

Solutions IMO 1995

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This document contains solutions to the Solutions IMO 1995 problems, written by me during my preparation for the International Mathematical Olympiad.

The content reflects my own understanding and problem-solving process. Some solutions may have been inspired by the work of others or required external help, in which case proper attribution is given (see [section 3](#)).

If you notice any errors or have suggestions for improvement, I would greatly appreciate hearing from you at samuelbaraujo19@gmail.com.

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1 Problems

1. Let a, b, c be positive real numbers. Prove that

$$\frac{a^3 + 3b^3}{5a + b} + \frac{b^3 + 3c^3}{5b + c} + \frac{c^3 + 3a^3}{5c + a} \geq \frac{2}{3}(a^2 + b^2 + c^2)$$

2 Solutions

2.1 Problem 2.

Problem Statement

Let a, b, c be positive real numbers. Prove that

$$\frac{a^3 + 3b^3}{5a + b} + \frac{b^3 + 3c^3}{5b + c} + \frac{c^3 + 3a^3}{5c + a} \geq \frac{2}{3}(a^2 + b^2 + c^2)$$

By Titu's lemma we have

$$\sum_{\text{cyc}} \frac{a^4}{a(5a + b)} \geq \frac{a^2 + b^2 + c^2}{6} = x \quad \text{and} \quad 3 \sum_{\text{cyc}} \frac{b^4}{b(5a + b)} \geq \frac{a^2 + b^2 + c^2}{2} = y$$

since $\sum_{\text{cyc}} 5ab + b^2 \leq \sum_{\text{cyc}} 5a^2 + ab \leq 6(a^2 + b^2 + c^2)$. Hence $x + y = \frac{2}{3}(a^2 + b^2 + c^2)$.

3 References