Session 2

EE 382V: Social Computing **Fall 2018**

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Lecturer: Vijay Garg *Scribe: Carmina Francia*

3.1 Stable Matching/Marriage

Matching has a lot of real world applications. We see it in markets, buyers/sellers, computer processes, residencies, and so on. So far we've used matching in the context of girls likes boys and boys like girls. These matchings can sometimes have *preferences*.

This is called **Stable matching/marriage**. It is finding a matching between two equal sets each following an ordered preference list.

For example, let's create two preference lists:

Table 3.1: Men Preference List

M1	W4	W1	W2	W3
M2	W2	W3	W1	W4
M3	W2	W4	W3	W1
M4	W3	W1	W4	W2

Table 3.2: Women Preference List

W1	M4	M1	M3	M2
W2	M1	M3	M2	M4
W3	M1	M2	M3	M4
W4	M4	M1	M3	M2

This is still a bipartite and a good matching would be when we don't have one woman assigned to two men and vice versa.

Blocking Pair

What kind of marriage would not last long? Let's say we have the following matchings: *(m,w)* and *(m', w')*. If m and w' are not engaged, but m prefers w' to w and w' prefers m to m' , then there's an incentive for them to break. This is called a **blocking pair**.

The goal of stable matching is to find a matching that has *no* blocking pair.

3.2 Gale and Shapley Algorithm

The idea is similar to DGS algorithm: people bidding and going for the most valuable item. We will have men proposing (bidders). The difference is the other set, in this case women, has a say. Women can reject men.

Data: men preference list and women preference list
Result: (m,w) matchings
Assuming number of men is equal to number of women;
Put all men in a list (initially everyone if free);
while *free list of men is not empty* **do**
    pick any free man;
    man proposes to most preferred woman that he has not proposed to;
    if *woman is free* **then**
        men is engaged with that woman;
    else
        woman rejects the less desirable man of the two men based on her preference list;
    end
    rejected man go back to free list (does not matter where in the list)
end

Key notes:

- *engaged* means man and woman are not married. Matchings can change as algorithm progresses.
- Women rejects based on their preference list
- if you're a *free* woman, you cannot say no. Only way to reject is if a better man comes.
- Once a woman is engaged, she cannot be free.
- Algorithm **ALWAYS** terminates and everybody is married.

This example is *man-optimal*. It is seen as worse from women's perspective. However, as the algorithm progresses, men decrease on their preference while women are safe or their preference increases.

3.3 Stable Matching Example

Let's walk through a man-optimal example...

Table 3.3: Men Preference List

M1	**W4**	W1	W2	W3
M2	*W2*	**W3**	W1	W4
M3	**W2**	W4	W3	W1
M4	*W3*	**W1**	W4	W2

Table 3.4: Women Preference List

W1	**M4**	M1	M3	M2
W2	*M1*	**M3**	M2	M4
W3	*M1*	**M2**	M3	M4
W4	*M4*	**M1**	M3	M2

1. Let's begin with *M1*. *W4* is free so *M1* and *W4* becomes engaged.
2. We move on to *M2*. *W2* is free and becomes engaged with *M2*.
3. We go to *M3*. *W2* is already engaged, so we check the woman's preference list. *W2* prefers *M3* over *M2*. So now, *M3* and *W2* are engaged. *M2* goes back to the free list.
4. We go to *M4*. *W3* is free and becomes engaged with *M4*.

5. We pick *M3*. The next woman on his preference list that he has not proposed to is *W3*. *W3* is engaged to *M4*. We check the *W3*'s preference list. *W3* prefers *M3* over *M4*. So *M3* and *W3* becomes engaged and *M4* goes back to the list.

6. We pick *M4*. *W1* is free so *M4* and *W1* becomes engaged.

The list is now empty and returns the matchings: *(M1, W4)*, *(M2, W3)*, *(M3, W2)*, *(M4, W1)*. Since we don't see any blocking pairs, this is a stable matching.

Now let's say it's a woman-optimal. Same steps apply except we would initialize with all women in the free list and pick a woman to start with. Once the list is empty and everybody is married/matched, you will notice that we come up with the same matchings:*(W1, M4)*, *(W2, M3)*, *(W3, M2)*, *(W4, M1)*.

Note: If the man-optimal and woman-optimal are *equal*, this is the **only stable matching**.