Chapter-2

Boolean algebra and logic Gates

Introduction

In 1854 George Boole introduced a systematic treatment of logic and developed for this purpose an algebraic system known as symbolic logic, or Boolean algebra.

Boolean algebra is a branch of mathematics and it can be used to describe the manipulation and processing of binary information. The two-valued Boolean algebra has important application in the design of modern computing systems.

Used to design and simplify the circuit of electronic devices.

Input and output is a member of set {0,1}

Basic element of circuit(gates) implements a Boolean operations.

Boolean Algebra

Boolean algebra is algebra for the manipulation of objects that can take on only two values, typically true and false.

- It is common to interpret the digital value 0 as false and the digital value 1 as true.

- A two-valued Boolean algebra is defined on a set of 2 elements B = {0, 1} with 3 binary operators OR (+), AND (•), and NOT (').

Boolean algebra and ordinary algebra

Boolean algebra is based on binary numbers where ordinary algebra is based on decimal number system.

Boolean algebra is used in digital electronics. Ordinary algebra is used in mathematics.

Basic operation in Boolean algebra are AND, OR and NOT. Basic operation of ordinary algebra are addition , subtraction , multiplication and division.

No coefficient and power are used in Boolean algebra as A+A=A.

Coefficient and power are used in ordinary algebra as A+A=2A.

Boolean algebra holds both distributive laws:

A+(B.C)=(A+B).(A+C)

A.(B+C)=(A.B)+(A.C)

Ordinary algebra holds only one distribution law:

A.(B+C)=(A.B)+(A.C)

Binary logic

Binary logic consists of binary variables and logical operations. The variables are designated by letters of the alphabet such as A, B, C, x, y, Z, etc., with each variable having two and only two distinct possible values: 1 and 0. There are three basic logical operations: AND, OR, and NOT.

Switching circuits and Binary Signals

Electronic digital circuits are sometimes called switching circuits because they behave like a switch, with the active element such as a transistor either conducting (switch closed) or not conducting (switch open). Instead of changing the switch manually, an electronic switching circuit uses binary signals to control the conduction or non-conduction state of the transistor.

Basic postulates

S.No.	Name of the Postulates	Postulate Equation
1	Law of Identity	$A + 0 = 0 + A = A$ $A . 1 = 1 . A = A$
2	Commutative Law	$(A + B) = (B + A)$ $(A . B) = (B . A)$
3	Distributive Law	$A . (B + C) = (A . B) + (A . C)$ $A + (B . C) = (A + B) . (A + C)$
4	Associative Law	$A + (B + C) = (A + B) + C$ $(A . B) . C = A . (B . C)$
5	Complement Law	$A + A' = 1$ $A . A' = 0$

Basic Theory of Boolean Algebra

S.No	Theorem	Statement	Equations
1	Duality Theorem	A boolean relation can be derived from another boolean relation by changing OR sign to AND sign and vice versa and complementing the 0s and 1s.	A + A' = 1 and A . A' = 0 are the dual relations.
2	DeMorgan's Theorem 1	Complement of a product is equal to the sum of its complement.	(A . B)' = A' + B'
3	DeMorgan's Theorem 2	Complement of a sum is equal to the product of the complement.	(A + B)' = A' . B'
4	Idempotency Theorem	–	A + A = A A . A = A
5	Involution Theorem	–	A'' = A
6	Absorption Theorem	–	A + (A . B) = A A . (A + B) = A
7	Associative Theorem	–	A + (B + C) = (A + B) + C A . (B . C) = (A . B) . C

The AND Operation

The AND operation is a binary operation, meaning that it needs two operands.

c = a AND b

Both a and b must be true for the result to be true.

AND		b: T	b: F
a	T	T	F
	F	F	F

The OR Operation

The OR operation is also a binary operation with two operands.

c = a OR b

If either a OR b is true, then the result is true.

OR		b	
		T	F
a	T	T	T
	F	T	F

The NOT Operation

The NOT operation is a unary operation with only one operand.

c = NOT (a)

It simply reverses the true or false value of the operand.

NOT		
a	T	F
	F	T

Boolean Function

A binary variable can take a value of 0 or, 1. A Boolean function is an expression formed with binary variables, the two binary operators OR and AND, unary operator NOT, parenthesis and an equal sign. For a given value of variables, the function either can 0 or 1.

E.g.

$F1 = xyz$

$F2 = x + y'z$

$F3 = x'y'z + x'yz + xy'$

$F4 = xy' + x'z$

Truth Table of Boolean functions:

A truth table shows the relationship, in tabular form, between the input values and the result of a specific Boolean operator or function on the input variables.

Prove the above theorem using truth table……..

Algebraic Manipulation

When a Boolean function is implemented with logic gates, each literal in the function designates an input to a gate and each term is implemented with a gate. The minimization of the number of literals and the number of terms results is a circuit with less equipment.

Simplify the following Boolean functions to a minimum number of literals.

$x(x' + y)$

$= xx' + xy$

$= 0 + xy$

$= xy$

$\mathbf{x + x'y}$ [Write law ñames]

$= (x + x')(x + y)$

$= 1(x + y)$

$= x + y$

$\mathbf{(x + y)(x + y')}$

$= x + xy + xy' + yy'$

$= x(1 + y + y')$

$= x$

$\mathbf{xy + x'z + yz}$

$= xy + x'z + yz(x + x')$

$= xy + x'z + xyz + x'yz$

$= xy(1 + z) + x'z(1 + y)$

$= xy + x'z$

Q. Prove that: $(x + y)(\overline{x} + y) = y$

Q. Prove the Boolean expression: $AB + AB'C + A'BC = AB + AC + BC$

Q. Minimize the expression: $AB + A\overline{C} + BC$ using theorem and properties of Boolean Algebra.

Q. Simplify the given Boolean expression

$AB'C+A'B'C+ABC$

$AB+A'BC+BC$

$PQ'+Q(P+Q)+P(P'+Q)$

$(X+Y)(XY'Z+XYZ+XY'Z')$

Complement of a function

The complement of a function F is F′ and is obtained from an interchange of 0's for 1's and1's for 0's in the value of F. The complement of a function may be derived algebraically through De Morgan's theorem.

(A+B+C)'=(A+X)' [suppose X=B+C]

=A'X' [DE Morgan's 2nd law]

=A'. (B+C)'

=A'. (B'.C')

=A'B'C'

Generalized theorems for finding complement:

(A+B+C+D+………Z)'=A'B'C'D'……..Z'

(A.B.C.D…………Z)'= A'+B'+C'+D'+……..Z'

Find the Complement of the functions F1 = x′yz′ + x′yz and F2 = x(y′z′ +yz).

Logic Gates

A logic gate is an electronic device that produces a result based on one or more input values.

- In reality, gates consist of one to six transistors, but digital designers think of them as a single unit.

- Integrated circuits contain collections of gates suited to a particular purpose.

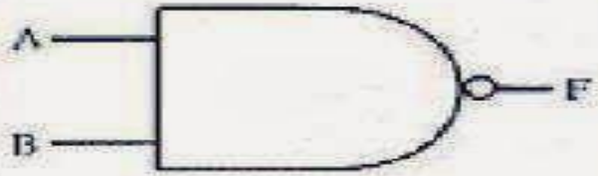
Logic gate can be categories as follows:

Basic Gate:

AND

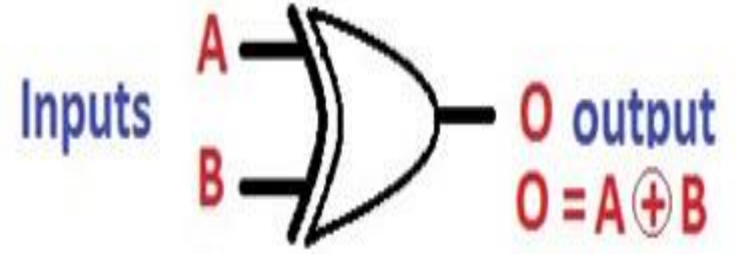
OR

NOT

Name	Graphic Symbol	Algebraic Function	Truth Table
AND		$F = A \cdot B$ or $F = AB$	A B | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = A + B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT		$F = \overline{A}$ or $F = A'$	A | F 0 | 1 1 | 0
NAND		$F = (\overline{AB})$	A B | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (\overline{A + B})$	A B | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0

Exclusive-OR gate(XOR)

It produces 0 when two inputs are same. Otherwise 1



Symbol

Inputs		Output
A	B	O
0	0	0
0	1	1
1	0	1
1	1	0

Truth table

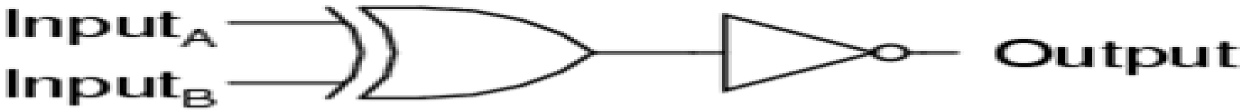
Exclusive-NOR gate

It is the complement of XOR gate. An XNOR gate produce 1 if its two inputs are same.

Exclusive-NOR gate

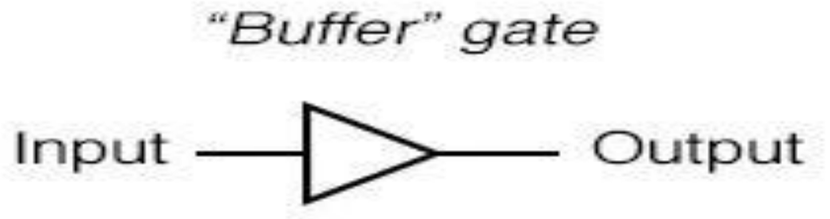
A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1

Equivalent gate circuit



Buffer gate

The Buffer Gate is a logic block that takes any input and compares the value to 0. If the input signal is zero, the output will be zero. If the input is non-zero, the output will be a 1.



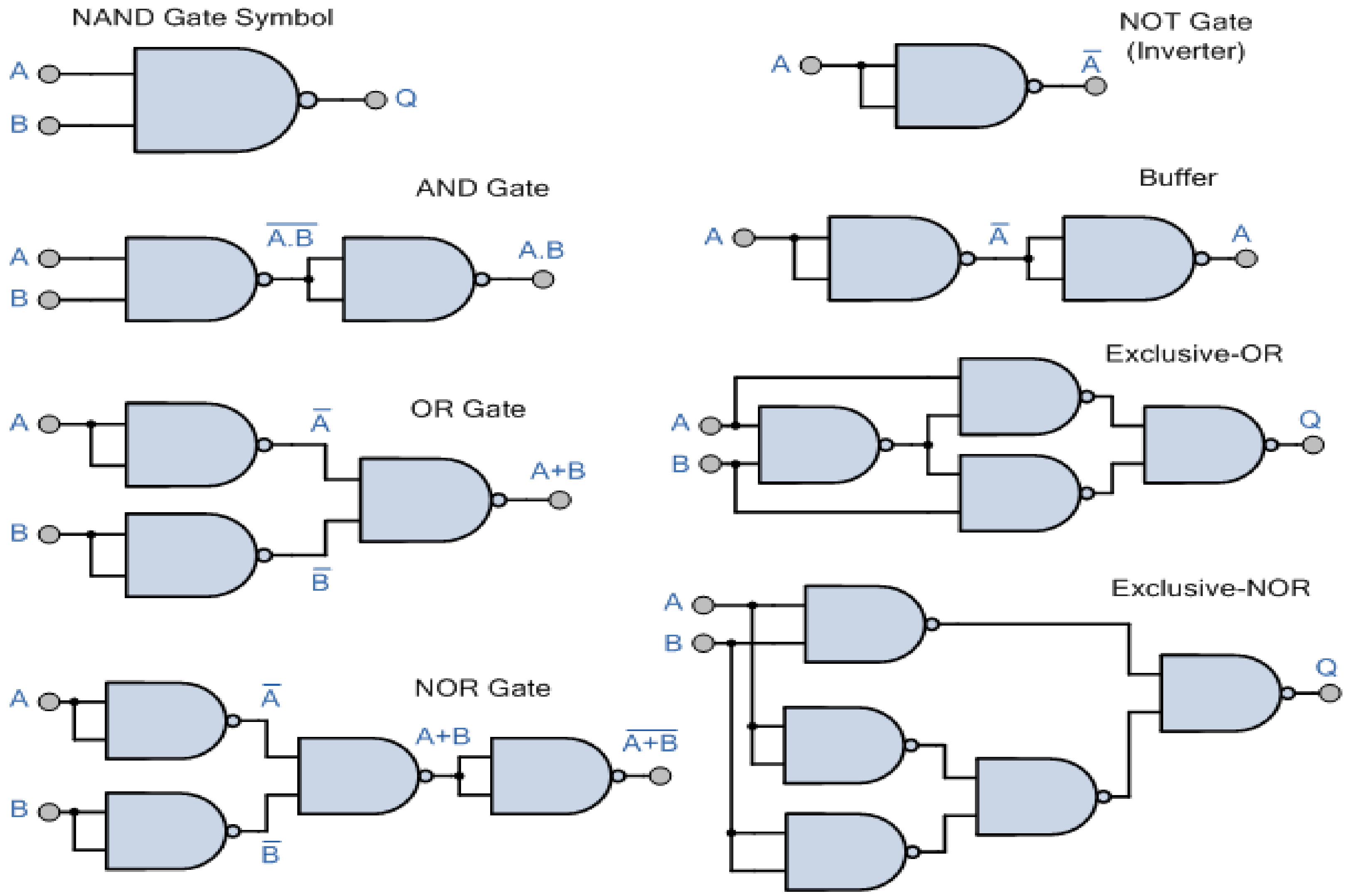
Input	Output
0	0
1	1

Universal gates

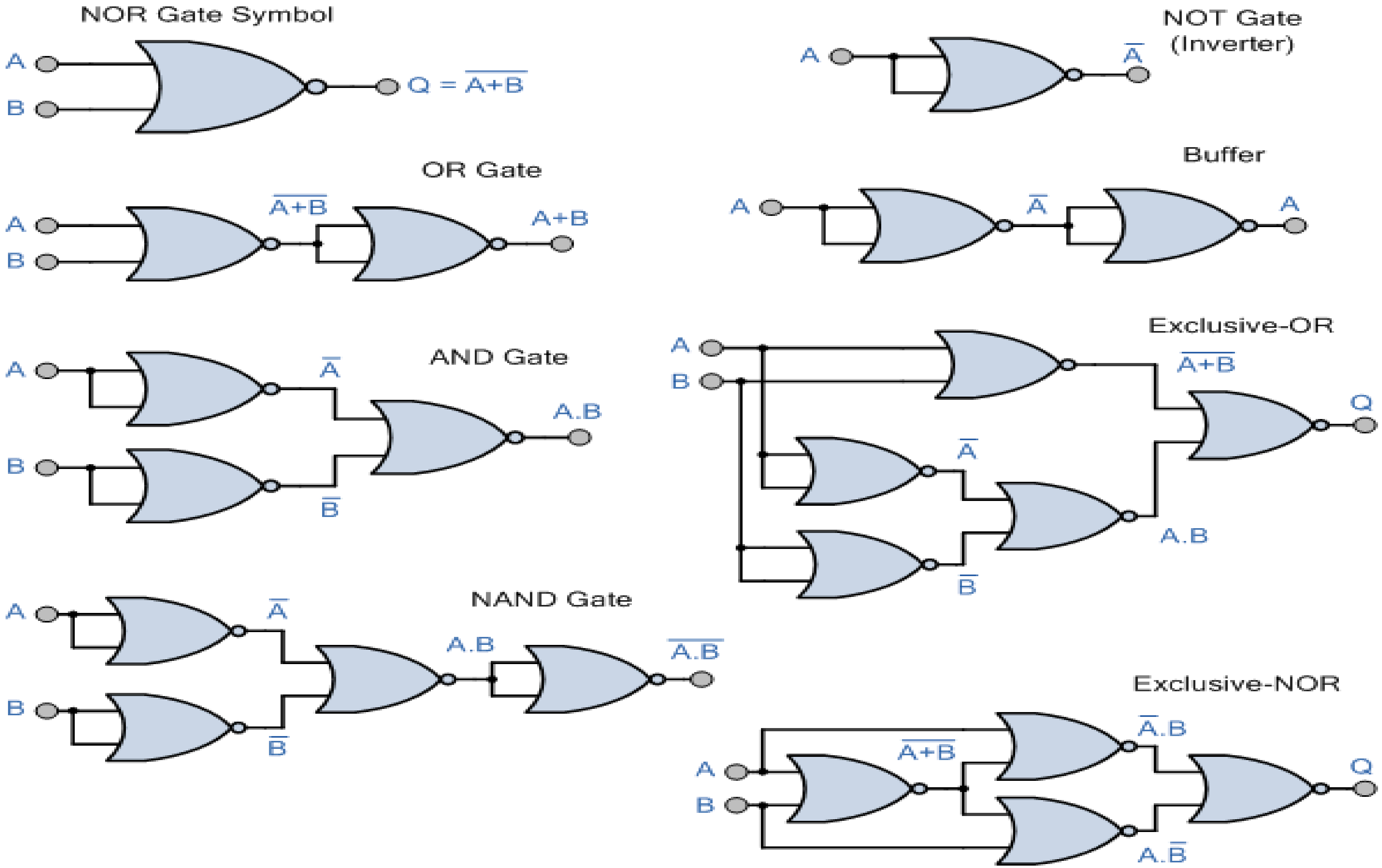
A universal gate is a gate which can implement any Boolean function without need to use any other gate type. The NAND and NOR gates are universal gates. In practice, this is advantageous since NAND and NOR gates are economical and easier to fabricate and are the basic gates used in all IC digital logic families.

NAND Gate is a Universal Gate

To prove that any Boolean function can be implemented using only NAND gates, we will show that the AND, OR, and NOT operations can be performed using only these gates.

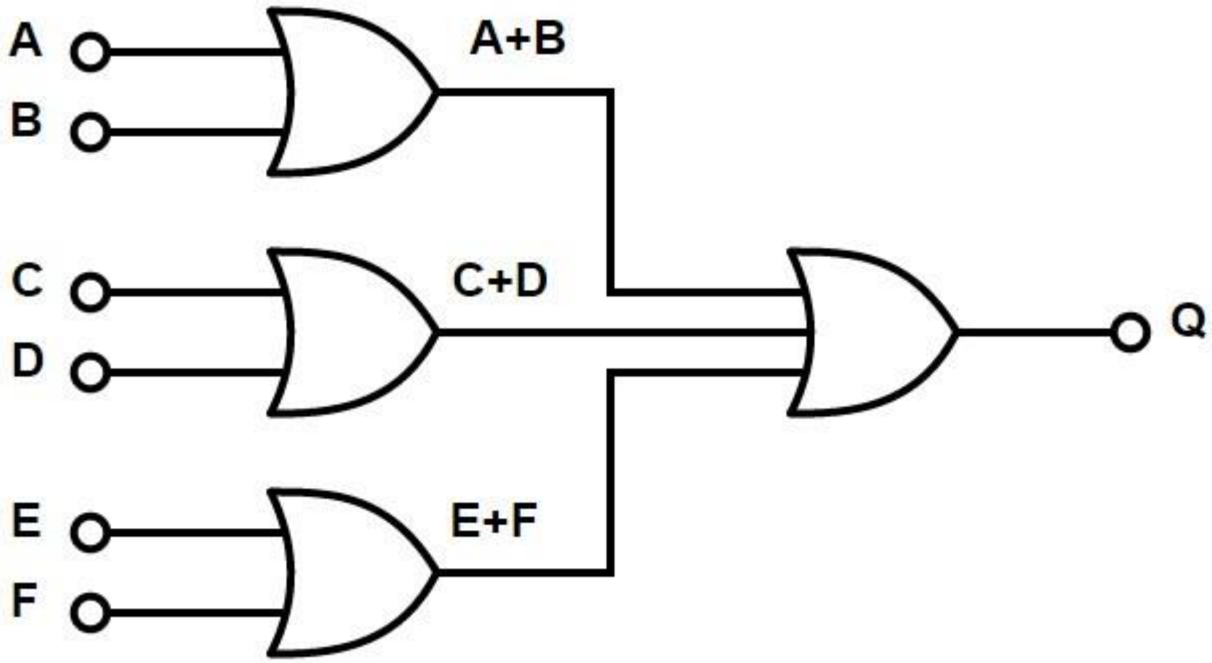

NAND Gate Symbol
A
B
Q
AND Gate
$\overline{A.B}$
A.B
OR Gate
$\overline{A}$
$\overline{B}$
A+B
NOR Gate
A+B
$\overline{A+B}$
NOT Gate
(Inverter)
Buffer
Exclusive-OR
Exclusive-NOR

NOR Gate is a Universal Gate



Extending gates to multiple inputs

The gates shown in Fig above, except for the inverter and buffer, can be extended to have more than two inputs. A gate can be extended to have multiple inputs if the binary operation it represents is commutative and associative.



Implementation of Boolean function using gates

Implement the following Boolean expression using gate.

$F1 = xyz'$

$F2 = x + y'z$

$F3 = x'y'z + x'yz + xy'$

Draw a logic gates that implements the following:

a. $F = AB + C D' + B' C$

b. $F = (A + B) (B' + C) (C' + D + E)$

Integrated Circuits

An Integrated circuit is an association (or connection) of various electronic devices such as resistors, capacitors and transistors fabricated to a semiconductor material such as silicon or germanium. It is also called as a chip or microchip. An IC can function as an amplifier, rectifier, oscillator, counter, timer and memory. Sometime ICs are connected to various other systems to perform complex functions.

Types of ICs

ICs can be categorized into two types

- Analog or Linear ICs
- Digital or logic ICs

Integrated Circuits

Analog or Linear ICs: They produce continuous output depending on the input signal. From the name of the IC we can deduce that the output is a linear function of the input signal. operational amplifier is one of the types of linear ICs which are used in amplifiers, timers and counters, oscillators etc.

Digital or Logic ICs: Unlike Analog ICs, Digital ICs never give a continuous output signal. Instead it operates only during defined states. Digital ICs are used mostly in microprocessor and various memory applications. Logic gates are the building blocks of Digital ICs which operate either at 0 or 1.

Advantages of ICs

In consumer electronics, ICs have made possible the development of many new products, including personal calculators and computers, digital watches, and video games.

They have also been used to improve or lower the cost of many existing products, such as appliances, televisions, radios, and high-fidelity equipment.

The logic and arithmetic functions of a small computer can now be performed on a single VLSI chip called a microprocessor.

Complete logic, arithmetic, and memory functions of a small computer can be packaged on a single printed circuit board, or even on a single chip.

Advantages of ICs

- Small physical size
- High speed
- Low power consumption
- Reduced cost
- High reliability
- Good performance

Levels of Integration

During 1959 two different scientists invented IC's. Jack Kilby from Texas Instruments made his first germanium IC during 1959 and Robert Noyce made his first silicon IC during the same year. But ICs were not the same since the day of their invention; they have evolved a long way. Integrated circuits are often classified by the number of transistors and other electronic components they contain:

SSI (small-scale integration): Up to 100 electronic components per chip

MSI (medium-scale integration): From 100 to 3,000 electronic components per chip.

LSI (large-scale integration): From 3,000 to 100,000 electronic components per chip

VLSI (very large-scale integration): From 100,000 to 1,000,000 electronic components per chip

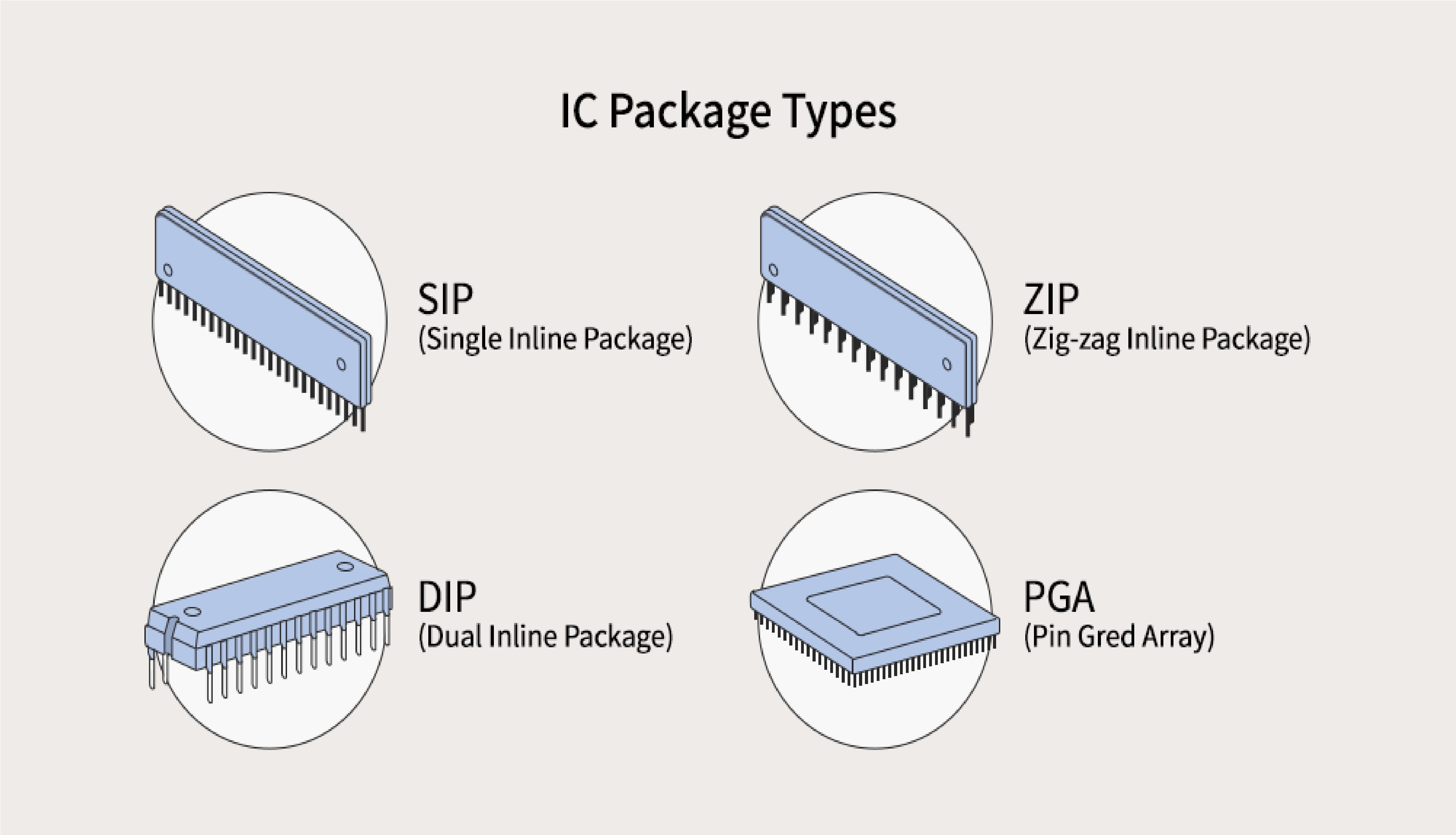
ULSI (ultra large-scale integration): More than 1 million electronic components per chip

SIP (Single In-line Package)

A single in-line package is an electronic device package which has one row of connecting pins. It is not as popular as the dual in-line package (DIP) which contains two rows of pins, but has been used for packaging RAM chips and multiple resistors with a common pin. SIPs group RAM chips together on a small board. The board itself has a single row of pin-leads that resembles a comb extending from its bottom edge, which plug into a special socket on a system or system-expansion board. SIPs are commonly found in memory modules. SIP is not to be confused with SIPP which is an archaic term referring to Single In-line Pin Package which was a memory used in early computers.

DIP (Dual In-line Package)

Dual in-line package (DIP) is a type of semiconductor component packaging. DIPs can be installed either in sockets or permanently soldered into holes extending into the surface of the printed circuit board. DIP is relatively broadly defined as any rectangular package with two uniformly spaced parallel rows of pins pointing downward, whether it contains an IC chip or some other device(s), and whether the pins emerge from the sides of the package and bend downwards. A DIP is usually referred to as a DIPn, where n is the total number of pins.

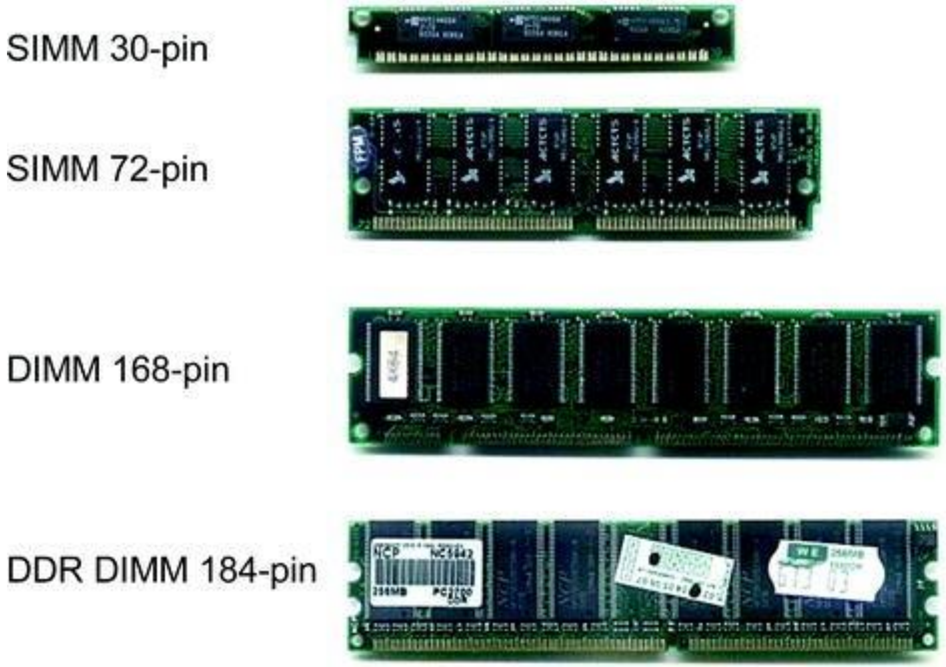

IC Package Types
SIP
(Single Inline Package)
ZIP
(Zig-zag Inline Package)
DIP
(Dual Inline Package)
PGA
(Pin Gred Array)

SIMM (Single In-line Memory Module)

Short for Single In-line Memory Module, SIMM is a circuit board that holds six to nine memory chips per board, the ninth chip usually an error checking chip (parity/non parity) and were commonly used with Intel Pentium or Pentium compatible motherboards. SIMMs are rarely used today and have been widely replaced by DIMMs. SIMMs are available in two flavors: 30 pin and 72 pin. 30-pin SIMMs are the older standard, and were popular on third and fourth generation motherboards. 72-pin SIMMs are used on fourth, fifth and sixth generation PCs.

DIMM (Dual In-line Memory Module)

Short for Dual In-line Memory Module, DIMM is a circuit board that holds memory chips. DIMMs have a 64-bit path because of the Pentium Processor requirements. Because of the new bit path, DIMMs can be installed one at a time, unlike SIMMs on a Pentium that would require two to be added. They come in both 168-pin and 184-pin versions. Below is an example image.



IC digital logic Families

Digital logic family refers to the specific circuit technology to which digital integrated circuits belong. Family has its own basic electronic circuit upon which more complex digital circuits and components are developed. The basic circuit in each technology is a NAND, NOR, or an inverter gate. The electronic components used in the construction of the basic circuit are usually used as the name of the technology. Different logic families have been introduced commercially. Some of most popular are:

TTL (transistor-transistor logic): The TTL family evolved from a previous technology that used diodes and transistors for the basic NAND gate. This technology was called DTL for diode-transistor logic. Later the diodes were replaced by transistors to improve the circuit operation and the name of the logic family was changed to TTL.

Various TTL Families

Name	ns	mW
• Standard TTL	10	10
• Low-power TTL	33	1
• High-speed TTL	6	22
• Schottky TTL	3	19
• Low-power Schottky TTL	10	2

Propagation delay and power dissipation per gate of various TTL families

IC digital logic Families

ECL (emitter-coupled logic): Emitter-coupled logic (ECL) circuits provide the highest speed among the integrated digital logic families. ECL is used in systems such as supercomputers and signal processors, where high speed is essential. The transistors in ECL gates operate in a non-saturated state, a condition that allows the achievement of propagation delays of 1 to 2 nanoseconds.

MOS (metal-oxide semiconductor): The metal-oxide semiconductor (MOS) is a unipolar transistor that depends upon the flow of only one type of carrier, which may be electrons (n-channel) or holes (p-channel), this is in contrast to the bipolar transistor used in TTL and ECL gates, where both carriers exist during normal operation. A p-channel MOS is referred to as PMOS and an n-channel as NMOS. NMOS is the one that is commonly used in circuits with only one type of MOS transistor.

IC digital logic Families

CMOS (complementary metal-oxide semiconductor): Complementary MOS (CMOS) technology uses one PMOS and one NMOS transistor connected in a complementary fashion in all circuits. The most important advantages of MOS over bipolar transistors are the high packing density of circuits, a simpler processing technique during fabrication, and a more economical operation because of the low power consumption.

IIL (Integrated Injection Logic): Integrated injection logic (IIL, I^2L, or I2L) is a class of digital circuit technology built with multiple collector bipolar junction transistors (BJT). When introduced it had speed comparable to TTL yet was almost as low power as CMOS, making it ideal for use in VLSI (and larger) integrated circuits. Although the logic voltage levels are very close (High: 0.7V, Low: 0.2V), I2L has high noise immunity because it operates by current instead of voltage. Sometimes also known as Merged Transistor Logic.

Characteristics of digital logic families (Technology Parameters)

For each specific implementation technology, there are details that differ in their electronic circuit design and circuit parameters. The most important parameters used to characterize an implementation technology are:

Fan-in

For high-speed technologies, fan-in, the number of inputs to a gate, is often restricted on gate primitives to no more than four or five. This is primarily due to electronic considerations related to gate speed. To build gates with larger fan-in, interconnected gates with lower fan-in are used during technology mapping.

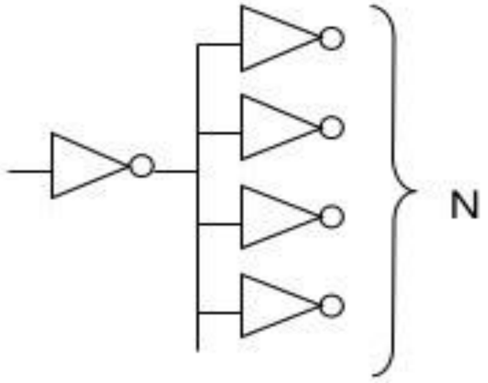
Characteristics of digital logic families (Technology Parameters)

Fan-out specifies the number of standard loads driven by a gate output i.e. Fan-out is a measure of the ability of a logic gate output to drive a number of inputs of other logic gates of the same type. Maximum Fan-out for an output specifies the fan-out that the output can drive with out exceeding its specified maximum transition time. Standard loads may be defined in a variety of ways depending upon the technology. For example: the input to a specific inverter can have load equal to 1.0 standard load. If a gate drives six such inverters, then the fan-out is equal to 6.0 standard loads

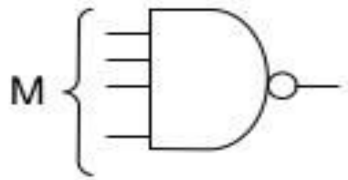
Characteristics of digital logic families (Technology Parameters)

Fan-Out and Fan-In

- Fan-out – number of load gates connected to the output of the driving gate
  - gates with large fan-out are slower



- Fan-in – the number of inputs to the gate
  - gates with large fan-in are bigger and slower



Characteristics of digital logic families (Technology Parameters)

Propagation delay:

The signals through a gate take a certain amount of time to propagate from the inputs to the output. This interval of time is defined as the propagation delay of the gate. This is symbolized as t_{pd}. Propagation delay is measured in nanoseconds (ns). 1 ns is equal to 10-9 of a second. The signals that travel from the inputs of a digital circuit to its outputs pass through a series of gates. The sum of the propagation delays through the gates is the total delay of the circuit.

Characteristics of digital logic families (Technology Parameters)

Power Dissipation: Every electronic circuit requires a certain amount of power to operate. The power dissipation is a parameter expressed in milliwatts (mW) and represents the amount of power needed by the gate. The number that represents this parameter does not include the power delivered from another gate; rather, it represents the power delivered to the gate from the power supply. An IC with four gates will require, from its power supply, four times the power dissipated in each gate.

The amount of power that is dissipated in a gate is calculated as:

P_D (Power Dissipation) = $V_{cc} * I_{cc}$

Where V_{cc} = supply voltage and

I_{cc} = current drawn by the circuit

Power Dissipation

The current drain from the power supply depends on the logic state of the gate. The current drawn from the power supply when the output of the gate is in the high-voltage level is termed I_{CCH}. When the output is in the low-voltage level, the current is I_{CCL}. The average current is

I_{cc} (avg)= ($I_{CCH} + I_{CCL}$)/2

And used to calculate the average power dissipation as:

P_D (avg) = $V_{cc} * I_{cc}$ (avg)

Example: A standard TTL NAND gate uses a supply voltage V_{CC} of 5V and has current drains I_{CCH} = 1 mA and I_{CCL} = 3 mA.

The average current is (3 + 1)/2 = 2 mA.

The average power dissipation is 5 x 2 = 10 mW.

An IC that has four NAND gates dissipates a total of 10 x 4 = 40 mW.

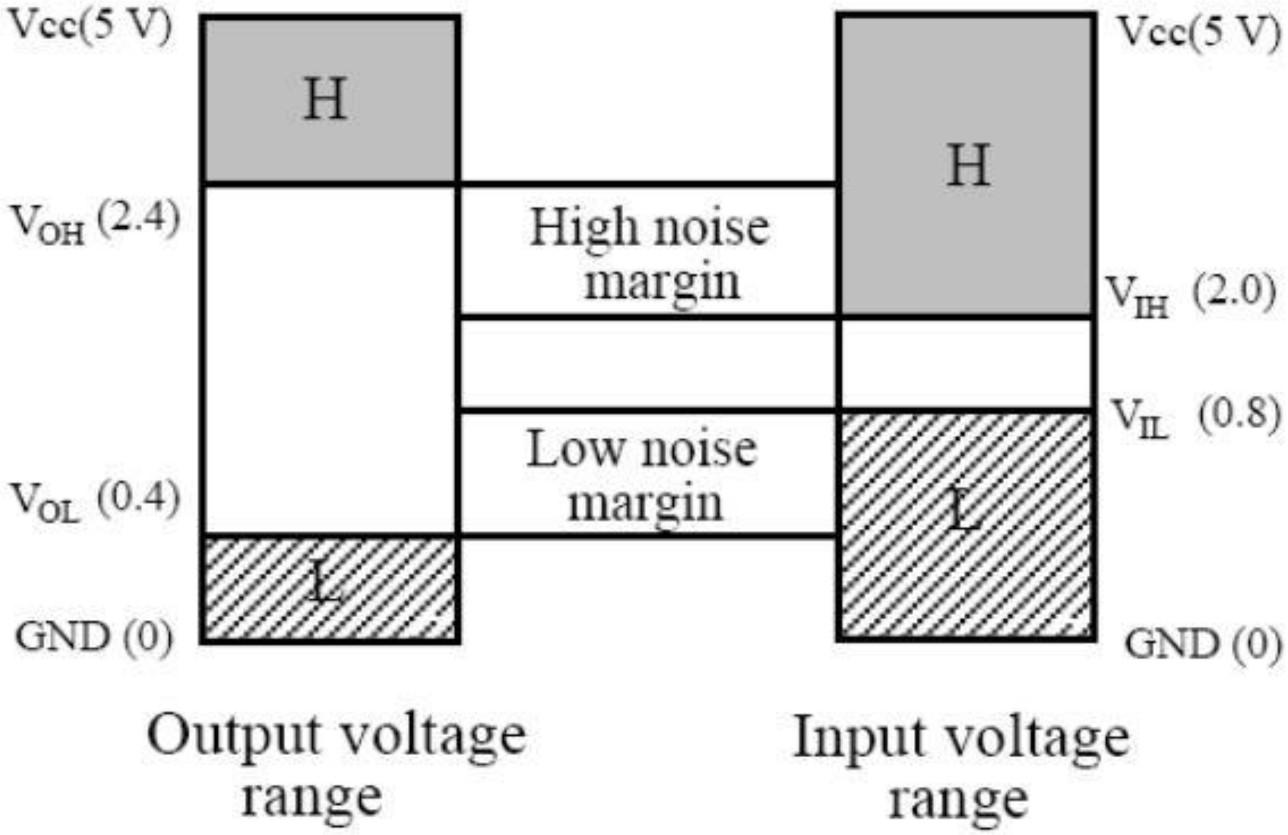
Characteristics of digital logic families (Technology Parameters)

Noise Margin: Undesirable or unwanted signals (e.g. voltages, currents etc.) on the connecting wires between logic circuits are referred to as noise. There are two types of noise to be considered:

- DC noise is caused by a drift in the voltage levels of a signal.
- AC noise is a random pulse that may be created by other switching signals.

Thus, noise is a term used to denote an undesirable signal that is superimposed upon the normal operating signal. Noise margin is the maximum noise voltage added to an input signal of a digital circuit that does not cause an undesirable change in the circuit output. The ability of circuits to operate reliably in a noise environment is important in many applications. Noise margin is expressed in volts and represents the maximum noise signal that can be tolerated by the gate.

Characteristics of digital logic families (Technology Parameters)



Characteristics of digital logic families (Technology Parameters)

In fig, V_{OL} is the maximum voltage that the output can be when in the low-level state. The circuit can tolerate any noise signal that is less than the noise margin (V_{IL} - V_{OL}) because the input will recognize the signal as being in the low-level state. Any signal greater than V_{OL} plus the noise-margin figure will send the input voltage into the indeterminate range, which may cause an error in the output of the gate. In a similar fashion, a negative-voltage noise greater than V_{OH} - V_{IH} will send the input voltage into the indeterminate range.

The parameters for the noise margin in a standard TTL NAND gate are V_{OH} = 2.4 V, V_{OL} = 0.4 V, V_{IH} = 2 V, and V_{IL} = 0.8 V. The high-state noise margin is 2.4 - 2 = 0.4 V, and the low-state noise margin is 0.8 - 0.4 = 0.4 V.

Positive Logic and Negative logic

In the binary system, the digits 1 and 0 are called binary digits. To represent these two bits, we use two different voltage levels. These voltages are called logic levels.

Generally higher voltage HIGH represents 1 and low voltage LOW represents 0. This is called positive logic. Another system that represents 1 with LOW and 0 with HIGH is called negative logic. In a practical digital circuit, however HIGH and LOW can be any voltage between a specified minimum and maximum voltages.

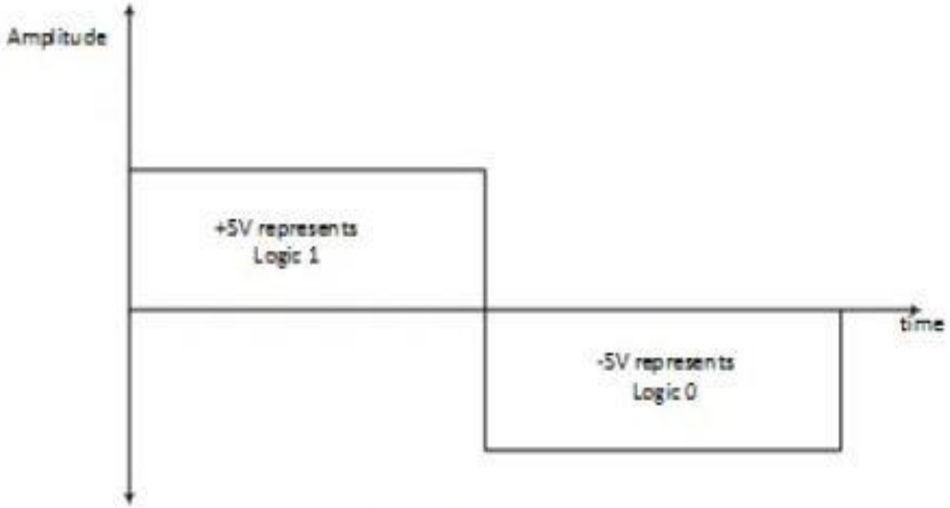


Fig. 2 a. Positive logic representation

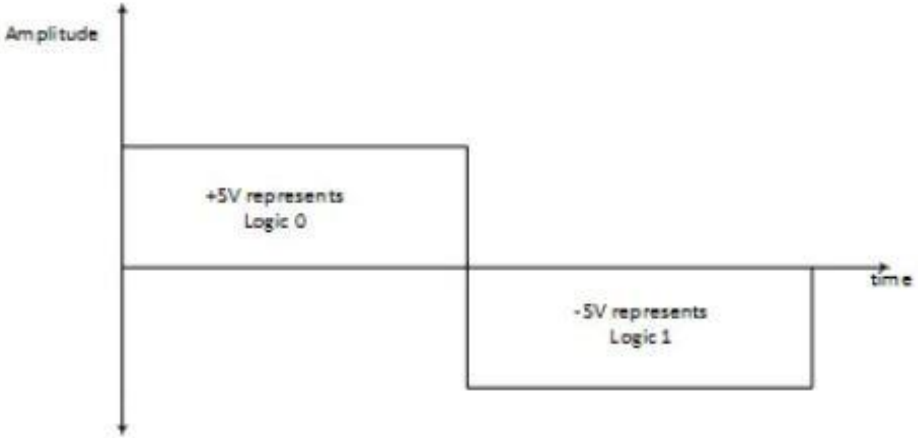


Fig. 3 a. Negative logic representation

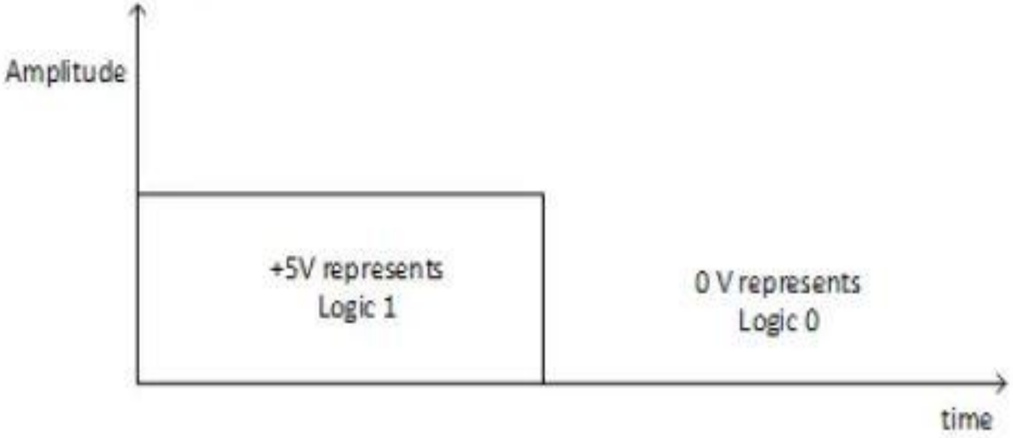


Fig. 2 b. Positive logic representation

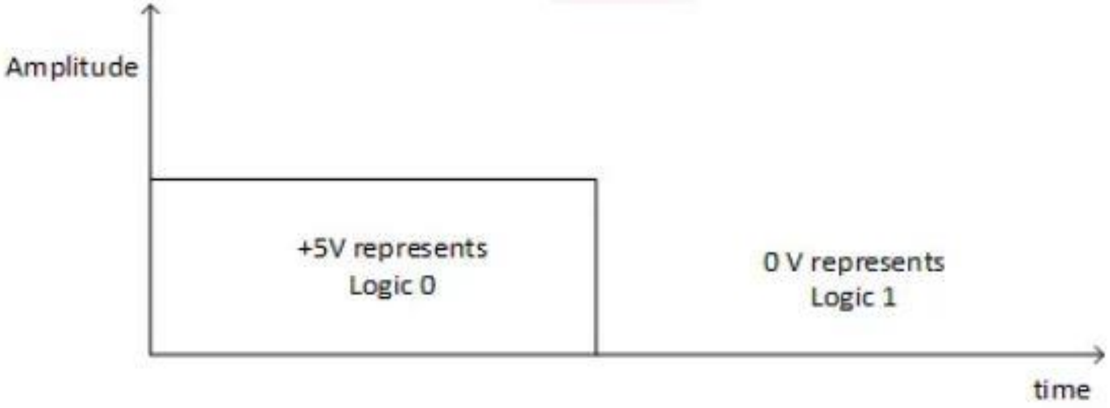
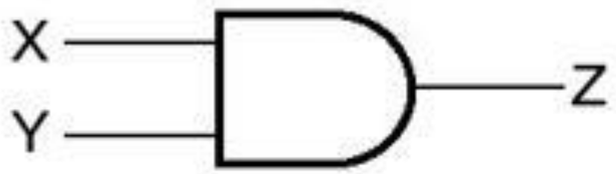


Fig. 3 b. Negative logic representation

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

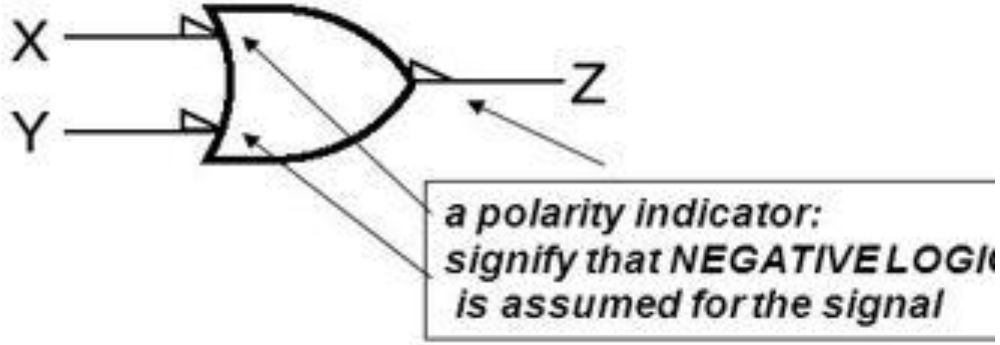
(c) Truth table for positive logic



(d) Positive-logic AND gate

X	Y	Z
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic



(f) Negative-logic OR gate

Comparison

Logic ⇒ Family	**TTL**	**ECL**	**I^2L**	**CMOS**
Basic gate	NAND	OR	NOR	NOR
Fan out	10	25	8	>50
Pd per gate (mW)	10-22	40-55	0.1-0.2	0.01-1.0
Noise immunity	Very Good	Good	Nominal	Very Good
Tpd per gate (ns)	6-10	1-4	10-20	70