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Title:	Implementation of Bayes Theorem.
Date of Performance:	
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Marks:	
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Aim: Implementation of Bayes Theorem.

Objective: To study about how to use Bayes Theorem in reasoning process.

Theory:

Bayes' theorem is also known as Bayes' rule, Bayes' law, or Bayesian reasoning, which determines the probability of an event with uncertain knowledge.

In probability theory, it relates the conditional probability and marginal probabilities of two random events.

Bayes' theorem was named after the British mathematician Thomas Bayes. The Bayesian inference is an application of Bayes' theorem, which is fundamental to Bayesian statistics.

What is Bayes' Theorem?

Bayes theorem (also known as the Bayes Rule or Bayes Law) is used to determine the conditional probability of event A when event B has already occurred.

The general statement of Bayes' theorem is "The conditional probability of an event A, given the occurrence of another event B, is equal to the product of the event of B, given A and the probability of A divided by the probability of event B." i.e.

$$P(A|B) = P(B|A)P(A) / P(B)$$

where,

$P(A)$ and $P(B)$ are the probabilities of events A and B

$P(A|B)$ is the probability of event A when event B happens

$P(B|A)$ is the probability of event B when A happens

Bayes' Theorem comprises four key components:

1. **Prior Probability (P(A)):** This is the initial probability or belief in an event A before considering any new evidence. It represents what we know or assume about A based on prior knowledge.
2. **Likelihood (P(B|A)):** The likelihood represents the probability of observing evidence B given that the event A is true. It quantifies how well the evidence supports the event.
3. **Evidence (P(B)):** Evidence, also known as the marginal likelihood, is the probability of observing evidence B, regardless of the truth of A. It serves as a normalizing constant, ensuring that the posterior probability is a valid probability distribution.

4. Posterior Probability (P(A|B)): The posterior probability represents the updated belief in event A after considering the new evidence B. It answers the question, "What is the probability of A being true given the observed evidence B?"

Applying Bayes' rule:

Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(B), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$\textbf{P(cause | effect)} = \frac{P(effect|cause)\, P(cause)}{P(effect)}$$

Example:
Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability P(King|Face), which means the drawn face card is a king card.

Solution:

$$\textbf{P(king | face)} = \frac{P(Face|king) * P(King)}{P(Face)} \quad ....(i)$$

P(king): probability that the card is King= 4/52= 1/13

P(face): probability that a card is a face card= 3/13

P(Face|King): probability of face card when we assume it is a king = 1

Putting all values in equation (i) we will get:

$$P(king|face) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})} = \textbf{1/3, it is a probability that a face card is a king card.}$$

Following are some applications of Bayes' theorem:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

Code:

```
# Function to implement Bayes' Theorem
def bayes_theorem(prior_A, likelihood_B_given_A, prior_B):
    # Bayes' theorem formula
    posterior_A_given_B = (likelihood_B_given_A * prior_A) / prior_B
    return posterior_A_given_B

# Example: Calculate P(King|Face)
# P(King): Probability that the card is a king
prior_King = 4 / 52

# P(Face): Probability that a card is a face card
prior_Face = 12 / 52  # 3 face cards (J, Q, K) per suit, 4 suits total

# P(Face|King): Probability of face card given it's a king
likelihood_Face_given_King = 1  # If it's a king, it is also a face card

# Applying Bayes' Theorem
posterior_King_given_Face = bayes_theorem(prior_King, likelihood_Face_given_King, prior_Face)

print(f"Posterior Probability P(King|Face): {posterior_King_given_Face:.4f}")
```

Output:

```
Posterior Probability P(King|Face): 0.3333
```

Conclusion:

In the given example, the program successfully applied Bayes' Theorem to calculate the posterior probability \(P(King|Face) \), updating the belief about drawing a king card given that the card is a face card. The decision-making process relies on prior knowledge and observed evidence, demonstrating how the theorem helps refine predictions based on available information. The conclusion shows the usefulness of Bayesian reasoning in decision-making under uncertainty, and its wide applicability in various fields, such as robotics, weather forecasting, and solving probability puzzles like the Monty Hall prob