USAJMO 2016

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This document contains solutions to the **USAJMO 2016** problems, written by me during my preparation for the International Mathematical Olympiad.

The content reflects my own understanding and problem-solving process. Some solutions may have been inspired by the work of others or required external help, in which case proper attribution is given (see section 3).

If you notice any errors or have suggestions for improvement, I would greatly appreciate hearing from you at samuelbaraujo19@gmail.com.

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1. Find, with proof, the least integer N such that if any 2016 elements are removed from the set $1, 2, ..., N$, one can still find 2016 distinct numbers among the remaining elements with sum N.

2 Solutions

Problem Statement

Find, with proof, the least integer N such that if any 2016 elements are removed from the set $1, 2, ..., N$, one can still find 2016 distinct numbers among the remaining elements with sum N.

Claim — badfals

Observation — adfadf

The least integer N that satisfies the statement is

$$\sum_{i=2017}^{4032} i = 6049 \cdot 1008 = 6097392.$$

Notice that if we form pairs of numbers from the set with equal sum, there will be at least 3024 such pairs. Even if at most 2016 pairs are destroyed, there will still remain at least 1008 pairs with equal sum, i.e., 2016 numbers. Each pair consists of the xth number from the left and the xth number from the right. This way, each pair has sum $6048 + 1$ as in $(1, 6048), (2, 6047), \ldots, (3024, 3025)$. $6049 \cdot 1008 = 6097392$.

N can't be less than $6,097,392$ because the least possible sum is

$$\sum_{i=1}^{2016} i = 2033136.$$

However, the first 2016 numbers of the set can be removed, changing the least possible sum to

$$\sum_{i=2017}^{4032} i = 6097392.$$

3 References