EE 382V: Social Computing	**Fall 2018**

Lecture 29: November 10

Lecturer: Vijay Garg — *Scribe: Bryson Banks*

29.1 Answers to Problems on Auctions

The following two exercises focus on second-price, sealed-bid auctions where a seller is selling some object that bidders each have their own independent, private valuations for.

29.1.1 Exercise 9-3

(a) Four possible outcomes of two bidders having valuations 0 or 1:

- $(v_1, v_2) = (0, 0)$: Bidder 1 or 2 wins and pays 0.
- $(v_1, v_2) = (0, 1)$: Bidder 2 wins and pays 0.
- $(v_1, v_2) = (1, 0)$: Bidder 1 wins and pays 0.
- $(v_1, v_2) = (1, 1)$: Bidder 1 or 2 wins and pays 1.

If we equally weight the probability of each of the four possible outcomes above, then the probability of each outcome is $\frac{1}{4}$. With this knowledge, we can then show the seller's expected revenue (ER_S) is also $\frac{1}{4}$.

$$ER_S = \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(1) = \frac{1}{4}$$

(b) Eight possible outcomes of three bidders having valuations 0 or 1:

- $(v_1, v_2, v_3) = (0, 0, 0)$: Bidder 1, 2, or 3 wins and pays 0.
- $(v_1, v_2, v_3) = (0, 0, 1)$: Bidder 3 wins and pays 0.
- $(v_1, v_2, v_3) = (0, 1, 0)$: Bidder 2 wins and pays 0.
- $(v_1, v_2, v_3) = (0, 1, 1)$: Bidder 2 or 3 wins and pays 1.
- $(v_1, v_2, v_3) = (1, 0, 0)$: Bidder 1 wins and pays 0.
- $(v_1, v_2, v_3) = (1, 0, 1)$: Bidder 1 or 3 wins and pays 1.
- $(v_1, v_2, v_3) = (1, 1, 0)$: Bidder 1 or 2 wins and pays 1.
- $(v_1, v_2, v_3) = (1, 1, 1)$: Bidder 1, 2, or 3 wins and pays 1.

If we equally weight the probability of each of the eight possible outcomes above, then the probability of each outcome is $\frac{1}{8}$. With this knowledge, we can then show the seller's expected revenue (ER_S) has increased to $\frac{1}{2}$ with the additional bidder.

$$ER_S = \frac{1}{8}(0) + \frac{1}{8}(0) + \frac{1}{8}(0) + \frac{1}{8}(1) + \frac{1}{8}(0) + \frac{1}{8}(1) + \frac{1}{8}(1) + \frac{1}{8}(1) = \frac{4}{8} = \frac{1}{2}$$

(c) Comparing the solutions for (a) and (b), we see that going from two bidders to three bidders increased the seller's expected revenue from $\frac{1}{4}$ to $\frac{1}{2}$. If we were to add in an additional fourth bidder, we would again expect to see the seller's expected revenue increase. The same again with a fifth bidder, and a sixth, and so on.

To prove this, we note that the seller will earn 0 anytime less than two bidders bid 1, and 1 otherwise. Thus, in this particular case the seller's expected revenue happens to be equal to the probability that at least two bidders bid 1, or 1 minus the probability that 1 or 0 bidders bid 1.

$$ER_S = P(2\ or\ more) = 1 - P(1\ or\ 0)$$

$P(1\ or\ 0)$ is equal to the number of outcomes where 1 or 0 bidders bid 1 divided by the total number of possible outcomes. Let n be the number of bidders. Then, there are n different outcomes where a single bidder bids 1, and 1 outcome where every bidder bids 0. Additionally, since each bidder has only 2 possible choices, we see there are 2^n total possible outcomes.

$$P(1\ or\ 0) = \frac{number\ of\ outcomes\ where\ 1\ or\ 0\ bidders\ bid\ 1}{total\ number\ of\ possible\ outcomes} = \frac{n+1}{2^n}$$

.

After substituting, we get a function of n, the total number of bidders, to easily calculate the seller's expected revenue for any positive integer number of bidders.

$$ER_S(n) = 1 - \frac{n+1}{2^n}$$

Analyzing the function, we see as n grows, so too does ER_S. We also see as n grows to infinity, ER_S converges to 1, never actually reaching 1.

$$ER_S(1) = 0,\ \ ER_S(2) = \frac{1}{4},\ \ ER_S(3) = \frac{1}{2},\ \ ER_S(4) = \frac{11}{16},\ \ ER_S(5) = \frac{13}{16},\ \ \ldots$$

29.1.2 Exercise 9-4

(a) We know a always bids its valuation, but b sometimes makes a mistake with its bid. When b's actual valuation is 0, half the time it bids 1 mistakenly. Below we outline the possible outcomes of valuations and bids.

- $(v_a, v_b) = (0, 0)$:
  - **Outcome 1.** $(b_a, b_b) = (0, 0)$: Bidder a (50%) or b (50%) wins and pays 0.
  - **Outcome 2.** $(b_a, b_b) = (0, 1)$: **Mistake bid.** Bidder b wins and pays 0.
- $(v_a, v_b) = (0, 1)$:
  - **Outcome 3.** $(b_a, b_b) = (0, 1)$: Bidder b wins and pays 0.
- $(v_a, v_b) = (1, 0)$: Bidder 1 wins and pays 0.
  - **Outcome 4.** $(b_a, b_b) = (1, 0)$: Bidder a wins and pays 0.
  - **Outcome 5.** $(b_a, b_b) = (1, 1)$: **Mistake bid.** Bidder a (50%) or b (50%) wins and pays 1.
- $(v_a, v_b) = (1, 1)$:
  - **Outcome 6.** $(b_a, b_b) = (1, 1)$: Bidder a or b wins and pays 1.

Given a bids 0 (Outcomes 1, 2, and 3), we see there is a $\frac{1}{8}$ probability it wins and pays 0, and a 0 probability it wins and pays 1. To calculate these values note that given a bids 0, outcomes 1 and 2 each have a $\frac{1}{4}$ probability of occurrence, while outcome 3 has a $\frac{1}{2}$ probability of occurrence.

$$P(a\ wins\ and\ pays\ 0 \mid a\ bids\ 0) = \frac{1}{4}(\frac{1}{2}) + \frac{1}{4}(0) + \frac{1}{2}(0) = \frac{1}{8}$$

$$P(a\ wins\ and\ pays\ 1 \mid a\ bids\ 0) = \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{2}(0) = 0$$

Given a bids 1 (Outcomes 4, 5, and 6), we see there is a $\frac{1}{4}$ probability it wins and pays 0, and a $\frac{3}{8}$ probability it wins and pays 1. To calculate these values note that given a bids 1, outcomes 4 and 5 each have a $\frac{1}{4}$ probability of occurrence, while outcome 6 has a $\frac{1}{2}$ probability of occurrence.

$$P(a\ wins\ and\ pays\ 0 \mid a\ bids\ 1) = \frac{1}{4}(1) + \frac{1}{4}(0) + \frac{1}{2}(0) = \frac{1}{4}$$

$$P(a\ wins\ and\ pays\ 1 \mid a\ bids\ 1) = \frac{1}{4}(0) + \frac{1}{4}(\frac{1}{2}) + \frac{1}{2}(\frac{1}{2}) = \frac{3}{8}$$

To determine if remaining true is still the dominant strategy for a given b's mistake bids, for each of the possible true valuations for a, we will use the probabilities calculated above to analyze whether staying true or lying is better.

Case that a's true valuation is 0:

When a's true valuation is 0, by staying true and bidding 0 we see using the probabilities above it only has a $\frac{1}{8}$ probability of winning and paying its valuation or less, but has 0 probability of winning and having to pay more than its valuation.

If instead it lies and bids 1, we see it has an increased $\frac{1}{4}$ probability of winning and paying its valuation or less, but it also now has a $\frac{3}{8}$ probability of winning and having to pay more than its valuation, which makes this strategy inadvisable.

Thus, it is better to stay true given a's valuation is 0.

Case that a's true valuation is 1:

When a's true valuation is 1, by staying true and bidding 1 we see using the probabilities above it has a $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$ probability of winning and paying its valuation or less.

If instead it lies and bids 0, we see it only has a $\frac{1}{8}$ probability of winning and paying its valuation or less.

Thus, it is better to stay true given a's valuation is 1.

Conclusion:

In each possible valuation for a, staying true remains the dominant strategy for a even considering it is aware of the mistake bids made by b.

(b) If we equally weight the probability of each of the four possible valuations above at $\frac{1}{4}$, and equally the possible bids in each valuation case, we can then show the seller's expected revenue (ER_S) is $\frac{3}{8}$.

$$ER_S = \frac{1}{4}(\frac{1}{2}(0) + \frac{1}{2}(0)) + \frac{1}{4}(0) + \frac{1}{4}(\frac{1}{2}(0) + \frac{1}{2}(1)) + \frac{1}{4}(1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

EE 382V: Social Computing **Fall 2018**

Chapter 6: Excercise 8, 9

Scribe: Kyle Sung

6.8 Problem Overview

The problem is a coordination game with the payoff matrix

1,1	0,0
0,0	4,4

6.8.1 Part a

Q: Find all pure-strategy Nash equilibria for this game.

A: (U, L) and (D, R) are both NE.

6.8.2 Part b

Q: Find the mix-strategy NE, and give an explanation.

A:

A plays U with prob p

B plays L with prob q

Expected payoff for A to play U: q*1 + (1-q)*0 = q

Expected payoff for A to play D: q*0 + (1-q)*4 = 4 - 4q

Indifference for B:

q = 4 - 4q, 5q = 4. q = 4/5.

Expected payoff for B to play L:

p*1 + (1-p)*0 = p

Expected payoff for B to play R:

p*0 + (1-p)*4 = 4 - 4p

Indifference for A:

p = 4 - 4p, 5p = 4, p = 4/5.

Mixed strategy NE: ((4/5,1/5), (4/5,1/5)) When player A expect player B to play L with the probability of 4/5, then A will try to match the probability as well.

6.8.3 Part c

Q: With Schelling's focal point idea, what equilibrium do you think is the best prediction of how the game will be played?

A: When a Schellings focal point can be expected, player A would expect player B to play R, since it has the better payoff of the two NE. Likewise for player B to expect A to play D. Thus (D, R) would be the converged focal point.

6.8 Problem Overview

Find all Nash equilibria given the payoff matrices:

6.8.1 Part a

8,4	5,5
3,3	4,8

Pure strategy NE:

(U,R)

Mixed strategy NE:

A plays U with prob p

B plays L with prob q

Expected payoff for A to play U:

q*8 + (1-q)*5 = 5 + 3q

Expected payoff for A to play D:

q*3 + (1-q)*4 = 4 - q

Indifference for B:

5 + 3q = 4 - q, 4q = -1. q = -14. i.e. B always plays R

Expected payoff for B to play L:

p*4 + (1-p)*3 = 1 + p

Expected payoff for B to play R:

p*5 + (1-p)*8 = 8 - 3p

Indifference for A:

1 + p = 8 - 3p, 4p = 7, p = 7/4. i.e. A always plays U

This just shows that if a pure strategy NE exists, the mixed strategy NE would be the same (i.e. U,R in this case)

6.8.2 Part b

0,0	-1,1
-1,1	2,-2

Pure strategy NE:

Does not exist

Mixed strategy NE:

A plays U with prob p

B plays L with prob q

Expected payoff for A to play U:

q*0 + (1-q)*-1 = -1 + q

Expected payoff for A to play D:

q*-1 + (1-q)*2 = -q + 2 - 2q = 2 - 3q

Indifference for B:

-1+q=2-3q,4q =3,q=3/4.

Expected payoff for B to play L:

p*0 + (1-p)*1 = 1 - p

Expected payoff for B to play R:

p*1 + (1-p)*-2 = -2 + 3p

Indifference for A:

1 - p = -2 + 3p, 3 = 4p, p = 34.

((3/4, 1/4), (3/4, 1/4))

EE 382C/361C: Multicore Computing **Fall 2016**

Lecture 1: November 16

Lecturer: Vijay Garg *Scribe: Guillermo León*

1.1 Chapter 6 Exercises

10. Given payoff matrix.

		Player B	
		L	R
Player A	U	3,3	1,2
	D	2,1	3,0

(a) All pure strategy Nash equilibria of this game.

Let's try first finding dominant strategies. In this game we can see that Player B has L as a strictly dominant strategy since $P_B(U, L) > P_B(U, R)$ and $P_B(D, L) > P_B(D, R)$. In other words, strategy L is always the strict best response to whichever strategy Player A chooses. Because this is the case, it is expected that Player B will go with L, therefore the best response from Player A to L is U because it maximizes her payoff. The only pure strategy Nash equilibrium is (U, L).

		Player B	
		L	R
Player A	U	3,3	1,2
	D	2,1	3,0

(b) Changing player A's payoff from (U, L) to not have pure-strategy Nash equilibrium.

No, as it was stated in the answer above, Player B is indifferent of the payoff of Player A. In the original case Player A does not have a dominant strategy, let's say that we modify $P_A(U, L) = 1$. In this case Player A now has a strictly dominant strategy D, however the best response from Player B is still to go with L. This only moves the Nash equilibrium from (U, L) to (D, L).

		Player B	
		L	R
Player A	U	**1**,3	1,2
	D	2,1	3,0

Finally let's suppose we change the payoff $P_A(D, L) = 3$, in this case we now have 2 Nash equilibria because U and D are both best responses to L from player B.

		Player B	
		L	R
Player A	U	3,3	1,2
	D	**3**,1	3,0

(c) Changing player B's payoff from (U, L) to not have pure-strategy Nash equilibrium.

Yes, since Player B has a strictly dominant strategy it is just a matter of removing it. If we set the payoff for player B for the pair of strategies U and L to 1 or 0. In this case neither of the players have a dominant strategy. Let's analyze then the best responses for each player and each strategy. Player A chooses U, then best response from player B is R, but if player B chooses R then best response from player A is D. Finally if player A chooses D, then the best response from B is L and when player B chooses L, the best response from player A is U, which is the initial strategy. Therefore there is no pure strategy Nash equilibrium.

		Player B	
		L	R
Player A	U	3,**1**	1,2
	D	2,1	3,0

11. Explain why the strategies used in an equilibrium of this game will not be dominated strategies.

Let's recall that strategies in a pure strategy Nash equilibrium are the best responses to each other from both players. More precisely, let's suppose that Player 1 chooses strategy S and Player 2 chooses T, then we say that (S, T) are in a Nash equilibrium if S is a best response to T, and T is a best response to S. Because they are considered best strategies, that means that they produce a payoff that is at least as good as any other strategy. In other words, let's suppose that strategy S is indeed a dominated strategy, this means that there is a strategy $S\prime$ with a greater payoff, therefore $S\prime$ is a best response to a given strategy T and because of this S cannot be part of the Nash equilibrium.

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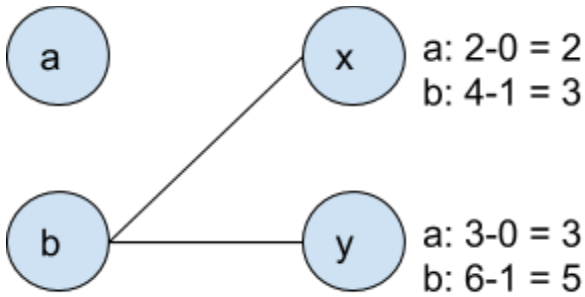
Chapter 10

1. Suppose we have a set of 2 sellers labeled a and b, and a set of 2 buyers labeled x and y. Each seller is offering a distinct house for sale, and the valuations of the buyers for the houses are as follows.

Buyer	Value for a's house	Value for b's house
x	2	4
y	3	6

Suppose that a charges a price of 0 for his house, and b charges a price of 1 for his house. Is this set of prices market-clearing? Give a brief (1-3 sentence) explanation; as part of your answer, say what the preferred-seller graph is with this given set of prices, and use this in your explanation.

No, this is not a market clearing price. This is because both buyers get a maximum payoff with seller b's house. The preferred seller graph has both buyers with a line leading to seller b.

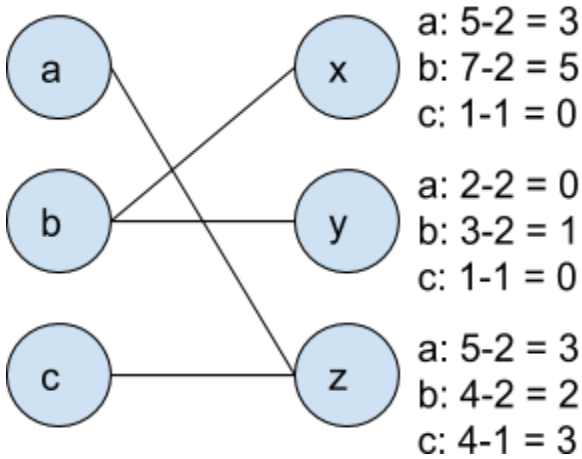


2. Suppose we have a set of 3 sellers labeled a, b, and c, and a set of 3 buyers labeled x, y, and z. Each seller is offering a distinct house for sale, and the valuations of the buyers for the houses are as follows.

Buyer	Value for a's house	Value for b's house	Value for c's house
x	5	7	1
y	2	3	1
z	5	4	4

Suppose that sellers a and b each charge 2, and seller c charges 1. Is this set of prices market-clearing? Give a brief explanation.

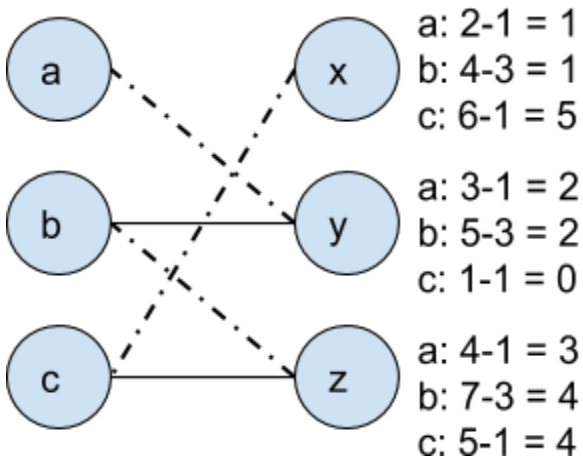
This is not a market clearing price, due to both buyers x and y having a preferred seller b.



3. Suppose we have a set of 3 sellers labeled a, b, and c, and a set of 3 buyers labeled x, y, and z. Each seller is offering a distinct house for sale, and the valuations of the buyers for the houses are as follows.

Buyer	Value for a's house	Value for b's house	Value for c's house
x	2	4	6
y	3	5	1
z	4	7	5

Suppose that sellers a and c each charge 1, and seller b charges 3. Is this set of prices market-clearing? Give a brief explanation.



Yes, this is a market clearing price, because all buyers can achieve their maximum payoff with the above shown matching (dotted lines).

1 10 Q4

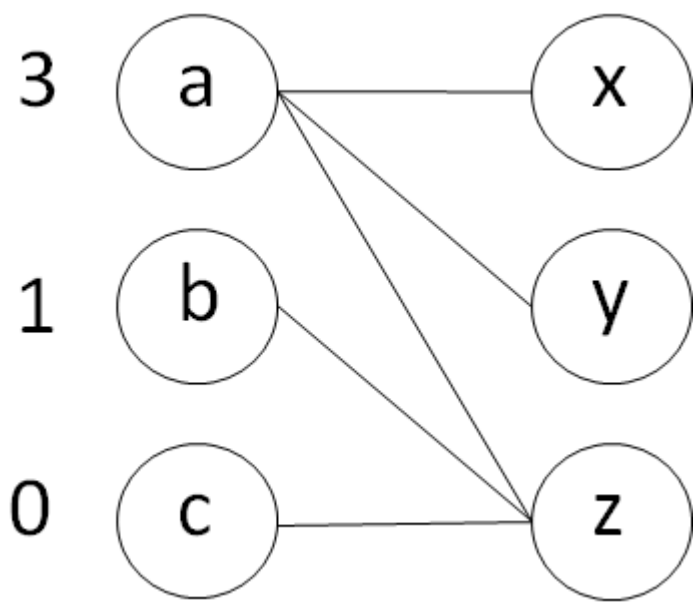
Sellers a, b, and c are selling their houses for prices of 3, 1, and 0, respectively.

Buyer x values house a at 12, house b at 9, and house c at 8.
Buyer y values house a at 10, house b at 3, and house c at 6.
Buyer z values house a at 8, house b at 6, and house c at 5.

Buyer x receives the maximum payoff by purchasing from Seller a.
Payoff with a = 12 - 3 = 9.

Buyer y receives the maximum payoff by purchasing from Seller a.
Payoff with a = 10 - 3 = 7.

Buyer z receives the maximum payoff by purchasing from either Seller a, b, or c.
Payoff with a = 8 - 3 = 5
Payoff with b = 6 - 1 = 5
Payoff with c = 5 - 0 = 5



This set of prices is not market clearing because Buyers x and y both want to purchase from Seller a in order to maximize their payoffs. Seller a should raise their price in the next round of the bipartite auction procedure.

2 10 Q5

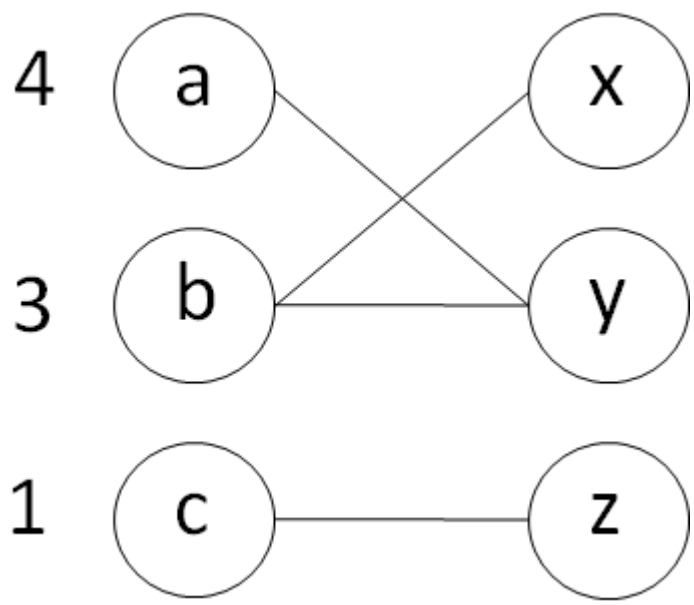
Sellers a, b, and c are selling their houses for prices 4, 3, and 1, respectively.

Buyer x values house a at 7, house b at 7, and house c at 4.
Buyer y values house a at 7, house b at 6, and house c at 3.
Buyer z values house a at 5, house b at 4, and house c at 3.

Buyer x receives the maximum payoff by purchasing from Seller b.
Payoff with b = 7 - 3 = 4.

Buyer y receives the maximum payoff by purchasing from Sellers a or b.
Payoff with a = 7 - 4 = 3.
Payoff with b = 6 - 3 = 3.

Buyer z receives the maximum payoff by purchasing from Seller c.
Payoff with c = 3 - 1 = 2



This set of prices is market clearing. Buyer x can purchase from Seller b, Buyer y can purchase from Seller a, and Buyer z can purchase from Seller c. Each buyer purchases from a unique seller and receives their maximum payoff.

Chapter 10

0.1 Question 6

In this question we have been given a set of 3 sellers a, b and c offering their houses for sale and a set of 3 buyers x, y and z having their own valuations for each of the houses. The figure below shows the valuations of each buyer for each of the houses:

Buyer	Value for a's house	Value for b's house	Value for c's house
x	6	3	2
y	10	5	4
z	7	8	6

Figure 1: Buyer's valuations

We first calculate the payoff of each buyer for all the houses.

As we know that,

Payoff for a buyer = Buyer's valuation of the house - Price of the house

Let P(x,a) be the payoff of a buyer x for the seller a

Let P(x,b) be the payoff of a buyer x for the seller b and so on..

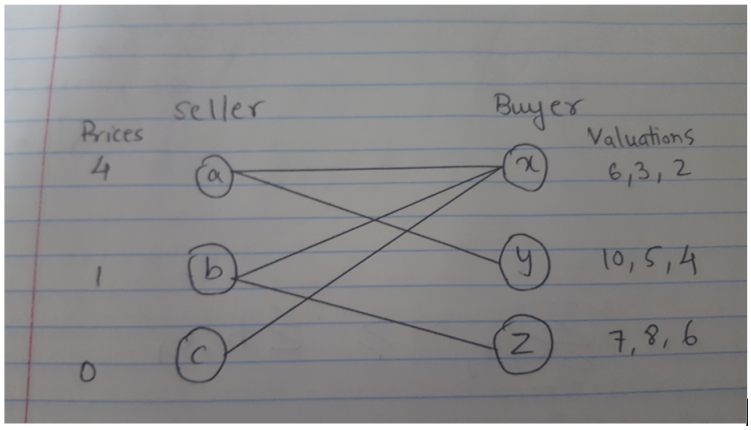
Therefore

P(x,a) = 2 , P(y,a) = 6 , P(z,a) = 3

P(x,b) = 2 , P(y,b) = 4, P(z,b) = 7

P(x,c) = 2 , P(z,b) = 4, P(z,c) = 6

The figure below shows a preferred-seller graph for the buyers and sellers where the buyers are connected to the sellers that give them the maximum payoff.



As we can see from the graph above that for buyer y, seller a is the preferred seller (since seller a gives this buyer the maximum payoff).

Also, for buyer z, seller b is the preferred seller (since seller b gives this buyer the maximum payoff).

Hence, buyer y must get the house from seller a and buyer z must get the house from seller b.

As we can see that there is no seller that gives a maximum payoff to the buyer x, since all the three sellers give an equal payoff of 2 to the buyer x. Hence, buyer x would get the house from seller c.

Also, since each house is now bought by a different buyer, the set of prices we have here are market-clearing prices. The market clearing value is [4,1,0]

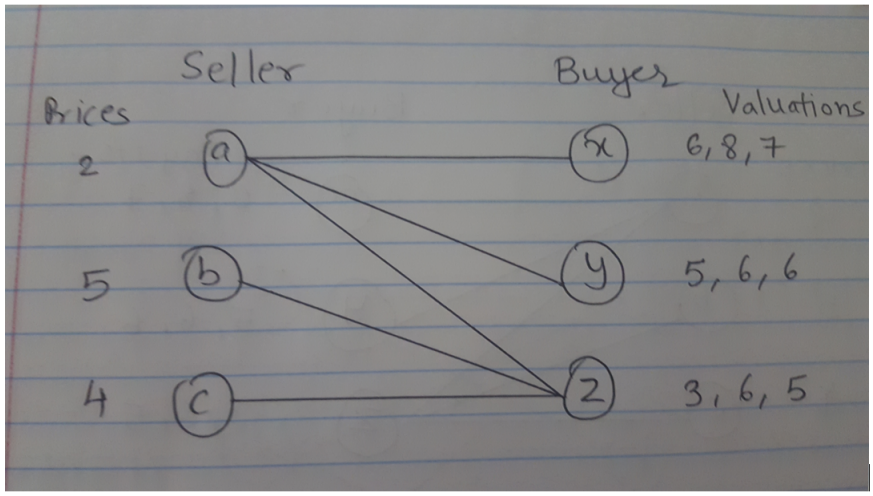
0.2 Question 7

In this question we have been given a set of 3 sellers a, b and c offering their houses for sale and a set of 3 buyers x, y and z having their own valuations for each of the houses. The figure below shows the valuations of each buyer for each of the houses:

Buyer	Value for a's house	Value for b's house	Value for c's house
x	6	8	7
y	5	6	6
z	3	6	5

Figure 2: Buyer's valuations

Based on the buyer's valuations for each houses and their actual prices, we match each buyer on the right with its preferred seller or sellers on the left. A preferred seller is the one which gives the highest payoff to a buyer.



Based on the above graph, we see that since seller 'a' is the preferred seller for both x and y, we know that the above set of prices are not market clearing since we have a constricted set S= x,y of buyers that prefer the same seller, N(S)=a.

Hence, to make these prices market clearing, we increase the price of a by 1. So, the new prices become, a=3, b=5 and c=4. Hence the next round of the bipartite auction procedure is shown as follows:

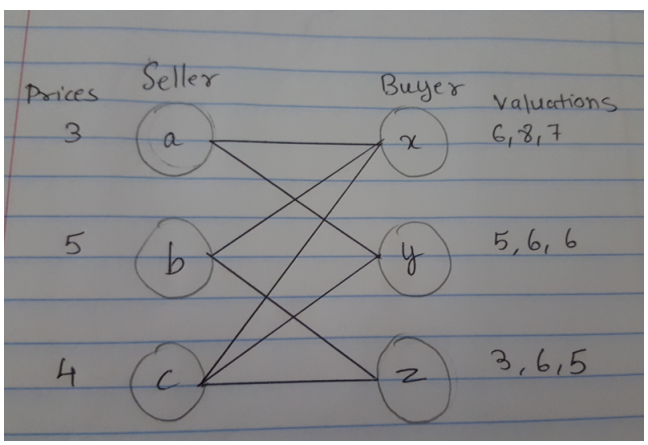


Figure 3: Bipartite graph after increasing the price for seller a

From the above graph we can see that buyer x has a payoff of 3 for all the houses, hence x could go with any of the three houses. Buyer y has a payoff of 2 with seller a and c. Hence, y could go with either seller a or c. Buyer z has a payoff of 1 for seller b and c. Hence, z could go with seller b or c.

Hence, there are three possible ways in which the buyers could be matched with the sellers:

1) [(x,c),(y,a),(z,b)] or

2) [(x,b),(y,a),(z,c)] or

3) [(x,a),(y,c),(z,b)]

Hence, the market clearing price for the above set of buyers and sellers is [3,5,4]

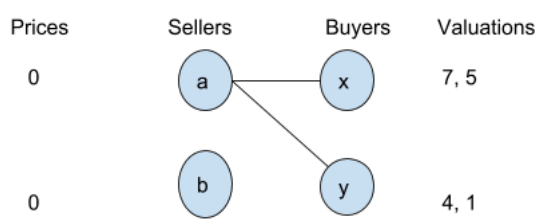
Chapter 10 - Question 8

Utkarsh Vardan

Given buyers and sellers are as follows :
buyers : a,b
sellers:x,y

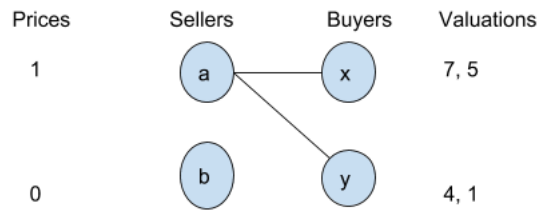
Buyer	Value for a's house	Value for b's house
x	7	5
y	4	1

Let the initial price be 0 for each seller:



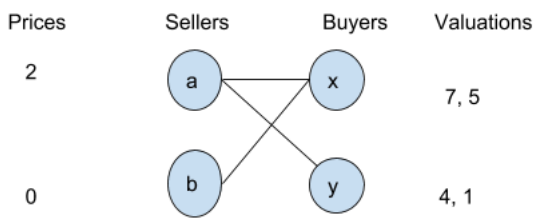
Round1
Set of buyers X and Y are constricted to neighbour a, This is not a perfect matching so seller a increases the price by 1 and the price of b remains 0.
The payoff for buyer x are as follows
a: (7-0)=7
b: (5-0)=5
The payoff of buyer y are as follows
a: (4-0)=4
b:(1-0)=1

Round2



Set of buyers X and Y are constricted to neighbour a,This is not perfect matching so seller a increases the price by 1 and the price of b remains 0.
The payoff for buyer x are as follows
a: (7-1)=6
b: (5-0)=5
The payoff of buyer y are as follows
a: (4-1)=3
b: (1-1)=0

Round3



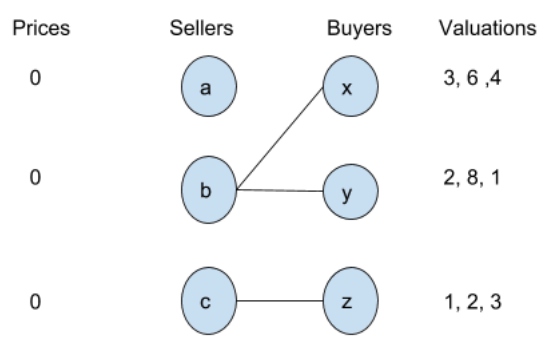
We have perfect match now. Hence the current price 2 and 0 is the market clearing price.
The payoff for buyer x are as follows
a: (7-2)=5
b: (5-0)=5
The payoff of buyer y are as follows
a: (4-2)=2
b: (1-2)=-1

Chapter10 - Question 9

Buyer	Value for a's house	Value for b's house	Value for c's house
x	3	6	4
y	2	8	1
z	1	2	3

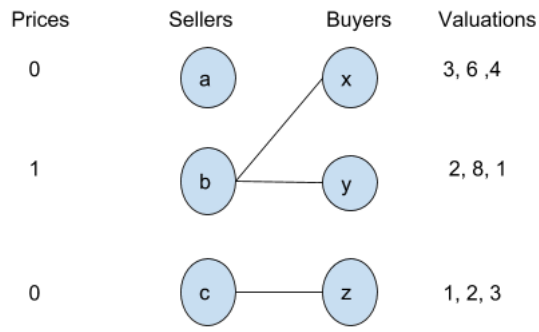
Given set of buyers and sellers are as follows:
Buyers x, y, z
Sellers a,b,c
Let the starting price for the sellers be 0.

Round1



Set of buyers consist of x and y forms a constricted set to the neighbour a. This is not a perfect matching. The price of b increases by 1 and the price of a and c remains 0.
The payoff for buyer x are as follows
a: (3-0)=3
b: (6-0)=6
c: (4-0)=4
The payoff of buyer y are as follows
a: (2-0)=2
b: (8-0)=8
c: (1-0)=1
The payoff of buyer z are as follows
a: (1-0)=1 b: (2-0)=2
c: (3-0)=3

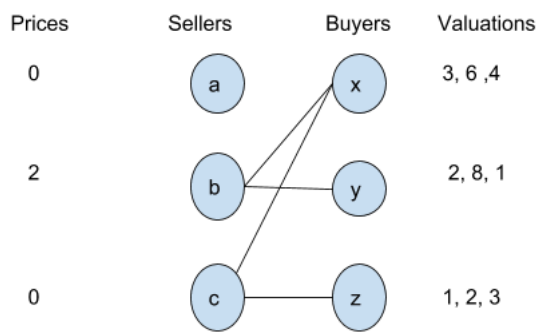
Round2



Set of buyers consist of x and y forms a constricted set to the neighbour a. This is not a perfect matching. Only price of b increases by 1.

The payoff for buyer x are as follows
a: (3-0)=3
b: (6-1)=5
c: (4-0)=4
The payoff of buyer y are as
a: (2-0)=2
b: (8-1)=7
c: (1-0)=1
The payoff of buyer z are as follows
a: (1-0)=1
b: (2-1)=1
c: (3-0)=3

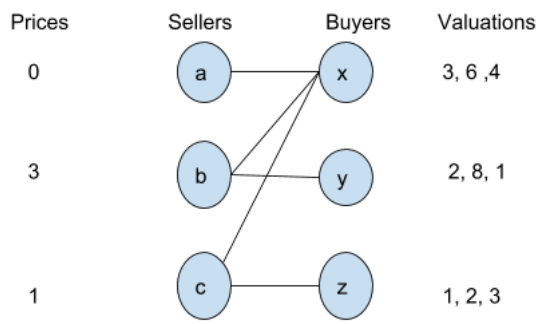
Round3



Set of buyers consist of x, y and Z forms a constricted set to the neighbour b and c. This is not a perfect matching. The price of b and c increases by 1 and the price of a and remains 0.
The payoff for buyer x are as follows
a: (3-0)=3
b: (6-2)=4

c: (4-0)=4
The payoff of buyer y are as
a: (2-0)=2
b: (8-2)=6
c: (1-0)=1
The payoff of buyer z are as follows
a: (1-0)=1
b: (2-2)=0
c: (3-0)=3

Round4



We have a perfect match now. Hence the current price 0,3,1 are the marker clearing price
The payoff for buyer x are as follows
a: (3-0)=3
b: (6-3)=3
c: (4-1)=3
The payoff of buyer y are as
a: (2-0)=2
b: (8-3)=5
c: (1-1)=0
The payoff of buyer z are as follows
a: (1-0)=1
b: (2-3)=-1
c: (3-1)=2

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Chapter 10: Excercise 11

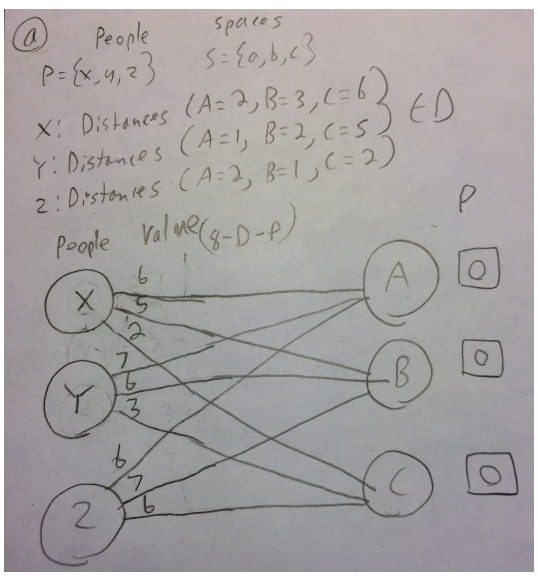
Lecturer: Vijay Garg — *Scribe: Alex Hoganson*

10.1 Problem Overview

The problem description indicates a set of People {x,y,z} and a set of parking spaces {a,b,c} that are up for auction around where people live. The value of a spot is calculated as a function of the inverse of the distance away from those spots(i.e. the shorter the spot to person x, the higher the valuation)

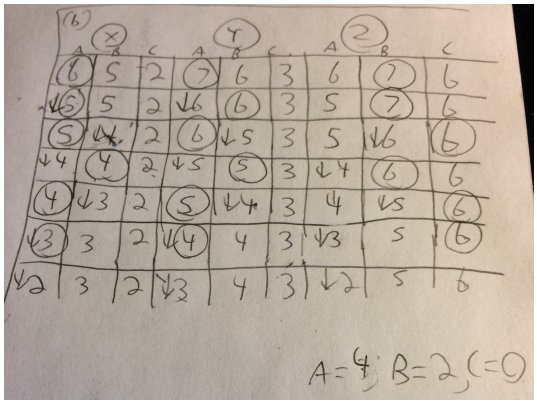
10.1.1 Part A

Provide a bipartite graph that represents this problem with buyers/sellers, valuations, and prices.



10.1.2 Part B

Find the market clearing price using the bipartite auction algorithm.



10.1.3 Part C

How do the prices in Part B denote the inherent attractiveness of the parking spaces to the subset of People {x,y,z}?

Answer: The Market clearing prices were {4,2,0}, and the higher price correlates with a spot's average distance away from each person in the people subset, it's general attractiveness. As the averaged distance decreases that in turn creates even more competition for those spots, such that the price needs to be raised in order to make buyers consider other alternatives instead of just those spots. As seen by the outlier parking spot C, there is no competition whatsoever for that spot due to its location only being close to one person, therefore keeping its price low.

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Chapter 10: Question 13

Lecturer: Vijay Garg *Scribe: Beth Richardson*

10.1 Question

Suppose you want to design an auction for the following type of situation: you have two identical copies of a valuable object, and there are four potential buyers for the object. Each potential buyer i wants at most one copy, and has a value v_i for either copy.

You decide to design the auction by analogy with the way in which we derived the single-item ascending-bid (English) auction from the general procedure for matching markets. In the present case, as there, you want to create a bipartite graph that encodes the situation, and then see what prices the bipartite graph auction procedure comes up with.

(a) Describe how this construction would work using an example with four potential buyers. In creating your example, first choose specific valuations for the potential buyers, and then show how the auction proceeds and what the market-clearing prices are.

(b) In the case of the single-item auction, the bipartite graph procedure yielded the simple rule from the ascending-bid (English) auction: sell to the highest bidder at the second-highest price. Describe in comparably simple terms what the rule is for the current case of two identical items (i.e. your description should not involve the terms bipartite, graph, or matching

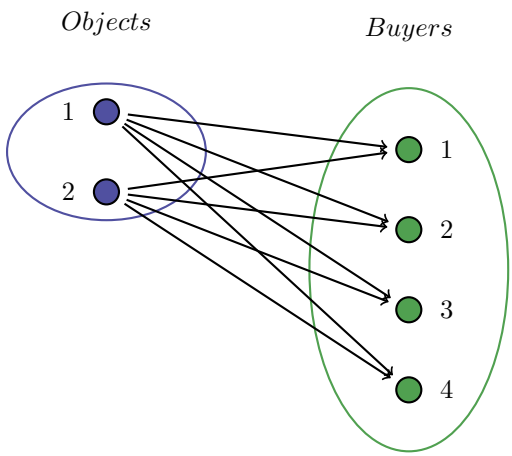
10.2 Answer - a

We have 2 copies of a valuable object and 4 potential buyers. The following buyers value the object at these valuations:

Buyer1	Buyer2	Buyer3	Buyer4
1	2	3	4

10.2.1 Auction Begins

At the beginning of the auction we would have the following bipartite graph:

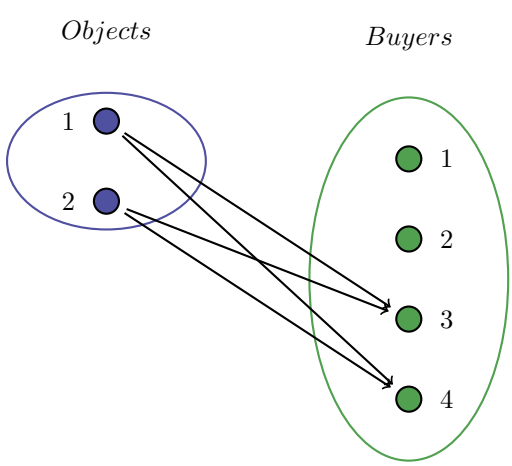


10.2.2 Auction Steps

In the first step of the auction, both sellers have a price of 0 and all buyers are willing to purchase the items. In the second step of the auction, both sellers have a price of 1 and buyers 2, 3 and 4 are willing to purchase the items. In the third step of the auction, both sellers have a price of 2 and buyers 3 and 4 are both willing to purchase both items. There are no further steps of the auction as both buyers then have purchased items and both items have been purchases.

10.2.3 Auction Ends

At the end of the auction we would have the following bipartite graph, in which Buyer 4 would buy the first copy of the object for the valuation of Buyer 2 (2). Buyer 3 would get the second copy of the object for the valuation of Buyer 2 (2).



10.3 Answer - b

In this auction, the top and second highest bidder both buy for the valuation of the third highest bidder.