Balkan MO 1986 Solution

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A collection of Balkam MO 1986 solutions, inspired by Evan Chen's style.

All solutions were written by me while preparing for the International Mathematical Olympiad (IMO).

If you spot any errors or have suggestions or comments, feel free to reach out!

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1 Problems

1. A line passing through the incenter I of the triangle ABC intersect its incircle at D and E and its circumcircle at F and G, in such a way that the point D lies between I and F. Prove that: $DF \cdot EG \geq r^2$.

2 Solutions

2.1 Problem 1.

Problem statement

A line passing through the incenter I of the triangle ABC intersect its incircle at D and E and its circumcircle at F and G, in such a way that the point D lies between I and F. Prove that: $DF \cdot EG \geq r^2$.

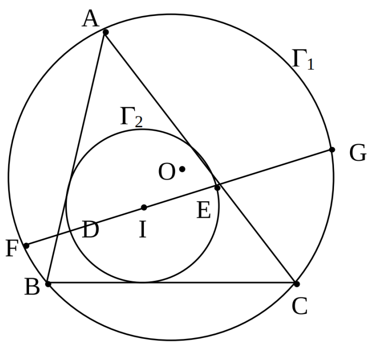

Figure 1: A figure illustrating the solution to the first problem. Source

$DF \cdot EG = (IF - ID)(IG - IE) = (IF - r)(IG - r)$ where r is the inradius. Hence
$IF \cdot IG - r(IF + IG) + r^2 = -Pot_{\Gamma_1} I - GFr + r^2$
$= R^2 - IO^2 - GFr + r^2$.
Since the distance IO between the incenter and circumcenter satisfies $IO = \sqrt{R^2 - 2Rr}$, it follows
$R^2 - IO^2 - GFr + r^2 = 2Rr - GFr + r^2 \Rightarrow 2Rr$
$- GFr + r^2 \geq r^2 \iff 2Rr \geq GFr$.
This inequality holds since GF is a chord of Γ_1.
$DF \cdot EG = r^2 \iff 2Rr = GFr$.

2.2 Problem 2.

2.3 Problem 3.

2.4 Problem 4.

3 References

This document was made possible thanks to the help and inspiration of the following resource:

[1] OBM - Olimpíada Brasileira de Matemática. *Potência de Ponto, Eixo Radical, Centro Radical e Aplicações*, 2017. Available in: https://www.obm.org.br/content/uploads/2017/01/eixos-2.pdf