MATH1001 Assignment4 Shen Hongshan 203629036 Engineering BEng(DS&E)

Q1 W_1 W_2 are subspaces of V.

① closed under addition:

$\forall u, v \in W_1+W_2$, $\exists w_1, w_1' \in W_1$, $w_2, w_2' \in W_2$ s.t. $u=w_1+w_2$, $v=w_1'+w_2'$

$\Rightarrow u+v=(w_1+w_1')+(w_2+w_2')$

W_1 is subspace $\Rightarrow w_1+w_1' \in W_1$

W_2 is subspace $\Rightarrow w_2+w_2' \in W_2$ $\Rightarrow u+v \in W_1+W_2$

② closed under scalar multiplication

$\forall u \in W_1+W_2$, $\exists w_1 \in W_1$, $w_2 \in W_2$ s.t. $u=w_1+w_2$

$\alpha u = \alpha(w_1+w_2) = \underbrace{\alpha w_1}_{\in W_1} + \underbrace{\alpha w_2}_{\in W_2} \Rightarrow \alpha u \in W_1+W_2$

③ contain zero vector

subspace $\Rightarrow 0 \in W_1$ $0 \in W_2 \Rightarrow 0=0+0 \in W_1+W_2$

Q2. $S \subseteq V$ Pf: $\text{span}(S)=S \Leftrightarrow S$ is vector subspace of V

$(\Rightarrow)$ Df: $\text{span}(S)=S \Rightarrow S :=$ set consisting all linear combinations of the vectors in S

$\Rightarrow \begin{cases} ① \text{ closed under addition} \\ ② \text{ closed under scalar multiplication} \end{cases}$

$\Rightarrow S$ is vector subspace of V

$(\Leftarrow)$ $S \subset V$.

① $S \subseteq \text{span}(S)$ by definition of span

② $\text{span}(S) \subseteq S$:

$\forall \alpha_1 s_1 + \alpha_2 s_2 + \cdots + \alpha_n s_n$, $\alpha_i \in \mathbb{R}$, $s_i \in S$,

S is vector subspace $\Rightarrow$ closed under addition, scalar multiplication

$\Rightarrow \alpha_1 s_1 + \cdots + \alpha_n s_n \in S \Rightarrow \text{span}(S) \subseteq S$

①② $\Rightarrow \text{span}(S)=S$

Q3 $\text{Mat}_{5\times5}$ vector space of 5×5 matrices

(a) S_1 is subspace

$B_{6\times5}$, $S_1=\{A \in \text{Mat}_{5\times5} \mid BA=0\}$

① $B0=0 \Rightarrow 0 \in S_1$ zero matrix

② Closed under addition

$\forall A_1, A_2 \in S_1$, $BA_1=0$, $BA_2=0$

$B(A_1+A_2)=BA_1+BA_2=0 \Rightarrow A_1+A_2 \in S_1$

② Closed under scalar multiplication

$\forall A_3 \in S_1$, $BA_3=0$ $B(\alpha A_3)=\alpha(BA_3)=0 \Rightarrow \alpha A_3 \in S_1$

(b) S_2 is not subspace

Let $A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ $A_1 + A_2 = I_5$

$\det(A_1) = \det(A_2) = 0 \Rightarrow A_1, A_2 \in S_1$

$\det(A_1 + A_2) = 1 \neq 0 \Rightarrow A_1 + A_2 \notin S_1$ contradict to "closed under addition"

(c) S_3 is subspace

$\forall A \in S_3$. $A^T = A \Rightarrow A$ symmetric

① closed under addition

$\forall A_1, A_2 \in S_3$ $(A_1 + A_2)^T = A_1^T + A_2^T = A_1 + A_2 \Rightarrow A_1 + A_2 \in S_3$

② scalar multiplication

$\forall A_3 \in S_3$ $\alpha \in \mathbb{R}$. $(\alpha A_3)^T = \alpha (A_3^T) = \alpha A_3 \Rightarrow \alpha A_3 \in S_3$

(d) S_4 is not subspace

Let $A = 0$ $B0 = 0 \Rightarrow 0 \notin S_4$ contridicts to "subspace contains zero vector"

Q4. $\left(\begin{array}{cc|c} 1 & 1 & 1 \\ -2 & 1 & 0 \\ 1 & -1 & 2 \\ 1 & -1 & 3 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 1 & 1 & 1 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$ ← inconsistent.

$\Rightarrow$ no $\alpha, \beta \in \mathbb{R}$. s.t $\alpha \begin{pmatrix} 1 \\ -2 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix}$ is not in the span.