

Solutions IMO 2001

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This document contains solutions to the Solutions IMO 2001 problems, written by me during my preparation for the International Mathematical Olympiad.

The content reflects my own understanding and problem-solving process. Some solutions may have been inspired by the work of others or required external help, in which case proper attribution is given (see [section 3](#)).

If you notice any errors or have suggestions for improvement, I would greatly appreciate hearing from you at samuelbaraujo19@gmail.com.

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1 Problems

1. Prove that for all positive real numbers a, b, c ,

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1.$$

2 Solutions

2.1 Problem 2.

Problem Statement

Prove that for all positive real numbers a, b, c ,

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1.$$

By Hölder,

$$\left(\sum_{\text{cyc}} \frac{a}{\sqrt{a^2 + 8bc}} \right) \left(\sum_{\text{cyc}} a \sqrt{a^2 + 8bc} \right) \geq (a + b + c)^2.$$

Hence, is it enough to prove that

$$(a + b + c)^2 \geq \sum_{\text{cyc}} a \sqrt{a^2 + 8bc} \iff 2(a^2b^2 + b^2c^2 + c^2a^2) \geq 2(a^2bc + ab^2c + abc^2),$$

which is clearly true by the Muirhead's inequality.

3 References