

# Solutions IMO 1995

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This document contains solutions to the Solutions IMO 1995 problems, written by me during my preparation for the International Mathematical Olympiad.

The content reflects my own understanding and problem-solving process. Some solutions may have been inspired by the work of others or required external help, in which case proper attribution is given (see [section 3](#)).

If you notice any errors or have suggestions for improvement, I would greatly appreciate hearing from you at [samuelbaraujo19@gmail.com](mailto:samuelbaraujo19@gmail.com).

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# 1 Problems

1. Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

## 2 Solutions

### 2.1 Problem 2.

#### Problem Statement

Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

**First solution** By Cauchy-Schwarz,

$$\left( \sum_{\text{cyc}} \frac{1}{a^3(b+c)} \right) \left( \sum_{\text{cyc}} a(b+c) \right) \geq \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)^2 = (ab+bc+ca)^2.$$

Hence, it is enough to prove that

$$(ab+bc+ca)^2 \geq \frac{3}{2} \left( \sum_{\text{cyc}} a(b+c) \right) \iff (ab+bc+ca)^2 \geq 3(ab+bc+ca)(abc)^{\frac{2}{3}},$$

which is true by AM-GM.

**Second solution** By Cauchy-Schwarz,

$$\left( \sum_{\text{cyc}} \frac{1}{a^3(b+c)} \right) \left( \sum_{\text{cyc}} a(b+c) \right) \geq \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)^2 = (ab+bc+ca)^2,$$

and by Titu's lemma,

$$\sum_{\text{cyc}} \frac{\left(\frac{1}{a}\right)^2}{a(b+c)} \geq \frac{ab+bc+ca}{2} \geq \frac{3(abc)^{\frac{2}{3}}}{2} = \frac{3}{2}$$

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### 3 References