

Orthogonal Equipartitions of 3-color Points in the Plane

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Introduction I

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tions

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Conclusions

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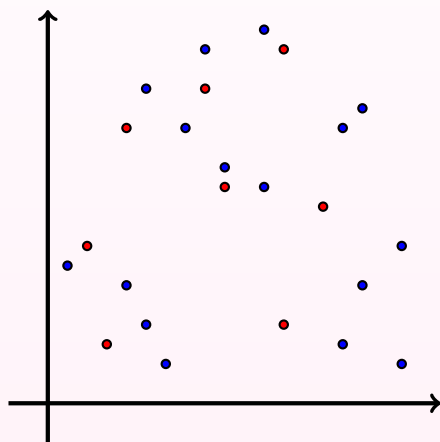


Figure: 2-color points in the plane in general position.

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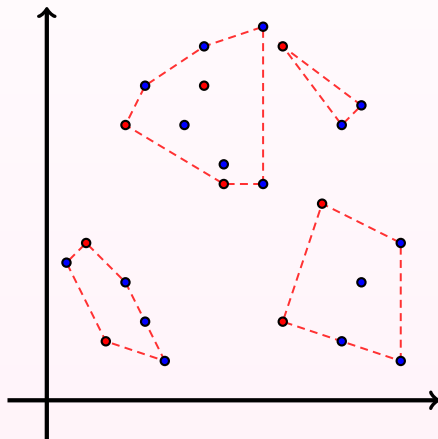


Figure: Balanced partition of convex sets.

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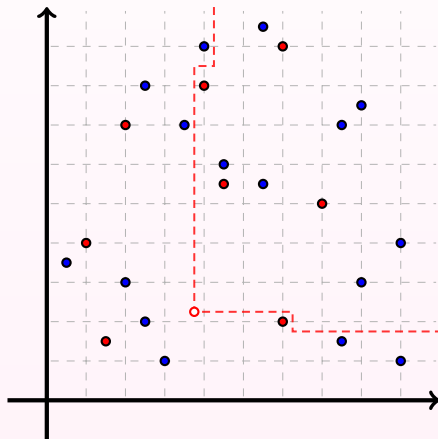


Figure: Balanced partition of semi-rectangular cut.

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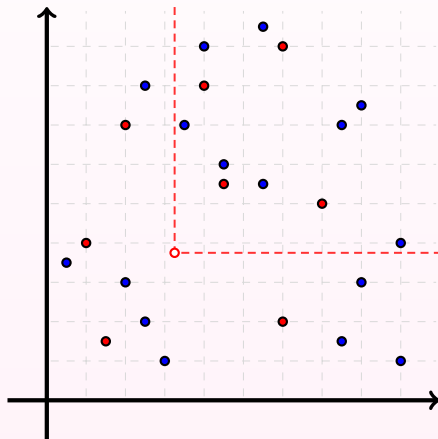


Figure: Balanced partition of 1 vertical and 1 horizontal segments.

Main Problem

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tions

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Upper bound
Lower bound

Conclusion

Problem 1

Let $k > 2$ be an integer. Given a set of k -color points in general position in the plane. What is a smallest number of line segments for equitable partitioning k -color into two regions?

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1.1.1. General

1.1.2. Special

Result

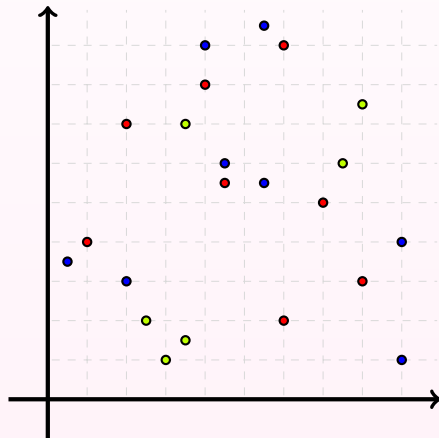


Figure: The 3-color points in general position in the plane.

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1. Line Segment

2. Equipartition

Result

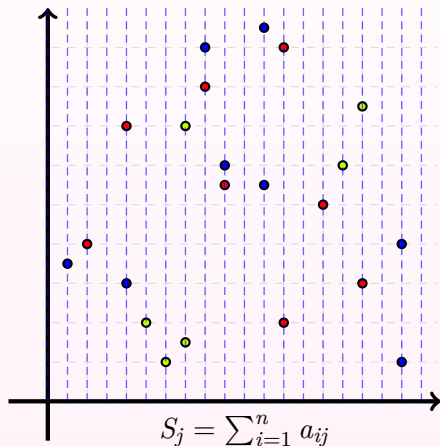


Figure: S_j is a sum of number of each color points along left side of a line $x = j$.

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A Line Segment
of Equipartitions

Result

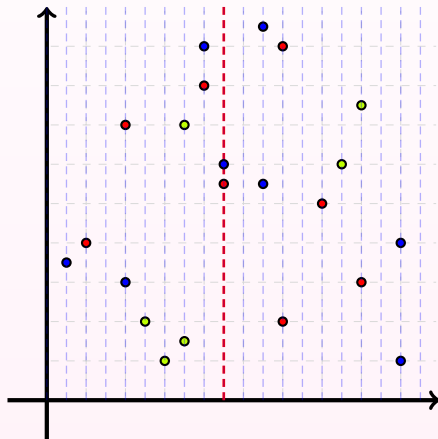


Figure: A first vertical line $x = j$.

Methodology V

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1. Data Generation
2. Data Analysis

Result

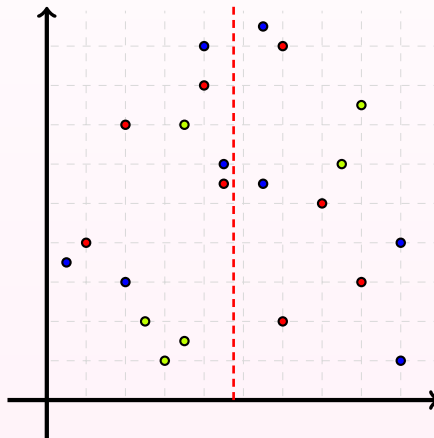


Figure: A first vertical line $x = j$.

Methodology VI

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1 Line Segment

2 Line Segment

Result

Verify a solution

A first vertical line $x = j$ is a solution when

$$\left| \frac{S_n}{2} - S_j \right| \leq \frac{1}{2},$$

for every color of points.

3 Line Segments I

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3 Line Segments

Conclusion

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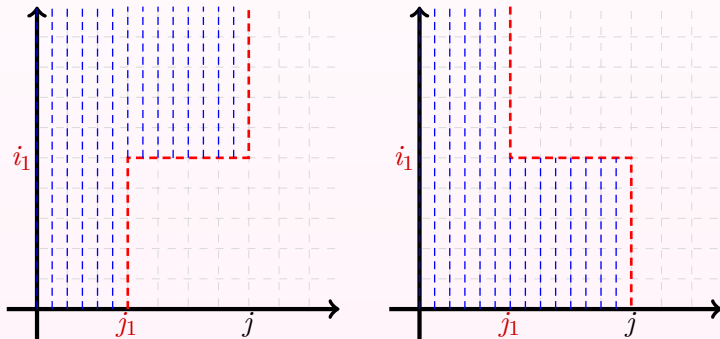


Figure: 2 vertical and 1 horizontal segments.

3 Line Segments II

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3 Line Segments

Complexity

Result

Convert into equation

$$V = \sum_{i=0}^{i_1} a_{ij} + \sum_{i=i_1}^n a_{ij_1} \quad \text{or} \quad V = \sum_{i=0}^{i_1} a_{ij_1} + \sum_{i=i_1}^n a_{ij}, \quad (1)$$

where $0 \leq j_1 \leq j$ for each color of points.

Verify a solution

This partition is a solution when

$$\left| \frac{S_n}{2} - V \right| \leq \frac{1}{2},$$

for every color of points.

3 Line Segments III

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3 Line Segments

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Recap

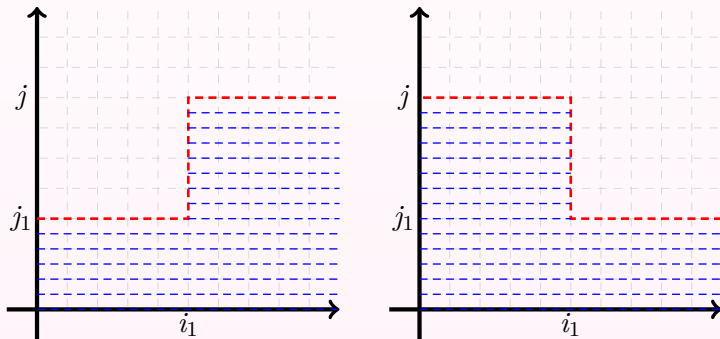


Figure: 1 vertical and 2 horizontal segments.

Some Cases of 5 Line Segments I

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4 Line Segments

5 Line Segments

Results

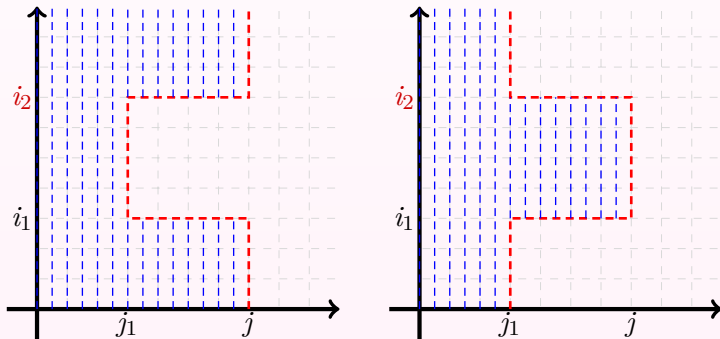


Figure: Some cases of 3 vertical and 2 horizontal segments.

Some Cases of 5 Line Segments II

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4 Line Segments

5 Line Segments

Results

Convert into equation

$$V = \sum_{i=0}^{i_1} a_{ij} + \sum_{i=i_1}^{i_2} a_{ij_1} + \sum_{i=i_2}^n a_{ij},$$

or

$$V = \sum_{i=0}^{i_1} a_{ij_1} + \sum_{i=i_1}^{i_2} a_{ij} + \sum_{i=i_2}^n a_{ij_1},$$

where $0 \leq j_1 \leq j$ for each color of points.

Some Cases of 5 Line Segments III

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4 Line Segments

5 Line Segments

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Verify a solution

This partition is a solution when

$$\left| \frac{S_n}{2} - V \right| \leq \frac{1}{2},$$

for every color of points.

Some Cases of 5 Line Segments IV

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4 Line Segments

5 Line Segments

Result

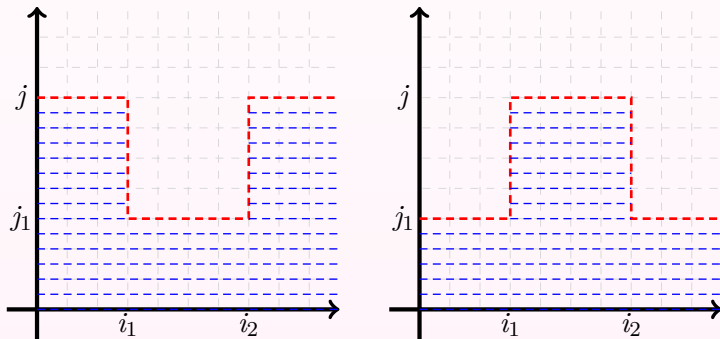


Figure: Some cases of 2 vertical and 3 horizontal segments.

Counterexample of 3 Line Segments

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1 Line Segment

2 Line Segments

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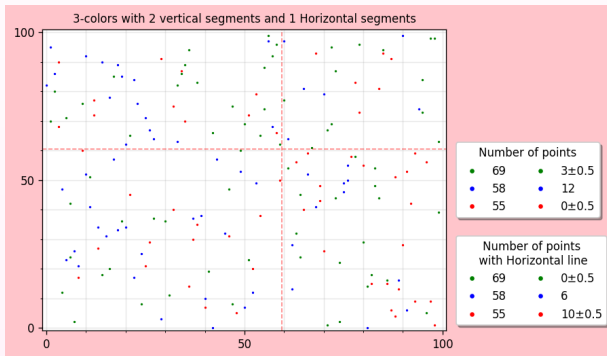


Figure: Counterexample of equipartitions with 3 line segments for 3 color point sets.

5 Line Segments for 3-color Points

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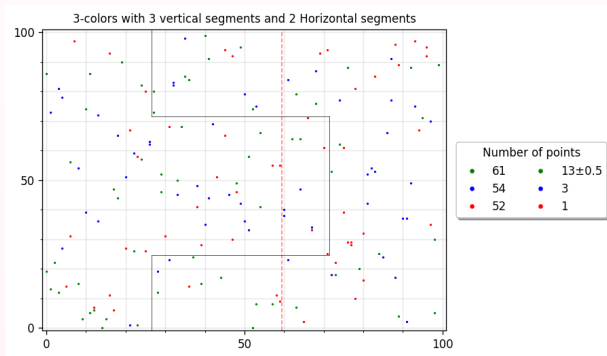


Figure: Equipartitions with 3 vertical and 2 horizontal segments for 3 color point sets when translated a value $x = j$.

Counterexample of some cases of 5 Line Segments

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1 Line Segment

2 Line Segments

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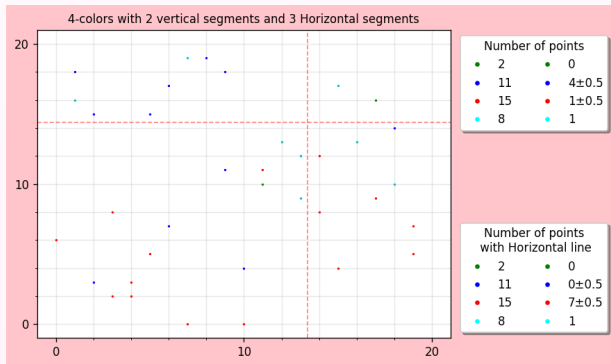


Figure: Counterexample of equipartitions with some cases of 5 line segments for 4 color point sets.

General Case of 5 Line Segments I

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1 Line Segment

2 Line Segments

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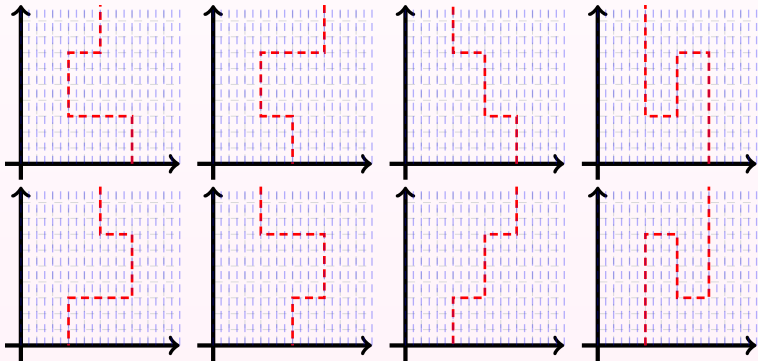


Figure: General case of 3 vertical and 2 horizontal segments.

General Case of 5 Line Segments II

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Convert into equation

$$V = \sum_{i=0}^{i_1} a_{ij_1} + \sum_{i=i_1}^{i_2} a_{ij_2} + \sum_{i=i_2}^n a_{ij_3},$$

where $i_1 \leq i_2$ for each color of points or

$$V = \sum_{i=0}^{i_1} a_{ij_1} - \sum_{i=i_1}^{i_2} a_{ij_2} + \sum_{i=i_2}^n a_{ij_3},$$

where $i_1 > i_2$ for each color of points.

General Case of 5 Line Segments III

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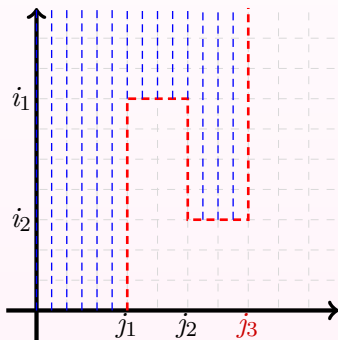
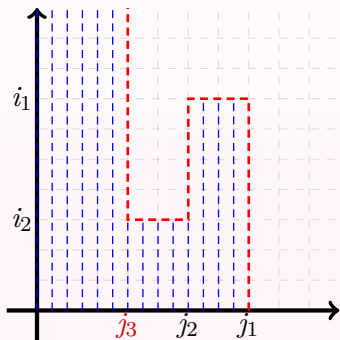


Figure: General case of 3 vertical and 2 horizontal segments with $i_1 > i_2$.

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Q & A 🤔 🤔

Application I

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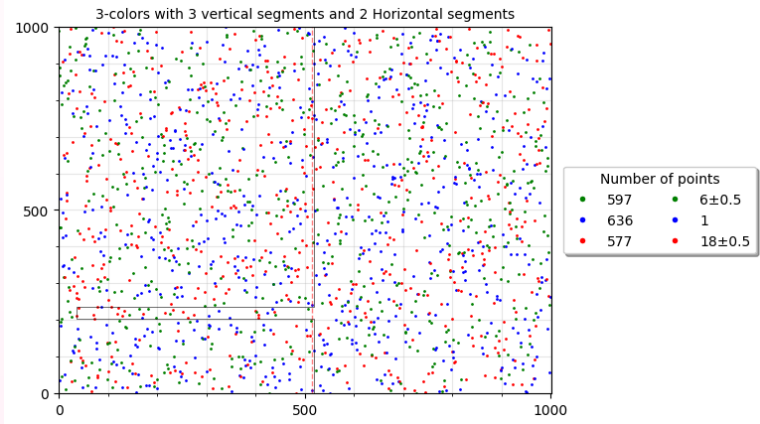
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1-Dimensional

2-Dimensional

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Application II

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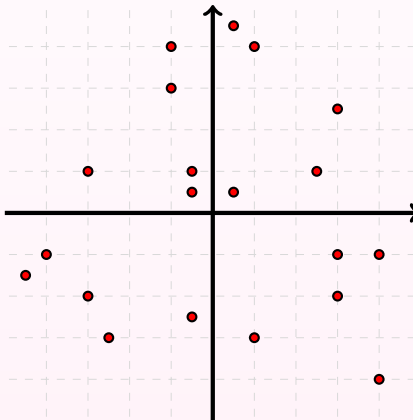
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