

The background of the entire page is a vibrant underwater scene. Various types of fish, ranging from small blue and yellow striped ones to larger, more detailed species like a shark and a turtle, are scattered throughout. Green, branching coral reefs of different sizes are interspersed among the fish. A large, multi-colored fish with vertical stripes is positioned in the lower-left foreground. The water is depicted with soft, circular brushstrokes in shades of blue.

The Olympiad Algebra Handbook

Samuel de Araújo Brandão

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Part I

Inequalities

Chapter 1

Basic Inequalities

1.1 Replacements

1.1.1 abc=1

If $abc = 1$, replacing a, b, c with $\frac{x}{y}, \frac{y}{z}, \frac{z}{x}$, respectively, usually homogenizes the inequality.

1.1.2 Ravi substitution

If a,b,c are said to be the sides of a triangle, replace them with $y + z$, $z + x, x + y$.

1.2 Schur's Inequality

Let a, b, c be nonnegative real numbers and $r > 0$ be positive real number. Then

$$\sum_{\text{cyc}} a^r(a^2 + bc) \geq \sum_{\text{cyc}} a^{r+1}(b + c).$$

Equality occurs if $a = b = c$ or $a = b$ and $c = 0$ and permutations.

For example, when $r = 1$:

$$\sum_{\text{cyc}}(a^3 + abc) \geq \sum_{\text{cyc}} a^2b.$$

An easier way to write the Schur's Inequality is the following:

$$\sum_{\text{cyc}} a^r(a - b)(a - c) \geq 0.$$

1.3 Hölder Inequality

1.4 Cauchy

1.4.1 Cauchy-Schwarz

1.4.2 Titu's Lemma