Physics 3340/8300-005 - "Computational Physics" - Fall 2013
Homework 9
Assigned: Monday, 4 November
Due: Friday, 8 November 11:59pm by Blackboard upload

Reading:

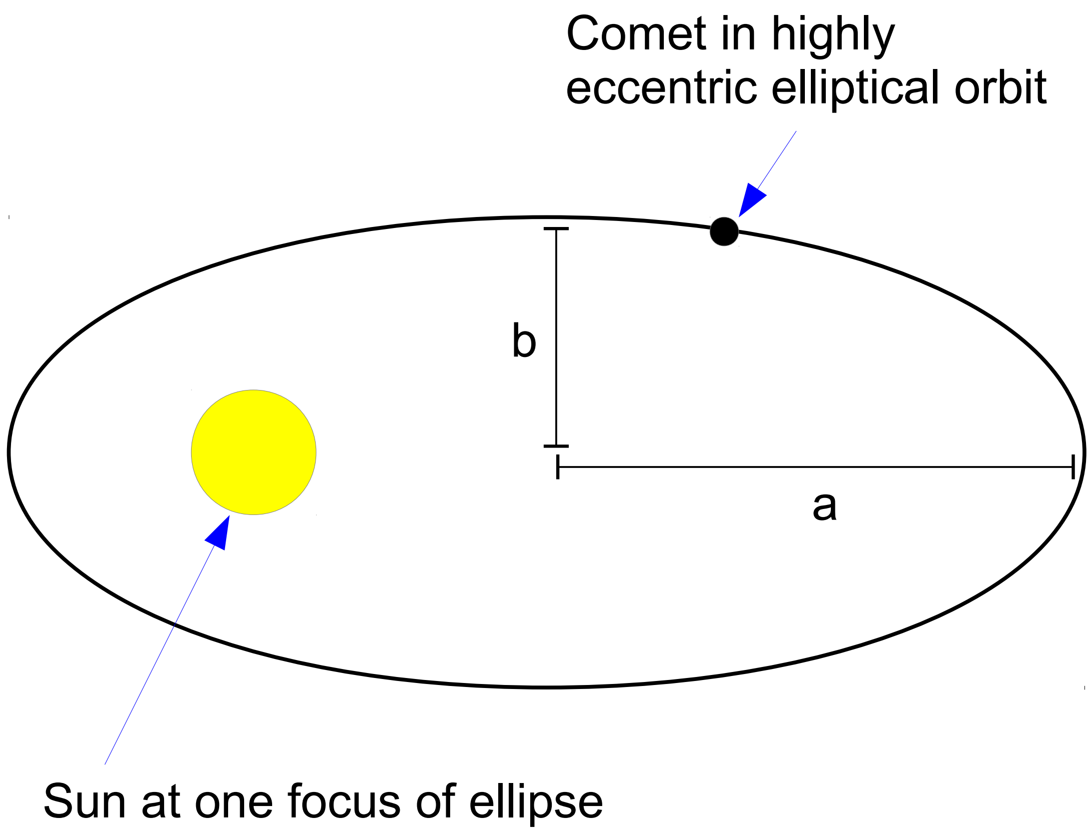
Cheney and Kincaid sections 9.1, 9.3, 5.1, 5.3 (6^{th} ed.: 12.1, 12.3, 5.1, 5.2, 6.1)
http://www.physics.smu.edu/fattarus/FittingLab.html
http://www.physics.smu.edu/fattarus/IntegrationLab.html

Problems:
For the following problems submit your solutions in your email report, with any C source code you write plus enough text or gnuplot graphics to trace the process of your work.

1) The following table lists measurements of the decay rate of radioactive iodine in counts per second as a function of time in minutes. Fit a line with the least squares method to the natural log of the decay rate and use the fit to find the half life. Plot the fit line together with the data file used to generate the fit in gnuplot. Note that as the decay count slows to low rates the fractional experimental error is typically greater than at higher rates.

Time (minutes)	**Decay rate (1/sec)**
4	398
36	156
68	69
100	24
132	12
164	3
196	2
218	1

2) A comet is constrained to a highly elliptical orbit around the sun, as pictured:

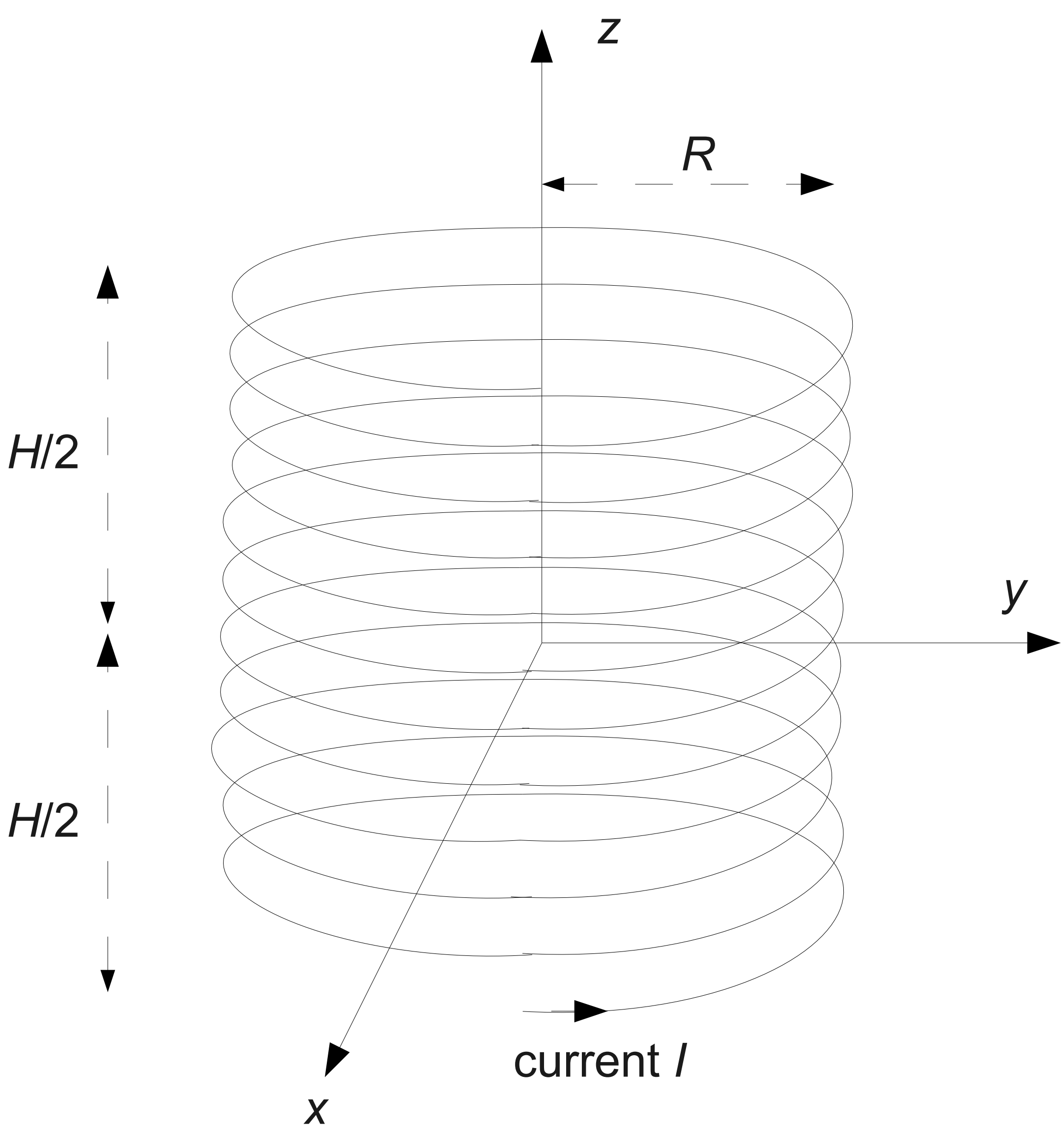


The dimension a is known as the “semi-major axis”, b is the “semi-minor axis”, and the eccentricity $\epsilon = \sqrt{1 - \frac{b^2}{a^2}}$ The circumference C around the orbit is $C = 4aE(\epsilon)$ where E is a complete elliptical integral of the second kind: $E(\epsilon) = \int_0^{\frac{\pi}{2}} \sqrt{1 - \epsilon^2 \sin^2(\theta)}\, d\theta$

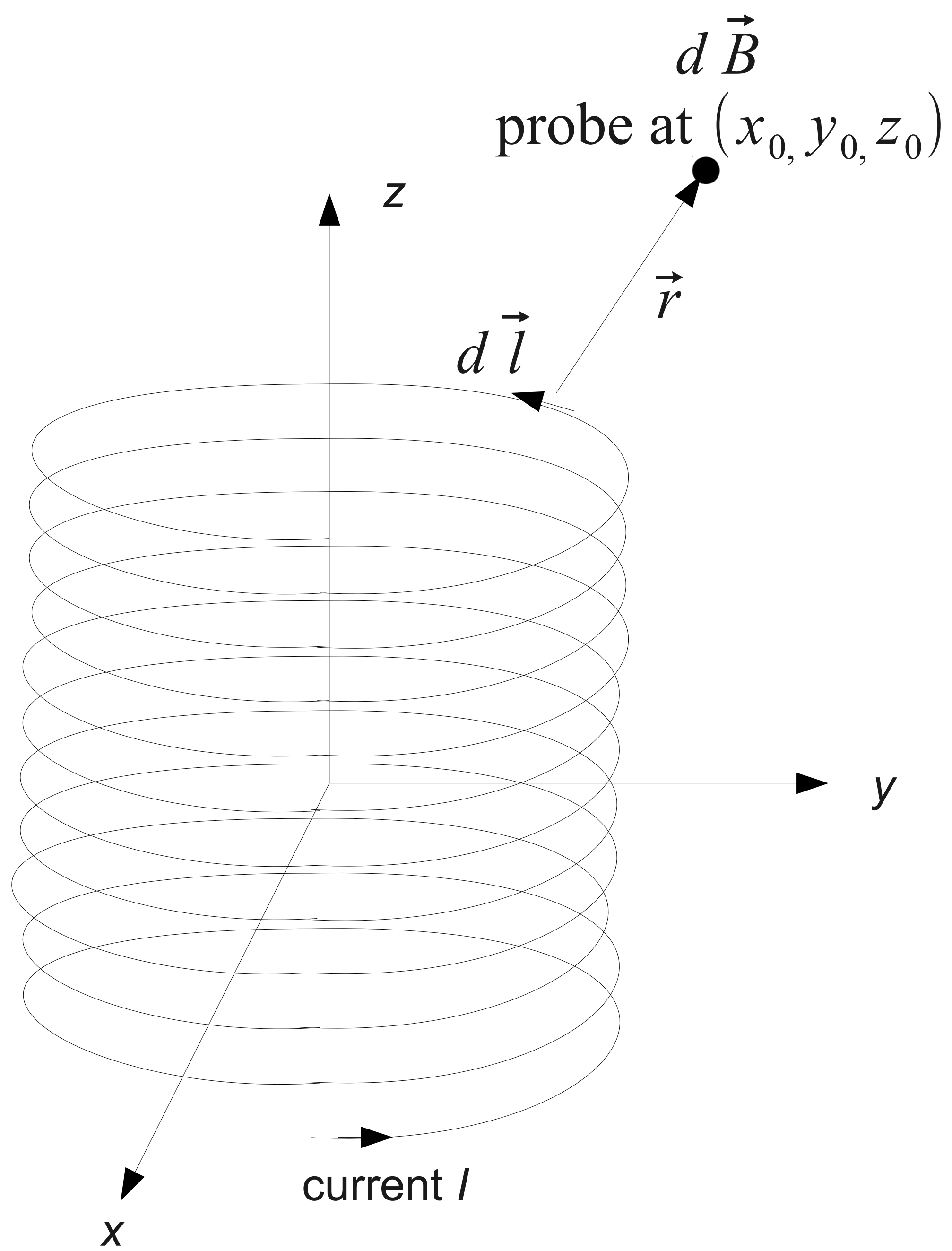
Write a C program patterned after the integrate_function.c main program in the class Integration Lab that computes the circumference of the comet's orbit. Use a semi-major axis of 2.5x10^{12} m and an eccentricity of 0.9. Compare your calculated circumference to that of a perfectly circular orbit of radius 2.5x10^{12} m. (By comparison, earth's orbit is almost circular $\epsilon \approx 0$ with radius 1.5x10^{11} m.)

3) The exhaust ejection velocity of a particular rocket engine is highest initially with a full supply of fuel, and then reduces significantly as fuel is consumed. The ejection velocity relative to the rocket v_{rel} is given by the relation $v_{rel} = \frac{2000}{1 + e^{-m_{fuel}/200}}$. Note that this relation is in terms of the **mass of fuel** in the rocket, **not** the total rocket mass. Use arbitrary units for the fuel mass and velocity. Neglecting losses due to air drag, find the final velocity of the rocket assuming it starts at rest, the **mass of the rocket without fuel** is 100, and the initial **mass of fuel** is 1000. Write a program patterned after the integrate_function.c main program in the class Integration Lab. Hint: be careful of the sign you use for the step size on the command line.

4) A cylindrical solenoid of radius R carrying current I is wound with N turns in a height of H oriented in an x,y,z coordinate system as:



The path of the conductor in the solenoid follows the parameterized equations
$x = R\cos(\theta)$ $\quad y = R\sin(\theta)$ $\quad z = a\theta$ $\quad$ for $-N\pi \leqslant \theta \leqslant N\pi$ $\quad$ where $a = \dfrac{H}{2\pi N}$ $\quad$ Use Simpson's rule for numerical integration with the Biot-Savart law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{l}\times\vec{r}}{r^3}$ and $\vec{B}(\vec{r}) = \int d\vec{B}$ to write a program patterned after the integrate_function.c main program in the class Integration Lab to find **only the z component** of the B field at an arbitrary probe location (x_0, y_0, z_0), as illustrated by:



Your program should accept coordinates for the probe point in meters on its command line. Write it for a solenoid of height 1 meter and radius 10cm, wound with 100 turns and carrying 1 ampere of current. Use at least 2000 sampling points over θ, and be sure to write the code so that there are an even number of discrete integration segments for Simpson's rule. Hint: Use the relation that the *z* component of the cross product of two vectors $\vec{a}\times\vec{b}$ with cartesian coordinates is $a_x b_y - b_x a_y$. Submit your program source code.

a) First verify your program's computation by running it with the probe placed at (0,0,0) at the center of the solenoid. Compare the *z* component of B with the value from the theoretical formula for an ideal infinite solenoid: $B_z = \mu_0 I n$ where *n* is the turns per unit length = *N*/*H*.

b) The ideal solenoid derivation assumes that the B field strength is uniform across the cross section of the cylinder. Test this assumption for the case of the real solenoid by running your program at six probe points inside the cylinder. Move the probe from its center position of (0,0,0) by varying either the x or y coordinate to -0.075, -0.05, -0.025, 0.025, 0.05 and 0.075. Compare the computed results with the center position result found in part a.

c) The ideal solenoid derivation assumes that the B field strength is also uniform along the axis of the solenoid. Test this assumption for the case of the real solenoid by running your program at five more points along its axis, at (0,0,0.1), (0,0,0.2), (0,0,0.3), (0,0,0.4), and (0,0,0.5). Comment on the result at a *z* coordinate of 0.5, where the probe is just level with the edge of the solenoid.

d) The ideal solenoid derivation assumes that the B field strength is zero outside the solenoid. Test this assumption for the case of the real solenoid by running your program for a point well outside the cylinder. Move the probe from its center position of (0,0,0) by varying either the x or y coordinate to 0.5. Comment on the magnitude and sign of the result.