Solutions IMO 2001

Samuel de Araújo Brandão

December 4, 2025

This document contains solutions to the Solutions IMO 2001 problems, written by me during my preparation for the International Mathematical Olympiad.

The content reflects my own understanding and problem-solving process. Some solutions may have been inspired by the work of others or required external help, in which case proper attribution is given (see section 3).

If you notice any errors or have suggestions for improvement, I would greatly appreciate hearing from you at samuelbaraujo19@gmail.com.

Contents

1 Problems

1. Prove that for all positive real numbers a, b, c,

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1.$$

2 Solutions

2.1 Problem 2.

Problem Statement

Prove that for all positive real numbers a, b, c,

$$\frac{a}{\sqrt{a^2+8bc}} + \frac{b}{\sqrt{b^2+8ca}} + \frac{c}{\sqrt{c^2+8ab}} \geq 1.$$

By Hölder,

$$\left(\sum_{\text{cyc}} \frac{a}{\sqrt{a^2+8bc}}\right)\left(\sum_{\text{cyc}} a\sqrt{a^2+8bc}\right) \geq (a+b+c)^2.$$

Hence, is it enough to prove that

$$(a+b+c)^2 \geq \sum_{\text{cyc}} a\sqrt{a^2+8bc} \iff 2(a^2b^2+b^2c^2+c^2a^2) \geq 2(a^2bc+ab^2c+abc^2),$$

which is clearly true by the Muirhead's inequality.

3 References