1.1 Chapter 9 Questions 1 & 2

1. The seller will run a sealed-bid, second-price auction. Your firm will bid in the auction, but it does not know for sure how many other bidders will participate in the auction. There will be either two or three other bidders in addition to your firm. All bidders have independent, private values for the good. Your firm's value for the good is c. What bid should your firm submit, and how does it depend on the number of other bidders who show up?

   Answer.
   For sealed-bid, second-price auctions, the number of bidders, n , does not affect what you should bid for $n \geq 2$. That is because regardless of the number of bidders, you are incentivized to bid your true value for the item.

   If you bid a price, b_a , which is above your true value, v , the only time that would change your realized payoff is if b_a wins when a bid of v would not have won. In this case, you would pay your opponent's price, which is above your true value. By paying that price, you realize a negative payoff because you overpaid for the item.

   If you bid a price, b_b , which is below your your true value, the only time the only time this bid would change your realized payoff is if b_b loses when your bid of v would have won. In this case, your payoff is zero because you lost when your payoff could have been $v - b_o \geq 0$ where b_o is your opponent's bid.

   In this entire analysis, your bid has only ever been dependent on one other competing bid, so you should bid your true value if there is at least one other bidder.

2. In this problem we will ask how the number of bidders in a second-price, sealed-bid auction affects how much the seller can expect to receive for his object. Assume that there are two bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both $1/2$. (If there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x .)

   (a) Show that the seller's expected revenue is $6/4$.

   Answer.
   In a closed-bid, second-price auction, the bidders are always incentivized to bid their true value. By case analysis, that means there is a $1/4$ chance both bids are 3, making the revenue 3; a $1/2$ chance one bid is 1 and the other 3, making the revenue 1 (because 1 is the second price); and a $1/4$ chance both bids are 1, making the revenue 1. By multiplying the chances by the revenue of each case and summing, $1/4 \cdot 3 + 1/2 \cdot 1 + 1/3 \cdot 1$, the expected revenue is $6/4$.

(b) Now let's suppose that there are three bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both $1/2$. What is the seller's expected revenue in this case?

Answer.
We perform a similar case analysis and find there is a $1/8$ chance all three bids are 3, making the revenue 3; a $3/8$ chance two bids are 3 and one is 1, making the revenue 3; a $3/8$ chance two bids are 1, and one is 3, making the revenue 1; and a $1/8$ chance all three bids are 1, making the revenue 1. Applying the formula for expected payoff, $1/8 \cdot 3 + 3/8 \cdot 3 + 3/8 \cdot 1 + 1/8 \cdot 1$, the expected revenue is 2.

(c) Briefly explain why changing the number of bidders affects the seller's expected revenue.

Answer.
The number of players changes the seller's expected revenue because it becomes more and more likely that two bids will be the high price, 3, when more players play. In fact, as $n \to \infty$, expected revenue goes to 3 in this scenario.