

Neural Network Model Evaluation for Neutron Star Equation of State Reconstruction

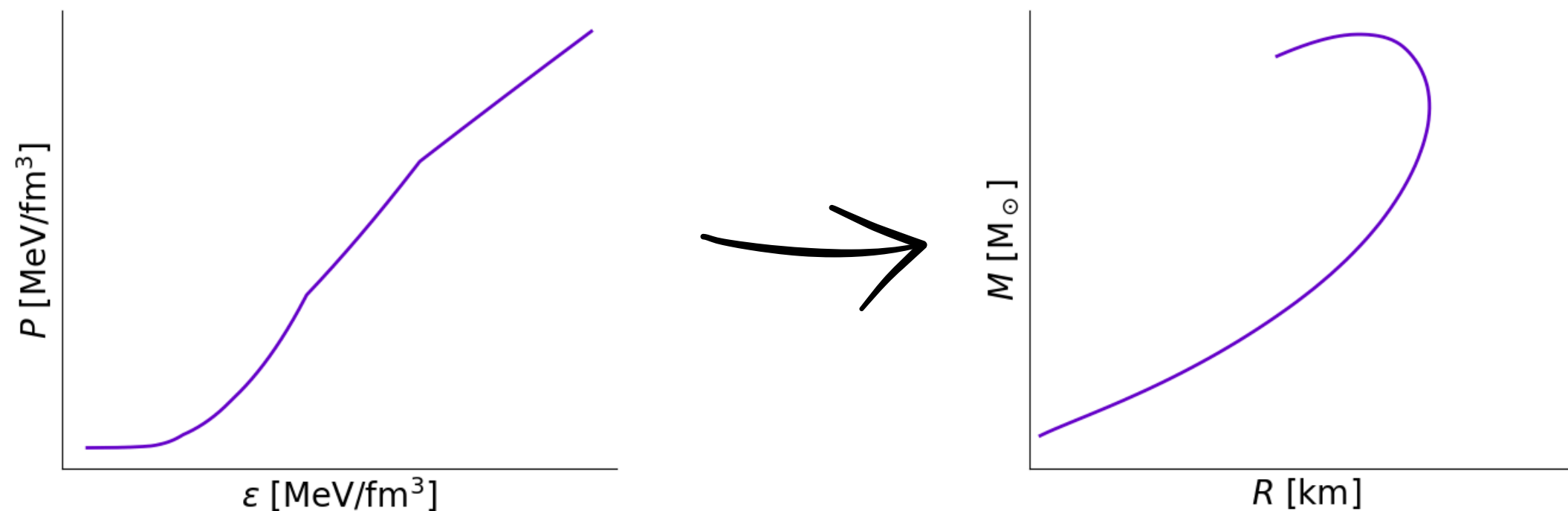
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Arthur E. B. Pasqualotto, Dyana C. Duarte**



- The TOV^{1,2} equations link the equation of state (EoS) of neutron stars to mass and radius observations.

$$\frac{dp}{dr} = - \frac{[\epsilon(r) + p(r)][m(r) + 4\pi r^3 p(r)]}{r[r - 2m(r)]}$$

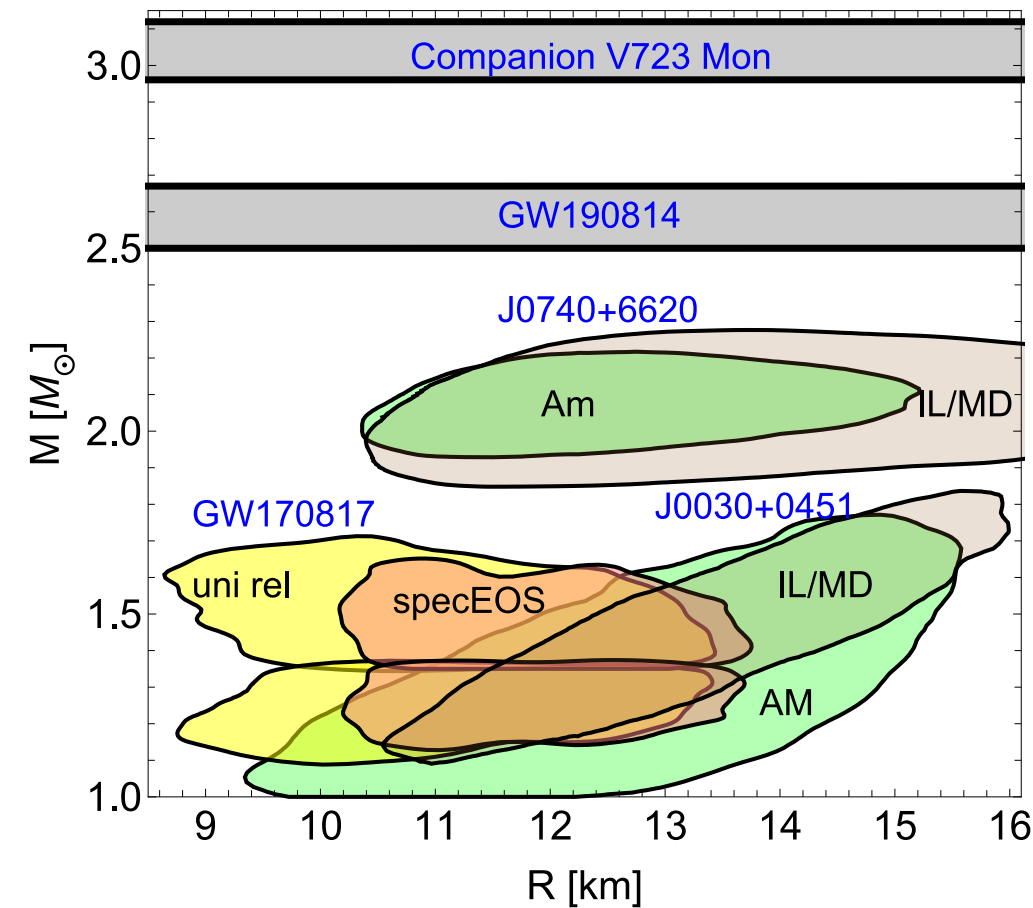
$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$



¹R. C. Tolman, Phys. Rev. **55**, 364-373 (1939)

²J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 364-373 (1939)

- The inverse of this map is non-trivial, specially when accounting for uncertainties;

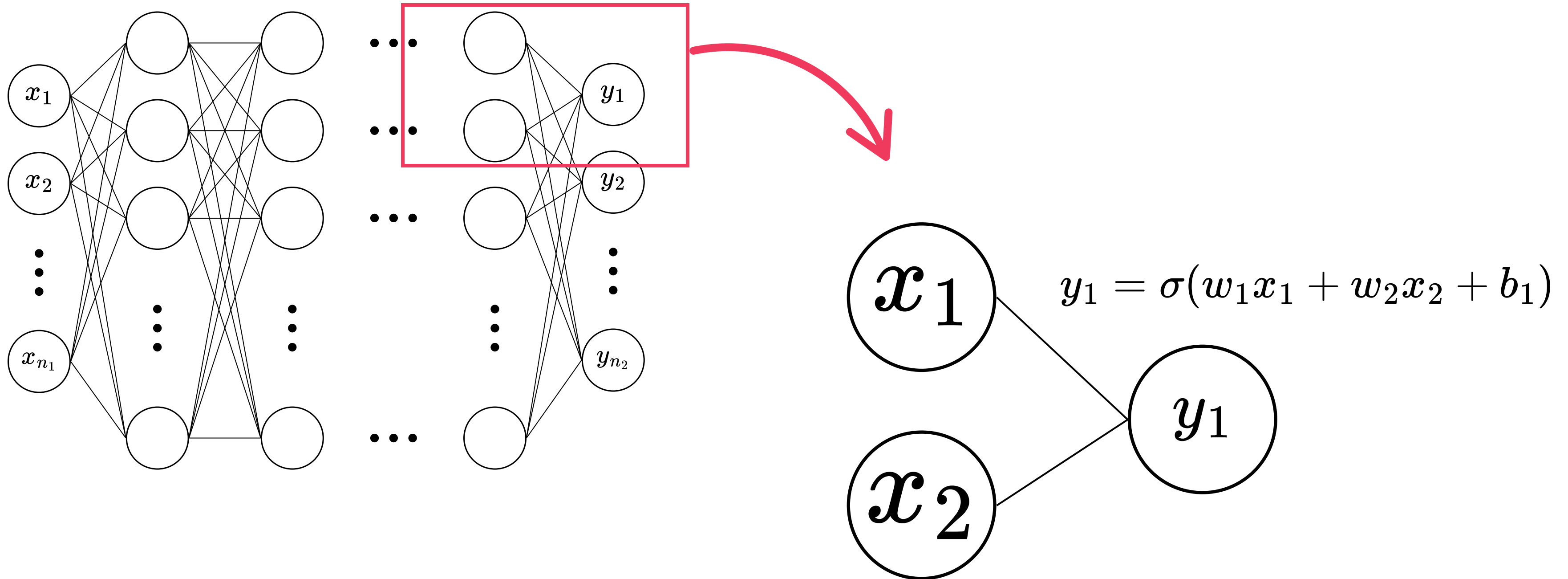


[arXiv:2106.03890v1](https://arxiv.org/abs/2106.03890v1) [astro-ph.HE]

- Several machine learning methods were proposed, including bayesian analysis and neural networks (see ³ for an overview).

³ [arXiv:2303.15136v2](https://arxiv.org/abs/2303.15136v2) [hep-ph]

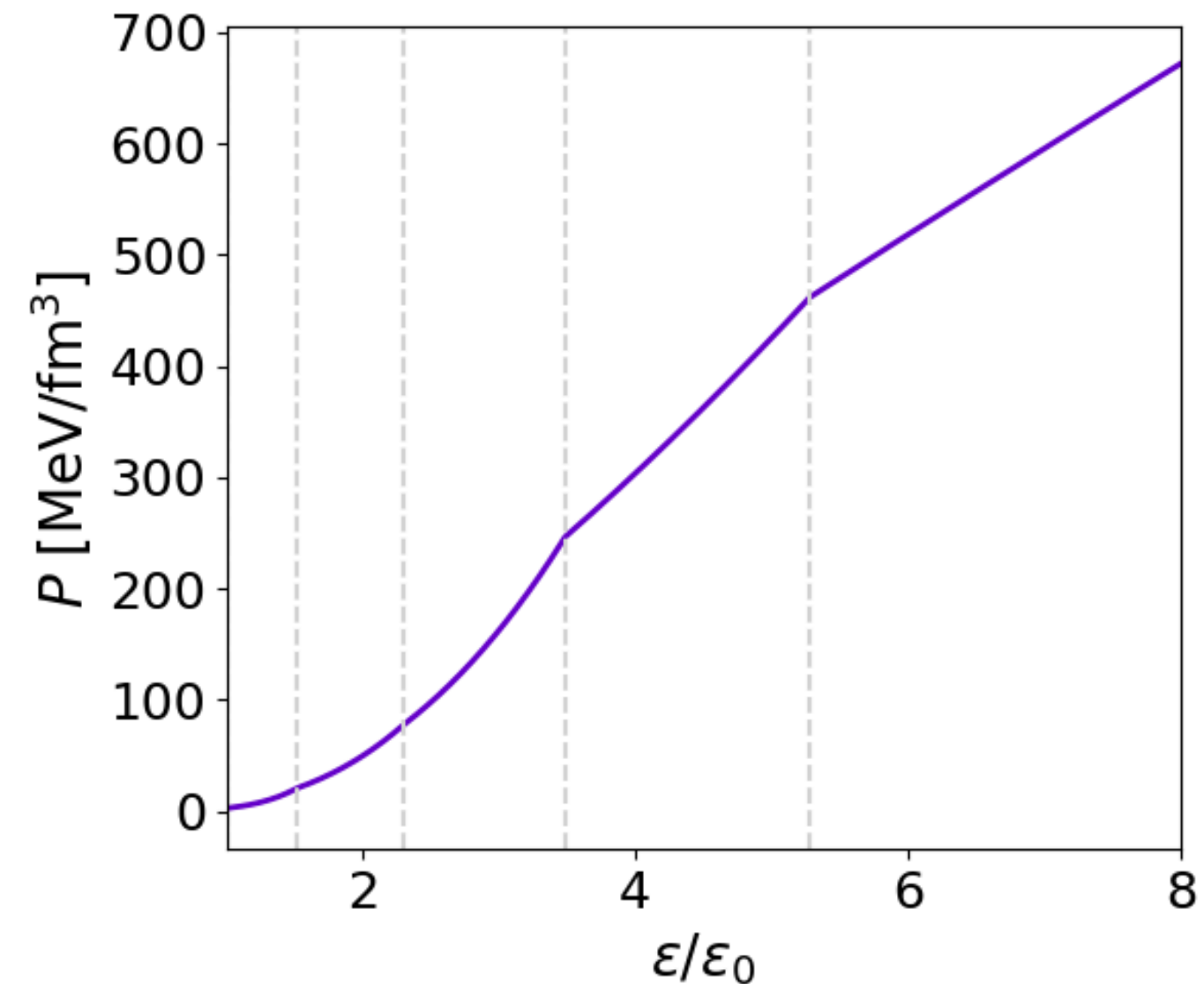
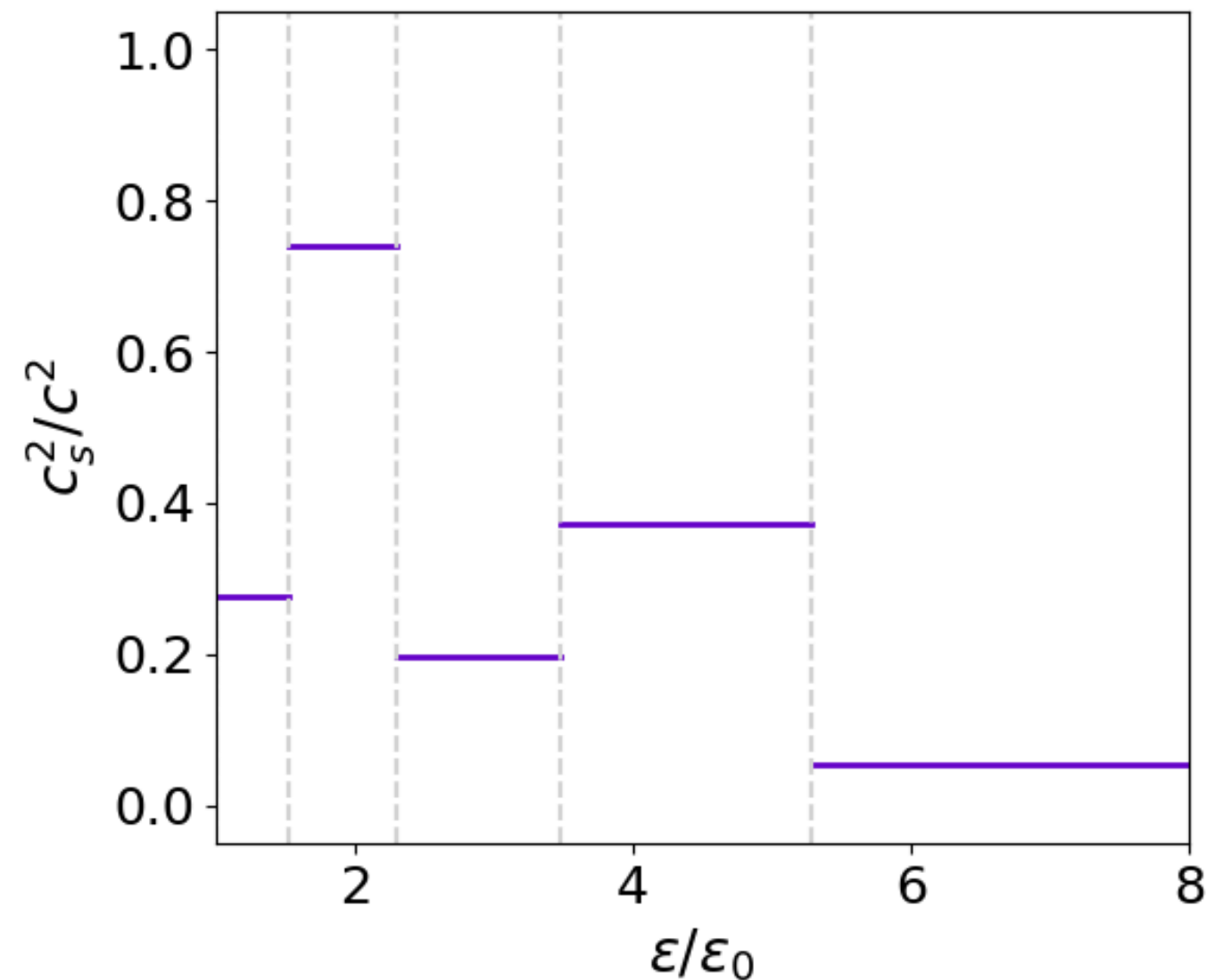
- Here we implement a neural network (NN) model from ^{4,5}. A NN can represent a given function with some set of parameters, here weights and biases.



⁴ [arXiv:1903.03400v3 \[nucl-th\]](#)

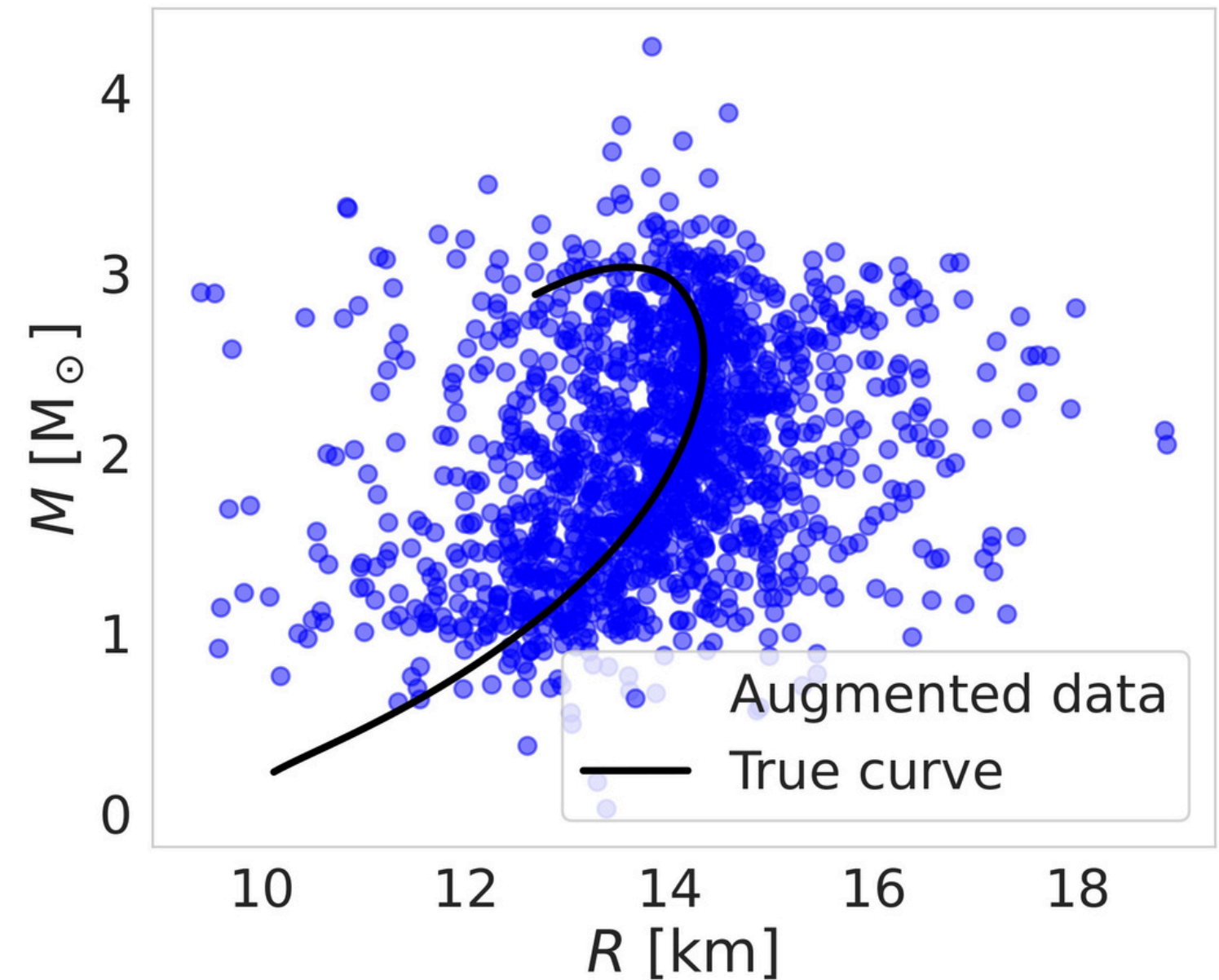
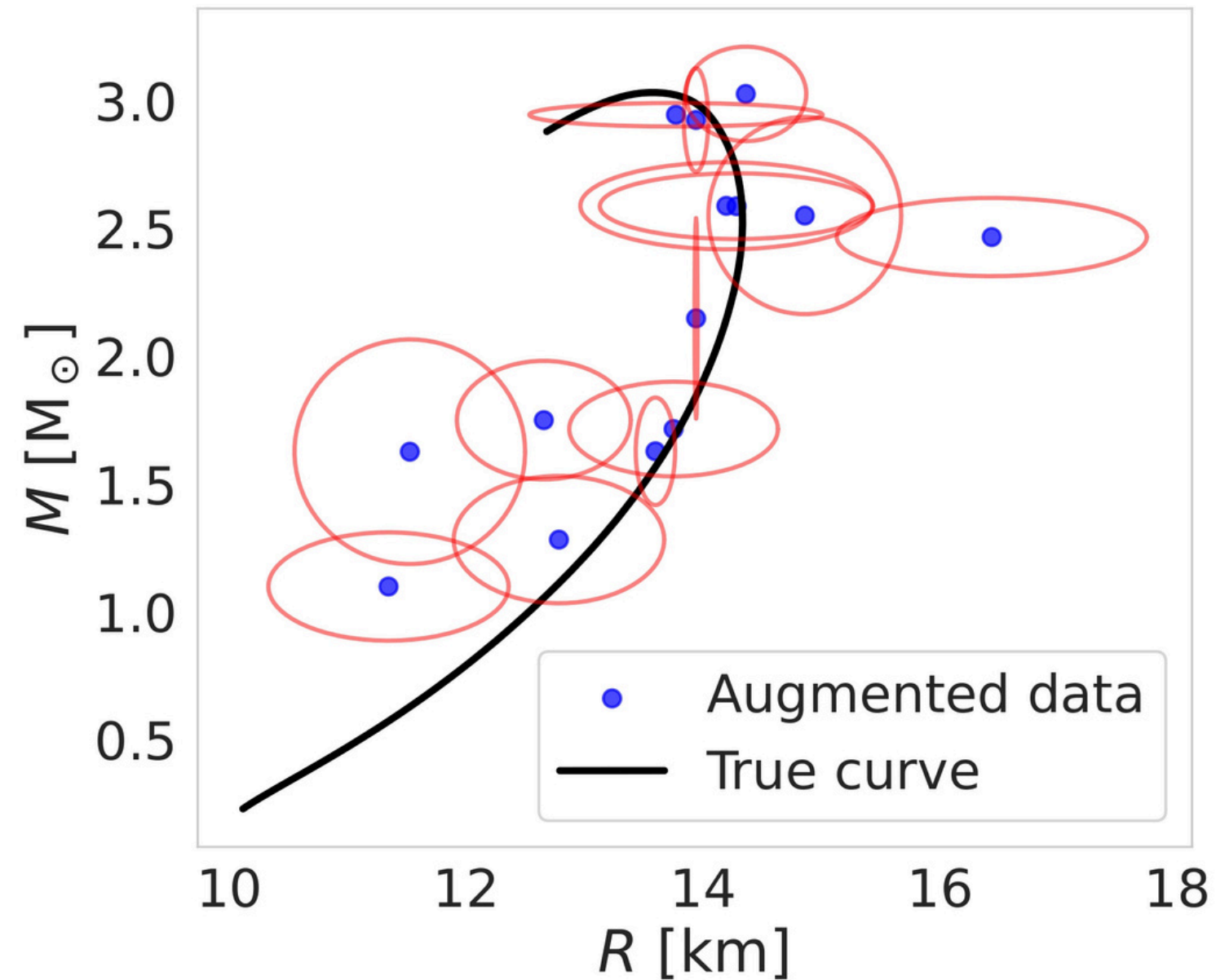
⁵ [arXiv:2401.12688v2 \[nucl-th\]](#)

- To fit the NN model parameters, we need training data;
- We assume polytropic equations of state $P = K\epsilon^\Gamma$
- Starting from a crust EoS, we can generate equations of state from the average speed of sound in 5 intervals, by matching the pressure and energy density at the boundary⁶:



⁶ [arXiv:2101.08156v1](https://arxiv.org/abs/2101.08156v1) [nucl-th]

- We solve the TOV equations for a given EoS and sample points from the M-R diagram obtained;



- We proceed to generate 4000 EoSs for two crust models, SLy and SKa^{7,8,9}, of which 200 are only used for monitoring performance (validation). A neural network model is trained for each crust model according to the table.

Layer	Number of neurons	Activation function
Input	d_{in}	N/A
Hidden layer 1	$4d_{\text{in}}$	ReLU
Hidden layer 2	$4d_{\text{in}}$	ReLU
Hidden layer 3	$2d_{\text{in}}$	ReLU
Hidden layer 4	$2d_{\text{in}}$	ReLU
Output	d_{out}	sigmoid

Neural network architecture used following ^{4,5}

⁷ [arXiv:astro-ph/0111092v2](#)

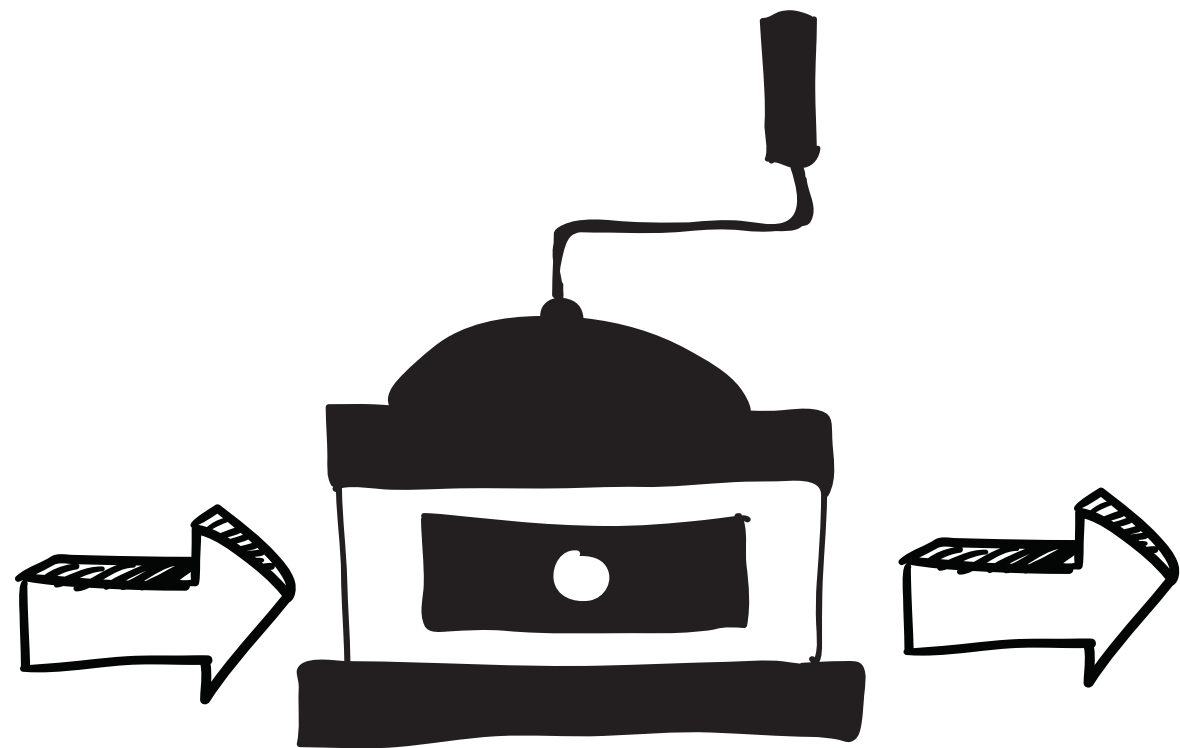
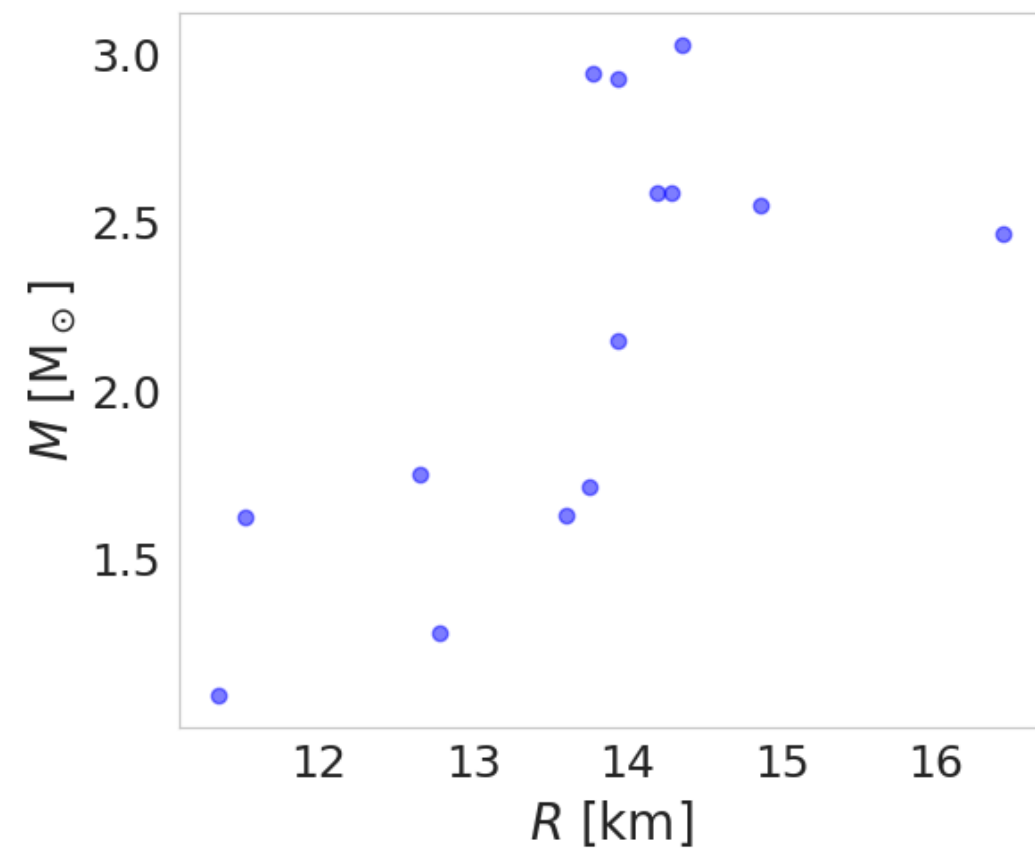
⁸ [arXiv:1504.04493v2](#) [nucl-th]

⁹ [arXiv:2203.03209v1](#) [astro-ph.HE]

$$d_{\text{in}} = 56$$

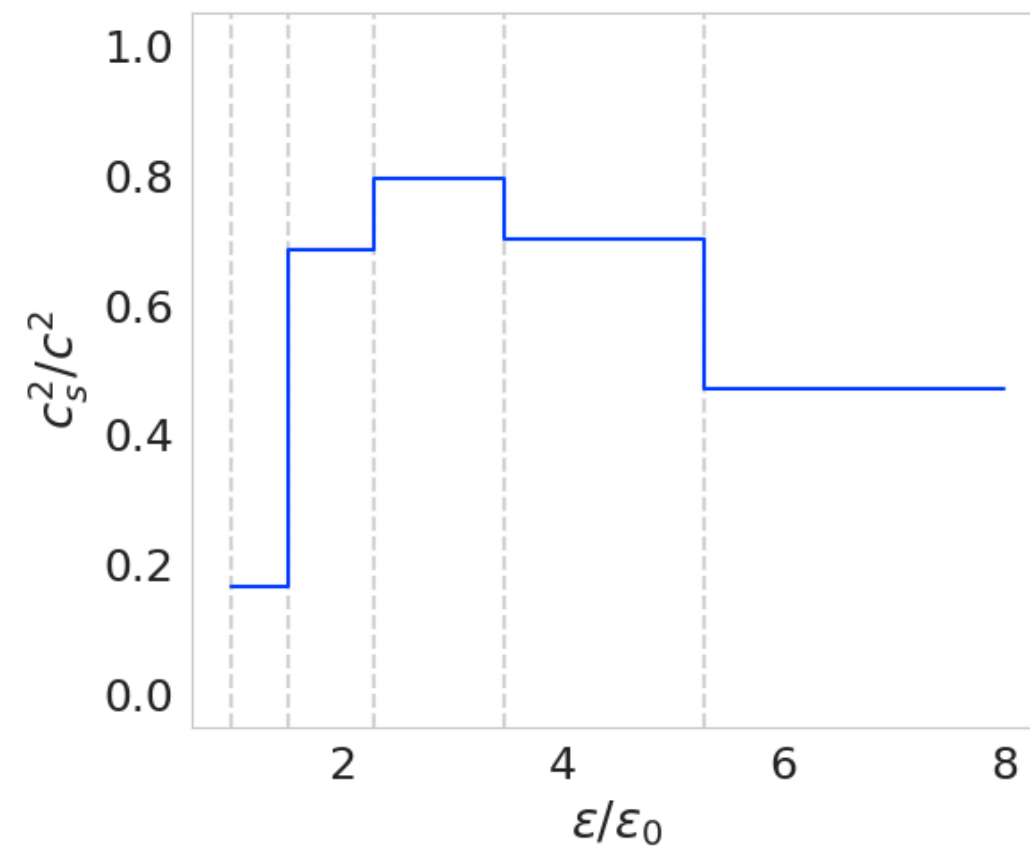
$$d_{\text{out}} = 5$$

M-R observations

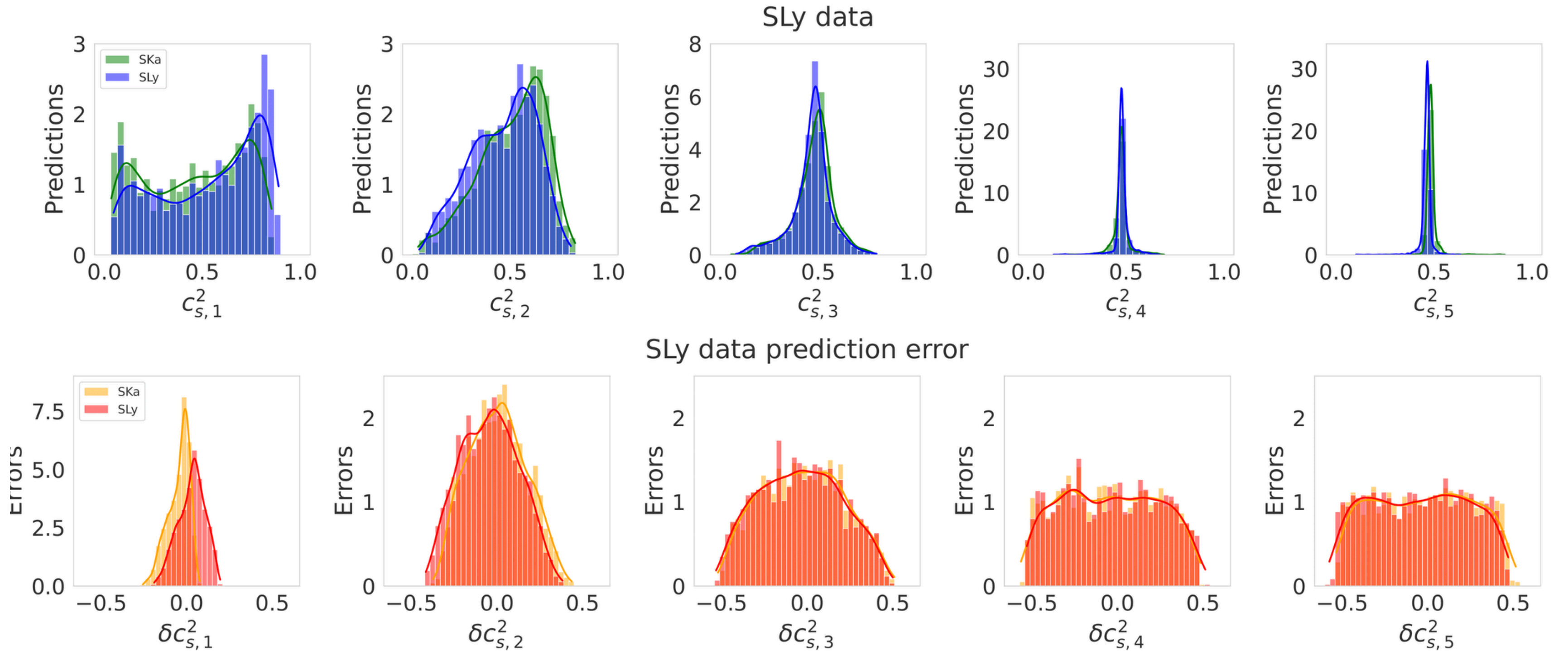


NN model

Parameterized equation of state

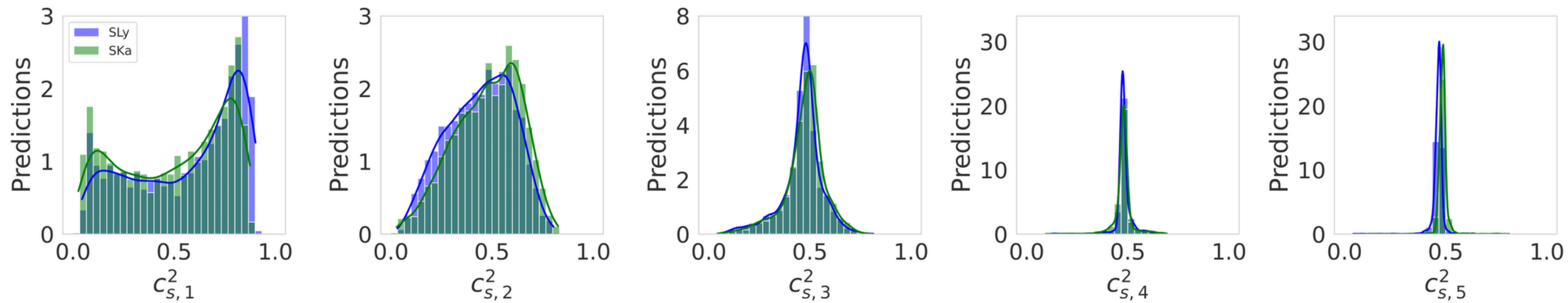


- Another 2000 EoS dataset is generated for both models, and evaluated to generate histograms of each prediction of the NNs:

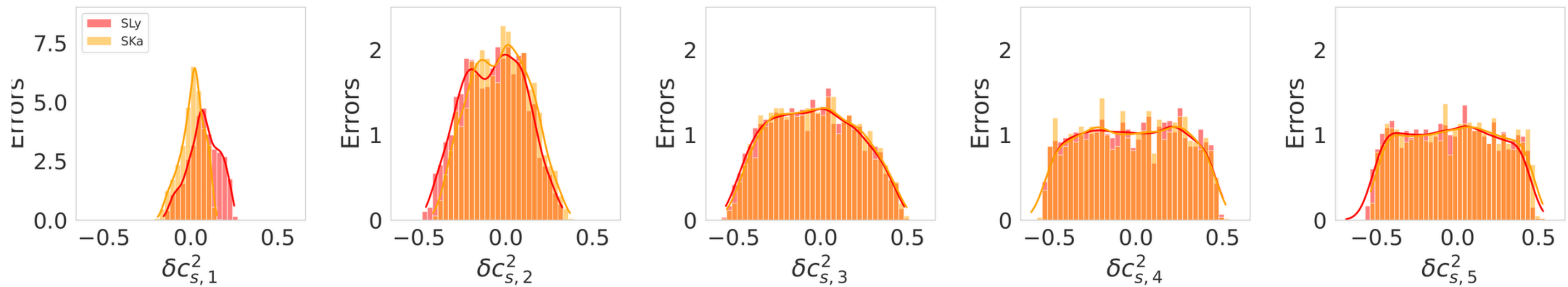


SLy and SKa models predictions and errors on SLy data

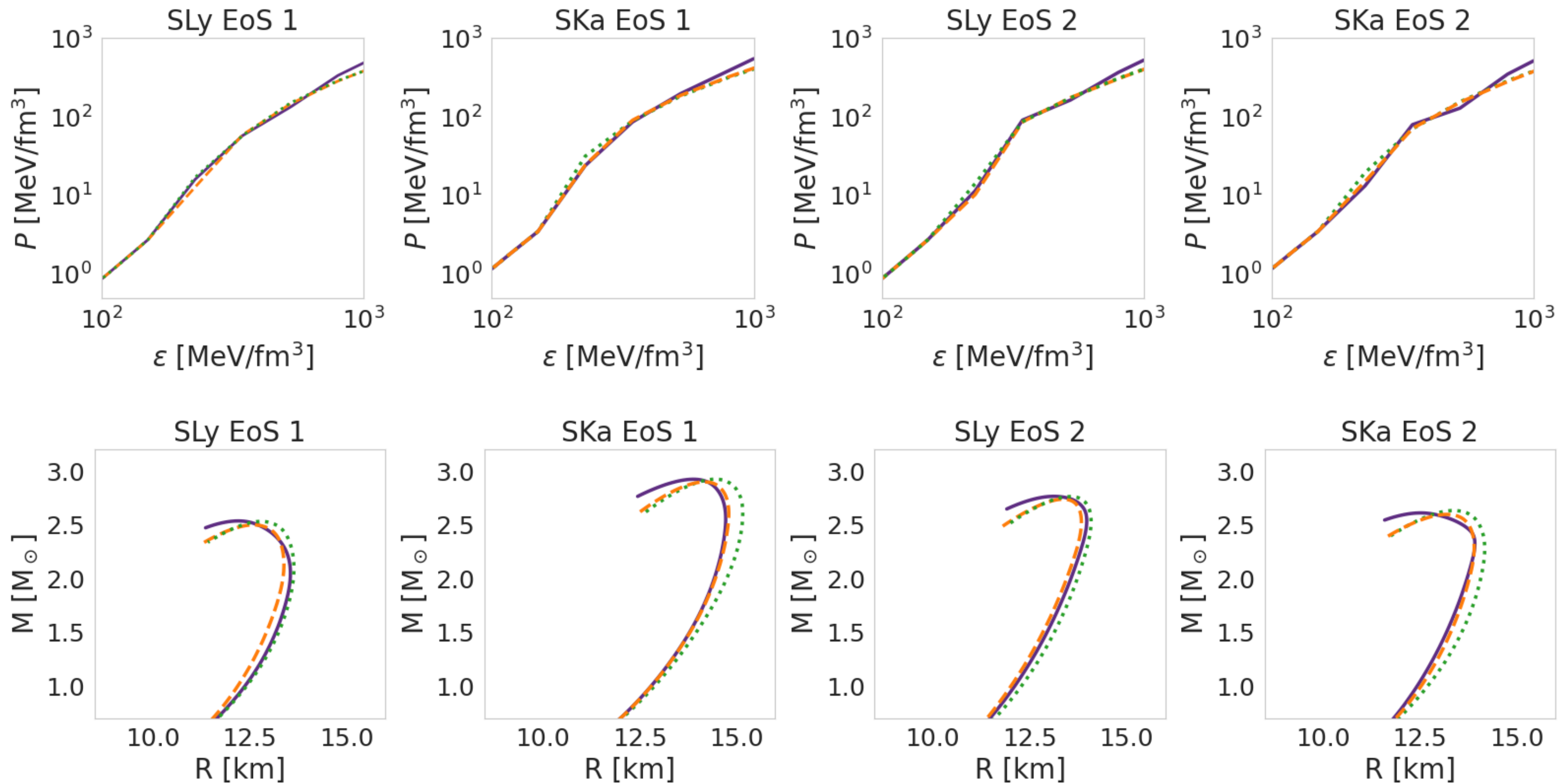
SKa data



SKa data prediction error



SLy and SKa models predictions and errors on SKa data



SLy and SKa models predictions and errors on SLy data