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Performance enhancement in two-stage innovation contests: Feedback and elimination schemes

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Abstract

Innovation is one of the driving forces of economic development and social progress, and the crowdsourcing contest is a well-established mechanism for encouraging innovation. This paper examines two incentive schemes in two-stage innovation contests: feedback and elimination. Feedback enhances the efforts by revealing the competitive status, and elimination intensifies the competition by removing less-qualified participants. We build a game theoretical model to investigate how the organizer should design the feedback and elimination schemes and then analyze the equilibrium efforts and optimal contest design in four-solver contests. The results suggest that the optimal design depends on the combined effects of the reward, effort sensitivity, and cost coefficiency. Elimination and nonelimination contests can be optimal under different conditions. Furthermore, we extend the equilibrium analysis to competitions with $n > 4$ contestants and investigate the optimal design with numerical studies. The most interesting result is that the elimination contest with feedback from the organizer is an ideal option for a budget-constrained enterprise that seeks an innovative solution from the public for a complex innovation project. Also, the optimal number of contestants in the second stage is not always two when feedback is combined with elimination.

Keywords: innovation contest; multistage contest; elimination and feedback; effort incentives; service co-production

1. Introduction

Competition is one of the classical mechanisms to incentivize innovation. Multiple studies have verified the effects of competition on technology development (e.g., Rathi, 2014; Marshall and Parra, 2019). Among the vast categories of mechanisms that enhance innovation by using competition, the crowdsourcing contest (or innovation contest) attracts increasing attention from scholars and practitioners. It can reduce the cost of research and development while broadening the sources of innovative products and designs (Vrolijk et al., 2021). Several studies have investigated reward schemes (e.g., Ales et al., 2017; Tian and Bi, 2021), contest duration (Korpeoglu et al., 2021), and contestant participation (e.g., Stouras et al., 2022; Tian et al., 2021; Tian, 2022).

Even with such rich literature, most of the research treats the crowdsourcing contest almost like a "black box," assuming that the contest organizer cannot observe, evaluate, and interfere with the participants' performance in the middle of the tournament. In this situation, the contest organizer loses control of the competition once it is launched, which is not always the case in practice. For example, the contest organizer can provide feedback to the contestants in crowdsourcing platforms like 99 Design and Kaggle. Observing the feedback, the contestants can optimize their solutions and enhance their performance. Using another option, the organizer can hold a two-stage contest and limit the number of participants in the final stage. For instance, only a certain number of finalists qualify to enter the final contest in the L'Oreal Innovation Competition and the Tianchi Big Data Competition. Interference mechanisms like feedback and elimination make the competition more intense and the contestants more devoted to the contests.

Mihm and Schlapp (2018) is the first study that considers the application of interference mechanisms. In their research, the organizer can provide feedback to the contestants in the middle of the competition based on observing the contestants' performance. Later on, Hou and Zhang (2021) discuss a contest design problem, where the organizer can evaluate the contestants' capabilities in the preliminary contest and select some qualified candidates to participate in the final contest. However, in practice, the contest organizer may apply both feedback and elimination to enhance the contestants' performance. Recently, Khorasani and Krishnan (2023) study the effects of screening in a two-stage contest, where the first stage is to distinguish whether a solver is viable, and the performance is valued only in the second stage. They find that screening can be a better choice when the first stage is costly for the solvers.

Different from Hou and Zhang (2021) and Khorasani and Krishnan (2023), which focus on the final-stage solution quality only, we consider the case that the organizer cares about the contestants' overall performance on both stages.

For instance, in the 1940s the Russian Army held an innovation contest to seek the best newly designed assault rifles. The army seeks the rifle with the highest overall quality over reliability and performance. The Kalashnikov rifle (which is later known as the AK-47) won the competition because it had the best overall performance (Britannica, 2023). In our paper, the organizer can take both feedback and elimination schemes to motivate the effort exertion. The contest organizer can choose from four types of contests: the nonelimination contest with no feedback (NCN), the nonelimination contest with public feedback (NCP), the elimination contest with no feedback (ECN), and the elimination contest with public feedback (ECP).

We derive the equilibrium efforts of the contest with a general number of participants and examine the best contest design analytically in a small-sized contest with four participants as Mihm and Schlapp (2018) and Gao et al. (2022). We investigate the four-participant contest rather than the three-participant contest because we can study both whether to eliminate and how many to eliminate in the two-stage contest. The results suggest that the optimal design depends on the combined effects of the parameters in the four-participant contest. The NCN is optimal when the seeker values the participants' average performance, and both NCN and ECP can be optimal when the seeker values the best performance. Furthermore, in a contest with more than four participants, our numerical studies reveal that ECP can be the optimal contest design when the cost coefficient is large enough. It indicates that applying both feedback and elimination schemes can be a cost-effective method when the contest has a large number of participants.

2. Literature review

The crowdsourcing contest has been treated as an effective tool to boost innovation, earning increasing attention (Chen et al., 2020). In our paper, we discuss the regulation configuration and the interference mechanisms in the contest. Thus, the paper is closely related to two streams of the literature: (i) innovation contest design and (ii) interference mechanisms in the contest.

2.1. Innovation contest design

In the crowdsourcing contest, the organizer seeks solutions from the public, which expands the opportunity to find the ideal solution for a project. Segev (2020) provides a thorough review of the innovation contest design literature, and Stouras et al. (2022) summarize the latest advances.

Past literature mainly focuses on how to set up contest regulations to boost effort exertion. For example, Terwiesch and Xu (2008) study the optimal number of participants and suggest that an open contest is optimal for an ideation project. Ales et al. (2017) and Tian and Bi (2021) study the award schemes of ideation and random trial contests and point out that the winner-take-all scheme and multiple-winner schemes can be optimal in different situations. Tian (2022) investigates contest regulation design with risk-averse participants, finding that the multiple-winner scheme is optimal. As a product usually has multiple components, the organizer may hold a multistage contest and test the performance of components in each stage. For example, Hu and Wang (2021) study the contest design problem on the product with two substitute attributes and investigate the optimal contest structure, and Chen et al. (2022) extend the discussion to products with both substitute and complementary attributes. Bimpikis et al. (2019) study two-stage dynamic contests, and Tian et al. (2022) investigate the contest design problem with stochastic arriving projects.

We contribute to the literature by investigating the two-stage contest design problem and discussing the application of both feedback and elimination to motivate the contestants' effort exertion. This study suggests that it may not be optimal to hold an open contest and invite as many participants as possible. Instead, a small group of participants can be the best option under certain circumstances.

2.2. Interference mechanisms in the contest

Aside from determining the contest regulations at the beginning of the contest, the contest organizer can interfere with the participants' behavior in the middle of the contest with certain mechanisms.

First, the organizer can provide feedback to the participants. The organizer can share information on a participant's performance, and the participants may alter their competition strategy based on the feedback. Yildirim (2005) is the first to investigate how information disclosure affects the participants' behavior, and Marinovic (2015) investigates the case that a firm provides feedback to the participants with continuous effort investments. Mihm and Schlapp (2018) introduce a game theory model to investigate the effects of feedback in the middle of the crowdsourcing contest. Second, elimination is another tool to incentivize participants in the multistage contest. The potential of being eliminated may make the contestants exert more effort. For example, Tullock et al. (1980) study the contest design problem with the rent-seeking model, and Deck and Kimbrough (2015) use experiments and data analysis to compare the participants' efforts in single- and multiple-stage contests. In Khorasani and Krishnan (2023), the effects of screening in a two-stage contest are studied, where the first stage is designed as a precontest to distinguish whether a solver is viable. They suggest that screening can be a better choice when the first stage is costly for the solvers. Recently, Hou and Zhang (2021) introduce an analytical model to study how to design a two-stage contest with elimination. In the study, the organizer holds a precontest, and only those contestants who perform well in it are allowed to participate in the formal contest.

We contribute to the literature by discussing the application of both feedback and elimination schemes, which is more complex than the nonelimination case (Mihm and Schlapp, 2018). Different from Hou and Zhang (2021), we consider the case that the organizer uses elimination to intensify the competition rather than just selecting qualified candidates. Our analysis reveals that when both elimination and feedback are applied, the solution qualities can be improved without any additional investment.

3. Model setup

In this section, we discuss the model of the crowdsourcing contest. We describe the problem in Section 3.1 and discuss different contest designs in Section 3.2. We formulate the solver's problem in Section 3.3 and discuss the seeker's problem in Section 3.4.

3.1. Problem description

An innovation contest provides new opportunities for firms to seek breakthrough designs of products and technologies. In this section, we develop a base model to examine the behavior of the organizer (seeker) and contestants (solvers) in crowdsourcing contests with multiple attributes and multiple stages. The seeker ("she") outsources innovation tasks to the public, and the solvers (generically, "he") make efforts to win the rewards. The number of solvers (n) is exogenously given. Let $\mathcal{S} = \{1, \dots, n\}$ denote the set of solvers in the whole contest. Without loss of generality, we consider an

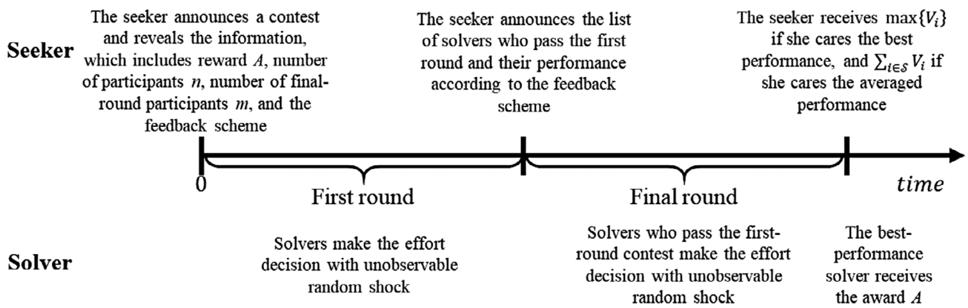

Fig. 1. Structure of the innovation contest with elimination and feedback

innovation contest with two substitute attributes indexed by a and b. Let $\mathcal{R} = \{a, b\}$ denote the set of attributes in the contest. The seeker may eliminate some solvers at the end of the first stage, such that only $m \in \{2, \ldots, n\}$ solvers enter the final stage. This matches the contest regulation in many big data contest platforms (e.g., the Tianchi Big Data Competition and Paddle Paddle Competition (AIStudio, 2023)), in which only a stated number of participants can enter the final-stage contest.

The solvers compete on both attributes in the NCN and NCP and attribute a and b in the first and final stages in the ECN and ECP, respectively.

In the elimination contest, because only one attribute is examined in each stage, we can take the attribute index as the stage index. Thus, set $\mathcal{R}$ is also the set of stages in the elimination contest. To facilitate the analysis, we define $\mathcal{S}_j$ as the set of players in stage $j \in \mathcal{R}$ of an elimination contest. Let A denote the total prize of the project, and we assume that there is only one winner at the end of the contest. It suggests that the seeker takes the winner-take-all scheme for the distribution of the reward, which is frequently discussed in the literature (e.g., Hu and Wang, 2021; Korpeoglu et al., 2021).

Figure 1 illustrates the structure of the innovation contest. The seeker announces the number of first-stage participants n and the number of final-stage participants m at the beginning of the contest. The seeker chooses from different elimination schemes to maximize the average performance (AP) or best performance of the solvers over the two specified attributes. We assume that the solvers will always choose to participate in the final-stage contest if they pass the first stage. With a given contest design, the solvers make their own effort decisions and submit the product to the seeker.

In our paper, we assume the seeker is looking for a solution for an ideation project as defined in Terwiesch and Xu (2008). The performance of the solver on each attribute is determined by two variables. The first variable is the *effort level* that the solver decides. Solver i makes his own effort decision $e_{i,j}$ in stage j based on the reward incentives. The second variable is the *random shock* that affects the performance of each player stochastically. The random shock depicts the random performances of the solvers when they create innovative products. We assume that the random shocks along the two attributes, denoted as ε_a and ε_b, are independent, and they are independently and identically distributed across all the solvers. Similar to Mihm and Schlapp (2018) and Hou and Zhang (2021), we assume that these random shocks follow the same uniform distribution Uniform$(-\frac{1}{2}, \frac{1}{2})$ with the cumulative distribution function (CDF) $\Psi(\cdot)$ and probability density function (PDF) $\psi(\cdot)$.

In the ideation project, the performance of a solver on each attribute is a weighted sum of the effort and random shock. If solver i makes effort $e_{i,j}$ in stage j, then we define his performance as $V_{i,j}$, given by $V_{i,j} = ke_{i,j} + \varepsilon_{i,j}$ $\forall i \in \mathcal{S}$ and $\forall j \in \mathcal{R}$, where $k > 0$ is the sensitivity of the effort. We assume that k is a given constant independent of the solvers and attributes. The seeker's interest is a single product with two attributes. The overall performance of solver i is the total performance on the two attributes, given by $V_i = V_{i,a} + V_{i,b}$ $\forall i \in \mathcal{S}$.

In addition, we assume that the solver incurs some cost when exerting the effort. The cost of exerting effort can be considered to be the time and money invested in developing the product. Following Mihm and Schlapp (2018), we assume that the solvers have the same cost function along with the two attributes, and it is given by $\gamma(e) = ce^2$, where $c > 0$ is the coefficient of the cost. To avoid unnecessary technical complications, we assume that c is large enough, such that the solver would not exert unlimited resources on the innovation project.

3.2. An overview of different contest designs

When organizing an innovation contest, the seeker may consider using either the elimination or feedback scheme to enhance the solvers' performance. There are four different contest designs that the seeker can choose from, which are NCN, NCP, ECN, and ECP. We then briefly introduce the concept of each type of contest.

(i) *NCN*. The seeker announces the number of participants at the beginning of each stage. She neither eliminates the solvers nor provides any feedback at the end of the first stage.
(ii) *NCP*. The seeker announces the number of participants at the beginning of each stage. She does not eliminate the solvers but provides feedback at the end of the first stage.
(iii) *ECN*. At the end of the first stage in the ECN, the seeker does not provide any feedback on the solvers' first-stage performance but just announces the list of solvers entering the final stage.
(iv) *ECP*. At the end of the first stage in the ECP, the seeker provides feedback on the solvers' first-stage performance to the public and announces the list of solvers entering the final stage. In this case, each solver learns his own and his competitor's first-stage performance.

3.3. The solver's payoff

In this section, we formulate the solver's problem under different contest designs. We use NN, NP, EN, and EP in the superscript to denote the variables of the NCN, NCP, ECN, and ECP, respectively. The solvers make their effort decisions to maximize their expected utility.

3.3.1. NCN

In the NCN, the firm launches a single contest on the performance of the two attributes. The solvers make an aggregate submission for two attributes instead of a separate solution for each attribute. Thus, the performance of the solver depends on the random shocks in both stages. To express the joint effect, we introduce a random variable η_i to express the difference of random shocks of solver

i, which is given by $\eta_i = \varepsilon_{i,a} - \varepsilon_{i,b}$. Define $\Phi(\eta_i)$ and $\phi(\eta_i)$ as the CDF and PDF of variable η_i respectively. We present the formulations of $\Phi(\eta_i)$ and $\phi(\eta_i)$ in Lemma A1 of Section A.1. The performance of solver i can then be expressed as $V_i = k(e_{i,a} + e_{i,b}) + \eta_i$.

The solvers make the effort decision based on their own beliefs regarding winning the contest. Because the solver with the best performance obtains the whole prize A, the payoff of solver i is

$$u_i^{NN}(e_{i,a}, e_{i,b}) = \begin{cases} A - \gamma(e_{i,a}) - \gamma(e_{i,b}), & \text{if solver } i \text{ wins,} \\ -\gamma(e_{i,a}) - \gamma(e_{i,b}), & \text{if solver } i \text{ loses.} \end{cases}$$

Let the winning probability of solver i be $\mathrm{P}(V_i > V_j | j \in \mathcal{S}, j \neq i)$. Solver i's expected utility is $\mathrm{E}[u_i^{NN}(e_{i,a}, e_{i,b})] = A \cdot \mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i) - \gamma(e_{i,a}) - \gamma(e_{i,b})$, where V_i is a function of $e_{i,a}$ and $e_{i,b}$.

3.3.2. NCP

In the NCP, the seeker announces the first-stage performance to all the solvers at the end of the first round. We formulate the solvers' problems in each stage respectively.

(i) *Final stage*. Because the solver with the best performance wins prize A, the payoff of solver i in the final stage if he exerts effort $e_{i,b}$ is given by

$$u_{i,b}^{NP}(e_{i,b}) = \begin{cases} A - \gamma(e_{i,b}), & \text{if } i \text{ wins,} \\ -\gamma(e_{i,b}), & \text{if } i \text{ loses.} \end{cases}$$

In the final stage, each solver perfectly knows the players' performance. Define the first-ranked solver in the first stage as (1), such that his performance is $V_{(1),a}$. Let $\Delta_i \in [0, 1]$ denote the difference between the performances of solvers i and (1) in the first stage, and it is given by $\Delta_i = V_{(1),a} - V_{i,a}$. With the public feedback, solver i in the final stage can fully observe the value of Δ_i at the end of the first stage. In this case, he makes the effort decisions based on the value of Δ_i in the final stage and the belief of Δ_i in the first stage.

Since solver i is able to observe the difference Δ_i, we denote the winning probability of solver i as $\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}_2, j \neq i, \Delta_i)$ when he makes an effort $e_{i,b}$. The expected utility of solver i can be expressed by $\mathrm{E}[u_{i,b}^{NP}(e_{i,b})] = A \cdot \mathrm{P}(V_i > V_j | i, j \in \mathcal{S}_2, j \neq i, \Delta_i) - \gamma(e_{i,b})$, where V_i is a function of $e_{i,b}$.

(ii) *First stage*. In the first stage, the solver makes the effort decision based on the expected payoff in the final stage. Let $\mathrm{E}[u_{i,b}^{NP}(e_{i,b})]$ be the expected final-stage payoff. The payoff of solver i in the first stage is given by

$$u_{i,a}^{NP}(e_{i,a}) = \begin{cases} \int_{-\infty}^{\infty} \mathrm{E}\Big[u_{i,b}^{NP}(e_{i,b})\Big] d\Delta_i - \gamma(e_{i,a}), & \text{if solver } i \text{ passes the first stage,} \\ -\gamma(e_{i,a}), & \text{if solver } i \text{ loses the first stage.} \end{cases}$$

Denote the winning probability of solver i as $\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i, \Delta_i)$ when he makes an effort $e_{i,b}$. The expected utility can be expressed by $\mathrm{E}[u_{i,b}^{NP}(e_{i,b})] = A \cdot \mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i, \Delta_i) \int_{-\infty}^{\infty} \mathrm{E}[u_{i,b}^{NP}(e_b^P(n, \Delta_i))] d\Delta_i - \gamma(e_{i,a})$, where V_i is a function of $e_{i,b}$ and Δ_i.

3.3.3. ECN

In the ECN, the firm launches two subcontests, each focusing on one attribute. The seeker does not announce the first-stage performance to all the solvers at the end of the first round but discloses who is qualified to participate in the final round subcontest. The solvers make the effort decision based on their own beliefs about entering the final stage and winning the game. We formulate the solvers' problems in each stage, respectively.

(i) *Final stage*. Because the solver with the best performance wins prize A, the payoff of solver i in the final stage if he exerts effort $e_{i,b}$ is given by

$$u_{i,b}^{EN}(e_{i,b}) = \begin{cases} A - \gamma(e_{i,b}), & \text{if solver } i \text{ wins,} \\ -\gamma(e_{i,b}), & \text{if solver } i \text{ loses.} \end{cases}$$

Denote the winning probability of solver i as $\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}_2, j \neq i)$ when he makes an effort $e_{i,b}$. Then the expected utility is $\mathrm{E}[u_{i,b}^{EN}(e_{i,b})] = A \cdot \mathrm{P}(V_i > V_j | i, j \in \mathcal{S}_2, j \neq i) - \gamma(e_{i,b})$, where V_i is a function of $e_{i,b}$.

(ii) *First stage*. In the first stage, the solver makes the effort decision based on the expected payoff in the final stage. Let $\mathrm{E}[u_{i,b}^{EN}(e_{i,b})]$ be the expected final-stage payoff. Then, the payoff of solver i in the first stage if he exerts effort $e_{i,a}$ is given by

$$u_{i,a}^{EN}(e_{i,a}) = \begin{cases} \mathrm{E}\left[u_{i,b}^{EN}(e_{i,b})\right] - \gamma(e_{i,a}), & \text{if solver } i \text{ passes the first stage,} \\ -\gamma(e_{i,a}), & \text{if solver } i \text{ loses the first stage.} \end{cases}$$

Denote the winning probability of solver i as $\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i)$ when he makes an effort $e_{i,a}$. Then the expected utility is $\mathrm{E}[u_{i,a}^{EN}(e_{i,a})] = \mathrm{E}[u_{i,b}^{EN}(e_{i,b})]\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i) - \gamma(e_{i,a})$, where V_i is a function of $e_{i,b}$.

3.3.4. ECP

In the ECP, the seeker will not only announce who is qualified to join the final-stage competition but also each player's performance in the first round. In this design, the solver makes the first-stage effort decision based on his own belief about entering the final stage. He makes the final-stage effort decision based on his confidence in winning and with the first-stage performance feedback.

(i) *Final stage*. Because the best performance solver wins prize A, the payoff of solver i in the final stage is given by

$$u_{i,b}^{EP}(e_{i,b}) = \begin{cases} A - \gamma(e_{i,b}), & \text{if } i \text{ wins,} \\ -\gamma(e_{i,b}), & \text{if } i \text{ loses.} \end{cases}$$

In the final stage of ECP, the winning probability of solver i is $\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}_2, j \neq i, \Delta_i)$ when he makes an effort $e_{i,b}$. Then the expected utility is $\mathrm{E}[u_{i,b}^{EP}(e_{i,b})] = A \cdot \mathrm{P}(V_i > V_j | i, j \in \mathcal{S}_2, j \neq i, \Delta_i) - \gamma(e_{i,b})$, where V_i is a function of $e_{i,b}$ and Δ_i.

(ii) *First stage*. In the first stage, the solver makes the effort decision based on the expected payoff in the final stage. Let $\mathrm{E}[u_{i,b}^{EP}(e_{i,b})]$ be the expected final-stage payoff. The payoff of solver i in the first stage is given by

$$u_{i,a}^{EP}(e_{i,a}) = \begin{cases} \int_{-\infty}^{\infty} \mathrm{E}\big[u_{i,b}^{EP}(e_{i,b})\big] d\Delta_i - \gamma(e_{i,a}), & \text{if solver } i \text{ passes the first stage,} \\ -\gamma(e_{i,a}), & \text{if solver } i \text{ loses the first stage.} \end{cases}$$

Thus the winning probability of solver i is $\mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i)$ when he makes an effort $e_{i,b}$, and the expected utility is $\mathrm{E}[u_{i,a}^{EP}(e_{i,a})] = \mathrm{P}(V_i > V_j | i, j \in \mathcal{S}, j \neq i) \int_{-\infty}^{\infty} \mathrm{E}[u_{i,b}^{EP}(e_{i,b})] d\Delta_i - \gamma(e_{i,a})$, where V_i is a function of $e_{i,a}$.

3.4. The seeker's profit

As in Terwiesch and Xu (2008), we assume the seeker cares about either the average or best performance of all the solvers. Let Π_{avg} and Π_{best} be the values of average performance and best performance, respectively, which are given by

$$\Pi_{avg} = \frac{\sum_{i=1}^{n}}{V_{i}} n, \tag{1}$$

$$\Pi_{best} = \max\{V_i\}\ i \in \mathcal{S}. \tag{2}$$

To maximize the profit, the seeker has to choose from NCN, NCP, ECN, and ECP to enhance the player's performance in the contest.

4. Solvers' efforts and performance in the contests

In this section, we discuss the solvers' solution effort and performance in the four types of contests. We discuss the equilibrium efforts in Section 4.1 and the solvers' expected performance in Section 4.2. Following the classical literature (e.g., Terwiesch and Xu, 2008; Ales et al., 2021), we care about the symmetric equilibrium for different types of contests.

4.1. Solvers' solution efforts

We start by analyzing the solvers' solution efforts under each type of contest. We do not need to specify the seeker's objective because the solver's strategies are independent of the seeker's goal.

4.1.1. NCN

In the benchmark case of NCN, the seeker does not release any information on the performance. As a result, each solver's two-attribute effort choice problem becomes a simultaneous, single-stage utility maximization problem.

Let NCN(n) denote the NCN with n solvers. Theorem 4.1 describes the equilibrium efforts in the NCN(n). To help express the equilibrium efforts, we define a constant $F^n_{(1)}$, whose formulation is presented in Equation (A1) of Section A.2.1.

Theorem 4.1. *The equilibrium efforts in the NCN(n) are* $e_a^{NN}(n) = e_b^{NN}(n) = \frac{kAF^n_{(1)}}{2c}$.

Regarding NCN, it suggests similar results as Hu and Wang (2021), that the solution efforts are identical across different stages because no solver receives any information about their performance. Therefore, the solvers' inherent uncertainty determines the winner. As is common in the literature, the equilibrium efforts increase in the size of award A and the sensitivity of the effort k but decrease in the effort cost c.

4.1.2. NCP

In the NCP, there is no elimination in the second stage. Let NCP(n) denote the NCP with n solvers. Theorem 4.2 describes the equilibrium efforts in the NCP(n). To help express the equilibrium efforts, we define three constants, $\bar{A}^n$, $\hat{H}^n_{(n)}$, and $H^n_{(1)}(\Delta\varepsilon_i)$. We present their formulations in (A2)–(A4) of Section A.2.1

Theorem 4.2. *The equilibrium efforts in the NCP (n) are* $e_a^{NP}(n) = \frac{k\bar{A}^n}{2c}\hat{H}^n_{(n)}$ *and* $e_b^{NP}(n, \Delta\varepsilon_i) = \frac{kA}{2c}H^n_{(1)}(\Delta\varepsilon_i)$.

Similar to Mihm and Schlapp (2018), the solution efforts in the NCP are nonidentical across different stages because solvers adjust their second-stage effort exertion based on the received public feedback. Theorem 4.2 also suggests that the equilibrium efforts increase in the sensitivity of the effort k but decrease in the effort cost c.

4.1.3. ECN

In the ECN, the final-stage solvers only know they have passed the first stage; they do not know what the seeker thought about their performance.

Let ECN(n, m) denote the ECN with n solvers in the first stage and m solvers in the final stage. The Theorem 4.3 describes the equilibrium efforts in the ECN(n, m). To help express the equilibrium efforts, we define constants $G^m_{(1)}$ and $\hat{G}^n_{(m)}$, whose formulations are presented in (A5) and (A6) of Section A.2.1

Theorem 4.3. *The equilibrium efforts in the ECN(n, m) are* $e_a^{EN}(n, m) = (\frac{kA}{2mc} - \frac{k^3A^2(G^m_{(1)})^2}{8c^2})\hat{G}^n_{(m)}$ *and* $e_b^{EN}(m) = \frac{kAG^m_{(1)}}{2mc}$.

Theorem 4.3 reveals that the equilibrium efforts in different stages of the elimination contest can be nonidentical rather than identical as in Mihm and Schlapp (2018). The equilibrium efforts in the final stage increase in the size of award A and the sensitivity of the effort k but decrease in the effort cost c.

4.1.4. ECP

In the ECP, the seeker eliminates $n - m$ solvers.

Let ECP(n, m) denote the ECP with n solvers in the first stage and m solvers in the final stage.

Theorem 4.4 describes the equilibrium efforts in the ECP(n, m). To help express the equilibrium efforts, we define constants $\bar{A}^m$, $\hat{H}^n_{(m)}$, and $H^m_{(1)}(\Delta\varepsilon_i)$, whose formulations are presented in (A7)–(A9) of Section A.2.1

Theorem 4.4. *The equilibrium efforts in the ECP(n, m) are $e_a^{EP}(n, m) = \frac{k\bar{A}^m}{2c}\hat{H}^n_{(m)}$ and $e_b^{EP}(m, \Delta\varepsilon_i) = \frac{kA}{2c}H^m_{(1)}(\Delta\varepsilon_i)$.*

Theorem 4.4 suggests that the solvers exert the same equilibrium effort in the first stage but different efforts in the final stage. Since there is no feedback at the beginning of the first stage, the solvers make the first-stage effort decisions based on their identical beliefs about their probability of winning. Therefore, the first-stage equilibrium effort is identical across the solvers. Thus, the performance difference only comes from the first-stage random shock (i.e., $\Delta_i = \Delta\varepsilon_i$), and the second-stage equilibrium effort depends on $\Delta\varepsilon_i$ instead of Δ_i.

In the final stage, feedback has changed the behavior of the solvers. The solvers make the effort exertion decision considering the performance difference. Different from Mihm and Schlapp (2018), in our model, the solvers do not spend the same effort in the final stage. A solver works hard if he is a leader after the first stage (i.e., when $\Delta\varepsilon_i$ is small) but invests little effort if he falls behind many competitors (i.e., when $\Delta\varepsilon_i$ is large). Each solver seeks to close the first-stage performance gap but does not exert further effort. All the solvers rely on the final-stage random shock to decide the final winner. The final-stage equilibrium efforts increase in the sensitivity of the effort k but decrease in the effort cost c.

4.2. Solvers' expected performance

In this section, we discuss the optimal contest design, when the seeker cares about the average and best performance.

Lemma 4.1 presents the solvers' average and best performance in different contests. To help express the best performance, we define constants $\bar{\eta}^n_{(1)}$, $\bar{\vartheta}^{n,n}_{(1)}$, and $\bar{\vartheta}^{n,m}_{(1)}$ whose formulations are presented in (A10)–(A12) of Section A.2.2.

Lemma 4.1. *Solvers' expected average and best performance in the four contests are*

(i) $\Pi^{NN}_{avg}(n) = \frac{k^2AF^n_{(1)}}{c}$ *and* $\Pi^{NN}_{best}(n) = \frac{k^2AF^n_{(1)}}{c} + \bar{\eta}^n_{(1)}$ *in the NCN (n);*

(ii) $\Pi^{NP}_{avg}(n) = \frac{k^2}{2c}(\bar{A}^n\hat{H}^n_{(n)} + A\int_{-\infty}^{\infty} H^n_{(1)}(\Delta\varepsilon_i)d\Delta\varepsilon)$ *and* $\Pi^{NP}_{best}(n) = \frac{k^2}{2c}(\bar{A}^n\hat{H}^n_{(n)} + A\int_{-\infty}^{\infty} H^n_{(1)}(\Delta\varepsilon_i)d\Delta\varepsilon) + \bar{\eta}^n_{(1)}$ *in the NCP(n);*

(iii) $\Pi^{EN}_{avg}(n, m) = \frac{k^2A}{mc}(\hat{G}^n_{(m)} + G^m_{(1)}) - \frac{k^4A^2(G^m_{(1)})^2\hat{G}^n_{(m)}}{8c^2}$ *and* $\Pi^{EN}_{best}(n, m) = \frac{k^2A}{mc}(\hat{G}^n_{(m)} + G^m_{(1)}) - \frac{k^4A^2(G^m_{(1)})^2\hat{G}^n_{(m)}}{8c^2} + \bar{\vartheta}^{n,m}_{(1)}$ *in the ECN(n, m);*

(iv) $\Pi^{EP}_{avg}(n, m) = \frac{k^2}{2c}(\bar{A}^m\hat{H}^n_{(m)} + A\int_{-\infty}^{\infty} H^m_{(1)}(\Delta\varepsilon_i)d\Delta\varepsilon)$ *and* $\Pi^{EP}_{best}(n, m) = \frac{k^2}{2c}(\bar{A}^m\hat{H}^n_{(m)} + A\int_{-\infty}^{\infty} H^m_{(1)}(\Delta\varepsilon_i)d\Delta\varepsilon) + \bar{\vartheta}^{n,m}_{(1)}$ *in the ECP(n, m).*

Due to the mathematical complexity of analyzing the best contest design, we investigate the optimal contest design with numerical studies in Sections 6.1 and 6.2. To obtain managerial implications on the contest design problem, we analytically prove the optimal contest design of a four-solver contest in Section 5.2.

5. Four-solver contest

Due to the complexity of analyzing the optimal contest design with a general number of participants, we focus on the simplified case with a given number of solvers, as in Mihm and Schlapp (2018) and Gao et al. (2022). We consider the case with four solvers (i.e., $n = 4$) because it is the minimum-size case in which the seeker can actually choose to eliminate some but not all participants in the middle of the contest. For example, in an ECN or ECP, the seeker can choose $m = 2$ or $m = 3$ solvers for the final stage. We chose the four-solver contest rather than the three-solver contest because we can study both whether to eliminate and how many to eliminate when $n = 3$.

5.1. Solvers' solution efforts

We start by analyzing the solvers' solution efforts under the four-solver contest. The following lemma describes the solvers' equilibrium efforts in different contests.

Lemma 5.1. *Solvers' equilibrium efforts in the four-solver contest are*

(i) $e_a^{NN}(4) = e_b^{NN}(4) = \frac{43kA}{140c}$ *in the NCN(4).*
(ii) $e_a^{NP}(4) = \frac{23kA}{120c} - \frac{69A^2k^4}{448c^2}$ *and* $e_b^{NP}(4) = \frac{3kA}{2c}(\frac{1}{3} + \Delta\varepsilon_i + \Delta\varepsilon_i^2)$ *in the NCP(4).*
(iii) $e_a^{EN}(4, 2) = \frac{kA}{4c} - \frac{k^3A^2}{8c^2}$ *and* $e_b^{EN}(2) = \frac{kA}{4c}$ *in the ECN(4,2),* $e_a^{EN}(4, 3) = \frac{kA}{6c} - \frac{k^3A^2}{8c^2}$ *and* $e_b^{EN}(3) = \frac{kA}{6c}$ *in the ECN(4,3).*
(iv) $e_a^{EP}(4, 2) = \frac{kA(4c-k^2A)}{36c^2}$ *and* $e_b^{EP}(2, \Delta\varepsilon_i) = \frac{kA}{2c}(1 - \Delta\varepsilon_i)$ *in the ECP(4,2), and* $e_a^{EP}(4, 3) = \frac{kA(15c-8k^2A)}{180c^2}$ *and* $e_b^{EP}(3, \Delta\varepsilon_i) = \frac{kA}{2c}(1 - \Delta\varepsilon_i^2)$ *in the ECP(4,3).*

Lemma 5.1 is a direct result of Theorems 4.1–Theorem 4.4. We then analyze the optimal contest design based on Lemma 5.1.

5.2. Optimizing seeker's profit

In this section, we analyze the seeker's expected profit under different contest designs.

We discuss the optimal contest design optimizing the average and best performance in Proposition 5.1.

Proposition 5.1. *The following properties hold for a four-solver contest:*

(i) *When the seeker cares about the average performance, it is optimal to hold NCN(4).*

(ii) *When the seeker cares about the best performance, if* $A \geq \frac{3(\sqrt{128442017}-11055)c}{12320k^2}$*, it is optimal to hold NCN(4); otherwise, it is optimal to hold ECP(4,3).*

Proposition 5.1 suggests that NCN(4) is optimal in a four-solver contest for any parameters A, k, and c when the seeker values the average performance. Elimination and feedback are insufficient to make the solvers exert more effort than in the four-solver NCN. However, this property is not always true for any $n \geq 3$. By the numerical study in Section 6.1, we find that it can be optimal to hold an ECP when n is large enough. Thus, elimination, feedback, and the number of solvers jointly affect the optimal contest design.

Moreover, Proposition 5.1 also suggests that the parameters jointly affect the optimal contest design when the seeker cares about the best performance. NCN(4) is optimal if the seeker has enough budget to offer a considerable reward, while ECP(4,3) is ideal for the budget-constrained seeker to maximize the best performance. When holding an ECP(4,3), feedback and elimination jointly improve the solver's effort, but the expected value of random shock only diminishes slightly in this case. Hence, the solver's overall best performance is enhanced. Thus, ECP(4,3) is better than NCN(4).

Because elimination and feedback are two important tools for incentive effort exertion (see, e.g., Deck and Kimbrough, 2015; Mihm and Schlapp, 2018; Hou and Zhang, 2021), we compare the individual effects of feedback and elimination on the solvers' average performance in the contest. We conduct the comparison by examining the average performance of the solvers in the NCN (i.e., contest with no feedback nor elimination), NCP (i.e., contest with public feedback only), and ECN (i.e., contest with elimination only). We do not discuss ECP (i.e., contest with elimination and feedback) since it jointly applies elimination and feedback schemes.

Proposition 5.2.

(i) *The following properties hold when the seeker cares about the average performance:*
  (ia) *The average performance of NCP is lower than NCN.*
  (ib) *The average performance of ECN is lower than NCN.*
  (ic) *The average performance of ECN is lower than NCP when the award is small enough (i.e.,* $0 < A \leq \frac{88c}{195k^2}$*); otherwise, NCP is lower.*

(ii) *The following properties hold when the seeker cares about the best performance:*
  (iia) *The best performance of NCP is lower than NCN.*
  (iib) *The best performance of ECN is lower than NCN when the award is large enough (i.e.,* $A \geq \frac{2(\sqrt{11624965}-3245)c}{5775k^2}$*); otherwise, ECN is higher.*
  (iic) *The best performance of ECN is lower than NCP when the award is at a medium level (i.e.,* $\frac{(33220-4\sqrt{64906105})c}{10725k^2} \leq A \leq \frac{(2\sqrt{568502}+242)c}{2415k^2}$*); otherwise, ECN is higher.*

Proposition 5.2 reveals that the feedback-only scheme is worse than the no-feedback-no-elimination scheme when the seeker values either average or best performance. However, compared with the contest with elimination only, the no-feedback-no-elimination scheme is better when the seeker values the average performance, or if the award is large enough when the seeker values the best performance. Moreover, compared with the contest with elimination only (i.e., ECN), the contest with public feedback only (i.e., NCP) is better if the award is small enough when the seeker

Table 1
Optimal contest design maximizing the average performance with different award A

	A	n 3	7	11	15	19	23
Optimal design	0.20	NCN(3)	NCP(7)	ECP(11,7)	ECP(15,8)	ECP(19,9)	ECP(23,9)
	0.60	NCN(3)	NCN(7)	NCN(11)	ECP(15,4)	ECP(19,4)	ECP(23,4)
	1.00	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,2)	ECP(23,2)
	1.40	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,2)	ECP(23,2)
	1.80	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	ECP(23,2)
Π^*_{avg}	0.20	0.27	0.53	0.17	0.18	0.18	0.18
	0.60	0.80	0.58	0.46	0.45	0.45	0.45
	1.00	1.33	0.96	0.76	0.65	0.65	0.66
	1.40	1.87	1.34	1.07	0.91	0.83	0.83
	1.80	2.40	1.73	1.38	1.17	1.04	0.95

values the average performance, or if the award is at the medium level when the seeker values the best performance. Thus, the feedback-only scheme better suits the seeker who does not have a large budget to offer a generous award A.

6. Numerical studies

In this section, we present numerical studies on the optimal contest design with a general number of participants ($n \geq 3$). Based on the formulas, we discuss the optimal contest design maximizing the average performance in Section 6.1 and the best performance in Section 6.2.

In the numerical study, we investigate the contest design problem with $n \in \{3, 7, 11, 15, 19, 23\}$ and $m \in \{2, \ldots, n\}$. Similar to Körpeoğlu and Cho (2017), we configure the default parameters as $A = k = 1$, $c = 0.5$. In our test, we keep the parameters A, k, and c at default values and investigate the optimal contest design as the remaining parameters change.

6.1. *Contests optimizing the average performance*

In this section, we study the optimal contest design when the seeker cares about the average performance.

First, we present the optimal contest design with different award A in Table 1.

We can tell from the table that either the NCN or ECP can be optimal when the award varies, which is different from Proposition 5.1 (i). Award A and the number of solvers n jointly affect the optimal contest design. NCN is optimal when the number of solvers is small and the award is large enough. The optimal number of participants in the final stage of the ECP varies from 2 to $n - 1$. For the optimal value, we can tell that the average performance increases as reward A grows but decreases as the number of solvers n grows; this result is common in the literature (e.g., Terwiesch and Xu, 2008; Ales et al., 2017). It fits the intuition well that the solvers work harder when a better

Table 2
Optimal contest design maximizing the average performance with different effort sensitivity k

	k	n					
		3	7	11	15	19	23
Optimal design	0.20	ECN(3,2)	ECN(7,2)	NCP(11)	NCP(15)	NCP(19)	NCP(23)
	0.60	NCN(3)	NCN(7)	ECP(11,6)	ECP(15,6)	ECP(19,6)	ECP(23,6)
	1.00	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,2)	ECP(23,2)
	1.40	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
	1.80	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
Π^*_{avg}	0.20	0.78	0.92	2.22	25.27	296.16	3517.31
	0.60	0.48	0.35	0.29	0.30	0.30	0.30
	1.00	1.33	0.96	0.76	0.65	0.65	0.66
	1.40	2.61	1.88	1.50	1.28	1.13	1.03
	1.80	4.32	3.11	2.48	2.11	1.88	1.70

reward is offered. The competition gets more intense with more participants, making the solvers more conservative in effort exertion.

Next, we analyze the optimal contest with different effort sensitivities k in Table 2.

We can see that any of NCN, NCP, ECN, and ECP could be the optimal contest design. NCN is optimal when k is large, and ECN(n, 2) is optimal when k is small. NCP is optimal when k is small, and n is large. ECP is optimal with a medium value of k. The average performance increases in k because the performance is assumed to be proportional to k.

The result suggests that the optimal contest design also depends on the performance function. NCN is optimal when the performance depends more on the effort exertion than random shock (i.e., when k is large) because the nonelimination contest allows for a smaller expected random shock than the elimination contest. When k is small enough, NCP is optimal with larger n, which implies that the individual effect of public feedback is stronger than the individual effect of elimination when there is a large number of participants. When k is at the medium level, the joint application of elimination and public feedback is better. When k is large enough, neither the elimination nor public feedback can be optimal.

Third, we analyze the effect of cost coefficient c on the optimal contest in Table 3.

Like before, NCN and ECP are optimal with different cost coefficients c. Here, NCN is optimal when c is small, while ECP is optimal when c is large. The average performance decreases as the cost coefficient c and the number n increase, which means the average performance is negatively correlated with c. This effect is a natural result of the effort cost because the solvers will avoid exerting extra effort when the cost coefficient is high.

We summarize the conditions when the ECP is optimal by our numerical tests in Remark 6.1.

Remark 6.1. When the seeker cares about the average performance, the ECP is optimal in any of the following situations:

(i) The seeker is budget-constrained and cannot offer a generous award;
(ii) The solvers' performance depends more on the random shock than the effort exertion;
(iii) The innovative project is complex with a large cost coefficient;
(iv) The seeker aims to find an innovative solution from a large number of solvers.

Table 3
Optimal contest design maximizing the average performance with different cost coefficient c

	c	n 3	7	11	15	19	23
Optimal design	0.20	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
	0.60	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	ECP(23,2)
	1.00	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,2)	ECP(23,2)
	1.40	NCN(3)	NCN(7)	NCN(11)	ECP(15,3)	ECP(19,3)	ECP(23,3)
	1.80	NCN(3)	NCN(7)	NCN(11)	ECP(15,4)	ECP(19,4)	ECP(23,4)
Π^*_{avg}	0.20	6.67	4.80	3.82	3.26	2.89	2.63
	0.60	2.22	1.60	1.27	1.09	0.96	0.92
	1.00	1.33	0.96	0.76	0.65	0.65	0.66
	1.40	0.95	0.69	0.55	0.51	0.51	0.51
	1.80	0.74	0.53	0.42	0.42	0.42	0.43

Table 4
Optimal contest design maximizing the best performance with different award A

	A	n 3	7	11	15	19	23
Optimal design	0.20	NCN(3)	NCP(7)	ECP(11,9)	ECP(15,10)	ECP(19,12)	ECP(23,13)
	0.60	NCN(3)	NCN(7)	NCN(11)	ECP(15,7)	ECP(19,8)	ECP(23,9)
	1.00	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,5)	ECP(23,5)
	1.40	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
	1.80	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
Π^*_{best}	0.20	0.62	1.08	0.82	0.87	0.91	0.93
	0.60	1.15	1.13	1.09	1.13	1.17	1.19
	1.00	1.68	1.51	1.40	1.34	1.32	1.34
	1.40	2.22	1.89	1.70	1.60	1.53	1.48
	1.80	2.75	2.28	2.01	1.86	1.76	1.69

In short, ECP is a low-cost and high-efficient option for the seeker looking for an innovative solution to a complex project from the public.

6.2. *Contests optimizing the best performance*

In this section, we study the optimal contest design optimizing the best performance. We start by analyzing the optimal contest with different awards in Table 4.

We can tell from the table that parameters A and n jointly affect the optimal contest design. Similar to Proposition 5.1 (ii), either ECP or NCN or NCP can be optimal. ECP is optimal when award A is small, and the number of solvers n is large. The reason is that ECP(n, m) has a greater expected random shock than the NCN(n). When the reward is small and the number of solvers is large, the performance of a solver depends more on the random shock than the effort exertion, which makes the ECP optimal. We can also find that the optimal number of participants in the

Table 5
Optimal contest design maximizing the best performance with different effort sensitivity k

	k	n 3	7	11	15	19	23
Optimal design	0.20	NCN(3)	NCP(7)	NCP(11)	NCP(15)	NCP(19)	NCP(23)
	0.60	NCN(3)	NCN(7)	ECP(11,8)	ECP(15,9)	ECP(19,10)	ECP(23,11)
	1.00	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,5)	ECP(23,5)
	1.40	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
	1.80	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
Π^*_{best}	0.20	0.40	0.76	2.86	25.96	296.87	3518.06
	0.60	0.83	0.90	0.94	0.99	1.02	1.05
	1.00	1.68	1.51	1.40	1.34	1.32	1.34
	1.40	2.96	2.43	2.13	1.96	1.85	1.77
	1.80	4.67	3.66	3.11	2.80	2.59	2.45

Table 6
Optimal contest design maximizing the best performance with different cost coefficient c

	c	n 3	7	11	15	19	23
Optimal design	0.20	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
	0.60	NCN(3)	NCN(7)	NCN(11)	NCN(15)	NCN(19)	NCN(23)
	1.00	NCN(3)	NCN(7)	NCN(11)	NCN(15)	ECP(19,5)	ECP(23,5)
	1.40	NCN(3)	NCN(7)	NCN(11)	ECP(15,6)	ECP(19,5)	ECP(23,5)
	1.80	NCN(3)	NCN(7)	ECP(11,6)	ECP(15,7)	ECP(19,8)	ECP(23,9)
Π^*_{best}	0.20	7.02	5.35	4.46	3.95	3.61	3.37
	0.60	2.57	2.15	1.91	1.77	1.68	1.62
	1.00	1.68	1.51	1.40	1.34	1.32	1.34
	1.40	1.30	1.24	1.18	1.19	1.22	1.24
	1.80	1.09	1.08	1.06	1.11	1.14	1.17

final stage of the ECP increases as the award decreases. The best performance increases in award A, consistent with Körpeoğlu and Cho (2017), which indicates that award is an influential parameter in the optimal contest design. The same as Table 1, the result reveals that NCN is an ideal choice when the seeker's budget is limited. NCN is a low-cost method to boost innovation.

Next, we compare the contests in maximizing the best performance with different effort sensitivities k in Table 5.

The table reveals that the optimal contest design is either NCP, ECP, or NCN, which is different from Table 2. ECP is optimal when k is small, and n is large because the solvers are more conservative in exerting their efforts in these cases. The best performance increases in k because it depends on the effort exertion. NCN is an ideal choice when the seeker hopes to incentivize the performance of a large group of solvers who can offer diversified solutions for the innovative project. NCP is optimal when k is relatively small, and n is relatively large, which indicates the public feedback-only scheme is better than the no-feedback-no-elimination scheme under this condition.

Furthermore, we compare different contest designs when the value of c varies in Table 6.

The table shows that either NCN or ECP can be optimal, as before. ECP is optimal when both the cost coefficient c and the number of solvers n are large. The solvers exert less effort in these cases and rely on the random shock to determine the winner. The best performance decreases in c because it depends on the effort exertion. This fits the intuition that the solvers become more conservative when the cost is higher. NCN is an effective way to incentivize solvers when the innovative project requires costly effort.

Remark 6.2 summarizes the conditions when the ECP is optimal for the seeker optimizing the best performance.

Remark 6.2. When the seeker cares about the best performance, the same as Remark 6.1, ECP is a cost-effective option for a complex innovation project with multiple solvers.

7. Conclusion

A crowdsourcing contest is an effective tool to boost innovation through holding a competition. Most of the previous studies treat the crowdsourcing contest as a black box—the contest organizer cannot control it after launching. However, we consider an innovation contest design problem with feedback and elimination schemes, which leads to a model of four types of contest design (i.e., ECP, ECN, NCP, and NCN). These four types of contest design allow the organizer to observe, evaluate, and interfere with the contestants' performance during the process of the contest.

In our model, the contest organizer seeks innovative solutions to an ideation project with two attributes (elimination and feedback). The organizer aims to find the best solution with the highest total qualities for the two attributes and use the winner-takes-all scheme to reward the winner. We derive the equilibrium efforts of the contestants and study the optimal game design analytically in a four-solver contest. Innovatively, we investigate the optimal contest design under a general n-solver case with numerical studies. Our results indicate that the ECP is the optimal choice when the organizer seeks an innovative solution for a complex project with a large cost-coefficient and limited budget. Therefore, ECP can be an ideal tool for small and medium enterprises with constrained budgets for technological innovation.

More generally, we find many other designs of the optimal number of final-stage participants depending on the joint effects of the parameters. For example, our analysis shows that the ECP (elimination, public feedback) is optimal when the award is low, and the NCN (no elimination, no feedback) is optimal when the organizer has enough budget to offer a generous reward. Such findings contribute to the general design of the contest with no elimination. Our study verifies the importance of organizer participation in the innovation process. The contest organizer can alter the contestants' performance in the ECP without any additional resource investment.

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Appendix A: Technical proofs

In this appendix, we analyze the properties of the contest design problem. We first discuss the distribution of the random shocks in Section A.1. We then investigate the equilibrium efforts and performance of a n-solver contest in Section A.2, and a four-solver contest in Section A.3.

A.1. Distribution of the random shocks

In this section, we analyze the distribution of variables η and $\Delta\varepsilon_i$. To begin, we discuss the CDF and PDF of random shock η in the following lemma.

Lemma A1. *The PDF* ($\phi(\eta)$) *and CDF* ($\Phi(\eta)$) *of variable* η *are* $\phi(\eta) = (1+\eta)\mathbb{1}_{\{-1\le\eta\le 0\}} + (1-\eta)\mathbb{1}_{\{0<\eta\le 1\}}$, *and* $\Phi(\eta) = \frac{(1+\eta)^2}{2}\mathbb{1}_{\{-1\le\eta\le 0\}} + (\frac{1}{2}+\frac{(2-\eta)\eta}{2})\mathbb{1}_{\{0<\eta\le 1\}} + \mathbb{1}_{\{\eta>1\}}$.

Proof of Lemma A1. Since $\eta = \varepsilon_a - \varepsilon_b$, we have $\varepsilon_a = \eta + \varepsilon_b$. Thus, we have $\phi(\eta+\varepsilon_b) = \begin{cases} 1 & -\frac{1}{2} \le \eta - \varepsilon_b < \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases} = \begin{cases} 1 & \eta - \frac{1}{2} < \varepsilon_b \le \eta + \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases}$ For the PDF, when $-1 \le \eta < 0$, we have $-\frac{1}{2} \le \varepsilon_b \le \eta + \frac{1}{2}$, then $\phi(\eta) = \int_{-\infty}^{+\infty}\psi(\varepsilon_b)\psi(\eta+\varepsilon_b)\mathrm{d}\varepsilon_b = \int_{-\frac{1}{2}}^{\frac{1}{2}+\eta} 1\mathrm{d}\varepsilon_b = 1+\eta$; When $0 < \eta \le 1$, we have $\eta - \frac{1}{2} \le \varepsilon_b \le \frac{1}{2}$, then $\phi(\eta) = \int_{-\infty}^{+\infty}\psi(\varepsilon_b)\psi(\eta+\varepsilon_b)\mathrm{d}\varepsilon_b = \int_{\eta-\frac{1}{2}}^{\frac{1}{2}} 1\mathrm{d}\varepsilon_b = 1-\eta$. When $\eta = 0$, $\phi(\eta) = 1$.

For the CDF, when $-1 \le \eta \le 0$, we have $\Phi(\eta) = \int_{-1}^{\eta}\phi(v)d_v = \int_{-1}^{\eta}(1+v)d_v = \frac{(1+\eta)^2}{2}$. When $\eta > 0$, we have $\Phi(\eta) = \int_{-1}^{\eta}\phi(v)d_v = \int_{-1}^{0}(1+v)d_v + \int_{0}^{\eta}(1-v)d_v = \frac{1}{2} + \frac{(2-\eta)\eta}{2}$. ■

The PDF of random variable $\Delta\varepsilon_i$ is discussed in the following lemma.

Lemma A2. *The CDF of* $\Delta\varepsilon_i$ *is* $(\Delta\varepsilon_i - \frac{1}{n}(\Delta\varepsilon_i)^n))\mathbb{1}_{\{0\le\Delta\varepsilon_i\le 1\}}$, *and the PDF of* $\Delta\varepsilon_i$ *is* $\psi_\Delta(\Delta\varepsilon_i) = (1-(\Delta\varepsilon_i)^{n-1})\mathbb{1}_{\{0\le\Delta\varepsilon_i\le 1\}}$.

Proof of Lemma A2. First, we assume that $\varepsilon_{1,a}, \dots, \varepsilon_{n,a}$ are all uniform distributions on $[-\frac{1}{2}, \frac{1}{2}]$. By the definition, $\mathrm{P}(\Delta\varepsilon_i \le x) = \mathrm{P}(\varepsilon_{(1),a} - \varepsilon_{i,a} \le x) = \mathrm{P}(\max\{\varepsilon_{1,a}, \dots, \varepsilon_{n,a}\} - \varepsilon_{i,a} \le x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{P}(\max\{\varepsilon_{1,a}, \dots, \varepsilon_{n,a}\} \le x + t|\varepsilon_{i,a} = t)dt$. Assume $\varepsilon_{1,a}$ is the largest random shock in $\{\varepsilon_{1,a}, \dots, \varepsilon_{n,a}\}$. Then we have $\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{P}(\max\{\varepsilon_{1,a}, \dots, \varepsilon_{n,a}\} \le x + t|\varepsilon_{i,a} = t)dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{P}(\varepsilon_{1,a} \le x + t)^{n-1}dt = \int_{-\frac{1}{2}}^{\frac{1}{2}-x} \mathrm{P}(\varepsilon_{1,a} \le x+t)^{n-1}dt + \int_{\frac{1}{2}-x}^{\frac{1}{2}} P(\varepsilon_{1,a} \le x+t)^{n-1}dt = \int_{-\frac{1}{2}}^{\frac{1}{2}-x}(x+t+\frac{1}{2})^{n-1}dt + x = (\frac{1}{n}(1-x^n)+x)\mathbb{1}_{\{0\le x\le 1\}}$. Therefore, the CDF of $\Delta\varepsilon_i$ is $\Psi_\Delta(\Delta\varepsilon_i) = (\frac{1}{n}(1-\Delta\varepsilon_i^n) + \Delta\varepsilon_i)\mathbb{1}_{\{0\le\Delta\varepsilon_i\le 1\}}$. As a result, we can derive the PDF of $\Delta\varepsilon_i$ as $\psi_\Delta(\Delta\varepsilon_i) = (1-\Delta\varepsilon_i^{n-1})\mathbb{1}_{\{0\le\Delta\varepsilon_i\le 1\}}$.

Second, if we assume that $\varepsilon_{1,a}, \dots, \varepsilon_{n,a}$ are all uniform distributions on $[-\frac{1}{2}, \frac{1}{2}]$, we have $\psi_\Delta(\Delta\varepsilon_i) = (1-(\Delta\varepsilon_i)^{n-1})\mathbb{1}_{\{0\le\Delta\varepsilon_i\le 1\}}$ and $\Psi_\Delta(\Delta\varepsilon_i) = \int_0^{\Delta\varepsilon_i}(1-x^{n-1})\mathbb{1}_{\{0\le x\le 1\}}dx = (\Delta\varepsilon_i - \frac{1}{n}(\Delta\varepsilon_i)^n))\mathbb{1}_{\{0\le\Delta\varepsilon_i\le 1\}}$. ■

A.2. Solvers' efforts and performance in the contest. In this section, we prove the solvers' solution efforts and performance in the contests.

Solvers' solution efforts. We analyze the equilibrium efforts under different contest designs.

(i) *NCN*. To help express the equilibrium effort, we define the following term:

$$F^n_{(1)} = (n-1)\int_{-1}^{1} \Phi(\eta)^{n-2}\phi(\eta)^2 d\eta. \tag{A1}$$

Proof of Theorem 4.1. Supposing that all the solvers except solver i make the equilibrium $e_a^{NP}(n)$ and $e_b^{NP}(n)$, define $\mathrm{P}(e_{i,a}^{NN}, e_{i,b}^{NN}|e_a^{NN}(n), e_b^{NN}(n), j \neq i)$ as the winning probability of solver i, if he makes efforts $e_{i,a}^{NN}$ and $e_{i,b}^{NN}$ on the two attributes. Then, $\mathrm{P}(e_{i,a}^{NN}, e_{i,b}^{NN}|e_a^{NN}(n), e_b^{NN}(n), j \neq i) = \mathrm{P}(V_i > V_j | j \in \mathcal{S}, i, j \neq i)$ is the winning probability of solver i. In the NCN, the winning probability of solver i can be expressed by $\mathrm{P}(V_i > V_j | j \in \mathcal{S}, j \neq i) = \mathrm{P}(ke_i + \varepsilon_i > ke_j + \varepsilon_j | j \in \mathcal{S}, j \neq i) = \mathrm{P}(ke_i - ke_j + \varepsilon_i - \varepsilon_j > 0 | j \in \mathcal{S}, j \neq i) = \mathrm{P}(\varepsilon_i - \varepsilon_j < ke_i - ke_j | j \in \mathcal{S}, j \neq i)$. Given $\eta = \varepsilon_i - \varepsilon_j$, we have the winning probability of solver i as $\int_{-\infty}^{+\infty} \Phi(2ke_{i,a} - 2ke_a^{NN}(n) + \eta)^{n-1}\phi(\eta)d\eta$.
We follow the solution approach introduced in Proposition 1 of Mihm and Schlapp (2018) to prove the theorem. In the symmetric equilibrium, solver i makes the same effort as other solvers, such that $e_{i,a} = e_{i,b}$. We have $A\mathrm{P}(e_{i,a}, e_{i,a}|e_a^{NN}(n), e_b^{NN}(n)) - 2\gamma(e_{i,a}) = A\int_{-\infty}^{+\infty} \Phi(2ke_{i,a} - 2ke_a^{NN}(n) + \eta)^{n-1}\phi(\eta)d\eta - 2\gamma(e_{i,a})$. Taking derivative on $e_{i,a}$, the first-order condition (FOC) yields $2kA(n-1)\int_{-\infty}^{+\infty} \Phi(2ke_{i,a} - 2ke_a^{NN}(n) + \eta)^{n-2}\phi(\eta)^2 d\eta = 4ce_{i,a}$. As all the solvers take the same equilibrium efforts in this situation (i.e,. $e_{i,a} = e_{i,b} = e_a^{NN} = e_b^{NN}$) and $\gamma''(\cdot) > 0$, we have $e_a^{NN}(n) = e_b^{NN}(n) = \frac{kA(n-1)\int_{-\infty}^{+\infty}\Phi(\eta)^{n-2}\phi(\eta)^2 d\eta}{2c} = \frac{kA(n-1)\int_{-1}^{1}\Phi(\eta)^{n-2}\phi(\eta)^2 d\eta}{2c} = \frac{kAF^n_{(1)}}{2c}$. ■

(ii) *NCP*. To facilitate the analysis of the equilibrium efforts, we introduce the following terms:

$$\bar{A}^n = \int_0^1 \mathrm{E}\left[u_{i,b}^{NP}(e_b^{NP}(n, \Delta\varepsilon_i))\right] d\Delta\varepsilon_i, \tag{A2}$$

$$\hat{H}^n_{(n)} = (n-1)\sum_{j=2}^{n-1}\int_{-\frac{1}{2}}^{\frac{1}{2}} ((1-\Psi(\varepsilon))^{j-2}\psi^2(\varepsilon)(C_{n-2}^{j-1} - \Psi(\varepsilon)C_{n-1}^{j-1}))d\varepsilon$$
$$+ (n-1)\int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(\varepsilon)^{n-2}\psi(\varepsilon)d\varepsilon, \tag{A3}$$

$$H^m_{(1)}(\Delta\varepsilon_i) = \int_{-\frac{1}{2}}^{\frac{1}{2}-\Delta\varepsilon_i} (n-1)\Psi(\Delta\varepsilon_i + \varepsilon)^{n-2}\psi(\Delta\varepsilon_i + \varepsilon)\psi(\varepsilon)d\varepsilon. \tag{A4}$$

Proof of Theorem 4.2. We prove the equilibrium efforts in each stage respectively.

(a) *The first stage*. In the first stage, assuming all the opponents except solver i make an equilibrium effort $e_a^{NP}(n)$, the probability of solver i entering the final stage if he makes an effort $e_{i,a}$, is given by $\mathrm{P}(e_{i,a}|e_a^{NP}(n)) = \mathrm{P}(V_i > V_j | j \in \mathcal{S}, j \neq i) = \mathrm{P}(ke_i + \varepsilon_i > ke_j + \varepsilon_j | j \in \mathcal{S}, j \neq i) = \mathrm{P}(ke_i - ke_j + \varepsilon_i - \varepsilon_j > 0 | j \in \mathcal{S}, j \neq i) = \mathrm{P}(\varepsilon_i - \epsilon_j < ke_i - ke_j | j \in \mathcal{S}, j \neq i) = \sum_{j=1}^{n} \int_{-\infty}^{\infty} C_{n-1}^{j-1} \Psi(\varepsilon + ke_{i,a} - ke_a^{NP}(n))^{n-j} (1 - \Psi(\varepsilon + ke_{i,a} - ke_a^{NP}(n)))^{j-1} \psi(\varepsilon) d\varepsilon$.
To maximize the utility of the solver, we need to maximize $\mathrm{P}(e_{i,a}|e_a^{NP}(n)) \int_{-\infty}^{\infty} \mathrm{E}[u_{i,b}^{NP}(e_b^P(n, \Delta_i))] d\Delta_i - \gamma(e_{i,a})$. Noting that $\bar{A}^n = \int_{-a}^{a} E[u(e_b^P(n, \Delta_i))] d\Delta_i$, take the derivative of $e_{i,a}$, and let $e_{i,a} = e_a^P(n)$, we have $2ce_{i,a} = \bar{A}^n \frac{\partial P[e_{i,a}|e_a^P(n)]}{\partial e_{i,a}}|_{e_{i,a}=e_a^P(n)}$. As $P[e_{i,a}|e_a^P(n)] = \sum_{j=1}^{n} \int_{-\infty}^{\infty} C_{n-1}^{j-1} (1 - \Psi(\varepsilon + ke_{i,a} - ke_a^P(n)))^{j-1} \psi(\varepsilon) d\varepsilon$ and $\gamma''(\cdot) > 0$, we can derive the equilibrium efforts by investigating the FOC.
Because the solvers take the equilibrium effort in the first stage, we obtain $e_{i,a}^{NP}(n) = \frac{\bar{A}^n}{2c} \frac{\partial P[e_{i,a}|e_a^P(n)]}{\partial e_{i,a}}|_{e_{i,a}=e_a^P(n)} = \frac{k\bar{A}^n}{2c}(n-1) \sum_{j=2}^{n-1} \int_{-\infty}^{\infty} (1 - \Psi(\varepsilon))^{j-2} \psi^2(\varepsilon) (C_{n-2}^{j-1} - \Psi(\varepsilon) C_{n-1}^{j-1}) d\varepsilon + \frac{k\bar{A}^n}{2c}(n-1) \int_{-\infty}^{\infty} \Psi(\varepsilon))^{n-2} \psi(\varepsilon) d\varepsilon = \frac{k\bar{A}^n}{2c}(n-1) \sum_{j=2}^{n-1} \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - \Psi(\varepsilon))^{j-2} \psi^2(\varepsilon) (C_{n-2}^{j-1} - \Psi(\varepsilon) C_{n-1}^{j-1})) d\varepsilon + \frac{k\bar{A}^n}{2c}(n-1) \int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(\varepsilon))^{n-2} \psi(\varepsilon) d\varepsilon = \frac{k\bar{A}^n}{2c} \hat{H}_{(n)}^n$.

(b) *The final stage*. Because the seeker announces the first-stage performance to all the solvers participating in the final stage, solver i in the final stage knows the performance difference Δ_i clearly. Let the equilibrium effort in the final stage of the elimination contest be $e_b^P(n, \Delta_i)$. Supposing all the solvers make the equilibrium effort except solver i. The winning probability of solver i in the final stage, if he makes the effort $e_{i,b}$, is given by $\mathrm{P}(e_{i,b}|e_b^P(n, \Delta_i), \Delta_i) = \mathrm{P}(V_i > V_j + \Delta_i | j \in \mathcal{S}, j \neq i) = \int_{-\infty}^{\infty} \Psi(\Delta_i + \varepsilon + ke_{i,b} - ke_b^P(n, \Delta_i))^{n-1} \psi(\varepsilon) d\varepsilon$.
In addition to Δ_i, we define the random variable $\Delta\varepsilon_{i,a} = \varepsilon_{(1),a} - \varepsilon_{i,a}$, where $\varepsilon_{(1),a}$ denotes the first-order statistic of $\{\varepsilon_i, a, i \in \{1, \ldots, n\}\}$. Define the PDF and CDF of variable $\Delta\varepsilon_i$ as $\psi_\Delta(\Delta\varepsilon_i)$ and $\Psi_\Delta(\Delta\varepsilon_i)$, respectively. We present the formulation of $\psi_\Delta(\Delta\varepsilon_i)$ and $\Psi_\Delta(\Delta\varepsilon_i)$ in Lemma A2 of Section A.1. Here, $\Delta\varepsilon_i$ plays an important role in our analysis of the equilibrium efforts. Different from Δ_i, $\Delta\varepsilon_i$ is a random variable indicating the difference between the random shocks only. By the definition, Δ_i and $\Delta\varepsilon_i$ may follow different distributions. However, as we only focus on the situation of symmetric equilibrium, these two random variables follow the same distribution.
By (a), we can tell that all the solvers take the same equilibrium effort in the first stage, therefore $\Delta_i = \Delta\varepsilon_i$. $2ce_{i,b} = A \int_{-\infty}^{\infty} \frac{(n-1)!}{(n-j)!(j-1)!} [(n-j) \Psi(\Delta\varepsilon_i + \varepsilon + ke_{i,b} - ke_b^{NP}(n, \Delta\varepsilon_i))^{n-j-1} \cdot \psi(\Delta\varepsilon_i + \varepsilon + ke_{i,b} - ke_b^{NP}(n, \Delta\varepsilon_i)) k (1 - \Psi(\Delta\varepsilon_i + \varepsilon + ke_{i,b} - ke_b^{NP}(n, \Delta\varepsilon_i)))^{j-1} + \Psi(\Delta\varepsilon_i + \varepsilon + ke_{i,b} - ke_b^{NP}(n, \Delta\varepsilon_i))^{n-j} (j-1) \cdot (1 - \Psi(\Delta\varepsilon_i + \varepsilon + ke_{i,b} - ke_b^{NP}(n, \Delta\varepsilon_i)))^{j-2} (-\psi(\Delta\varepsilon_i + \varepsilon + ke_{i,b} - ke_b^{NP}(n, \Delta\varepsilon_i))) k] \psi(\varepsilon) d\varepsilon$, where $j \neq 1$.
Thus, we have $e_{i,b} = \frac{kA}{2c} \int_{-\infty}^{\infty} \frac{(n-1)!}{(n-j)!(j-1)!} [(n-j) \Psi(\Delta\varepsilon_i + \varepsilon)^{n-j-1} \psi(\Delta\varepsilon_i + \varepsilon) (1 - \Psi(\Delta\varepsilon_i + \varepsilon))^{j-1} + \Psi(\Delta\varepsilon_i + \varepsilon)^{n-j} (j-1) (1 - \Psi(\Delta\varepsilon_i + \varepsilon))^{j-2} (-\psi(\Delta\varepsilon_i + \varepsilon))] \psi(\varepsilon) d\varepsilon = \frac{kA}{2c} \int_{-\infty}^{\infty} \psi(\varepsilon) \psi(\Delta\varepsilon_i + \varepsilon) [(n-1) C_{n-2}^{j-1} \Psi(\Delta\varepsilon_i + \varepsilon)^{n-j-1} (1 - \Psi(\Delta\varepsilon_i + \varepsilon))^{j-1} - (n-1) C_{n-2}^{j-2} \Psi(\Delta\varepsilon_i + \varepsilon)^{n-j} (1 - \Psi(\Delta\varepsilon_i + \varepsilon))^{j-2}] d\varepsilon = \frac{kA(n-1)}{2c} \int_{-\infty}^{\infty} \psi(\varepsilon) \psi(\Delta\varepsilon_i +$

$\varepsilon)\Psi(\Delta\varepsilon_i+\varepsilon)^{n-j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-2}[C_{n-2}^{j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))-C_{n-2}^{j-2}\Psi(\Delta\varepsilon_i+\varepsilon)]d\varepsilon = \frac{kA(n-1)}{2c}\int_{-\infty}^{\infty}\psi(\varepsilon)\psi(\Delta\varepsilon_i+\varepsilon)\Psi(\Delta\varepsilon_i+\varepsilon)^{n-j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-2}[C_{n-2}^{j-1}-C_{n-1}^{j-1}\Psi(\Delta\varepsilon_i+\varepsilon)]d\varepsilon$.

When $j=1$, we have $A\int_{-\infty}^{\infty}[(n-1)\Psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(n,\Delta\varepsilon_i))^{n-2}\psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(n,\Delta\varepsilon_i))k]\psi(\varepsilon)d\varepsilon = 2ce_{i,b}$. Suppose $e_{i,b}=e_b^{NP}(n,\Delta\varepsilon_i)$, we obtain $kA\int_{-\infty}^{\infty}(n-1)\Psi(\Delta\varepsilon_i+\varepsilon)^{n-2}\psi(\Delta\varepsilon_i+\varepsilon)\psi(\varepsilon)d\varepsilon = 2ce_{i,b}$. As the solvers take the equilibrium effort and $\gamma''(\cdot)>0$, we have $e_b^{NP}(n)=\frac{kA}{2c}\int_{-\infty}^{\infty}(n-1)\Psi(\Delta\varepsilon_i+\varepsilon)^{n-2}\psi(\Delta\varepsilon_i+\varepsilon)\psi(\varepsilon)d\varepsilon = \frac{kA}{2c}\int_{-\frac{1}{2}}^{\frac{1}{2}}(n-1)\Psi(\Delta\varepsilon_i+\varepsilon)^{n-2}\psi(\Delta\varepsilon_i+\varepsilon)\psi(\varepsilon)d\varepsilon = \frac{kA}{2c}H_{(1)}^n(\Delta\varepsilon_i)$.

■

(iii) *ECN*. To facilitate the analysis, we introduce the following terms:

$$G_{(j)}^n = \int_{-\frac{1}{2}}^{\frac{1}{2}} C_{n-1}^{j-1}\Psi(\varepsilon)^{n-j-1}(1-\Psi(\varepsilon))^{j-2}[(n-j)(1-\Psi(\varepsilon))-(j-1)\Psi(\varepsilon)]\psi(\varepsilon)^2 d\varepsilon, \tag{A5}$$

$$\hat{G}_{(m)}^n = \sum_{j=1}^{m} G_{(j)}^n. \tag{A6}$$

Proof of Theorem 4.3. We analyze the equilibrium efforts in the final stage and first stage, respectively.

(a) *The final stage*. The solvers do not need to have the highest ranking to enter the final stage. They can enter the final stage as long as their first-stage performance ranks among the m highest.
In the final stage, the winning probability of solver i if he makes effort $e_{i,b}$ is $\mathrm{P}(V_i>V_j|i,j\in\mathcal{S}_2, j\neq i)=\int_{-\infty}^{\infty}\Psi(\varepsilon+ke_{i,b}-ke_b^{EN}(m))^{m-1}\psi(\varepsilon)d\varepsilon$.
Let $e_b^{EN}(m)$ be the second-stage equilibrium effort in the ECN. Suppose all the solvers except solver i make the equilibrium effort. The expected payoff of solver i when exerting effort $e_{i,b}$ is $A\mathrm{P}(e_{i,b}|e_b^{EN}(m))-\gamma(e_{i,b})=A\mathrm{P}(V_i>V_j|i,j\in\mathcal{S}_2,j\neq i)=A\int_{-\infty}^{+\infty}\Psi(ke_{i,b}-ke_b^{EN}(m)+\varepsilon_{i,b})^{m-1}\psi(\varepsilon_{i,b})d\varepsilon_{i,b}-\gamma(e_{i,b})$. To maximize the utility, we take the derivative on $e_{i,b}$ and yield the FOC as $kA(m-1)\int_{-\infty}^{+\infty}\Psi(ke_{i,b}-ke_b^{EN}+\varepsilon_{i,b})^{m-2}\psi(\varepsilon_{i,b})d\varepsilon_{i,b}=2ce_{i,b}$. Plugging in the condition that solver i take the equilibrium effort $e_{i,b}=e_b^{EN}(m)$, we have $kA(m-1)\int_{-\infty}^{+\infty}\Psi(\varepsilon_{i,b})^{m-2}\psi(\varepsilon_{i,b})d\varepsilon_{i,b}=kA(m-1)\int_{-\frac{1}{2}}^{\frac{1}{2}}\Psi(\varepsilon_{i,b})^{m-2}\psi(\varepsilon_{i,b})d\varepsilon_{i,b}=kAG_{(1)}^m=2ce_b^{EN}(m)$. Therefore, we have $e_b^{EN}=\frac{kAG_{(1)}^m}{2c}$ because $\gamma''(\cdot)>0$.

(b) *The first stage*. Let $e_a^{EN}(n,m)$ be the first-stage equilibrium effort in the ECN. In the first stage of ECN, the probability of solver i entering the final stage if he makes an effort $e_{i,a}$ is $\mathrm{P}(e_{i,a}|e_a^{EN}(n,m))=\mathrm{P}(V_i\geq V_{(m)})=\sum_{j=1}^{m}\int_{-\infty}^{\infty}C_{n-1}^{j-1}\Psi(\varepsilon+ke_{i,a}-ke_a^{EN}(n,m))^{n-j}(1-\Psi(\varepsilon+ke_{i,a}-ke_a^{EN}(n,m)))^{j-1}\psi(\varepsilon)d\varepsilon$.
As solvers in the final stage exert the same equilibrium effort, we have $e_{i,b}=e_b^{EN}(m)$. Therefore, the expected award in the first stage is $\mathrm{E}[u_{i,b}^{EN}(e_b^{EN}(m))]=\frac{A}{m}-\gamma(e_b^{EN}(m))$. We can express the expected payoff of solver i in the first stage when exerting $e_{i,a}$ as $\mathrm{E}[u_{i,b}^{EN}(e_b^{EN}(m))]\mathrm{P}(e_{i,a}|e_a^{EN}(n,m))-\gamma(e_{i,a})=(\frac{A}{m}-c(e_b^{EN}(m))^2)\sum_{j=1}^{m}\int_{-\infty}^{\infty}C_{n-1}^{j-1}\Psi(\varepsilon+$

$ke_{i,a} - ke_a^{EN}(n,m))^{n-j}(1-\Psi(\varepsilon + ke_{i,a} - ke_a^{EN}(n,m)))^{j-1}\psi(\varepsilon)d\varepsilon - \gamma(e_{i,a})$. Take derivative on $e_{i,a}$ and plugging in the condition that $e_{i,a} = e_a^{EN}(n,m)$, the FOC yields: $k(\frac{A}{m} - c(e_b^{EN}(m))^2)\sum_{j=1}^m \int_{-\infty}^{+\infty} C_{n-1}^{j-1}\Psi(\varepsilon)^{n-j-1}(1-\Psi(\varepsilon))^{j-2}[(n-j)(1-\Psi(\varepsilon)) - (j-1)\Psi(\varepsilon)]\psi(\varepsilon)^2 d\varepsilon = k(\frac{A}{m} - c(e_b^{EN}(m))^2)\sum_{j=1}^m \int_{-\frac{1}{2}}^{\frac{1}{2}} C_{n-1}^{j-1}\Psi(\varepsilon)^{n-j-1}(1-\Psi(\varepsilon))^{j-2}[(n-j)(1-\Psi(\varepsilon)) - (j-1)\Psi(\varepsilon)]\psi(\varepsilon)^2 d\varepsilon = k(\frac{A}{m} - c(\frac{kAG_{(1)}^m}{2c})^2)\hat{G}_{(m)}^n = 2c(e_a^{EN}(n,m))$. Therefore, the optimal effort level is $e_a^{EN}(n,m) = (\frac{kA}{2mc} - \frac{k^3A^2(G_{(1)}^m)^2}{8c^2})\hat{G}_{(m)}^n$ because $\gamma''(\cdot) > 0$. ■

(iv) *ECP*. To facilitate the analysis of the equilibrium efforts, we introduce the following terms:

$$\bar{A}^m = \int_0^1 \mathrm{E}\big[u_{i,b}^{EP}(e_b^{EP}(m, \Delta\varepsilon_i))\big]d\Delta\varepsilon_i, \tag{A7}$$

$$\begin{aligned}\hat{H}_{(m)}^n =&(n-1)\sum_{j=2}^m \int_{-\frac{1}{2}}^{\frac{1}{2}} (\Psi(\varepsilon)^{n-j-1}(1-\Psi(\varepsilon))^{j-2}\psi^2(\varepsilon)(C_{n-2}^{j-1} - \Psi(\varepsilon)C_{n-1}^{j-1}))d\varepsilon \\ &+ (n-2)\int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(\varepsilon)^{n-2}\psi(\varepsilon)d\varepsilon,\end{aligned} \tag{A8}$$

$$H_{(1)}^m(\Delta\varepsilon_i) = \int_{-\frac{1}{2}}^{\frac{1}{2}-\Delta\varepsilon_i} (m-1)\Psi(\Delta\varepsilon_i + \varepsilon)^{m-2}\psi(\Delta\varepsilon_i + \varepsilon)\psi(\varepsilon)d\varepsilon. \tag{A9}$$

Proof of Theorem 4.4. We discuss the equilibrium effort in the first and final stages of the ECP respectively.

(a) *The first stage*. Let $e_a^{EP}(m)$ be the first-stage equilibrium effort in the ECP. In the first stage of ECP, the probability of solver i entering the final stage if he makes an effort $e_{i,a}$, is given by $\mathrm{P}(e_{i,a}|e_a^{EP}(n,m)) = \mathrm{P}(V_i \geq V_{(m)}) = \sum_{j=1}^m \int_{-\infty}^{\infty} C_{n-1}^{j-1}\Psi(\varepsilon + ke_{i,a} - ke_a^{EP}(n,m))^{n-j}(1-\Psi(\varepsilon + ke_{i,a} - ke_a^{EP}(n,m)))^{j-1}\psi(\varepsilon)d\varepsilon$.
For the utility, as $\bar{A}^m = \int_{-a}^{a} E[u(e_b^{EP}(m,\Delta_i))]d\Delta_i$, take the derivative of $e_{i,a}$, and let $e_{i,a} = e_a^{EP}(n,m)$, we have $2ce_{i,a} = \bar{A}^m \frac{\partial P[e_{i,a}|e_a^{EP}(n,m)]}{\partial e_{i,a}}|_{e_{i,a}=e_a^{EP}(n,m)}$. Notice that $P[e_{i,a}|e_a^{EP}(n,m)] = \sum_{j=1}^m \int_{-\infty}^{\infty} C_{n-1}^{j-1}\Psi(\varepsilon + ke_{i,a} - ke_a^{EP}(n,m))^{n-j}(1-\Psi(\varepsilon + ke_{i,a} - ke_a^{EP}(n,m)))^{j-1}\psi(\varepsilon)d\varepsilon$ and $\gamma''(\cdot) > 0$, we can derive the equilibrium efforts by investigating the FOC.
Because the solvers take the equilibrium effort in the first stage, we obtain $e_{i,a}^{EP}(n,m) = \frac{\bar{A}^m}{2c}\frac{\partial P[e_{i,a}|e_a^{EP}(n,m)]}{\partial e_{i,a}}|_{e_{i,a}=e_a^{EP}(n,m)} = \frac{k\bar{A}^m}{2c}(n-1)\sum_{j=2}^m \int_{-\infty}^{\infty} \Psi(\varepsilon)^{n-j-1}(1-\Psi(\varepsilon))^{j-2}\psi^2(\varepsilon)(C_{n-2}^{j-1} - \Psi(\varepsilon)C_{n-1}^{j-1}))d\varepsilon + \frac{k\bar{A}^m}{2c}(n-1)\int_{-\infty}^{\infty} \Psi(\varepsilon))^{n-2}\psi(\varepsilon)d\varepsilon = \frac{k\bar{A}^m}{2c}(n-1)\sum_{j=2}^m \int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(\varepsilon)^{n-j-1}(1-\Psi(\varepsilon))^{j-2}\psi^2(\varepsilon)(C_{n-2}^{j-1} - \Psi(\varepsilon)C_{n-1}^{j-1}))d\varepsilon + \frac{k\bar{A}^m}{2c}(n-1)\int_{-\frac{1}{2}}^{\frac{1}{2}} \Psi(\varepsilon))^{n-2}\psi(\varepsilon)d\varepsilon = \frac{k\bar{A}^m}{2c}\hat{H}_{(m)}^n$.

(b) *The final stage*. Because the seeker announces the first-stage performance to all the solvers participating in the final stage, solver i in the final stage knows the performance difference Δ_i clearly. Let the equilibrium effort in the final stage of the ECP be $e_b^{EP}(m,\Delta_i)$. Suppose all the solvers except solver i make the equilibrium effort. The winning probability of solver

i in the final stage, if he makes the effort $e_{i,b}$ is given by $\mathrm{P}(e_{i,b}|e_b^{EP}(m,\Delta_i),\Delta_i)=\mathrm{P}(V_i>V_j+\Delta_i|i,j\in\mathcal{S}_2,j\neq i)=\int_{-\infty}^{\infty}\Psi(\Delta_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta_i))^{m-1}\psi(\varepsilon)d\varepsilon$.
By (a), we can tell that all the solvers take the same equilibrium effort in the first stage, therefore $\Delta_i=\Delta\varepsilon_i$. $2ce_{i,b}=A\int_{-\infty}^{\infty}\frac{(m-1)!}{(m-j)!(j-1)!}[(m-j)\Psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i))^{m-j-1}\cdot\psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i))k(1-\Psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i)))^{j-1}+\Psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i))^{m-j}(j-1)\cdot(1-\Psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i)))^{j-2}(-\psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i)))k]\psi(\varepsilon)d\varepsilon$, where $j\neq 1$.
Thus, we have $e_{i,b}=\frac{kA}{2c}\int_{-\infty}^{\infty}\frac{(m-1)!}{(m-j)!(j-1)!}[(m-j)\Psi(\Delta\varepsilon_i+\varepsilon)^{m-j-1}\psi(\Delta\varepsilon_i+\varepsilon)(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-1}+\Psi(\Delta\varepsilon_i+\varepsilon)^{m-j}(j-1)(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-2}(-\psi(\Delta\varepsilon_i+\varepsilon))]\psi(\varepsilon)d\varepsilon=\frac{kA}{2c}\int_{-\infty}^{\infty}\psi(\varepsilon)\psi(\Delta\varepsilon_i+\varepsilon)[(m-1)C_{m-2}^{j-1}\Psi(\Delta\varepsilon_i+\varepsilon)^{m-j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-1}-(m-1)C_{m-2}^{j-2}\Psi(\Delta\varepsilon_i+\varepsilon)^{m-j}(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-2}]d\varepsilon=\frac{kA(m-1)}{2c}\int_{-\infty}^{\infty}\psi(\varepsilon)\psi(\Delta\varepsilon_i+\varepsilon)\Psi(\Delta\varepsilon_i+\varepsilon)^{m-j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-2}[C_{m-2}^{j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))-C_{m-2}^{j-2}\Psi(\Delta\varepsilon_i+\varepsilon)]d\varepsilon=\frac{kA(m-1)}{2c}\int_{-\infty}^{\infty}\psi(\varepsilon)\psi(\Delta\varepsilon_i+\varepsilon)\Psi(\Delta\varepsilon_i+\varepsilon)^{m-j-1}(1-\Psi(\Delta\varepsilon_i+\varepsilon))^{j-2}[C_{m-2}^{j-1}-C_{m-1}^{j-1}\Psi(\Delta\varepsilon_i+\varepsilon)]d\varepsilon$.
When $j=1$, we have $A\int_{-\infty}^{\infty}[(m-1)\Psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i))^{m-2}\psi(\Delta\varepsilon_i+\varepsilon+ke_{i,b}-ke_b^{EP}(m,\Delta\varepsilon_i))k]\psi(\varepsilon)d\varepsilon=2ce_{i,b}$. Suppose $e_{i,b}=e_b^{EP}(m,\Delta\varepsilon_i)$, we obtain $kA\int_{-\infty}^{\infty}(m-1)\Psi(\Delta\varepsilon_i+\varepsilon)^{m-2}\psi(\Delta\varepsilon_i+\varepsilon)\psi(\varepsilon)d\varepsilon=2ce_{i,b}$. As the solvers take the equilibrium effort and $\gamma''(\cdot)>0$, we have $e_b^{EP}(m)=\frac{kA}{2c}\int_{-\infty}^{\infty}(m-1)\Psi(\Delta\varepsilon_i+\varepsilon)^{m-2}\psi(\Delta\varepsilon_i+\varepsilon)\psi(\varepsilon)d\varepsilon=\frac{kA}{2c}\int_{-\frac{1}{2}}^{\frac{1}{2}}(m-1)\Psi(\Delta\varepsilon_i+\varepsilon)^{m-2}\psi(\Delta\varepsilon_i+\varepsilon)\psi(\varepsilon)d\varepsilon=\frac{kA}{2c}H_{(1)}^m(\Delta\varepsilon_i)$.

■

Solvers' expected performance. In this section, we discuss the contest designs optimizing seeker's expected average performance and best performance, respectively.

To help analyze the expected best performance, we introduce the following assistant functions to express the expected random shocks:

$$\Theta_i^n(x)=\int_0^x(\mathbb{1}_{\{0\le v\le 1\}}\int_0^v nC_{n-1}^{n-i}x^{n-i}(1-x)^{i-1}dx+\mathbb{1}_{\{1\le v\le 2\}}\int_{v-1}^1 nC_{n-1}^{n-i}x^{n-i}(1-x)^{i-1}dx)dv, \tag{A10}$$

$$\bar{\eta}_{(1)}^n=n\int_{-\frac{1}{2}}^{\frac{1}{2}}\eta\Phi(\eta)^{n-1}\phi(\eta)d\eta, \tag{A11}$$

$$\bar{\vartheta}_{(1)}^{n,m}=\left[\int_0^2(1-\prod_{i=1}^m\Theta_i^n(x))dx-1\right]. \tag{A12}$$

We first derive the winner's expected random shocks in Lemma A3.

Lemma A3. *The following properties hold for the winner's expected random shocks:*

(a) *In the NCN(n) and NCP(n), we have* $E[\eta_{(1)}^n]=\bar{\eta}_{(1)}^n$*;*

(b) *In the ECN(n, m) and ECP(n, m), we have* $E[\max_{1\le i\le m}\{\varepsilon_{i,a}+\varepsilon_{i,b}\}] = \bar{\vartheta}^{n,m}_{(1)}$.

Proof of Lemma A3. We prove the expected value of the nonelimination contest and elimination contest in (a) and (b), respectively.

(a) *Expected random shock in the nonelimination contest*. As the random shock η has a PDF $\phi(\eta)$, the PDF of the $(n-j+1)$-st order statistic of random shock η is given by $\phi^n_{(j)}(\eta) = nC^{j-1}_{n-1}\Phi(\eta)^{n-j}(1-\Phi(\eta))^{j-1}\phi(\eta)$. We have $\mathrm{E}[\eta^n_{(1)}] = \int_{-\infty}^{\infty}\eta\phi^n_{(j)}(\eta)d\eta = n\int_{-\infty}^{\infty}\eta\Phi(\eta)^{n-1}\phi(\eta)d\eta = n\int_{-1/2}^{1/2}\eta\Phi(\eta)^{n-1}\phi(\eta)d\eta = \bar{\eta}^n_{(1)}$.
(b) *Expected random shock in the elimination contest*. Given that there are n people, and m of them enter the final stage. In order to simplify the calculation, we first assume that all random shocks are uniformly distributed from 0 to 1. Thus, we have $\varepsilon_{i,a} \sim nC^{n-i}_{n-1}x^{n-i}(1-x)^{i-1}\mathbb{1}_{\{0\le x\le 1\}}$ and $\varepsilon_{i,b} \sim \mathbb{1}_{\{0\le x\le 1\}}$. Thus, we can compute the PDF of $\varepsilon_{i,a}+\varepsilon_{i,b}$, which is $\theta^n_i(v) = \mathbb{1}_{\{0\le v\le 1\}}\int_0^v nC^{n-i}_{n-1}x^{n-i}(1-x)^{i-1}dx + \mathbb{1}_{\{1\le v\le 2\}}\int_{v-1}^1 nC^{n-i}_{n-1}x^{n-i}(1-x)^{i-1}dx$. Then we have $\mathrm{E}[\max_{1\le i\le m}\{\varepsilon_{i,a}+\varepsilon_{i,b}\}] = \int_0^{+\infty}\mathrm{P}(\max_{1\le i\le m}\{\varepsilon_{i,a}+\varepsilon_{i,b}\}\ge x)dx = \int_0^2(1-\prod_{i=1}^m P(\{\varepsilon_{i,a}+\varepsilon_{i,b}\}\le x))dx = \int_0^2(1-\prod_{i=1}^m\Theta^n_i(x))dx$, when $\varepsilon_{i,a},\varepsilon_{i,a}\sim \mathrm{Unif}(0,1)$. Second, if we assume $\varepsilon_{i,a},\varepsilon_{i,a}\sim \mathrm{Unif}\left(-\frac{1}{2},\frac{1}{2}\right)$, we can derive the expected random shock as $\mathrm{E}[\max_{1\le i\le m}\{\varepsilon_{i,a}+\varepsilon_{i,b}\}] = \int_0^2(1-\prod_{i=1}^m\Theta^n_i(x))dx - 1$. At last, if we assume $\varepsilon_{i,a},\varepsilon_{i,a}\sim \mathrm{Unif}\left(-\frac{1}{2},\frac{1}{2}\right)$, we can get $\mathrm{E}[\max_{1\le i\le m}\{\varepsilon_{i,a}+\varepsilon_{i,b}\}] = [\int_0^2(1-\prod_{i=1}^m\Theta^n_i(x))dx - 1] = \bar{\vartheta}^{n,m}_{(1)}$.

■

Proof of Lemma 4.1. By the definition in eq. 1, the average performance is just the sum of equilibrium efforts on the two attributes. We then prove this lemma by the value of the equilibrium efforts discussed in Theorems 4.1–4.4.

By the definition in eq. 2, the best performance is the sum of the equilibrium efforts and the random shock. We then prove this lemma by plugging in the value of the equilibrium efforts discussed in Theorems 4.1–4.4 and the expected random shocks in Lemma A3. ■

A.3. Four-solver contest. In this section, we analyze the solvers' solution efforts and the seeker's profit.

Solvers' solution efforts. We first derive the solvers' equilibrium efforts of different four-solver contests.

Proof of Lemma 5.1. The proof follows Theorems 4.1–4.4 by plugging in $n = 4$. ■

Optimizing seeker's profit. We analyze the expected average performance in the four-solver contest in Lemma A4.

Lemma A4. *The solvers' expected average performance in the four-solver contests is*

(a) $\Pi_{avg}^{NN}(4) = \frac{43k^2A}{70c}$ *in the NCN(4);*
(b) $\Pi_{avg}^{NP}(4) = \frac{k^2A(3448c-1035k^2A)}{6720c^2}$ *in the NCP(4);*
(c) $\Pi_{avg}^{EN}(4,2) = \frac{k^2A(4c-k^2A)}{8c^2}$ *in the ECN(4,2) and* $\Pi_{avg}^{EN}(4,3) = \frac{k^2A(8c-3k^2A)}{24c^2}$ *in the ECN(4,3);*
(d) $\Pi_{avg}^{EP}(4,2) = \frac{k^2A(121c-10k^2A)}{360c^2}$ *in the ECP(4,2) and* $\Pi_{avg}^{EP}(4,3) = \frac{k^2A(135c-16k^2A)}{360c^2}$ *in the ECP(4,3).*

Proof of Lemma A4. We prove this lemma by plugging in n and m into Lemma 4.1. ■

Then, we prove the expected average and best performance of different four-solver contests.

Because there are different configurations according to the number of solvers eliminated in the elimination contest, we start by analyzing the design of the elimination contest in Lemma A5.

Lemma A5. *In a four-solver optimizing the average performance, the following properties hold:*

(a) *When conducting an ECN, it is optimal to organize the ECN(4,2);*
(b) *When conducting an ECP, if* $A \geq \frac{7c}{3k^2}$*, it is optimal to organize the ECP(4,2); otherwise, it is optimal to organize the ECP(4,3);*

Lemma A5 suggests that the seeker should eliminate as many first-stage solvers as possible in the ECN. Similar results can be seen in Hou and Zhang (2021), which indicates that it is optimal to eliminate as many as possible to maximize the second-stage performance. On the other hand, the proposition suggests that the optimal number of final-stage participants in ECP depends on the combined effects of A, c, and k. Our analysis shows that it is optimal to have three solvers in the final stage when the reward is small enough, which has not been discussed in the literature.

Proof of Lemma A5.

(a) As $\Pi_{avg}^{EN}(4,2) - \Pi_{avg}^{EN}(4,3) = \frac{k^2A(4c-k^2A)}{8c^2} - \frac{k^2A(8c-3k^2A)}{24c^2} = \frac{k^2A}{6c^2} > 0$, we have $\Pi_{avg}^{EN}(4,2) > \Pi_{avg}^{EN}(4,3)$ for any A, c, and k.
(b) As $\Pi_{avg}^{EP}(4,2) - \Pi_{avg}^{EP}(4,3) = \frac{k^2A(121c-10k^2A)}{360c^2} - \frac{k^2A(135c-16k^2A)}{360c^2} = \frac{k^2A(-7c+3k^2A)}{180c^2}$, we have $\Pi_{avg}^{EN}(4,2) \geq \Pi_{avg}^{EN}(4,3)$ when $A \geq \frac{7c}{3k^2}$, and $\Pi_{avg}^{EN}(4,2) \leq \Pi_{avg}^{EN}(4,3)$ otherwise.
(c) As $\Pi_{avg}^{NN}(4) - \Pi_{avg}^{NP}(4) = \frac{43k^2A}{70c} - \frac{k^2A(3448c-1035k^2A)}{6720c^2} = \frac{k^2A(680c+1035k^2A)}{6720c^2} > 0$, we have $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{NP}(4)$ for sure.
(d) As $\Pi_{avg}^{NN}(4) - \Pi_{avg}^{EN}(4,2) = \frac{43k^2A}{70c} - \frac{k^2A(4c-k^2A)}{8c^2} = \frac{k^2A(32c+35k^2A)}{280c^2} > 0$, we have $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{EN}(4,2)$ for sure. Recall that $\Pi_{avg}^{EN}(4,2) > \Pi_{avg}^{EN}(4,3)$ in (i), we have $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{EN}(4,2) > \Pi_{avg}^{EN}(4,3)$ for any parameters A, k, and c.

■

We next analyze the design of the elimination contest, for which multiple configurations exist. Lemma A6 describes the optimal elimination contest design.

Lemma A6. *The following properties hold for the four-solver elimination contest optimizing the best performance:*

(a) *When conducting an ECN, if* $A \geq \frac{20291c}{115500k^2}$, *it is optimal to organize the ECN(4,2); otherwise, it is optimal to organize the ECN(4,3);*
(b) *When conducting an ECP, if* $A \geq \sqrt{\frac{216071c^2}{2310k^4}} + \frac{7c}{6k^2}$ *it is optimal to organize the ECP(4,2); otherwise, it is optimal to organize the ECP(4,3).*

Lemma A6 suggests that the optimal number of solvers eliminated depends on the joint effect of A, k, and c. It is optimal to eliminate more solvers in both the ECN and ECP when the reward is high enough.

Proof of Lemma A6.

(a) As $\Pi_{best}^{EN}(4,2) - \Pi_{best}^{EN}(4,3) = \frac{k^2A(4c-k^2A)}{8c^2} + \frac{4751}{11550} - \frac{k^2A(8c-3k^2A)}{24c^2} - \frac{305351}{693000} = \frac{k^2A}{6c^2} - \frac{20291}{693000}$, we have $\Pi_{avg}^{EN}(4,2) \geq \Pi_{avg}^{EN}(4,3)$ when $A \geq \frac{20291c}{115500k^2}$.
(b) As $\Pi_{best}^{EP}(4,2) - \Pi_{best}^{EP}(4,3) = \frac{k^2A(121c-10k^2A)}{360c^2} - \frac{4751}{11550} - \frac{k^2A(135c-16k^2A)}{360c^2} - \frac{305351}{693000} = \frac{k^2A(-7c+3k^2A)}{180c^2} - \frac{20291}{693000}$, we have $\Pi_{best}^{EN}(4,2) \geq \Pi_{best}^{EN}(4,3)$ when $A \geq \sqrt{\frac{216071c^2}{2310k^4}} + \frac{7c}{3k^2}$ and $\Pi_{best}^{EN}(4,2) \leq \Pi_{best}^{EN}(4,3)$ otherwise.

■

Then we derive the expected best performance of different four-solver contests in the following lemma.

Lemma A7. *The solvers' expected best performance in the four-solver contests are*

(a) $\Pi_{best}^{NN}(4) = \frac{43k^2A}{70c} + \frac{1069}{2520}$ *in the NCN(4).*
(b) $\Pi_{best}^{NP}(4) = \frac{k^2A(3448c-1035k^2A)}{6720c^2} + \frac{1069}{2520}$ *in the NCP(4);*
(c) $\Pi_{best}^{EN}(4,2) = \frac{k^2A(4c-k^2A)}{8c^2} + \frac{4751}{11550}$ *in the ECN(4,2);* $\Pi_{best}^{EN}(4,3) = \frac{k^2A(8c-3k^2A)}{24c^2} + \frac{305351}{693000}$ *in the ECN(4,3);*
(d) $\Pi_{best}^{EP}(4,2) = \frac{k^2A(121c-10k^2A)}{360c^2} + \frac{4751}{11550}$ *in the ECP(4,2);* $\Pi_{best}^{EP}(4,3) = \frac{k^2A(135c-16k^2A)}{360c^2} + \frac{305351}{693000}$ *in the ECP(4,3).*

Proof of Lemma A7. We prove this lemma by plugging in n and m into Lemma 4.1. ■

Proof of Proposition 5.1.

(i) By (c) and (d) of Lemma A5, we have $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{NP}(4)$, $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{EN}(4,2)$ for any parameters. The seeker chooses the contest design by solving the following maximization problem:

$$\max\left\{\Pi_{avg}^{NN}(4), \Pi_{avg}^{NP}(4), \Pi_{avg}^{EP}(4,2), \Pi_{avg}^{EP}(4,3)\right\}. \tag{A13}$$

Comparing the expected average performance presented in Lemma A4, the optimal solution of problem eq. A13 is $\Pi_{avg}^{NN}(4)$ for any parameters A, k, and c.

(ii) According to (a) and (b) in Lemma A6, we have $\Pi_{best}^{EN}(4,2) > \Pi_{best}^{EN}(4,3)$ if $A \geq \frac{20291c}{115500k^2}$; $\Pi_{best}^{EP}(4,2) > \Pi_{best}^{EP}(4,3)$ if $A \geq \sqrt{\frac{216071c^2}{2310k^4}} + \frac{7c}{6k^2}$. The seeker makes the contest design decision by solving the following maximization problem:

$$\max\left\{\Pi_{best}^{NN}(4), \Pi_{best}^{NP}(4), \Pi_{best}^{EN}(4,2), \Pi_{best}^{EN}(4,3), \Pi_{best}^{EP}(4,2), \Pi_{best}^{EP}(4,3)\right\} \quad \text{(A14)}$$

(a) As $\Pi_{best}^{NN}(4) - \Pi_{best}^{EN}(4,2) = \frac{43k^2A}{70c} + \frac{1069}{2520} - \frac{k^2A(4c-k^2A)}{8c^2} - \frac{4751}{11550} = \frac{k^2A(32c+35k^2A)}{280c^2} + \frac{1783}{138600} > 0$, we have $\Pi_{best}^{NN}(4) > \Pi_{best}^{EN}(4,2)$ for sure.

(b) As $\Pi_{best}^{EP}(4,3) - \Pi_{best}^{EN}(4,3) = \frac{k^2A(135c-16k^2A)}{360c^2} + \frac{305351}{693000} - \frac{k^2A(8c-3k^2A)}{24c^2} - \frac{305351}{693000} = \frac{k^2A(15c+29k^2A)}{360c^2} > 0$, we have $\Pi_{best}^{EP}(4,3) > \Pi_{best}^{EN}(4,3)$ for sure.

(c) As $\Pi_{best}^{NN}(4) - \Pi_{best}^{EP}(4,2) = \frac{43k^2A}{70c} + \frac{1069}{2520} - \frac{k^2A(121c-10k^2A)}{360c^2} - \frac{4751}{11550} = \frac{55k^2A(701c+70k^2A)}{138600c^2} + \frac{1783}{138600} > 0$, we have $\Pi_{best}^{NN}(4) > \Pi_{best}^{EP}(4,2)$ for sure.

(d) As $\Pi_{best}^{NN}(4) - \Pi_{best}^{NP}(4) = \frac{43k^2A}{70c} + \frac{1069}{2520} - \frac{k^2A(3448c-1035k^2A)}{6720c^2} - \frac{1069}{2520} = \frac{k^2A(136c+207k^2A)}{1344c^2} > 0$, we have $\Pi_{best}^{NN}(4) \geq \Pi_{best}^{NP}(4)$ for sure.

(e) As $\Pi_{best}^{NN}(4) - \Pi_{best}^{EP}(4,3) = \frac{43k^2A}{70c} + \frac{1069}{2520} - \frac{k^2A(135c-16k^2A)}{360c^2} - \frac{305351}{693000} = \frac{k^2A(603c+112k^2A)}{2520c^2} - \frac{158}{9625} > 0$, we have $\Pi_{best}^{NN}(4) \geq \Pi_{best}^{EP}(4,3)$ when $A \geq \frac{3(\sqrt{128442017}-11055)c}{12320k^2}$, and $\Pi_{best}^{NN}(4) < \Pi_{best}^{EP}(4,3)$ otherwise.

The optimal solution of eq. A14 is $\Pi_{best}^{NN}(4)$ when $A \geq \frac{3(\sqrt{128442017}-11055)c}{12320k^2}$, and $\Pi_{best}^{EP}(4,3)$ otherwise. Therefore, NCN(4) is optimal when $A \geq \frac{3(\sqrt{128442017}-11055)c}{12320k^2}$, and ECP(4,3) is optimal otherwise. ■

Proof of Proposition 5.2.

(i) By the Proof of Proposition 5.1 (i), when the seeker values the average performance, we have $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{NP}(4)$, and $\Pi_{avg}^{NN}(4) > \Pi_{avg}^{EN}(4,2) > \Pi_{avg}^{EN}(4,3)$, for sure. Moreover, as $\Pi_{avg}^{NP}(4) - \Pi_{avg}^{EN}(4,2) = \frac{k^2A(3448c-1035k^2A)}{6720c^2} - \frac{k^2A(4c-k^2A)}{8c^2} = \frac{k^2A(88c-195k^2A)}{6720c^2}$, we have $\Pi_{avg}^{NP}(4) \geq \Pi_{avg}^{EN}(4,2)$ when $A \leq \frac{88c}{195k^2}$, and $\Pi_{avg}^{NP}(4) \leq \Pi_{avg}^{EN}(4,2)$ otherwise.

(ii) By the Proof of Proposition 5.1 (ii), when the seeker values the best performance, we have $\Pi_{best}^{NN}(4) > \Pi_{best}^{EN}(4,2)$ for sure. Meanwhile, we have $\Pi_{best}^{EN}(4,2) > \Pi_{best}^{EN}(4,3)$ if $A \geq \frac{20291c}{115500k^2}$, and $\Pi_{best}^{NN}(4) > \Pi_{best}^{EN}(4,3)$ if $A \geq \frac{2\sqrt{11624965}-3245)c}{5775k^2}$. Moreover, we have $\Pi_{best}^{NN}(4) \geq \Pi_{best}^{NP}(4)$ for sure. Additionally, we have (a) $\Pi_{best}^{NP}(4) \geq \Pi_{best}^{EN}(4,2)$ when $A \leq \frac{2(\sqrt{568502}+242)c}{2145k^2}$ (b) $\Pi_{best}^{NP}(4) \geq \Pi_{best}^{EN}(4,3)$ when $\frac{4(8305-\sqrt{64906105})c}{10725k^2} \leq A \leq \frac{4(8305+\sqrt{64906105})c}{10725k^2}$. Therefore, the optimal solution among NCP(4), ECN(4,2), and ECN(4,3) is NCP(4) when $\frac{(33220-4\sqrt{64906105})c}{10725k^2} \leq A \leq \frac{(2\sqrt{568502}+242)c}{2415k^2}$, ECN(4,2) is optimal when $A > \frac{(2\sqrt{568502}+242)c}{2415k^2}$, and ECN(4,3) is optimal when $0 < A < \frac{(33220-4\sqrt{64906105})c}{10725k^2}$.

■