# Orthogonal Equipartitions of 3-color Points in the Plane

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# Introduction I

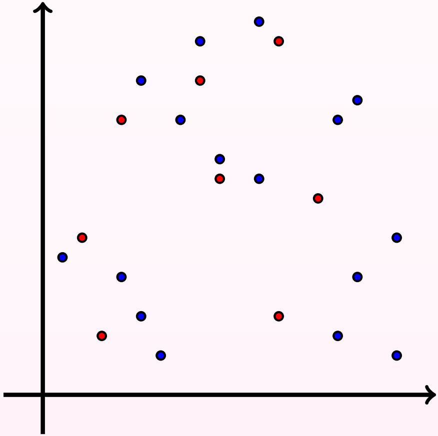


Figure: 2-color points in the plane in general position.

# Introduction II

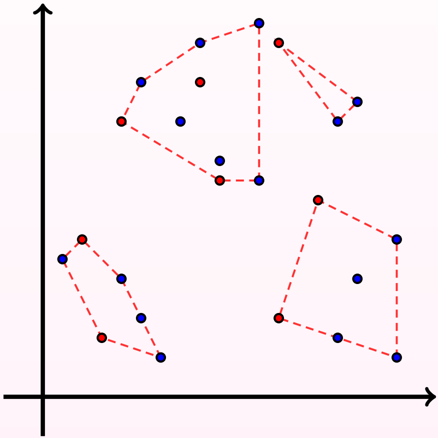


Figure: Balanced partition of convex sets.

# Introduction III

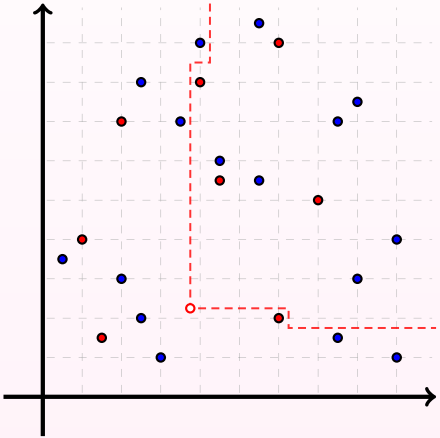


Figure: Balanced partition of semi-rectangular cut.

## Introduction IV



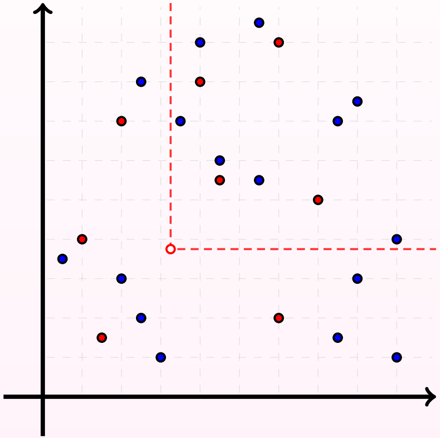


Figure: Balanced partition of 1 vertical and 1 horizontal segments.

# Main Problem

**Problem 1**

Let $k > 2$ be an integer. Given a set of $k-$color points in general position in the plane. What is a smallest number of line segments for equitable partitioning $k-$color into two regions?

# Methodology I

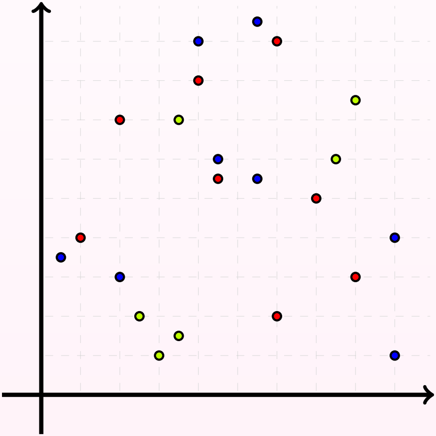


Figure: The 3-color points in general position in the plane.

# Methodology II

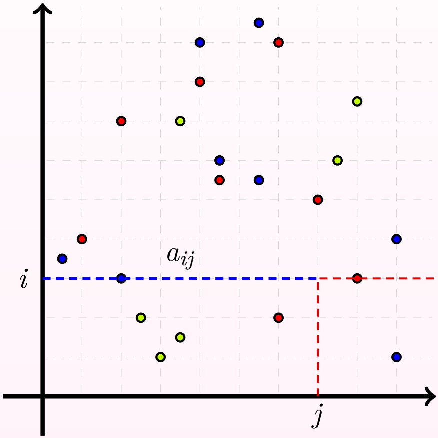



Figure: $a_{ij}$ is the number of each color points that lie on $y = i$ from $x = 0$ to $x = j$

# Methodology III

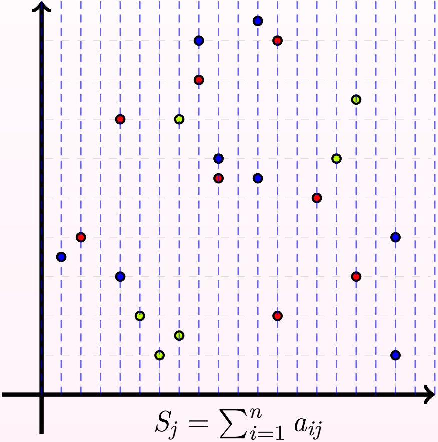



Figure: $S_j$ is a sum of number of each color points along left side of a line $x = j$.

# Methodology IV

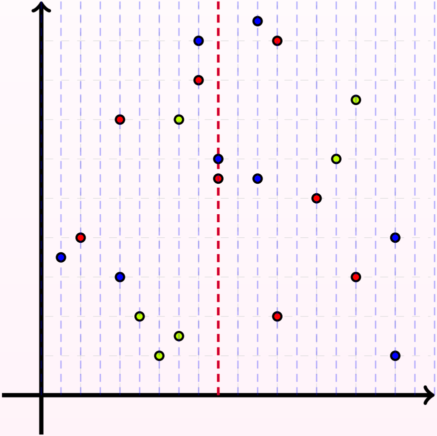


Figure: A first vertical line $x = j$.

# Methodology V

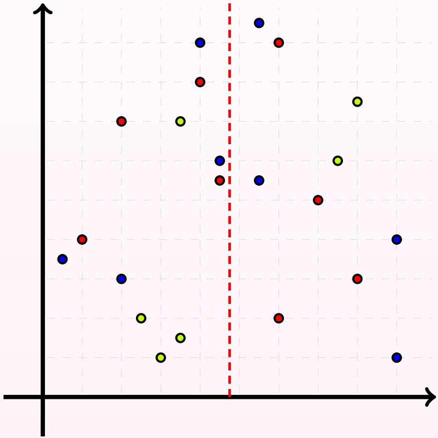


Figure: A first vertical line $x = j$.

# Methodology VI



## Verify a solution

A first vertical line $x = j$ is a solution when

$$\left| \frac{S_n}{2} - S_j \right| \leq \frac{1}{2},$$

for every color of points.

# 3 Line Segments I

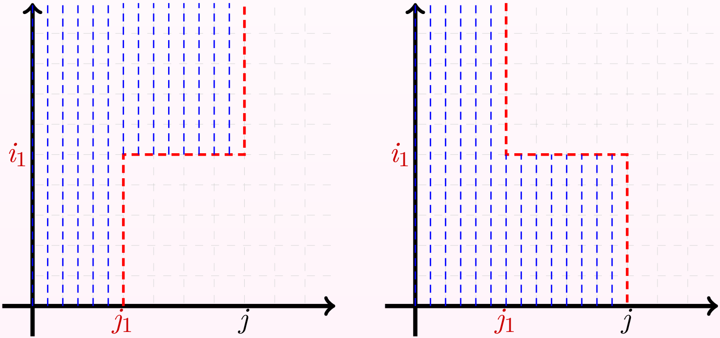



Figure: 2 vertical and 1 horizontal segments.

# 3 Line Segments II



## Convert into equation

$$V = \sum_{i=0}^{i_1} a_{ij} + \sum_{i=i_1}^{n} a_{ij_1} \quad \text{or} \quad V = \sum_{i=0}^{i_1} a_{ij_1} + \sum_{i=i_1}^{n} a_{ij}, \tag{1}$$

where $0 \leq j_1 \leq j$ for each color of points.

## Verify a solution

This partition is a solution when

$$\left| \frac{S_n}{2} - V \right| \leq \frac{1}{2},$$

for every color of points.

# 3 Line Segments III

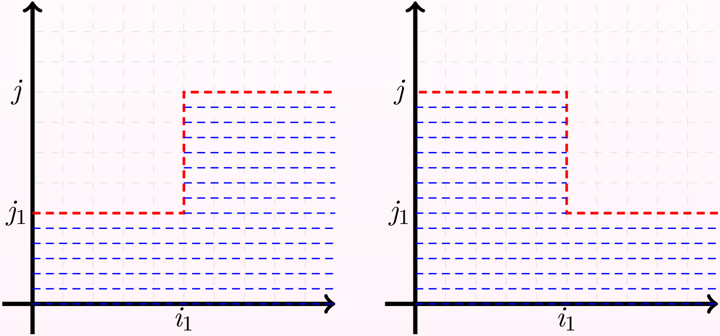



Figure: 1 vertical and 2 horizontal segments.

# Some Cases of 5 Line Segments I



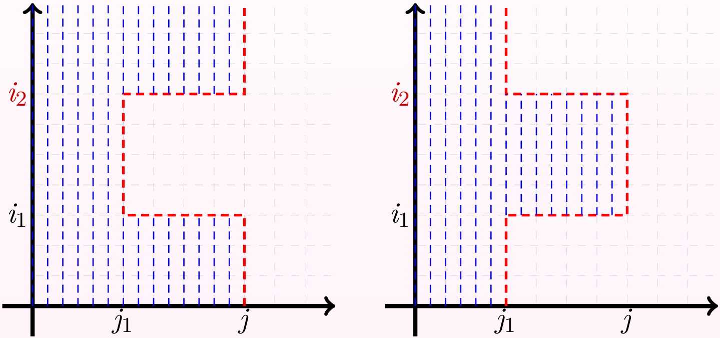



Figure: Some cases of 3 vertical and 2 horizontal segments.

# Some Cases of 5 Line Segments II

## Convert into equation

$$V = \sum_{i=0}^{i_1} a_{ij} + \sum_{i=i_1}^{i_2} a_{ij_1} + \sum_{i=i_2}^{n} a_{ij},$$

or

$$V = \sum_{i=0}^{i_1} a_{ij_1} + \sum_{i=i_1}^{i_2} a_{ij} + \sum_{i=i_2}^{n} a_{ij_1},$$

where $0 \leq j_1 \leq j$ for each color of points.

## Some Cases of 5 Line Segments III



### Verify a solution

This partition is a solution when

$$\left|\frac{S_n}{2} - V\right| \leq \frac{1}{2},$$

for every color of points.

# Some Cases of 5 Line Segments IV



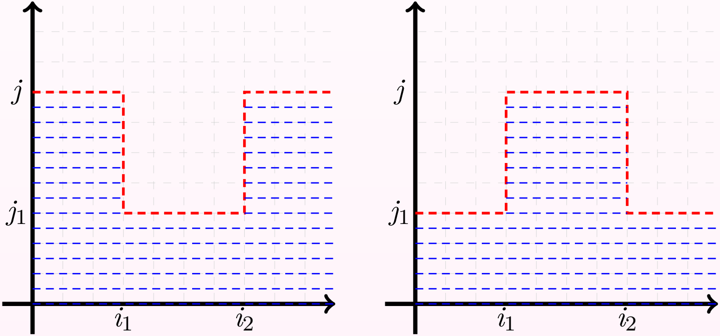



Figure: Some cases of 2 vertical and 3 horizontal segments.

# Counterexample of 3 Line Segments

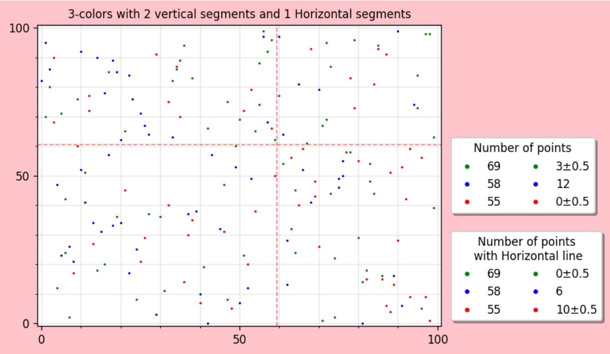



Figure: Counterexample of equipartitions with 3 line segments for 3 color point sets.

# 5 Line Segments for 3-color Points

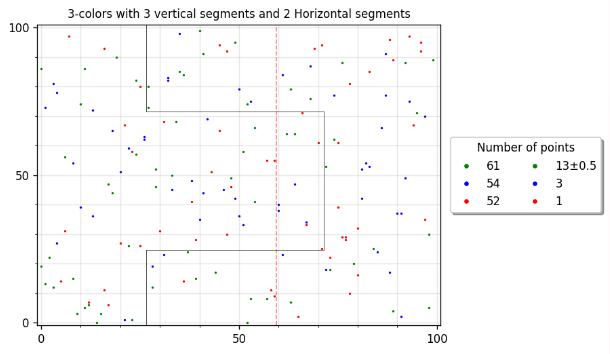



Figure: Equipartitions with 3 vertical and 2 horizontal segments for 3 color point sets when translated a value $x = j$.

# Counterexample of some cases of 5 Line Segments



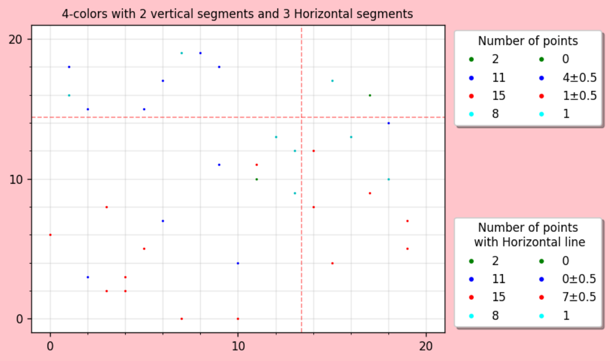



Figure: Counterexample of equipartitions with some cases of 5 line segments for 4 color point sets.

# General Case of 5 Line Segments I

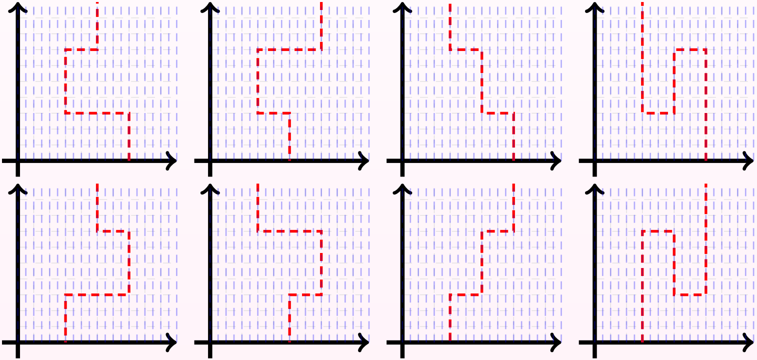


Figure: General case of 3 vertical and 2 horizontal segments.

# General Case of 5 Line Segments II



## Convert into equation

$$V = \sum_{i=0}^{i_1} a_{ij_1} + \sum_{i=i_1}^{i_2} a_{ij_2} + \sum_{i=i_2}^{n} a_{ij_3},$$

where $i_1 \leq i_2$ for each color of points or

$$V = \sum_{i=0}^{i_1} a_{ij_1} - \sum_{i=i_1}^{i_2} a_{ij_2} + \sum_{i=i_2}^{n} a_{ij_3},$$

where $i_1 > i_2$ for each color of points.

# General Case of 5 Line Segments III

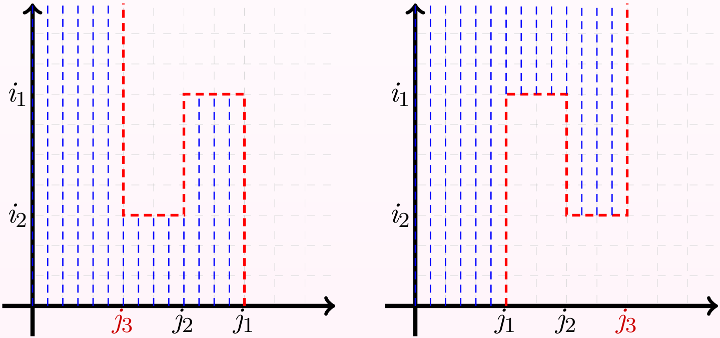
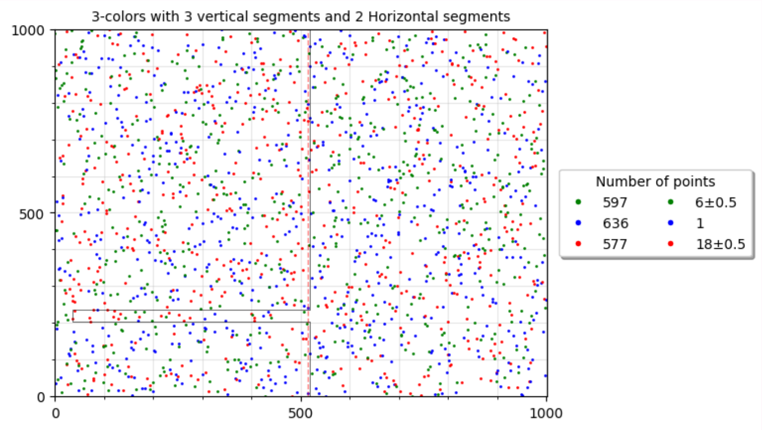



Figure: General case of 3 vertical and 2 horizontal segments with $i_1 > i_2$.

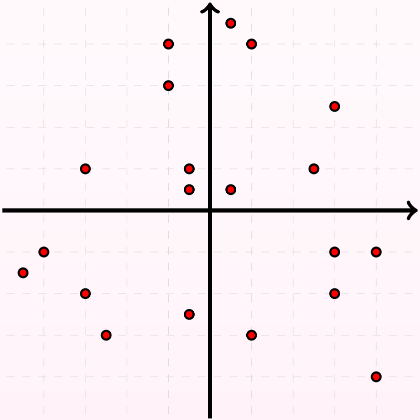
# Q & A 🤔🤔

# Application I



# Application II





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