Solutions IMO Shortlist 2010

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December 2, 2025

This document contains solutions to the **Solutions IMO Shortlist 2010** problems, written by me during my preparation for the International Mathematical Olympiad.

The content reflects my own understanding and problem-solving process. Some solutions may have been inspired by the work of others or required external help, in which case proper attribution is given (see section 3).

If you notice any errors or have suggestions for improvement, I would greatly appreciate hearing from you at samuelbaraujo19@gmail.com.

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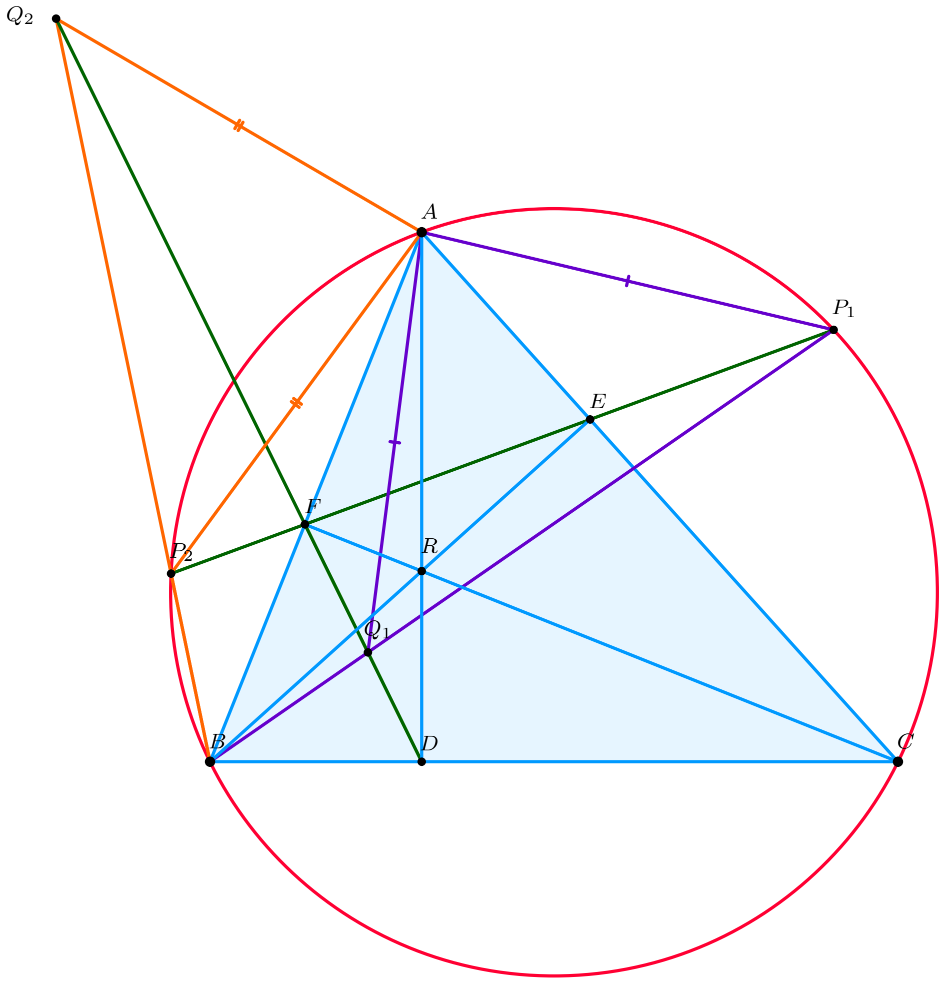
1 Problems

1. Let ABC be an acute triangle with D, E, F the feet of the altitudes lying on BC, CA, AB respectively. One of the intersection points of the line EF and the circumcircle is P. The lines BP and DF meet at point Q. Prove that $AP = AQ$.

2 Solutions

Problem Statement

Let ABC be an acute triangle with D, E, F the feet of the altitudes lying on BC, CA, AB respectively. One of the intersection points of the line EF and the circumcircle is P. The lines BP and DF meet at point Q. Prove that $AP = AQ$.



Let $\measuredangle$ denote directed angles mod $180°$. The line EF meets the circumcircle at two points. Directed angles allow us to treat both at once, so we fix one of them.

Our goal is to show $\measuredangle PQA = \measuredangle APQ$, i.e., that $\triangle APQ$ is isosceles. We will first prove that A, F, P, Q are concyclic.

Since $\triangle DEF$ is an orthic triangle, $\measuredangle CFA = \measuredangle ADC = 90°$. Therefore, $FACD$ is cyclic, culminating in $\measuredangle ACD = \measuredangle BFD$. Besides that, A, F, B are colinear, as are D, F, Q. Hence, $\measuredangle AFQ = \measuredangle BFD = \measuredangle ACD$. However, $APCB$ is cyclic, so $\measuredangle APB = \measuredangle ACB = \measuredangle ACD$. Thus, $\measuredangle APQ = \measuredangle AFQ$. Which means that $AFPQ$ is a cyclic quadrilateral.

$FECB$ is a cyclic quadrilateral either, by the same $FACD$'s reason. Therefore, putting together everything we've seen so far:

$$\measuredangle PQA = \measuredangle PFA = \measuredangle EFB = \measuredangle ECB = \measuredangle ACB = \measuredangle APB.$$

Hence, $AP = AQ$.

3 References