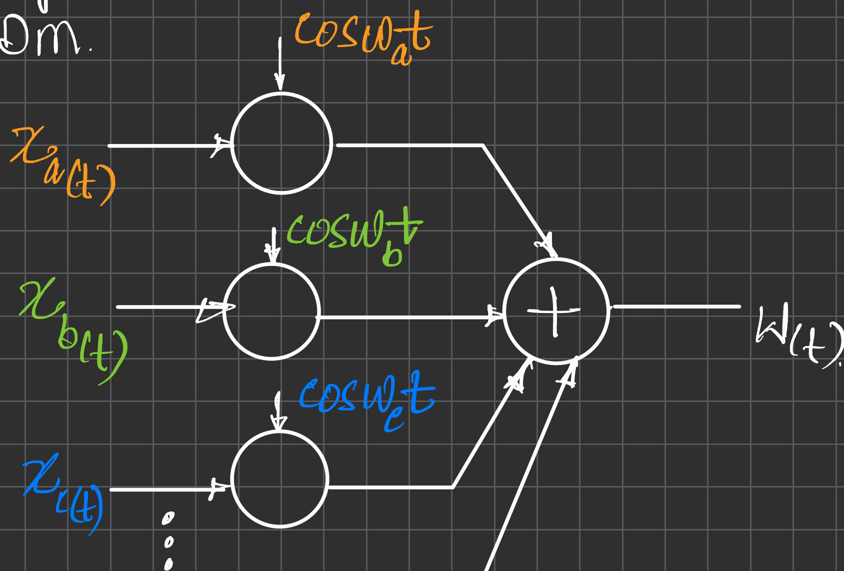


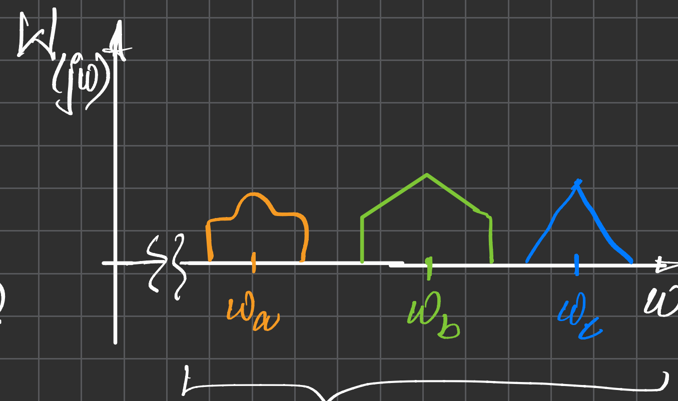
Time Division Multiplexing

Freq. Division Multiplexing (FDM)

(max freq ω_m) $\rightarrow x(t) \rightarrow \otimes \rightarrow x\cos\omega_c t$, with $\cos\omega_c t$ into $\otimes$; $\omega_c \ggg \omega_m$ *

- To simultaneously transmit multiple signals over a single wideband channel
- If the individual voice signals, which are overlapping in freq., have their freq. content shifted by means of sinusoidal AM, for example — so that the spectra of the modulated signals no longer overlap. *
- They can be transmitted simultaneously over the same channel.
- FDM.





FDM.

Example: Radio channels.

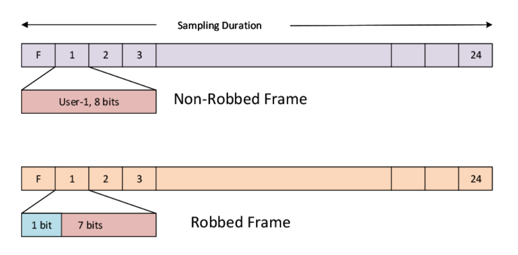
What is Multiplexing?

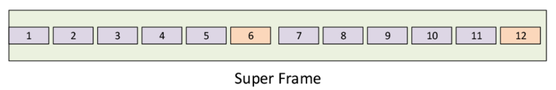
Time Division Multiplexing

- T1-carrier system has been adopted in USA, Canada, and Japan
- Accommodate 24 voice signals
- The voice signals are filtered with low-pass filter (cutoff 3400 Hz)
- The filtered signals are sampled at 8KHz
- The μ-law companding technique is used with $\mu = 255$
- Sampling rate of 8KHz
- Each non-robbed frame has twenty four 8-bit words from each user plus a single framing bit for synchronization
- Each frame consists of a total 193 bits. Each frame is of duration 125 μ sec, correspondingly, the bit rate is 1.544 Mbps

Robbed/Non-Robbed Frames

- In a robbed frame, 1 bit per user is used for signaling purposes
- A super frame is composed of 12 frames and two frames are robbed (i.e., every 6th frame is a robbed frame)
- Total duration of a super frame is $12 \times 125\mu s$





Rate Calculations

$24 \times 8 = 192 + 1 = 193$; $193/125\mu = 1.544$ Mbps

1 framing bit in each frame is used for synchronization.
8000 frames/sec as $f_s = 8k$ samp/s
$1 \times 8000 = 8000$ kbps.

- Data rate per frame for all users
- Data rate per frame for synchronization
- Data rate per non-robbed frame for a user data → Nonrobbed: 8 bits/frame. ⇒ $8 \times 8000 = 64$ Kbps. for a user
- Data rate per robbed frame for a user data → Robbed: 7 bits/frame. ⇒ $7 \times 8000 = 56$ kbps.
- Data rate per super frame of a user
- Signaling data rate

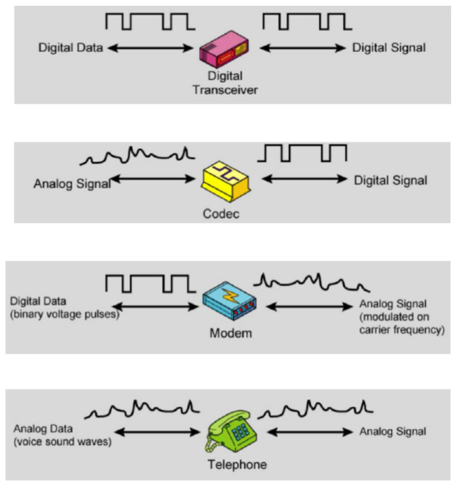
Every sixth frame is robbed for signalling.

$8000/6 \times 24 = 32$ kbps.
24 = channels

Data in a superframe $= \underbrace{10 \times 8}_{\text{Non Rob.}} + \underbrace{2 \times 7}_{\text{Robbed}} = 94$

$8000/12 \times 94 = 62.67$ kbps.

Analog vs Digital (Data and Transmissions)



Thank You
Happy Learning
☺