

MATH2101

1. matrix & determinant

2. system of linear equation

3. vector subspace

4.

5.

vector space

- Df: addition, scalar multiplication s.t. several property
- zero vector
- Exp: column vector, matrices: $\text{Mat}_n \times m$ function

vector subspace

- Pf: vector subspace zero vector exist closed under addition closed under scalar multiplication
- "zero vector space" is also "zero vector subspace"
- vector subspace \Rightarrow vector space
- $\text{span}(S) \Rightarrow$ vector subspace

span: $\text{span}(S)$

- $\text{span}(S) :=$ minimal vector space containing all vectors in S
- $\text{col}(A) :=$ column space $= \text{span}(\{v_1, v_2, \dots, v_n\})$
- $v \in \text{col}(A) \Leftrightarrow v = Ax$
- spanning set: $\text{Span}(S) = V$
- Pf: $\text{span}(S) = V \Leftrightarrow Ax = u$ (any u in V) A invertible \Rightarrow consistent \Rightarrow span

linear independence

- dependence: exist non-zero
- independence: any $\{v_i\}$, only trivial
- Pf: linear independence homo system linear Eqn \Rightarrow RREF

Df: linearly independence + span

basis and dimension

- dimension
- standard basis (e_i)
- basis of spanning set
- S independence $\Rightarrow S$ is basis of $\text{span}(S)$
- S dependent \Rightarrow basis?
- S smaller than basis $\Rightarrow \text{span}(S) \neq V$
- S larger than basis $\Rightarrow S$ dependent
- $v =$ unique combination of basis
- existence (by Df basis) + uniqueness
- (assume basis is finite set, though having counter exp - poly)
- basis has largest # of id vectors $\Leftrightarrow | \text{id set} | \leq | \text{basis} |$
- dimension determines $| \text{basis} |$ \Leftrightarrow 2 bases have same size
- Df: $\dim(V) := | \text{basis} |$
- $\dim(\mathbb{R}^n) = n$
- $S \text{ id} \Rightarrow \dim(\text{span}(S)) = | S |$
- $\dim(\text{vector subspace}) \leq \dim(\text{vector space})$
- # of linearly dependent vector bounded by $\dim(\text{vector space})$
- add v (in V) if not in $\text{span}(S)$
- S dependent, $\text{span}(S) = V \Rightarrow$ exist some v in S , st. $\text{span}(S \setminus \{v\}) = V$

construction of a basis

- extension
- $| S | = \dim(V)$ & S independence $\Rightarrow S$ is a basis
- reduction
- $\text{span}(V) ???$

compare: linearly independence spanning set dimension

ways to calculate matrix multiplication $A \cdot B$

element(i, j) = $\text{row}_i(A) \cdot \text{col}_j(B)$
sum numbers together respectively
 $\text{row}_i(C) = a_{i1} \cdot \text{row}_1(B) + a_{i2} \cdot \text{row}_2(B) + \dots$
A i-th row as parameter
 $\text{col}_j(C) = b_{1j} \cdot \text{col}_1(A) + b_{2j} \cdot \text{col}_2(A) + \dots$
B j-th col as parameter

multiply elementary matrix to represent row operation

- EA : row operation
- AE: col operation

convert unknown to known (given condition)

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