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Profit allocation in investment-based crowdfunding with investors of dynamic entry times

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ABSTRACT

Even distribution is a normal profit allocation mechanism for investment-based crowdfunding projects on many platforms. In other words, the investors with the same pledging funds will be paid evenly when the investment ends. The even allocation mechanism works well under the assumption that the investors arrive at the platform simultaneously. However, in practice, the investors are sequential, therefore, the stories are different when considering the dynamic entry times of the investors. In this paper, we study ways to design appropriate profit allocation mechanisms to enhance the success rate of an investment-based crowdfunding project. The basic model focuses on the two-investor case, where only two investors with dynamic entry times are considered. The profit allocation mechanism is shown to have great impacts on the pledging probabilities of investors, as well as the success rate of a project. After that, we shift our focus to the two-cohort case, where dynamic investors are assumed to arrive at the platform as two sequential cohorts. By taking the sizes of each cohort into consideration, we are able to analyze the success rate of a project under various practical situations. Finally, we implement some numerical experiments to generalize our studies to the situations where (i) there are more than two pledging periods for the investors, (ii) the herding effect of the investors is considered, and (iii) the valuations of the investors are assumed to be normally distributed. Our main results still hold under these general situations.

1. Introduction

It is well recognized that small start-ups and entrepreneurs encounter great difficulties while seeking finance from banks or venture capitalists (Cassar 2004; Cosh, Cumming, & Hughes 2009), especially during their initial stages. Complementing traditional financing options, crowdfunding emerged as an innovative form of seeking finance from people and networks, with a low-barrier (Bouncken, Komorek, & Kraus 2015; Mollick & Nanda 2015).

As the focus of our study, investment-based crowdfunding is one type of crowdfunding where investors can receive financial profits such as equity, interest, revenue, and loyalty as the return (Belleflamme, Omrani, & Peitz 2015).[1] One attribute that reveals the importance of investment-based crowdfunding projects is their amounts of funding. Investment-based crowdfunding has experienced dramatic growth since the Jumpstart Our Business Start-ups (JOBS) Act was passed in the USA in 2012 (Ahlers, Cumming, Günther, & Schweizer 2015). As reported in Massolution (2013), the average funding size in investment-based crowdfunding is more than 100 times larger than the size in donation-based crowdfunding. In addition, according to Barnett (2015), the World Bank has also estimated that the total funding size of investment-based crowdfunding would reach $90 billion by 2020 and surpass the size of venture capital. Moreover, the monetary return also makes investment-based crowdfunding different from other types of crowdfunding. For example, the products offered by reward-based crowdfunding are usually innovative products which are new to the market, so investors must pledge in the project to receive the specific product. However, there can be more competitions in investment-based crowdfunding because the investors are only seeking for monetary return which can be provided by any potential project. Therefore, given their importance and difficulty, we decided to choose investment-based crowdfunding projects as the focus of our research.

Crowdfunding platforms make it possible for small firms and entrepreneurs to simplify and decentralize their funding processes. By communicating with potential investors directly through the

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[1] What differentiates the type of a crowdfunding project is the distinctive form of return that the investors will receive. There are some other types of crowdfunding such as reward-based crowdfunding and donation-based crowdfunding where investors pledge for specific products and moral satisfaction, respectively.

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internet, entrepreneurs can introduce their proposals in a better manner and raise funds from a large number of individuals (Schwienbacher & Larralde 2010).

On an investment-based crowdfunding platform, a typical crowdfunding project will announce a funding target, along with a unit pledging price, a funding deadline, a proposal that specifies how the funds will be used, and a profit allocation mechanism. Then the investors will come to the project with dynamic entry times and decide whether to pledge or not respectively. The funding part succeeds only when the total amount of investment exceeds the target within the given period. If the project fails, all the funds raised will be returned to the investors. This mechanism is known as "All-or-nothing", while there also exists the "Keep-it-all" mechanism on some crowdfunding platforms where entrepreneurs can take the raised money regardless of whether the target is reached or not. The "Keep-it-all" mechanism has rarely been studied by previous literature. Moreover, among the five most popular crowdfunding platforms, only one platform allows this mechanism (Gedda, Nilsson, Såthén, & Søilen 2016), and we will only discuss our works based on the "All-or-nothing" mechanism in the crowdfunding market.

After raising enough funds, the entrepreneur will execute the proposal and final earnings will be allocated to investors, according to the profit allocation mechanism, in return. During the period of crowdfunding, investors make their decisions based on their pledges to the project and their valuations of the financial return from the proposal.

It is clear that successful crowdfunding projects can benefit all participants: entrepreneurs can get enough funds to start their businesses; investors can make use of spare cash for promising investments; and the platform can earn commission fees from the organization. However, because of uncertainty and asymmetric information, about two-thirds of the total number of projects have failed at the crowdfunding stage[2]. This indicates the urgent necessity of investigations on enhancing success rates of investment-based crowdfunding projects.

It is shown that the success rate of a project is significantly affected by its performance in the early stage (e.g., see Du, Hu, & Wu 2017; Mollick & Kuppuswamy 2014). On the one hand, lesser investment in the early stage not only puts more funding pressure on the later stages, but also weakens the investing willingness of later investors. Many existing studies(e.g., see Belleflamme et al. 2015; Li & Duan 2016) have suggested the existence of positive network externality and negative time effect in crowdfunding, that is, the portion of the target already reached has a positive influence, while the time remaining has a negative influence on later investors. Therefore, a surge of new pledges may appear around the time when the targets of crowdfunding projects are reached (e.g., see Wu, Shi, & Hu 2015). On the other hand, investors arriving in the early stages are usually less willing to participate for many reasons such as lack of information, observational learning and incurring higher waiting cost. Du et al. (2017) concludes that, among all the failed projects, 88.34% ended up raising lesser than 20% of their original targets. Similarly, Mollick and Kuppuswamy (2014) observes that the crowdfunding projects either succeed or fail by large margins, and the average percentage of raised funds is only 8% among all the failed projects. Apart from the potential low-quality of these projects, the low pledging willingness in the early stage may also be a crucial reason why these projects failed eventually.

In the past, to motivate early investors to improve success rates of crowdfunding projects, entrepreneurs were encouraged to make some sacrifice, including offering free gifts and lowering pledging prices (e.g., see Du et al. 2017; Kauffman, Lai, & Ho 2010). However, first, due to the lack of initial capital, offering free gifts may put more pressure on entrepreneurs. Second, the competition in investment-based crowdfunding is so intense that each entrepreneur prefers to set the pledging price at the lowest level. Once the initial pledging price is lowered further, the total amount of funds raised decreases, and the proposal is more likely to fail.

In this paper, instead of sacrificing the entrepreneurs themselves, we are interested in reallocating final profits earned from the proposal according to the dynamic entry times of investors. Intuitively, we assign more profits to early investors so that their waiting costs are balanced out and the resulting pledging probabilities are raised. Note that more profits allocated to (higher pledging probabilities of) early investors means fewer profits remain for (lower pledging probabilities of) the late ones. To enhance the overall success rate of a crowdfunding project, it is of utmost importance to provide the entrepreneur with appropriate profit allocation mechanisms. Our main contributions are summarized as follows.

First, to the best of our knowledge, this paper is the first attempt to analytically study the profit allocation mechanism to enhance the success rates of investment-based crowdfunding projects. Most literature on crowdfunding, especially investment-based crowdfunding, is empirical, and existing efforts on motivating investors focus on offering additional benefits and price discounts. Our study helps entrepreneurs design an optimal profit allocation mechanism to maximize the success rate without offering additional benefits during the project.

Second, we develop static models to analyze the pledging behavior of investors with dynamic entry times, and we characterize the "waiting cost" to explain the inequity between investors at different stages in crowdfunding projects. The main results show that because of the waiting cost, investors who arrive early are less willing to pledge money. It also shows that the entrepreneur should motivate early investors to enhance the success rate of the project. In addition, the extra return given to early investors as an incentive should increase with the waiting cost.

Third, as a generalization, we consider the difference in the number of investors who group as cohorts with different time of entry. We find that investors in different-sized cohorts are not equally sensitive with changes in profit allocation, and the entrepreneur should motivate investors in smaller cohorts to enhance the success rate of his crowdfunding project. This property, together with the effect of the waiting cost, decides the profit allocation strategy of the entrepreneur. In addition, we also provide managerial guidance on how the entrepreneur should adjust the optimal profit allocation mechanism when other factors in the market change.

Last, to enrich our research, we conduct a series of numerical experiments to extend our model by considering multiple periods and the herding effect. Our results show that the return allocated to the investors in multiple periods should decrease with their entry times, and the herding effect increases the extent of asymmetric allocation, i.e., the entrepreneur should allocate even more return to early investors. Moreover, our results reveal that the herding effect strengthens both the importance and the influence of allocating more return to early investors. We also test the robustness of our model with normally distributed valuations of the investors on the crowdfunding project.

The rest of this paper is structured as follows. The following section reviews relevant literature. We describe the basic problem in Section 3. In Section 4, we analyze the profit allocation mechanism using a primary model where there are only two potential investors. Section 5 generalizes the results of Section 4 by studying a two-cohort model where there are two cohorts of investors. Section 6 offers numerical examples to extend our

[2] Source:https://www.entrepreneur.com/article/269663

model and assess the robustness. The conclusions are shown in Section 7.

2. Literature review

Although crowdfunding is a relatively new phenomenon with nascent related research, the rapid growth of all kinds of crowdfunding platforms, as well as enormous economic benefits brought by them every year, have intrigued more and more researchers.

On the analytical side[3], Belleflamme, Lambert, and Schwienbacher (2014) gives instructions on choosing between pre-order crowdfunding and equity crowdfunding under different conditions. Similar to our research, they also study the pledging behaviors of investors, while under the situation where the entrepreneur is tapping into a certain crowd with known valuations, and the equity crowdfunding serves as an alternative to the reward-based crowdfunding. Therefore, there is no uncertainty of success and the project will either definitely fail or succeed, depending on the price and target. Hu, Li, and Shi (2015) develops a two-period model to study how pricing and product design strategies in crowdfunding differ from traditional financing. Moreover, their studies help entrepreneurs choose the suitable pricing strategies according to different targets, while we focus on improving the success rate of the project with a fixed target. Du et al. (2017) finds that the entrepreneur should contingently add a stimulus for enhancing the success rate. They focus on studying the optimal time point to stimulate investors with additional benefits (e.g., offering free samples) during the funding process, while our research aims to help entrepreneurs design an optimal profit allocation before the project is started. There are also other studies on the advantages of reward-based crowdfunding mechanism such as Chen, Gal-Or, and Roma (2017) and Chakraborty and Swinney (2016). Our work studies investment-based crowdfunding mechanism that has seldom been studied analytically. It is well recognized that a good success rate lies at the core of crowdfunding. We focus on enhancing the success rate by designing a profit allocation mechanism without offering additional benefits in crowdfunding projects.

As a supplement, crowdfunding is related to many fields of literature. For example, the "All-Or-Nothing" mechanism, in which money is refunded when the entrepreneur fails to collect enough pledges within a certain period, is similar to the common provision-point mechanism used by researchers to study private provisions of public goods (e.g., see Bagnoli & Lipman 1989; Palfrey & Rosenthal 1988). However, everyone can benefit from the provision of public goods once a project is built, while in crowdfunding, people must invest in the project to receive their return, thereby making the free-riding effect in the provision of public goods less essential.

Another stream of research similar to crowdfunding is group buying, wherein a qualified number of committed purchasers can get special discount on products. Tran and Desiraju (2017) and Yan, Zhao, and Lan (2017) study the impact of asymmetric information on group buying from the perspective of the manufacturer and the retailer. Hu, Shi, and Wu (2013) suggests that sellers disclose the cumulative sign-up information to later customers to increase success rates. Moreover, Wu et al. (2015) reveals the threshold effect that the sign-up behavior of customers accumulates right before and after the target is reached. This is consistent with the discovery that we have underlined, namely, that pledging probabilities of investors are higher in the later stages, where the threshold is about to be reached and the risk is much lower. A study on group buying that is similar to ours is Kauffman et al. (2010). They introduced demand externalities and concluded that motivating early consumers to join in on group buying efficiently improves the performance of projects. However, they explored the incentive mechanisms based on offering an extra and attractive discount to the first few participants or those who arrived within a short period of time, as soon as the project began. Group buying shares more similarities with reward-based crowdfunding than with investment-based crowdfunding. Group buying projects are often offered by well-established companies that launch these projects to advertise their brands and expand market share. It is easy for these large companies to give up profit to attract customers. But investment-based crowdfunding projects are always associated with new ventures and small start-ups that are in urgent need of initial funds. Therefore, our studies provide entrepreneurs with a new method to improve the success rate which only needs to redesign the profit allocation.

3. Problem description

On an investment-based crowdfunding platform, an entrepreneur will launch a project with a detailed proposal, a target amount of funds, a unit pledging price for each investor, and a specified profit allocation mechanism when the proposal is implemented. Then, the investors will arrive at the platform with sequential entry times, and decide whether to pledge or not by maximizing their own expected utilities. After that, the project closes. If the project succeeds (i.e., the target is achieved), the entrepreneur will implement his proposal, and the investors will get paid according to the preset profit allocation mechanism after the implementation. Otherwise, the platform will return the pledged money to the investors and the entrepreneur will not be able to receive anything.

Owing to the refunding policy, the objective of the entrepreneur is to increase the success rate of the crowdfunding project as far as possible. In particular, once the target amount of funds and the unit pledging price are predetermined, the profit allocation mechanism would be the remaining key factor that would affect the success rate of a project. This is the main focus of our paper.

As a first attempt to tackle the profit allocation mechanism in investment-based crowdfunding, this paper will restrict itself to the two-cohort situation, that is, the investors group as two cohorts, arriving in two specific periods. This two-period assumption is widely used to study the crowdfunding process (e.g., see Hu et al. 2015; Jing & Xie 2011; Liang, Ma, Xie, & Yan 2014). In fact, many of our results can be generalized to the case of multiple cohorts. For example, in Section 4.3 we conclude that the entrepreneur should motivate investors in the early cohort, and the return given to this cohort increases with the waiting cost. This conclusion is consistent with our numerical example which studies the multiple-period case in Section 6.1, and the numerical results show that the return given to each cohort decreases with its time of entry, that is, the later a cohort arrives, the lesser return will be allocated to it. In the basic model that is presented in Section 4, we focus on the two-investor case, where each cohort contains only one investor. In Section 5, we generalize our results to the two-cohort model.

Fig. 1 shows the basic procedures involved in two-investor crowdfunding. To be specific, the unit pledging price is p, the target amount of funds is $P = 2p$, and there are two potential investors I_1 and I_2. In each period t_i $(i = 1, 2)$, investor I_i arrives and makes his pledging decision. At the end of period t_2, the project closes.

[3] There are also many empirical studies on the characteristics that might influence the success rate of crowdfunding projects, including geographic distance among investors (Agrawal, Catalini, & Goldfarb 2011; Burtch, Ghose, & Wattal 2013), herding behavior (Berkovich 2011; Herzenstein, Dholakia, & Andrews 2011), financial intermediaries (Berger & Gleisner 2009), the funding purpose(Mach, Carter, & Slattery 2014), the existence of home bias (Lin & Viswanathan 2015), types of projects (Belleflamme, Lambert, & Schwienbacher 2013), choices of return offered in projects (Wang, Yang, Kang, & Hahn 2016), perverse incentives in crowdfunding (Hildebrand, Puri, & Rocholl 2016).

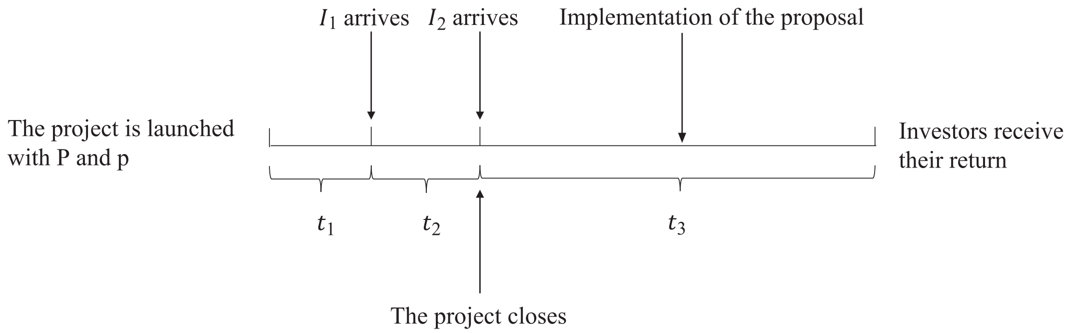


Fig. 1. Procedures of the two-investor case.

Table 1
Notations used in the problem description.

P	The target amount of funds in the project
p	The unit pledging price for each investor
t_i	The pledging period of the crowdfunding project, $i \in \{1, 2\}$
I_i	The investor arriving at period t_i, $i \in \{1, 2\}$
t_3	The implementing period of the proposal in the crowdfunding project
V_i	The rate of return from this proposal estimated by investor I_i, $i \in \{1, 2\}$
Δ	The risk-free rate of return of the market during period t_2
R	The risk-free rate of return of the market during period t_3

If either I_1 or I_2 chooses not to pledge, the project fails. Otherwise, the project succeeds and the entrepreneur implements the proposal during the period t_3. After the implementation of the proposal, the investors get their return at the end of period t_3. Note that t_3 is usually much longer than t_1 and t_2.

While making pledging decisions, each investor would maximize his own utility by comparing the expected return from pledging (ERP) with the expected return from not pledging (ERNP). To measure the ERP, we denote the valuation of I_i $(i = 1, 2)$ on the proposal as $V_i \times P$, where V_i can be regarded as the valuation rate of return of the proposal estimated by I_i. Then, the ERP of I_i is simply his share of $V_i \times P$ under some given profit allocation mechanism. For the valuation rate V_i, we assume that V_i $(i = 1, 2)$ are i.i.d., with a uniform distribution over interval [0, A] to tackle the heterogeneity of different investors. The assumption of uniform distribution can be found in other literature such as Belleflamme et al. 2014 where the marginal utilities of individuals are uniformly distributed between [0,1]. Furthermore, the valuation rates of the investors are assumed to be private, while their distributions are known to each other and the entrepreneur. Such assumptions are also widely used in crowdfunding studies (e.g., see Hu et al. 2015). Moreover, V_i is the expected valuation which has already taken into account the default risk that the implemented proposal may fail to deliver the promised return even if the crowdfunding project succeeds.

To measure the ERNP, by denoting the risk-free rate of return of the market during period t_3 as R, each investor can get a risk-free return of $R \times p$ during period t_3 with fixed investment p. Besides, note that I_1 pledges earlier and waits t_2 longer than I_2 until the project closes. Let $\Delta = 1 + \delta$ be the risk-free rate of return of the market during period t_2, where δ can be viewed as the rate of waiting cost for I_1. Thus, the risk-free return of I_1 would be $(1 + \delta) \times R \times p$ during periods t_2 and t_3 if he chooses not to pledge. By comparing the ERP with ERNP, an investor can make his own pledging decision. We now formally summarize the notations described above in Table 1.

4. Analyses of the profit allocation mechanism

It is clear that different profit allocation mechanisms lead to different pledging strategies for investors, and in turn, decide the success rates of crowdfunding projects. In this section, we will focus on the two-investor case where there are only two potential investors.

In most existing research, the profit allocation mechanism is simply even distribution among all investors despite of their dynamic entry times, which is referred to as an even allocation mechanism in our paper. We will generalize the results by allocating the profits among the investors unevenly. To be formal, for a given profit allocation mechanism $(\alpha, 1 - \alpha)$, we let the share of return allocated to I_1 be α $(0 < \alpha < 1)$, and consequently, the share of return allocated to I_2 can be written as $1 - \alpha$. For example, when $\alpha > 0.5$, the early investor will always receive more equity per dollar than the later investor no matter how much the proposal gains.

4.1. Pledging strategies of the investors

We first study the impacts of the profit allocation mechanism on the pledging strategies of investors by backward induction. The details are shown as follows.

When I_2 arrives during period t_2, he can observe the pledging decision made by I_1. If I_1 did not pledge, I_2 will walk away directly, since the target P cannot be met and the project will definitely fail. Otherwise, the project will succeed as long as I_2 pledges. On the one hand, since the valuation rate of return of I_2 on the proposal is V_2, the resulting ERP is given by $(1 - \alpha) \times V_2 \times P = 2p \times (1 - \alpha) \times V_2$. On the other hand, the ERNP of I_2 with investment p is simply $R \times p$ during period t_3. In this case, I_2 will pledge only when his ERP surpasses ERNP, that is,

$$2p \times (1 - \alpha) \times V_2 > R \times p, \text{ which is equivalent to } V_2 > R/2(1 - \alpha).$$

By noting that V_2 is uniformly distributed over interval [0, A], we can claim that when I_1 pledged, the pledging probability of I_2, denoted as q_2, is $1 - R/2A(1 - \alpha)$.

When I_1 arrives during period t_1, although he has no information on the pledging decision of I_2, he can speculate the pledging strategy of I_2 due to the awareness of the distribution of V_2. To be specific, the pre-condition for I_2 to pledge is that I_1 pledges and the pledging probability is q_2. In this case, on one hand, the ERP of I_1 can be written as $q_2 \times \alpha \times V_1 \times P + (1 - q_2) \times R \times p = q_2 \times 2\alpha \times V_1 \times p + (1 - q_2) \times R \times p$, where the former part is the expected return when I_2 pledges, and the latter part is the expected return when I_2 does not pledge and I_1 is refunded. On the other hand, the ENRP of I_1 with investment p is $R \times (1 + \delta) \times p$, which includes risk-free returns during both periods t_2 and t_3. Thus, I_1 will pledge only when

$$2\alpha \times V_1 \times p \times q_2 + (1 - q_2) \times R \times p > R \times (1 + \delta) \times p,$$

which is equivalent to

$$V_1 > (\delta + q_2) \times R/(2\alpha \times q_2).$$

Therefore, we can claim that the pledging probability of I_1, denoted as q_1, is $1-(\delta+q_2)\times R/(2\alpha\times q_2\times A)$.

Since the (crowdfunding) project succeeds only when both investors pledge, the success rate of the project, denoted as S, is $q_1\times q_2$. By letting $r=R/A,$ we can express the pledging probabilities of the investors and the success rate of the project as

$$q_1 = 1-\frac{\delta r(1-\alpha)}{2\alpha(1-\alpha)-\alpha r}-\frac{r}{2\alpha},$$
$$q_2 = 1-\frac{r}{2(1-\alpha)},\ \text{and}\ S=q_1\times q_2, \text{respectively.}$$

The ratio $r=R/A$ can be regarded as a factor reflecting the competitiveness of the risk-free market over the proposal provided by the entrepreneur. Moreover, in practice, r also refers to the competitions from other projects on the crowdfunding platform. When making pledges, investors can always deviate and choose to pledge any other project on the platform, and R can be regarded as the expected rate of return that investors can receive from other projects. In this case, we will still assume $R\le A$; otherwise, there is no need to study because even the investor with the highest valuation on it will not pledge and the project is doomed to fail. Therefore, the ratio r in our paper refers to the comprehensive performance of the crowdfunding market. When r is high, the crowdfunding market is so competitive that the investors are not interested in this proposal offered by the project, and when r is low, the results reverse.

4.2. Feasibility of a project

One of the most important steps for an entrepreneur before starting a crowdfunding project on a platform is to check the feasibility of his crowdfunding project, that is, the positivity of the success rate of a project. From the expressions of q_1 and q_2, we can see that the success rate is decided by r, δ, and α, where r and δ are exogenous, while α can be adjusted by the entrepreneur.

It is important to remember that $r=R/A$ reflects the competitiveness of the risk-free market over the proposal in the crowdfunding project. We now study the feasibility of a project from the perspective of r. Lemma 1 shows that there exists a tolerance bound on r, above which the project is destined for failure with given δ and α.

Lemma 1. *Under a given profit allocation mechanism $(\alpha,1-\alpha)$, the project is feasible only when $r<\bar{r}(\alpha,\delta)$, where $\bar{r}(\alpha,\delta)=1+(1-\alpha)\delta-[1+(1-\alpha)^2\delta^2+2(1-\alpha)(\delta-2\alpha)]^{1/2}$.*

Lemma 1 indicates that the entrepreneur will start a crowdfunding project only when $r<\bar{r}(\alpha,\delta)$.

Since the length of the pledging period t_2 (i.e., the value of δ) is hard to reduce in practice, it is desired to study the monotonicity of $\bar{r}(\alpha,\delta)$ in α, and the results are shown in Proposition 1. For the sake of simplicity, we will write $\bar{r}(\alpha,\delta)$ as $\bar{r}$ in short when the context is not confusing, and the same operations are applied to all other functions throughout this paper.

Proposition 1. *For given δ, function $\bar{r}$ is unimodal in α and the maximum tolerance bound, denoted as $\bar{r}^*$, is equal to $\frac{2(\delta+2-2\sqrt{\delta})}{4+\delta^2}$*

The unimodality of $\bar{r}$ in α can be interpreted as follows. Regardless of the dependence of the pledging decisions, the pledging probabilities of I_1 and I_2 are increasing in α and $1-\alpha,$ respectively. However, since the feasibility (positivity of the success rate) of a project is decided by the product of the two pledging probabilities, a straightforward result is that the monotonicity of $\bar{r}$ coincides with the monotonicity of $\alpha(1-\alpha)$ in α, that is, $\bar{r}$ is a unimodal function of α. Apparently, we can conclude the maximum tolerance bound according to Proposition 1.

Proposition 1 shows that, for any given δ, if $r>\bar{r}^*$, crowdfunding is infeasible, no matter how the entrepreneur will allocate the profits to the investors. In particular, when $\delta=0,$ the maximum tolerance bound is equal to 1. This indicates that when period t_2 is so short that the waiting cost of I_1 is close to 0, the necessary condition for a positive success rate is simply $R<A(r<1)$, that is, the return rate of the proposal has a chance to surpass the return rate of the risk-free market.

4.3. Success rate of a project

The previous subsection provides a necessary condition (a tolerance bound $\bar{r}^*$ on r) under which a project has a chance to succeed. In this part, we will focus on the case where $r<\bar{r}^*$, that is, the project is feasible under some allocation mechanism, and study how the success rate of a project will change with different profit allocation mechanisms.

It is important to remember that in Section 4.1 we have shown that the pledging probabilities of the two investors and the success rate of the project are

$$q_1 = 1-\frac{\delta r(1-\alpha)}{2\alpha(1-\alpha)-\alpha r}-\frac{r}{2\alpha},$$
$$q_2 = 1-\frac{r}{2(1-\alpha)},\ \text{and}\ S=q_1\times q_2,\ \text{respectively.}$$

From the expressions of q_1 and q_2, we can find that q_2 decreases in α while the monotonicity of q_1, as well as S, in α is unknown. To this end, we have Theorem 1 showing the monotonicity of S in α.

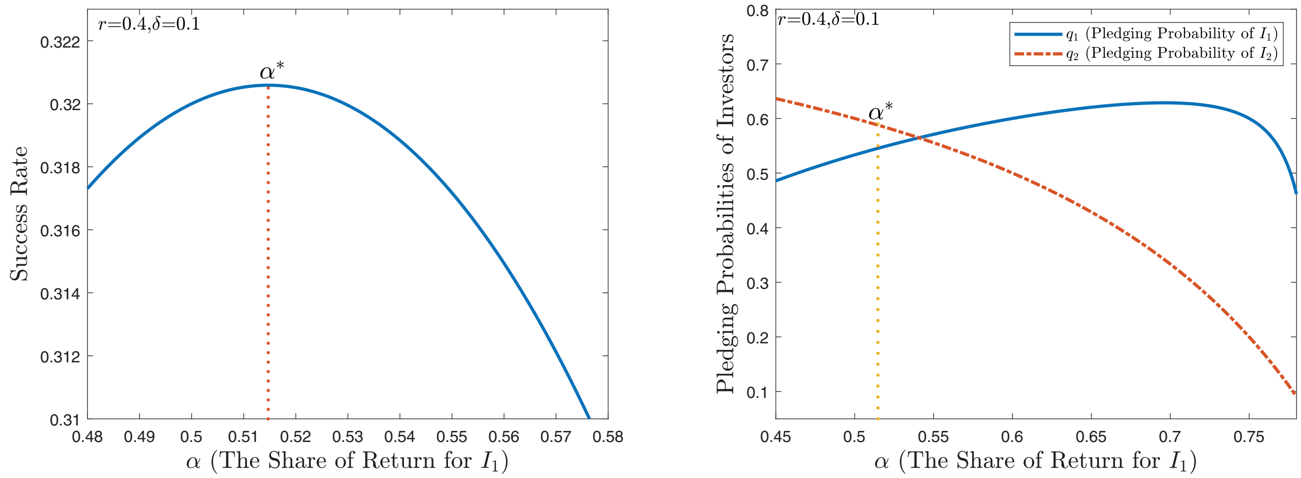
Theorem 1. *The success rate S is unimodal in α and reaches its maximum at α^*, where α^* is equal to $(2+2\delta-r-[(2-r)(2+2\delta-r)]^{1/2})/2\delta$ and larger than 1/2.*

The unimodality of S is expected. We can interpret this in a manner similar to what we did after Proposition 1. Suffice to say that the monotonicity of S is consistent with the monotonicity of $\alpha(1-\alpha)$ in α. For any given pair of δ and r, the entrepreneur is able to maximize the success rate of his crowdfunding project by letting α equal α^*. In addition, the intuition behind $\alpha^*>1/2$ is that the entrepreneur should compensate I_1 for his waiting cost during period 2. Compared with $\alpha=1/2,$ which maximizes $\alpha(1-\alpha),$ the entrepreneur should motivate investor I_1 with a greater return. Therefore, we can claim that the entrepreneur should always take sides with the first investor to maximize S.

Since the entrepreneur should compensate I_1 with a greater return for his waiting cost instead of allocating the return evenly, it's desired to figure out how the pledging probabilities of investors change under the optimal allocation $(\alpha^*,1-\alpha^*)$ and the results are shown in Proposition 2.

Proposition 2. *The pledging probability of I_1 is unimodal in α and reaches its maximum at $\alpha^1>\alpha^*$. Therefore, compared to the even allocation, the pledging probability of I_1 increases under the optimal profit allocation $(\alpha^*,\ 1-\alpha^*)$ while the pledging probability of I_2 decreases.*

We first interpret the monotonicities of the pledging probabilities of two investors. We have found that $q_2(\alpha)$ decreases in α from its expression, which is intuitive due to the decreased share of return allocated to I_2 when α increases. However, Proposition 2 reveals that the pledging probability of I_1 is unimodal in α instead of simply increasing. Remind that the success of a crowdfunding project requires the pledges from enough investors, therefore, I_1 must consider the pledging willingness of I_2. When α^* becomes too large, the pledging probability of I_2 is too small and I_1 is less willing to pledge despite the increased share of return allocated to him. This indicates that the decisions of investors in crowdfunding are affected by others, which is different



(a) Success Rate (b) Pledging Probability of Investors

Fig. 2. Success Rate and pledging probability of investors in the profit allocation mechanism.

from traditional trading or financing where investors usually make their decisions independently.

As we can see from Proposition 2, the pledging probability of I_2 decreases under the optimal allocation because $\alpha^* > 0.5$. This reveals that some of the later investors will turn to other projects after their share of return from this project being decreased to $1 - \alpha^*$. In the same way, the increase in the pledging probability of I_1 indicates that the optimal allocation will attract more investors to pledge in the early stage. According to Theorem 1, the overall success rate of the project increases under the optimal profit allocation $(\alpha^*, 1 - \alpha^*)$. Therefore, the entrepreneur should implement the profit allocation mechanism although it will inevitably lose part of the later investors.

We now use a numerical example to illustrate how α affects the pledging probabilities of the investors and the success rate of the project. The results are shown in Fig. 2, where $\delta = 0.1$, $r = 0.4$, the horizontal axes represent α, and the vertical axes represent the sucess rate and the pledging probability, respectively. Fig. 2 (a) confirms the monotonicity of S in α, and the optimal share of return for I_1 is larger than 0.5 which is consistent with Theorem 1. In Fig. 2 (b), the dotted line which is decreasing represents $q_2(\alpha)$, and the solid line which is unimodal associates with $q_2(\alpha)$.

As we can see from Theorem 1, the optimal α^* to maximize the success rate S is decided by both, r and δ. We now show the monotonicity of α^* in r and δ in Proposition 3.

Proposition 3. *The optimal α^* for S increases in both, δ and r.*

It is important to bear in mind that the risk-free return of I_1 and I_2 are $\left(1+\delta\right) \times R \times p$ and $R \times p$, respectively. Compared with I_2, investor I_1 incurs an additional waiting cost of $\delta \times R \times p$. Therefore, the entrepreneur is suggested to allocate more return to I_1 when δ or r increases. We refer to the increase of α^* in δ as the effect of waiting cost, and the δ-effect for short. Note that δ reflects the disadvantageous position of early investors, and it may include many aspects such as waiting cost, lack of information and observational learning. The δ-effect encourages the entrepreneur to compensate early investors for these disadvantages. Proposition 3 reveals that, although period t_3 is usually longer than t_2 and δ is a relatively small value, we should not neglect the importance of compensating I_1 because of the combined impact of $R \times \delta \times p$.

5. Two-cohort model

In Section 4, we studied the basic case, where there are only two potential investors arriving at the platform sequentially. In this section, we will extend our investigations to a general case where there are two cohorts of potential investors.

The main changes in the two-cohort model can be concluded as follows. We denote the two sequential cohorts arriving at the platform during periods t_1 and t_2 as C_1 and C_2, respectively. Let $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$ be the respective shares of return allocated to C_1 and C_2 by the entrepreneur. For each cohort C_i $(i = 1, 2)$, there are N_i identical investors: each of whom (1) has the same valuation rate of V_i^N on the proposal, which is uniformly distributed over [0, A] and (2) expects an average share of return of $\alpha_i \times V_i^N \times P/N_i$. The assumption of the identical valuations within each cohort can be found in existing literature (e.g. see Hu et al. 2015; Hu et al. 2013), and this simplification enables us to focus on the interactions among investors in different fundraising stages.

It is expected that the two-cohort model shares some similar results with the two-investor model. For example, the δ-effect still holds, that is, when δ increases, the entrepreneur needs to compensate the first cohort by allocating them more shares of return. However, the optimal profit allocation mechanism might change because of the emergence of the scale-effect of the cohorts.

We can interpret the intuition of the scale-effect in the two-cohort model as follows. For each unit of additionally allocated profit, the investors in the smaller cohort individually gain more, and thus increase faster in terms of pledging probability, than those in the larger cohort. Remind that the success of the crowdfunding project requires the pledges from all investors, therefore the entrepreneur can enhance the overall success rate by subsidizing the smaller cohort. These intuitions can be addressed by the following example. Suppose that there are two cohorts C_1 and C_2 containing N_1 and N_2 investors, respectively. When the entrepreneur decides to motivate C_1 by allocating them an extra return of x, the average return allocated to each investor in C_1 is increased by x/N_1, while the average return of each investor in C_2 is decreased by x/N_2. Thus, the investors in different cohorts are not equally sensitive with the same change of α. To take advantage of such unequal sensitivity, the scale-effect suggests that the entrepreneur should take sides with the smaller cohort while maximizing the success rate of his crowdfunding project. The scale-effect, together with the δ-effect, decides the incentive strategy of the entrepreneur in the two-cohort case.

From the problem setting, it is clear that the pledging strategies of different investors within the same cohort are identical. Similar to the two-investor model, to investigate the optimal profit allocation mechanism in the two-cohort case, we first analyze the pledging strategies of each cohort by backward induction.

When C_2 arrives, the investors in this cohort only pledge if C_1 has pledged. One the one hand, if C_1 pledged, since the valuation rate of return of C_2 on the proposal is V_2^N, the ERP for each investor in C_2 is given by $(N_1+N_2)\times p\times(1-\alpha)\times V_2^N/N_2$. On the other hand, the ERNP of each investor in C_2 with investment p is $R\times p$ during period t_3. In this case, investors in C_2 will pledge only when the ERP surpasses ERNP, that is,

$$V_2^N > N_2\times R/[(N_1+N_2)(1-\alpha)].$$

To conclude, when C_1 pledged, the pledging probability of C_2, denoted as q_2^N, is equal to $1-N_2\times R/[(N_1+N_2)(1-\alpha)A]$.

When C_1 arrives in period t_1, investors in C_1 know that the pre-condition for C_2 to pledge is that C_1 pledges and the pledging probability is q_2^N. On the one hand, the ERP of each investor in C_1 can be written as $q_2^N\times p\times(N_1+N_2)\alpha\times V_1^N/N_1+(1-q_2^N)\times R\times p$, where the former part is the expected return when C_2 pledges, and the latter part is the expected return when C_2 does not pledge. On the other hand, the ERNP of each investor in C_1 with investment p is $R\times(1+\delta)\times p$, which includes the risk-free returns in both periods t_2 and t_3. Thus, investors in C_1 will pledge only when the ERP is larger than the ERNP, that is,

$$V_1^N > N_1\times(\delta+q_2^N)R/[(N_1+N_2)\times q_2^N\times\alpha]$$

To conclude, the pledging probability of C_1, denoted as q_1^N, is equal to $1-N_1\times(\delta+q_2^N)R/[(N_1+N_2)\times q_2^N\times\alpha\times A]$.

Let $\rho=N_1/(N_1+N_2)$ and S_N denote the success rate of the project in the two-cohort situation. Then, we have

$$q_1^N = 1-\frac{(1-\alpha)\delta\rho r}{\alpha((1-\alpha)-(1-\rho)r)}-\frac{\rho r}{\alpha},$$
$$q_2^N = 1-\frac{(1-\rho)r}{(1-\alpha)},\ \text{and}\ S_N=q_1^N\times q_2^N.$$

Note that the two-investor model is a special case of the two-cohort model where $\rho=1/2$. The results are consistent with what we derived in the basic model.

There also exists a tolerance bound $\bar{r}_N$ on r, above which the crowdfunding project is infeasible. It is clear that $\bar{r}_N$ is decided by r, δ, ρ and α. By changing the value of α, we are able to adjust the tolerance bound. In addition, we can still show that function $\bar{r}_N$ is unimodal in α. The detailed explanations are omitted for the sake of simplicity. We present Corollary 1 as a conclusion.

Corollary 1. *In the two-cohort model, the tolerance bound $\bar{r}_N$ is unimodal in α, and the maximum tolerance bound is $\bar{r}_N^*=(1+\delta\times\rho-2\sqrt{\delta\times\rho(1-\rho)})/[(1-\delta\times\rho)^2+4\delta\times\rho^2]$.*

When a crowdfunding project is feasible ($r<\bar{r}_N^*$), we can maximize its success rate by choosing an optimal profit allocation mechanism. By denoting the optimal share of return allocated to C_1 as α_N^*, we have Theorem 2 which shows the profit allocation strategy of the entrepreneur.

Theorem 2. *The success rate S_N in the two-cohort model reaches its maximum at α_N^*, which is equal to $\frac{1}{2}$ when $\rho=1/(2+\delta)$, and is equal to*

$$\frac{(1+\delta)\rho-(1-\rho)\rho r}{(2+\delta)\rho-1}-\frac{1}{(2+\delta)\rho-1}$$
$$\times\left[(1-2\rho+\rho^2)\rho^2r^2-(1-\rho)(\delta\rho+1)\rho r+(1+\delta)(1-\rho)\rho\right]^{1/2}.$$

when $\rho\neq 1/(2+\delta)$

Proposition 4. *The entrepreneur should adjust the optimal profit allocation mechanism when ρ, δ and r changes:*

(i) The optimal share of return α_N^* allocated to C_1 increases in δ.

(ii) The optimal share of return α_N^* allocated to C_1 increases in r when $\rho>1/(2+\delta)$, and decreases in r when $\rho<1/(2+\delta)$.

As we can see from Theorem 2, the optimal α_N^* is jointly decided by δ, and r. Propositions 4 describes the monotonicity of α_N^* in δ, r. Intuitively, the result of Proposition 4 (i) coincides with the δ-effect. It is straightforward that the entrepreneur needs to compensate investors in the first cohort with more return when their waiting cost increases.

Unlike the basic model, where the optimal share of return allocated to the first investor is simply increasing in r, the monotonicity of α_N^* in r is complicated in the two-cohort case. We can explain the result of Proposition 4 (ii) as follows. First, when ρ is large, the cumulated δ-effect of C_1 is massive due to its large size. It is important to remember that the δ-effect results in an additional waiting cost of $\delta\times R\times p$ for each investor in the first cohort, and thus, if r increases, the entrepreneur tends to compensate the first cohort with more return to enhance the success rate of the project, and therefore, α_N^* is increased. Second, when ρ is small, the cumulated δ-effect of C_1 is minor. If r increases, since the proposal is less attractive to all the investors, the entrepreneur prefers to give more return to C_2 (the cohort with more investors) to enhance the success rate, therefore, α_N^* is decreased.

Following Proposition 4 (ii), we can investigate the detailed profit allocation strategy of the entrepreneur under different values of ρ. The results are shown in Theorem 3.

Theorem 3. *There exists a cohort ratio threshold $\rho^*=\big(1+\delta-r\big)/\big(2+\delta-2r\big)>1/2$ such that:*

(i) If $\rho=\rho^*$, then $\alpha_N^*=\rho$, that is, the entrepreneur will not motivate any cohort;

(ii) If $0<\rho<\rho^*$, then $\alpha_N^*>\rho$, that is, the entrepreneur should motivate C_1;

(iii) If $\rho^*<\rho<1$, then $\alpha_N^*<\rho$, that is, the entrepreneur should motivate C_2.

It is important to remember that the δ-effect indicates that the entrepreneur takes sides with the first cohort. Furthermore, due to the scale-effect, the entrepreneur tends to motivate the smaller cohort. Thus, we can claim that there exists a ratio threshold ρ^* at which the effects of scale and waiting cost cancel each other out, and ρ^* is larger than 1/2. When $\rho<\rho^*$, the entrepreneur will motivate the first cohort, while when $\rho>\rho^*$, the entrepreneur will motivate the second cohort. In particular, when $\rho=1/2<\rho^*$, we have that $\alpha_N^*>\rho=1/2$, which is consistent with the result in Theorem 1.

We now illustrate the results of Proposition 4 (ii) and Theorem 3 through a numerical example in Fig. 3. In the rectangular coordinates, the vertical axis represents the share of return allocated to C_1, and the horizontal axis represents the ratio of cohort C_1. The diagonal dotted line represents the straight line of $\alpha=\rho$ on which the entrepreneur motivates neither cohort, and the return is evenly distributed to each investor. The solid curve associates with the optimal α_N^* for different values of ρ. It is clear that if $\rho<\rho^*$, the solid line is above the dotted line, that is, $\alpha_N^*>\rho$, thus, the entrepreneur should motivate C_1 to maximize the success rate of the project. On the contrary, if $\rho>\rho^*$, we have that $\alpha_N^*<\rho$ and the entrepreneur should motivate C_2. According to Fig. 3, one

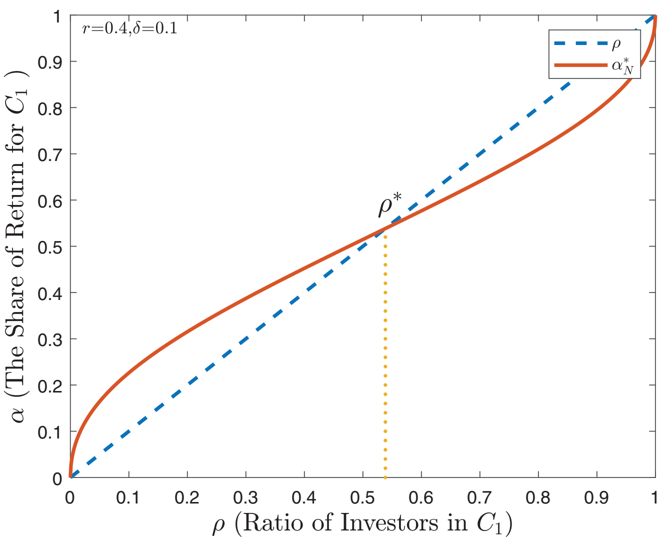


Fig. 3. The optimal α to maximize the success rate with different values of ρ.

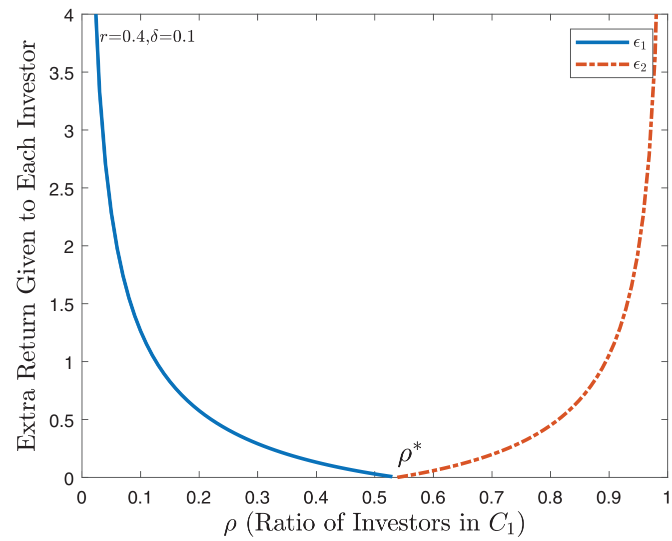


Fig. 4. Additional incentive allocated to each investor with different values of ρ.

can easily decide the optimal profit allocation mechanism to maximize the success rate for a given crowdfunding project.

As we can see from Theorems 1 and 3, the profit allocation strategies in the two-investor and two-cohort models are different due to the existence of the scale-effect. In order to eliminate the impacts of scales, we now study how the extra return received by each investor changes with ρ. The results are shown in Proposition 5. For preparation, according to Theorem 3, when $\rho < \rho^*$, the first cohort is motivated and each investor in C_1 gets an extra incentive of $\epsilon_1 = (\alpha_N^*(\rho, \delta, r) - \rho)/\rho$, while when $\rho > \rho^*$, the second cohort is motivated and each investor in C_2 gets an extra incentive of $\epsilon_2 = (\rho - \alpha_N^*(\rho, \delta, r))/\big(1 - \rho\big)$.

Proposition 5. *Let ρ^* be the ratio threshold given in Theorem 3, we have that the following:*

(i) if $\rho < \rho^*$, then $\epsilon_1 > 0$ and decreases in ρ; (ii) if $\rho > \rho^*$, then $\epsilon_2 > 0$ and decreases in $1 - \rho$.

Proposition 5 indicates that in order to maximize the success rate of the project, if cohort C_i is motivated, the average-extra return received by an individual investor in C_i always decreases in the size of C_i. To be specific, it is shown that ϵ_1 is decreasing in ρ and ϵ_1 is decreasing in $1 - \rho$. This is exactly the scale-effect that we introduced in the beginning of this section, that is, the entrepreneur takes sides with a cohort of smaller size. In particular, when $\rho = \rho^*$, we have that $\epsilon_1 = \epsilon_2 = 0$, which indicates that the entrepreneur will motivate neither cohort.

We still adopt the numerical example used in Fig. 3 to illustrate the results of Proposition 5. In Fig. 4, the horizontal axis represents the size ratio of C_1, and the vertical axis represents the average-extra incentive received by an investor. The left-hand side and right-hand side curves denotes the "$\rho \sim \epsilon_1$" and "$\rho \sim \epsilon_2$" functions, respectively. These two functions intersect at point $(\rho^*, 0)$ at which no incentive mechanism is applied and the success rate of the project is maximized.

In practice, many entrepreneurs prefer to motivate a small group of early investors in their projects. For example, many projects on Kickstarter, one of the largest crowdfunding websites, choose to offer "Early Bird Specials" to some early-stage individuals. The intuitions behind these actions are intricate, many researchers (e.g., Adam, Wessel, & Benlian 2019; Hooghiemstra & de Buysere 2016) believe that the "Early Bird Specials" can ease off the δ-effect to motivate the early-stage individuals and strengthen the herding effect to attract more later-stage individuals. Note that the early-stage backers are usually of smaller group sizes, according to the scale effect, the entrepreneurs would choose to motivate the smaller group (i.e., the early-stage group) to enhance the success rate. This strengthens the intuitions behind such "Early Bird Specials" mechanisms.

6. Numerical experiments

To assess the robustness of our results, a set of numerical experiments are implemented in this section to study the effects of profit allocation mechanism in more general situations. In Sections 6.1 and 6.2, we show the situation when there are more than two periods in crowdfunding projects and take the herding effect of the investors into consideration. Moreover, as an extension to the assumption in previous models that the valuations of investors are uniformly distributed, we further examine the case when the valuations of the investors are assumed to be normally distributed in Section 6.3.

6.1. Multi-period

We have concluded in Theorem 1 that the entrepreneur should motivate I_1 with a greater return because of the waiting cost. However, in practice, the entrepreneur may divide the whole pledging stage into multiple periods rather than only two. When there are more than two periods, investors with dynamic entry times will face different waiting costs, Section 6.1 studies how to assign the profit to investors to maximize the success rate in this case.

Assume that there are n investors $I_1, I_2, \ldots, I_n$ arriving in n different periods, and denote the share of return for I_i as α_i^n. Consistent with Section 3, we denote the rate of waiting cost of each period as δ. Then, investor I_i needs to wait for $n - i$ periods before the project closes, and the total rate of waiting cost for him is $(n - i) \times \delta$.

Similar to Section 4.1, we can use backward induction to conclude the pledging probability of I_i, denoted as q_i^n, and the success rate S_n:

$$q_i^n = 1 - \frac{[(n-i) \times \delta + \prod_{j=i+1}^{n} q_j^n] \times r}{n \times \alpha_i^n \times \prod_{j=i+1}^{n} q_j^n},$$

$$\sum_{1}^{n} \alpha_i^n = 1, \text{ and } S_n = \prod_{i=1}^{n} q_i^n$$

In the numerical experiments, we let $n = 5$ (given that the common length of a crowdfunding project is in months, dividing the

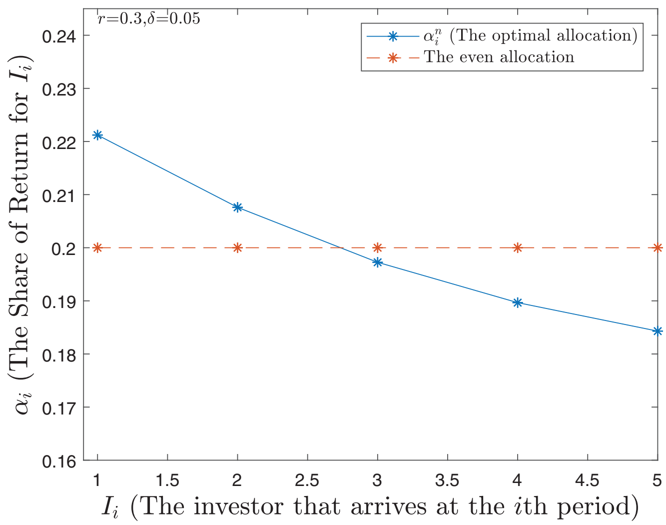


Fig. 5. The optimal profit allocation under five-period crowdfunding.

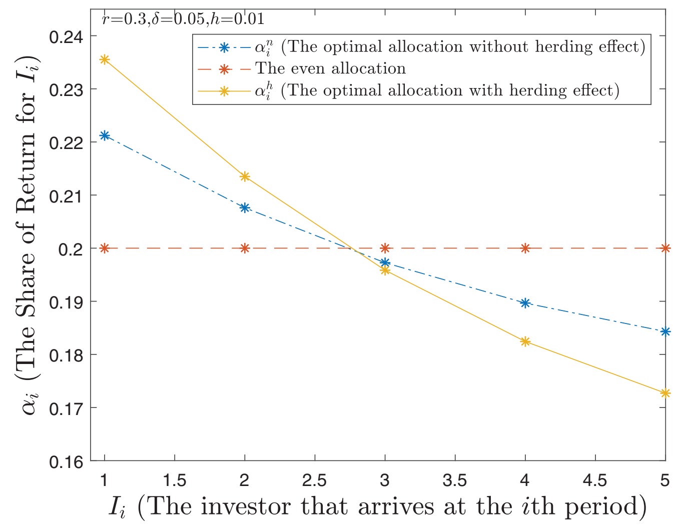


Fig. 6. The optimal profit allocation under five-period crowdfunding with herding effect.

whole period into 5 parts are enough in most situations). The numerical results are shown in Fig. 5.

In Fig. 5, the horizontal axis represents the investor and the vertical axis represents the share of return allocated to each investor; the solid curve represents the optimal profit allocation, maximizing the success rate of the project, for I_i, $i = 1, 2, \ldots, 5$, and the horizontal dotted line is simply the case with even profit allocation to each investor.

According to Fig. 5, the return allocated to I_1 and I_2 in the optimal profit allocation increases compared with the even allocation mechanism, while the share of return allocated to the last three investors is less than the average. Moreover, the share of return allocated to the investors decreases with their entry times. The results are consistent with what we have concluded in Proposition 3.

6.2. Herding effect

Some existing studies (e.g., see Belleflamme et al. 2015; Li & Duan 2016) have shown the existence of positive network externality. In the herding literature, researchers (e.g., see Herzenstein et al. 2011; Lee & Lee 2012) also claimed that investors exhibit herding behaviors in online commerce while facing information asymmetry. Therefore, the utility of an investor may be affected by the decisions of others, and the number of pledged investors can have a positive influence on the later investors. In this part, we will incorporate the herding effect in our studies.

Denote the herding effect of each unit of pledge on an investor as H, then when I_i arrives and finds that there are $i-1$ units of confirmed pledges, the total increase on his utility will be $(i-1)H$. Similar to Section 6.1, by letting $h = \frac{H}{P \times A}$ and denoting the share of return for I_i as α_i^h, we can derive the pledging probability of I_i with herding effect, denoted as q_i^h, and the resulting success rate of the project S_h:

$$q_i^h = 1 - \frac{[(n-i) \times \delta + \prod_{j=i+1}^n q_j^h] \times r}{n \times \alpha_i^h \times \prod_{j=i+1}^n q_j^h} + \frac{(i-1) \times h}{n \times \alpha_i^h \times \prod_{j=i+1}^n q_j^h},$$

$$\sum_1^n \alpha_i^h = 1 \text{ and } S_h = \prod_{i=1}^n q_i^h$$

We still let $n = 5$ in our experiments, and the numerical results are shown in Fig. 6, where the horizontal axis represents the investor and the vertical axis represents the share of return allocated to I_i; the solid curve represents α_i^h, i.e., the optimal profit allocation for the investors; the horizontal dotted line is still the case with even profit allocation; and the piece-wise-dotted line represents α_i^n, i.e., the optimal profit allocation for the investors with no herding effect.

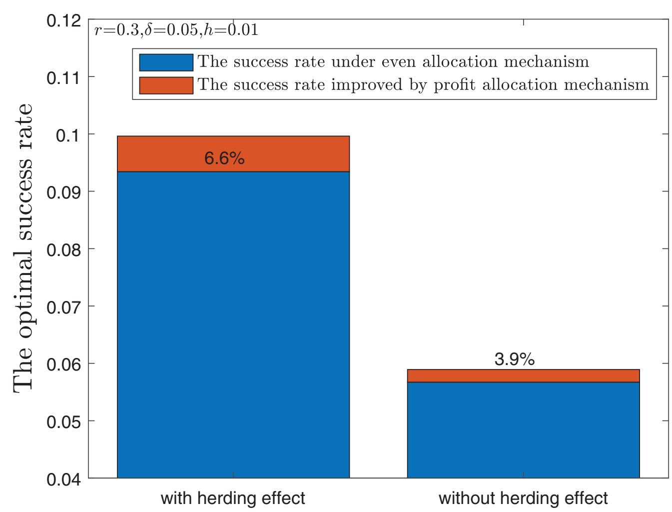


Fig. 7. The maximum success rate with and without herding effect.

As shown in Fig. 6, the existence of the herding effect does not affect the monotonicity of α_i^h in i, i.e., the entrepreneur should still allocate more returns to the earlier investors. In fact, by comparing α^h with α^n, we can see that the herding effect further strengthens the importance of the early investors, and the entrepreneur should allocate even more share of returns to them.

Moreover, as we can see from Fig. 7, with the herding effect, the success rate of the project is higher under some given profit allocation mechanism. Particularly, the improvement of the success rate by adopting the optimal profit allocation mechanism, instead of the even allocation method, also increases. When there is no herding effect, the optimal success rate of the project by adopting the optimal profit allocation is increased by 3.9%, while it is improved by 6.6% when herding effect exists.

To conclude, the existence of the herding effect strengthens the influence and importance of early investors. When the entrepreneur designs an optimal profit allocation to motivate these early investors, the increase of their pledging probabilities will have a positive effect on all the later investors. The herding effect, together with the effect of waiting cost (the key motivation of

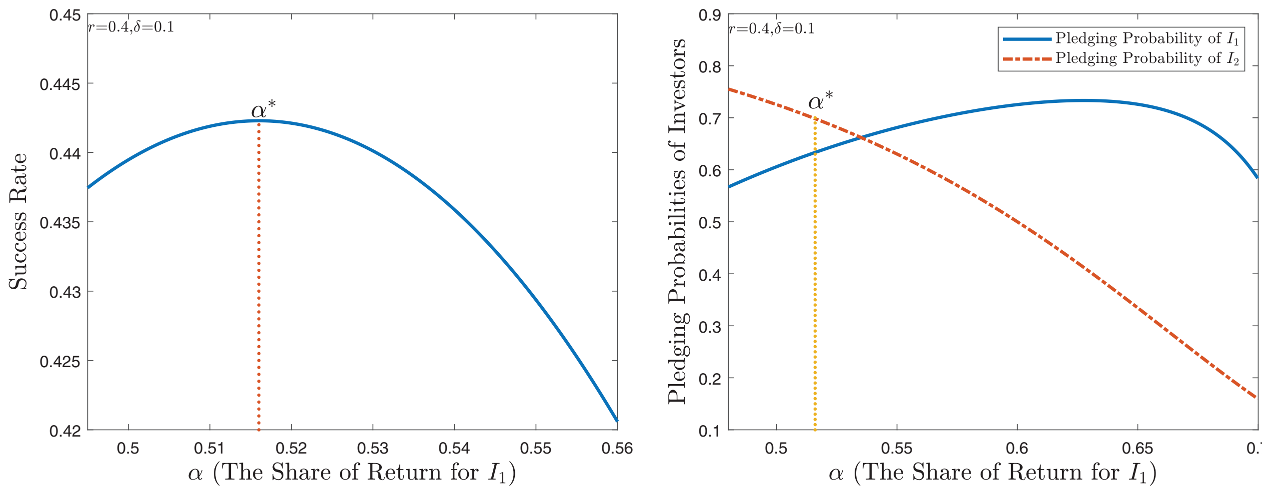


Fig. 8. Pledging probability of investors and success rate with normally distributed valuations.

asymmetry profit allocation in previous sections), encourages the entrepreneur to allocate more returns to the early investors.

6.3. Normal distribution

In Sections 4 and 5, we assumed that the valuations of the investors are uniformly distributed over internal [0,1], and studied the cases of two-investor and two-cohort, respectively. To assess the robustness of our results, we now replace the assumption of uniform distribution with a normal distribution $N(\frac{1}{2}, \frac{1}{6})$ over [0,1]. The values of mean μ and standard variation σ are chosen to ensure that $[\mu - 3\sigma, \mu + 3\sigma] \subseteq [0, 1]$. For simplification, under the assumption of normally distributed valuations, we only show the numerical results of the two-investor case, and the numerical results we derived for the two-cohort case are consistent with the theoretical results in Section 5.

Similar to Section 4, by denoting the cumulative distribution function of $N(\frac{1}{2}, \frac{1}{6})$ as $\psi'(x)$, we can analyze the behaviors of investors by comparing their expected return from pledging and the expected risk-free return. We can express the pledging probability of I_i, denoted as q_i', and the success rate S' as

$$q_1' = 1 - \psi'\left(\frac{\delta r(1-\alpha)}{2\alpha(1-\alpha) - \alpha r} - \frac{r}{2\alpha}\right),$$

$$q_2' = 1 - \psi'\left(\frac{r}{2(1-\alpha)}\right), \text{ and } S' = q_1' \times q_2', \text{ respectively.}$$

The numerical results of S' and q_i' in α are shown in Fig. 8. It is clear that the shapes of the curves are similar to those in Fig. 2. Specifically, the pledging probability of I_1 is unimodal in the share of return allocated to him; the success rate is unimodal in α. These numerical results are consistent with Proposition 2 and Theorem 1. Therefore, the entrepreneur should still compensate the early investor with more share of return in the profit allocation mechanism

7. Conclusion

Crowdfunding is emerging as an important source of finance for small start-ups and new entrepreneurs, and its market size has grown enormously in recent years. Note that success rate is the core problem in crowdfunding, especially in investment-based crowdfunding, where investors receive a financial return. It is well recognized that performance in the early stage of a crowdfunding project is crucial to its success, while investors are less willing to take on the higher risk of pledging earlier. Therefore it is intuitive to offer incentives to investors.

Instead of offering additional benefits during the project to motivate investors like in past literature, this paper studies how an entrepreneur should maximize the success rate with the profit allocation mechanism in investment-based crowdfunding. In our study, we stressed the need to provide the appropriate profit allocation to investors with dynamic entry times to enhance the success rate. Our main results show that the existence of the waiting cost, that is, the δ-effect, encourages the entrepreneur to motivate early investors in order to maximize the success rate. However, the entrepreneur also needs to take into account the difference in the sizes of cohorts arriving at different points in time, that is, the scale-effect. The smaller the cohort, the more suitable it is to be motivated. Our results suggest that the entrepreneur takes both, the scale-effect and the δ-effect into consideration while deciding which cohort to motivate. For example, different from the two-investor case, when too many investors arrive in the early stages of crowdfunding, the entrepreneur may choose to motivate the investors coming in later stages, instead.

Moreover, our analyses provide managerial guidance on how the entrepreneur should adjust his optimal profit allocation mechanism according to changes in the market. First, no matter which cohort is motivated, each investor in this cohort should receive more return as the incentive when this cohort becomes smaller (the scale-effect becomes stronger). Second, the entrepreneur should give early investors a greater return when their additional waiting cost increases (the δ-effect becomes stronger). Third, when the risk-free market becomes more competitive over the crowdfunding proposal than before, if the number of investors in the later cohort is very large, the entrepreneur should give them a greater return. Fourth, when there are multiple periods in the project, the share of return allocated to investors in each period should gradually decrease with their entry times. Last, entrepreneurs should increase the extent of asymmetry in profit allocation and allocate more return to early investors when taking the herding effect into consideration.

Crowdfunding, as an important source of finance, needs more attention in future research. One limitation of our research is that we simplify the study by assuming that the valuations of investors are distributed uniformly, while the valuations can be far more complex or even affected by the description and advertisement of entrepreneurs. Further, we did not consider the occasion that investors may strategically delay their pledges. We conduct our studies in a single-project situation, while the efficiency of improving the success rate may be influenced if other projects also adopt profit allocation mechanism. Therefore, it is of interests to further study the general equilibrium resulted from competitions in a more realistic scenario. Moreover, the arrival of investors can be stochastic, so the number of investors is uncertain in reality, and there is also the possibility of overfunding, which can be analyzed in the future.

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Appendix A. Proofs

Proof of Lemma 1. The project is feasible only when the pledging probabilities of both investors are positive. Apparently, $1 > q_2 > 0$ holds when $0 < r < 2(1-\alpha)$. In addition, we find out that $1 > q_1 > 0$ holds when $r^2 - 2[(1-\alpha)(1+\delta)+\alpha]r + 4\alpha(1-\alpha) > 0$, this quadratic polynomial of r is equal to $4\alpha(1-\alpha) > 0$ when $r = 0$; and $-4(1-\alpha)^2\delta < 0$ when $r = 2(1-\alpha)$, respectively, so there exists one root within $(0, 2(1-\alpha))$ and this root is $1 + (1-\alpha)\delta - [1+(1-\alpha)^2\delta^2 + 2(1-\alpha)(\delta-2\alpha)]^{1/2} < 2(1-\alpha)$. Suffice to say that the pledging probabilities of both investors are positive when $r < 1 + (1-\alpha)\delta - [1+(1-\alpha)^2\delta^2 + 2(1-\alpha)(\delta - 2\alpha)]^{1/2}$. Consequently, $\bar{r}(\alpha,\delta) = 1 + (1-\alpha)\delta - [1+(1-\alpha)^2\delta^2 + 2(1-\alpha)(\delta-2\alpha)]^{1/2}$ and the project is feasible when $r < \bar{r}(\alpha,\delta)$. □

Proof of Proposition 1. To analyze the monotonicity of $\bar{r}(\alpha,\delta)$ in α, we take the derivative of $\bar{r}(\alpha,\delta)$ with respect to α and yield:

$$\frac{\partial \bar{r}(\alpha,\delta)}{\partial \alpha} = \frac{(1-\alpha)(\delta^2+2)+(\delta-2\alpha)}{\sqrt{1+(1-\alpha)^2\delta^2+2(1-\alpha)(\delta-2\alpha)}} - \delta$$

We set $f_1(\alpha) = (1-\alpha)(\delta^2+2)+(\delta-2\alpha) - [1+(1-\alpha)^2\delta^2 + 2(1-\alpha)(\delta-2\alpha)]^{1/2}\delta$, then

$$\frac{\partial \bar{r}(\alpha,\delta)}{\partial \alpha} = 0 \Leftrightarrow f_1(\alpha) = 0 \Leftrightarrow$$
$$\alpha = (2+\delta(1+\delta-\sqrt{\delta}))/(4+\delta^2)$$

We can prove that function $f_1(\alpha)$ is strictly decreasing in α

$$\begin{aligned}\frac{df_1(\alpha)}{d\alpha} &= -\delta^2 - 4 + \frac{(1-\alpha)(\delta^2+2)+(\delta-2\alpha)}{\sqrt{1+(1-\alpha)^2\delta^2+2(1-\alpha)(\delta-2\alpha)}} \\ &< -\delta^2 - 4 + \frac{\delta^2+\delta+2}{1+\delta} \quad (\text{because } 0 < a < 1) \\ &< -\delta^2 - 4 + 4 < 0\end{aligned}$$

Define $\bar{\alpha} = \alpha = (2+\delta(1+\delta-\sqrt{\delta}))/(4+\delta^2)$, according to the monotonicity of $f_1(\alpha)$ in α, we can conclude that when $\alpha < \bar{\alpha}$, $f_1(\alpha) > 0$, so $\frac{\partial \bar{r}(\alpha,\delta)}{\partial\alpha} > 0$. In the same way, $\frac{\partial \bar{r}(\alpha,\delta)}{\partial\alpha} < 0$ when $\alpha > \bar{\alpha}$. Thus, for a given δ, $\bar{r}$ is unimodal in α and reached its maximum when $\alpha = \bar{\alpha}$.

Just conclude $\bar{r}(\bar{\alpha},\delta)$ and we have the maximum tolerance bound $\bar{r}^* = \frac{2(\delta+2-2\sqrt{\delta})}{4+\delta^2}$. □

Proof of Theorem 1. Taking the derivative of S with respect to α yields:

$$\frac{\partial S}{\partial \alpha} = \frac{r}{4\alpha^2(1-\alpha)^2}[2\delta\alpha^2 - (4+4\delta-2r)\alpha + 2 + 2\delta - r]$$

Define $f_2(\alpha) = 2\delta\alpha^2 - (4+4\delta-2r)\alpha + 2 + 2\delta - r$, $\frac{df_2(\alpha)}{d\alpha} = 2r - 4 < 0$. Note that $f_2(0) = 2+2\delta-r > 0$ and $f_2(1) = r-2 < 0$, then there exists a maximum point in (0,1) and is equal to $\alpha^* = (2+2\delta-r)/2\delta - [(2-r)(2+2\delta-r)]^{1/2}/2\delta$. We can conclude that function S is unimodal in α. In addition, $f_2(1/2) = \delta/2 > 0$, so $\alpha^* > 1/2$. □

Proof of Proposition 2. Take the derivative of $q_1(\alpha)$ with repest to α yields:

$$\frac{\partial q_1(\alpha)}{\partial \alpha} = \frac{r\left(4\alpha^2(1+\delta) + 4\alpha[r-2(1+\delta)] + (-2+r)[r-2(1+\delta)]\right)}{2\alpha^2(2-2\alpha-r)^2}$$

it's easy to conclude that when $r < \bar{r}^*$, $q_1(\alpha)$ is unimodal in α and reaches its maximum at α^1 where: $\alpha^1 = \frac{2-r+2\delta-\sqrt{r\delta(2-r+2\delta)}}{2(1+\delta)}$ and we can compare α^1 with α^* by analyzing if $\frac{\partial q_1(\alpha^*)}{\partial\alpha} > 0$:

$$\frac{\partial q_1(\alpha^*)}{\partial \alpha} = \frac{2r\delta^2[2(1+\delta)-r]\left([2(2+\delta)-(2+\delta^2)r] - 2\sqrt{(2-r)(2+2\delta-r)}\right)}{[2+2\delta-r-\sqrt{(2-r)(2+2\delta-r)}]^2 \times [2+r\delta-r+\sqrt{(2-r)(2+2\delta-r)}]^2}$$

Remind that $r < \bar{r}^*$ is equal to $(4+\delta^2)r^2 - 4(2+\delta)r + 4$, which is sufficient to prove that $[2(2+\delta)-(2+\delta^2)r] > 2\sqrt{(2-r)(2+2\delta-r)}$. Therefore, $\frac{\partial q_1(\alpha^*)}{\partial\alpha} > 0$, and $\alpha^1 > \alpha^*$. Finally we can conclude that q_1 increases from 1/2 to $\alpha^* > 1/2$, and $q_1(\alpha^*) > q_1(0.5)$. □

Proof of Proposition 3. Taking derivative of α^* with respect to δ and r respectively yields:

$$\frac{\partial \alpha^*}{\partial \delta} = \frac{(2-r)(2+\delta-r-\sqrt{(2-r)(2+2\delta-r)})}{2\delta^2\sqrt{(2-r)(2+2\delta-r)}},$$
$$\frac{\partial \alpha^*}{\partial r} = \frac{2+\delta-r-\sqrt{(2-r)(2+2\delta-r)}}{2\delta\sqrt{(2-r)(2+2\delta-r)}}$$

Note that $2+\delta-r = [(2-r)+(2+2\delta-r)]/2$, so $(2+\delta-r)^2 > (2-r)(2+2\delta-r)$ and $2+\delta-r-\sqrt{(2-r)(2+2\delta-r)} > 0$. Apparently, $\frac{\partial\alpha^*}{\partial\delta}$ and $\frac{\partial\alpha^*}{\partial r}$ are both positive, α^* increases in δ and r. □

Proof of Corollary 1. To make the project feasible:

$q_2^N > 0$ holds when $r < (1-\alpha)/(1-\rho)$

$$q_1^N > 0 \text{ holds when } f_3(r) = (1-\rho)\rho r^2 - [(1-\rho)\alpha + (1-\alpha)\delta\rho + (1-\alpha)\rho]r + \alpha(1-\alpha) > 0$$

$$f_3(0) = \alpha(1-\alpha) > 0,\ f_3\left(\frac{1-\alpha}{1-\rho}\right) = -(1-\alpha)\delta\rho r < 0$$

Therefore, there must be one left root of $f_3(r)$ in $(0, (1-\alpha)/(1-\rho))$. The project is feasible when $r < \bar{r}_N = \bar{r}_N(\alpha) = ((1-\alpha)(1+\delta)\rho + (1-\rho)\alpha - [((1-\alpha)(1+\delta)\rho + (1-\rho)\alpha)^2 - 4\alpha(1-\alpha)\rho(1-\rho)]^{1/2}/2(1-\rho)\rho$.

Taking the derivative of $\bar{r}_N$ with respect to α yields:

$$\frac{\partial \bar{r}_N}{\partial \alpha} = \frac{1}{2(1-\rho)\rho} \times f_4(\alpha)$$

$$f_4(\alpha) = 1 - (2+\delta)\rho - \frac{\alpha\left(1+\delta^2 \times \rho^2 + 2\rho \times \delta(2\rho-1)\right) - \rho(1-\delta+\delta \times \rho(3+\delta))}{\alpha^2\left(1+\delta^2 \times \rho^2+2\rho \times \delta(2\rho-1)\right) - 2\alpha\left(1+\delta^2 \times \rho + \delta(3\rho-1)\right) + (1+\delta)^2\rho^2}$$

$$\frac{df_4(\alpha)}{d\alpha} = -\frac{4\delta \times (1-\rho)^2 \times \rho^2}{\left[\alpha^2\left(1+\delta^2 \times \rho^2 + 2\delta \times \rho(2\rho-1)\right) - 2\alpha \times \rho\left(1+\delta^2\rho+\delta(3\rho-1)\right) + (1+\delta)^2\rho^2\right]^{3/2}} < 0$$

$$f_4(0) = \frac{2(1-\rho)}{(1+\delta)} > 0,\ f_4(1) = -2\rho < 0$$

Note that $f_4(\alpha)$ is decreasing in α and there must exist a point satisfying $f_4(\alpha) = 0$, therefore $\bar{r}_N$ is unimodal in α. Since the expression of $\bar{r}_N$ is very complex, we can conclude the maximum tolerance bound in another way. Note that the project is feasible when $f_3 > 0$, we transform f_3 in the form of α and $f_3(\alpha) = -\alpha^2 + (1 - r + 2\rho r + \delta\rho r)\alpha + (1-\rho)\rho r^2 - \delta\rho r - \rho r$. The project is feasible only when this function has roots, that is, the discriminant $\Delta = (1 + \delta^2\rho^2 - 2\delta\rho + 4\rho^2\delta)r^2 - (2 + 2\delta\rho)r + 1$ is positive. (Note that all the Δ in our appendix is the discriminant of a polynomial instead of the risk-free factor Δ in our model.) The discriminant is positive only when $r < (1 + \delta \times \rho - 2\sqrt{\delta \times \rho(1-\rho)})/[(1 - \delta \times \rho)^2 + 4\delta \times \rho^2]$, therefore the maximum tolerance bound if $\bar{r}_N^* = (1 + \delta \times \rho - 2\sqrt{\delta \times \rho(1-\rho)})/[(1 - \delta \times \rho)^2 + 4\delta \times \rho^2]$. □

Proof of Theorem 2. To maximize the success rate, we conclude S and the derivative of S with respect to α as follows:

$$S_N = [\alpha^2 + \rho r(1 + \delta - (1-\rho)r) - \alpha(1 - (1 - (2+\delta)\rho)) + (1 + \delta - (1-\rho)r)\rho r]/\alpha(\alpha - 1)$$

$$\frac{\partial S_N}{\partial \alpha} = \frac{r}{\alpha^2(1-\alpha)^2} \times f_5(\alpha)$$

$$f_5(\alpha) = \rho(1 + \delta - (1-\rho)r) - 2\rho(1 + \delta - (1-\rho)r)\alpha + ((2+\delta)\rho - 1)\alpha^2$$

$$f_5(0) = \rho(1 + \delta - (1-\rho)r) > 0, \;\; f_5(1) = (1 - \rho r)(\rho - 1) < 0$$

There must exist roots of $f_5(\alpha)$ in (0,1) according to intermediate value theorem. When $\rho = 1/(2+\delta)$, $f_5(\alpha)$ is linear and $\alpha = 1/2$ is its only root, so $\alpha = 1/2$ is the maximum point. When $\rho < 1/(2+\delta)$, $f_5(\alpha)$ is concavely quadratic and maximize at its larger root:

$$\alpha_N^* = \frac{(1+\delta)\rho - (1-\rho)\rho r}{(2+\delta)\rho - 1} - \frac{1}{(2+\delta)\rho - 1}[(1 - 2\rho + \rho^2)\rho^2 r^2 - (1-\rho)(\delta\rho + 1)\rho r + (1+\delta)(1-\rho)\rho]^{1/2}$$

When $\rho > 1/(2+\delta)$, $f_5(\alpha)$ is convexly quadratic and maximize at its smaller root, we can conclude that it is also α_N^*. □

Proof of Proposition 4. To prove (i), we take the derivative of α_N^* with respect to δ:

$$\frac{\partial \alpha_N^*}{\partial \delta} = \frac{\rho(1-\rho)(1-\rho r)}{2[(2+\delta)\rho - 1]^2} \times \left[-2 + \frac{1 + \delta \times \rho - 2\rho(1-\rho)r}{\sqrt{\rho(1-\rho)(1 - \rho \times r)[1 + \delta - (1-\rho)r]}}\right]$$

It's easy to prove that $1 + \delta \times \rho - 2\rho(1-\rho)r > 1 + \delta \times \rho - 2\rho(1-\rho) > 0$ always holds for $\delta > 0$ and $\rho \in (0, 1)$. In addition, $[1 + \delta \times \rho - 2\rho(1-\rho)r]^2 - 4\rho(1-\rho)(1 - \rho \times r)[1 + \delta - (1-\rho)r] = [(2+\delta)\rho - 1]^2 > 0$, therefore, $-2 + \frac{1+\delta\times\rho - 2\rho(1-\rho)r}{\sqrt{\rho(1-\rho)(1-\rho\times r)[1+\delta-(1-\rho)r]}}$ is positive and α_N^* increases in δ.

Moreover, we prove (ii) and take the derivative of α_N^* with respect to r:

$$\frac{\partial \alpha_N^*}{\partial r} = \frac{\rho(1-\rho)}{2[(2+\delta)\rho - 1]} \times \left[-2 + \frac{1 + \delta\rho - 2\rho r + 2\rho^2 r}{\sqrt{(1-\rho)(1-\rho r)(1 + \delta - (1-\rho)r)\rho}}\right]$$

It is obvious that $-2 + \frac{1+\delta\times\rho-2\rho(1-\rho)r}{\sqrt{\rho(1-\rho)(1-\rho\times r)[1+\delta-(1-\rho)r]}} > 0$, when $0 < \rho < 1/(2+\delta)$, $\frac{\partial \alpha_N^*}{\partial r} < 0$. On the contrary, when $1 > \rho > 1/(2+\delta)$, $\frac{\partial \alpha_N^*}{\partial r} > 0$. □

Proof of Theorem 3. We have proved in the proof of Theorem 2 that:

$$S_N = [\alpha^2 + \rho r(1 + \delta - (1-\rho)r) - \alpha(1 - (1 - (2+\delta)\rho)) + (1 + \delta - (1-\rho)r)\rho r]/\alpha(\alpha - 1)$$

$$\frac{\partial S_N}{\partial \alpha} = \frac{r}{\alpha^2(1-\alpha)^2} \times f_5(\alpha)$$

$$f_5(\alpha) = \rho(1 + \delta - (1-\rho)r) - 2\rho(1 + \delta - (1-\rho)r)\alpha + ((2+\delta)\rho - 1)\alpha^2$$

$$f_5(\rho) = (1-\rho) \times \rho \times [(1 + \delta - r) - (2 + \delta - 2r)\rho]$$

Since α_N^* is the only maximum point of function S_N within (0,1), $\frac{\partial S_N(\alpha_N^*)}{\partial \alpha} = 0$, therefore we can conclude whether α_N^* is larger than ρ with the positivity of $\frac{\partial S_N(\rho)}{\partial \alpha}$. It is shown that when $\rho = \frac{1+\delta-r}{2+\delta-2r}$, $f_5(\rho) = 0$, therefore $\frac{\partial S_N(\rho)}{\partial \alpha} = 0$ and $\alpha_N^* = \rho$. When $\rho < \frac{1+\delta-r}{2+\delta-2r}$, $f_5 > 0$, $\frac{\partial S_N(\rho)}{\partial \alpha} > 0$, ρ is on the left side of α_N^*, so $\alpha_N^* > \rho$; in the same way, when $\rho > \frac{1+\delta-r}{2+\delta-2r}$, $\alpha_N^* < \rho$. □

Proof of Proposition 5. (i) When $\rho < \rho^* = \frac{1+\delta-r}{2+\delta-2r}$ and the first cohort is motivated, that is, $\alpha_N^* > \rho$ and $\epsilon_1 > 0$:

$$\frac{\partial \epsilon_1}{\partial \rho} = \frac{\partial(\alpha_N^* - \rho)/\rho}{\partial \rho} = \frac{A_1 - B_1 * C_1}{2\rho[(2+\delta)\rho - 1]^2\sqrt{\rho(1-\rho)(1-\rho r)[1 + \delta - (1-\rho)r]}}$$

$$A_1 = -1 - 6\rho(-1 + r) + r - 2\rho^3 r^2 + 2\rho^2(-2 + 2r + r^2) + \delta^2\rho\left[3 + \rho^2 r - 2\rho(1 + r)\right] - \delta[1 + 3\rho(-3 + r) + 2\rho^3(-1 + r)r + \rho^2(6 + r - 2r^2)]$$

$$B_1 = \left[\delta^2\rho + (2 - r)\rho + \delta\rho(3 - r)\right]$$

$$C_1 = 2\sqrt{\rho}(1-\rho)(1-\rho r)[1 + \delta - (1-\rho)r]$$

Thus, we only need to proof $A_1 - B_1 \times C_1 \le 0$, We can write $A_1 - B_1 \times C_1$ as $A_1 - B_1 \times D_1 - B_1 \times (C_1 - D_1)$, where $D_1 = 1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r$, according to our proof in the earlier proposition, obviously $C_1 \le D_1$, $B_1 > 0$, so $B_1 \times (C_1 - D_1) \le 0$, and we can conclude $A_1 - B_1 \times D_1 = -[-1 + (2+\delta)\rho]^2 \times [1 + \delta + (-1+\rho)r] \le 0$ after simplification. So $A_1 - B_1 \times C_1 \le 0$ is equivalent to $(A_1 - B_1 \times D_1)^2 \ge B_1^2 \times (C_1 - D_1)^2$.

$$(A_1 - B_1 \times D_1)^2 - B_1^2 \times (C_1 - D_1)^2 = [-1 + (2+\delta)\rho]^4 \times [1 + \delta + (-1+\rho)r]^2 - (1+\delta)^2 \times \rho^2 \times (2 + \delta - r)^2 \times [1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r - 2\sqrt{\rho(1-\rho)(1-\rho r)[1 + \delta - (1-\rho)r]}]^2$$

Implementing the formula for the difference of squares:

Since

$$[-1 + (2+\delta)\rho]^2 \times [1 + \delta + (-1+\rho)r] + (1+\delta) \times \rho \times (2 + \delta - r) \times \left[1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r - 2\sqrt{\rho(1-\rho)(1-\rho r)[1 + \delta - (1-\rho)r]}\right] > 0$$

Thus, we only need to prove:

$$M = [-1 + (2+\delta)\rho]^2 \times [1 + \delta + (-1+\rho)r] - (1+\delta) \times \rho \times (2 + \delta - r) \times \left[1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r - 2\sqrt{\rho(1-\rho)(1-\rho r)[1 + \delta - (1-\rho)r]}\right]$$
$$= [-1 + (2+\delta)\rho]^2 \times \left[[1 + \delta + (-1+\rho)r] - \frac{(1+\delta) \times \rho \times (2+\delta-r)}{1+\delta\times\rho - 2\rho\times r + 2\rho^2\times r + 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right] \ge 0$$

We divide our proof into two parts:

Part I When $0 < \rho < \frac{1}{2+\delta}$, because $(1+\delta+(-1+\rho)r)-(1+\delta)\times\rho\times(2+\delta-r) = -(-1+(2+\delta)\rho)(1+\delta-r)$, then $(1+\delta+(-1+\rho)r) > (1+\delta)\rho(2+\delta-r)$ under this condition.

To prove $M>0$, we scale M as follow:

$$
\begin{aligned}
M \geq M_1 &= [-1+(2+\delta)\rho]^2 \times \\
&\left[(1+\delta)\times\rho\times(2+\delta-r) - \frac{(1+\delta)\times\rho\times(2+\delta-r)}{1+\delta\times\rho-2\rho\times r+2\rho^2\times r+2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right] \\
&= [-1+(2+\delta)\rho]^2\times(1+\delta)\times\rho\times(2+\delta-r)\times \\
&\left[1-\frac{1}{1+\delta\times\rho-2\rho\times r+2\rho^2\times r+2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right]
\end{aligned}
$$

$$
\begin{aligned}
M \geq 0 \Leftarrow M_1 > 0 &\Leftarrow 1+\delta\times\rho-2\rho\times r+2\rho^2\times r+2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]} > 1 \\
&\Leftarrow \rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]-(\delta\times\rho-2\rho\times r+2\rho^2\times r)^2 > 0 \\
&\Leftarrow 4\delta(-1+\rho)+\delta^2\times\rho+4(-1+\rho+r-\rho\times r) < 0 \\
&\Leftarrow \rho < \frac{4+4\delta-4r}{(2+\delta)^2-4r} \\
&\Leftarrow \frac{4+4\delta-4r}{(2+\delta)^2-4r} > \frac{1}{2+\delta} \\
&\Leftarrow r < 1 < \frac{(2+\delta)^2}{4(1+\delta)}
\end{aligned}
$$

Consequently $M \geq 0$ and $\frac{\partial(\alpha_N^*-\rho)/\rho}{\partial\rho} \geq 0$. Thus, we can conclude $\frac{\partial\epsilon_1}{\partial\rho} \geq 0$ when $0 < \rho < \frac{1}{2+\delta}$.

Part II When $\frac{1}{2+\delta} < \rho < \frac{1+\delta-r}{2+\delta-2r}$, then we have $(1+\delta+(-1+\rho)r) > \rho\times(2+\delta-r)$ under this condition. To prove $M>0$, we scale M as follow:

$$
\begin{aligned}
M > M_2 &= [-1+(2+\delta)\rho]^2 \times \\
&\left[\rho\times(2+\delta-r) - \frac{(1+\delta)\times\rho\times(2+\delta-r)}{1+\delta\times\rho-2\rho\times r+2\rho^2\times r+2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right] \\
&= [-1+(2+\delta)\rho]^2\times\rho\times(2+\delta-r)\times \\
&\left[1-\frac{(1+\delta)}{1+\delta\times\rho-2\rho\times r+2\rho^2\times r+2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right]
\end{aligned}
$$

$$
\begin{aligned}
M \geq 0 \Leftarrow M_2 > 0 &\Leftarrow 1+\delta\times\rho-2\rho\times r+2\rho^2\times r+2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]} > 1+\delta \\
&\Leftarrow 4\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]-(\rho-1)^2(\delta+2\rho\times r)^2 > 0 \\
&\Leftarrow \delta^2(\rho-1)+4\rho(1-r)+4\delta\times\rho\times(1-r) > 0 \\
&\Leftarrow \frac{\delta^2}{(4+4\delta+\delta^2-4r-4\delta\times r)} < \rho < 1 \\
&\Leftarrow \frac{\delta^2}{(4+4\delta+\delta^2-4r-4\delta\times r)} < \frac{1+\delta-r}{2+\delta-2r} \\
&\Leftarrow r < 1 < \frac{4+8\delta+3\delta^2}{4+4\delta}
\end{aligned}
$$

Consequently $M \geq 0$ and $\frac{\partial(\alpha_N^*-\rho)/\rho}{\partial\rho} \geq 0$. Thus, we can conclude $\frac{\partial\epsilon_1}{\partial\rho} \geq 0$ when $\frac{1}{2+\delta} < \rho < \frac{1+\delta-r}{2+\delta-2r}$. So far we have proved that when C_1 is motivated, $\frac{\partial\epsilon_1}{\partial\rho} \geq 0$, and we next prove the case when C_2 is motivated.

(ii) When $\rho > \rho^* = \frac{1+\delta-r}{2+\delta-2r}$ and the second cohort is motivated, that is, $\alpha_N^* < \rho$ and $\epsilon_2 > 0$:

$$
\frac{\partial\epsilon_2}{\partial\rho} = \frac{\partial(\rho-\alpha_N^*)/(1-\rho)}{\partial\rho} = -\frac{A_2-B_2*C_2}{2(1-\rho)[(2+\delta)\rho-1]^2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}
$$

$$
\begin{aligned}
A_2 &= 1-r+2\rho^3r^2+\delta^2\rho(1-2\rho+\rho^2r)-4\rho^2(1-r+r^2) \\
&\quad -2\rho(-1+r-r^2)+\delta\left[1+3\rho^2(-2+r)+2\rho^3r+3\rho(1-r)\right] \\
B_2 &= (1-\rho)(2-r+\delta) \\
C_2 &= 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}
\end{aligned}
$$

Thus, we only need to prove $A_2 - B_2 \times C_2 \le 0$. We can write $A_2 - B_2 \times C_2$ as $A_2 - B_2 \times D_2 - B_2(C_2 - D_2)$, where $D_2 = 1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r$, according to our proof in an earlier proposition, obviously $C_2 \le D_2$, $B_2 > 0$, so $B_2(C_2 - D_2) \le 0$, and we can conclude $A_2 - B_2 \times D_2 = -[(2+\delta)\rho - 1]^2(1 - \rho \times r) \le 0$ after simplification. So $A_2 - B_2 \times C_2 \le 0$ is equivalent to $(A_2 - B_2 \times D_2)^2 \ge B_2^2(C_2 - D_2)^2$.

$$(A_2 - B_2 \times D_2)^2 - B_2^2(C_2 - D_2)^2 = [(2+\delta)\rho - 1]^4(1-\rho r)^2 - (1-\rho)^2(2+\delta-r)^2 \times \left[1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r - 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}\right]^2$$

Implementing the formula for the difference of square:

Since

$$[(2+\delta)\rho - 1]^2(1-\rho r) + (1-\rho)(2+\delta-r) \times \left[1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r - 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}\right] > 0$$

Thus, we only need to prove

$$\begin{aligned} M_3 &= [(2+\delta)\rho - 1]^2 \times (1 - \rho \times r) - (1-\rho)(2+\delta-r) \\ &\quad \times \left[1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r - 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}\right] \\ &= [(2+\delta)\rho - 1]^2 \times \\ &\quad \left[(1-\rho \times r) - \frac{(1-\rho)(2+\delta-r)}{1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r + 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right] \ge 0 \end{aligned}$$

When $\rho > \rho^* = \frac{1+\delta-r}{2+\delta-2r}$, we have $(1-\rho)(2+\delta-r) < 1 - \rho \times r$.

$$\begin{aligned} M_3 > M_4 &= [(2+\delta)\rho - 1]^2 \times \\ &\quad \left[(1-\rho)(2+\delta-r) - \frac{(1-\rho)(2+\delta-r)}{1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r + 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right] \\ &= [(2+\delta)\rho - 1]^2 \times (1-\rho) \times (2+\delta-r) \times \\ &\quad \left[1 - \frac{1}{1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r + 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]}}\right] \end{aligned}$$

$$\begin{aligned} M_3 > 0 \Leftarrow M_4 > 0 &\Leftarrow 1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r + 2\sqrt{\rho(1-\rho)(1-\rho r)[1+\delta-(1-\rho)r]} > 1 \\ &\Leftarrow 1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r + 2(1-\rho)\sqrt{\rho(2+\delta-r)[1+\delta-(1-\rho)r]} > 1 \\ &\quad (\text{Because } (1-\rho)(2+\delta-r) < 1 - \rho \times r \text{ when } \rho > \rho^* = \frac{1+\delta-r}{2+\delta-2r}) \\ &\Leftarrow 1 + \delta \times \rho - 2\rho \times r + 2\rho^2 \times r + 2(1-\rho)[1+\delta-(1-\rho)r] > 1 \\ &\quad (\text{Because } \rho(2+\delta-r) > [1+\delta-(1-\rho)r] \text{ when } \rho > \rho^* = \frac{1+\delta-r}{2+\delta-2r}) \\ &\Leftarrow \rho < 1 < \frac{2\delta+2-2r}{\delta+2-2r} \end{aligned}$$

Consequently $M_3 \ge 0$ and $\frac{\partial(\rho - \alpha_N^*)/(1-\rho)}{\partial \rho} \ge 0$. Thus, we can conclude that $\frac{\partial \epsilon_2}{\partial \rho} \ge 0$ when $\rho > \rho^* = \frac{1+\delta-r}{2+\delta-2r}$. □

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