MATH 2101 LINEAR ALGEBRA I, FALL 2025 – ASSIGNMENT 1

Due date: 12 September (Friday), 2025 10:00pm.

- To receive *full credits*, the solution has to be *clear* and provide sufficient explanations.
- All solutions have to be turned in HKU moodle in the format of *PDF file*.
- Please include your *Name, UID, Faculty, Major (if declared)* in your solution.

1. Let U be an upper triangular matrix. Suppose all the diagonal entries of U are zero. Show that there exists an integer N such that for all $k > N$, $U^k = 0$.
2. Let K be the $n \times n$ matrix whose entries are 1 on the anti-diagonal entries and 0 elsewhere. The examples are: for $n = 2$,
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
for $n = 3$,
$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$
for $n = 4$,
$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$
Show that if U is a lower triangular $n \times n$ matrix, then KUK is an upper triangular matrix.
3. A matrix P is said to be a permutation matrix if for each row and column of P, there is precisely one non-zero entry in that row or column and such entry is one. Examples of permutation matrices include the identity matrix and the matrix K in Question 2. Other examples include
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
Let A be an $n \times n$ matrix and let P be an $n \times n$ permutation matrix. Show that the number non-zero entries in A is the same as the number of non-zero entries in PAP.