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Logistics cost optimisation and allocation for additive manufacturer make-to-order cooperation

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ABSTRACT

Additive manufacturing is a key technology in cloud and smart manufacturing due to its high level of automation and customisation. Advancements in manufacturing technology have reshaped the traditional make-to-order model, enabling better resource coordination, supply chain optimisation, and production flexibility. This paper tackles cooperative logistics optimisation for additive manufacturing by targeting systematic cost reduction and fair allocation. We develop a mixed-integer linear programming model that integrates production, inventory, and transportation costs while optimising order assignment, inventory transshipment, and centralised raw material procurement. Two coalition models for cost allocation are examined: one involving both manufacturers and retailers and another comprising only additive manufacturing manufacturers. The former may lead to empty core scenarios, whereas the latter guarantees a stable cost allocation mechanism. To address the empty core issue, we propose an exact row-generation algorithm along with a hybrid approach that combines linear and Lagrangian relaxations. Experiments reveal that the exact method suits small-scale problems, whereas the hybrid approach performs better on large-scale problems, offering valuable insights into logistics cost optimisation for cooperative make-to-order additive manufacturing.

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Logistics optimisation; cooperative game; optimal cost allocation problem; row generation; Lagrangian relaxation

1. Introduction

Additive manufacturing (AM), also known as 3D printing, is an industrial process that deposits materials layer by layer to create geometric 3D objects. It possesses notable characteristics such as design flexibility and rapid prototyping. Since the early 21st century, breakthroughs in AM equipment, advanced materials, and process optimisation have transformed this technology from experimental concept to industrial-scale feasibility. In fact, it has achieved widespread adoption in various fields, including the aerospace, automotive industry, medical sector, and so on (Delic and Eyers 2020; Patel and Gohil 2021).

AM introduces transformative shifts compared to traditional manufacturing methods. First, AM enables a broader range of individuals and businesses to design and manufacture highly complex, customised products, significantly lowering entry barriers. This accessibility drives demand for low-volume, customised items that were previously not economically viable under traditional mass-production paradigms. Second, AM inherently supports distributed manufacturing. Its standardised and reproducible processes facilitate collaborative production among manufacturers wherever raw materials are available, eliminating dependence on a single manufacturer. This collaboration makes production close to demand, reducing lead times and enhancing responsiveness to dynamic market demands. Although AM operates on a make-to-order (MTO) basis, it addresses the high costs and long lead times associated with traditional MTO models, which often require resource-intensive mold adjustments and production line reconfigurations. The medical sector exemplifies AM's potential for collaborative manufacturing, with applications ranging from patient-specific implants and orthotic devices to surgical instruments and diagnostic tools. For instance, hospitals are increasingly utilising AM to produce custom prosthetics and surgical guides on-demand (Culmone, Smit, and Breedveld 2019).

In addition to its differences from traditional MTO production models, collaborative AM systems introduce significant changes in logistics costs. First, since transportation costs for finished products are typically higher than those for raw materials, collaborative production among additive manufacturers offers substantial potential to reduce transportation expenses. Second, beyond one-time equipment costs and uniform material costs, AM production costs include material procurement

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expenses. Collaborative procurement of materials and raw material transfers can further optimise procurement costs. Third, given the stringent storage requirements for AM raw materials, inventory costs also constitute a significant component of total costs. Therefore, it is essential to examine the relationships between inventory costs, transportation costs, and raw material procurement costs in a collaborative AM system.

In the AM industry, in addition to additive manufacturers, there are two other key members: the AM equipment supplier and retailers. The AM equipment supplier acts as a headquarter, providing equipment and materials to the manufacturer, while retailers represent the demand side with orders to be fulfilled. Across the AM supply chain, the incentive to form collaborations with downstream manufacturers and retailers is not immediately obvious, as equipment suppliers often operate as oligopolies in material supply. However, by working together, both AM manufacturers and retailers can gain significant advantages. AM manufacturers and product retailers can coordinate their production and logistics plans under the leadership of a centralised decision maker. This centralised approach enables bulk purchasing, coordinated by the head office, to leverage economies of scale. It also facilitates inter-manufacturer material redistribution to mitigate localised shortages and optimise inventory utilisation. On the one hand, retailers gain access to a strong, stable, and cost-effective production channel. On the other hand, manufacturers can optimise their production plans based on demand information provided by retailers.

Although cooperation in additive manufacturing has attracted academic attention (Delic and Eyers 2020; Kapadia et al. 2022; Zehetner and Gansterer 2022), there is a notable lack of research on cooperative games specifically targeting AM. Unequal cost allocation tends to hinder the formation of stable alliances. Achieving stability in cooperation depends on establishing a benefit or cost allocation mechanism between manufacturers and retailers that theoretically removes each party's incentive to deviate from the larger alliance. Inappropriate or unfair cost allocations may motivate some manufacturers to depart from the largest alliances and form smaller, less efficient groups, ultimately impeding the achievement of optimal global cost reductions. Thus, achieving cooperation in AM technology fundamentally depends on the rational allocation of costs to support the formation and stability of the most efficient alliances. The cooperative game provides a valuable methodological framework for addressing the problem of cooperative stability.

This paper addresses the challenge of cost allocation in order assignment, order delivery, and raw material transportation and procurement cooperation among multi-site additive manufacturers and retailers. To foster collaboration and ensure coalition stability, we develop cost allocation methods that equitably allocate the total system cost upon the completion of production and distribution. Our key contributions are as follows.

First, we formulate the AM cooperative problem involving manufacturers and retailers as a mixed integer linear programming (MILP) model that integrates production, inventory, and transportation costs while optimising order assignment, inventory transshipment, and centralised raw material procurement. Based on this MILP model, we analyze the cooperative game of AM from the perspective of logistics cost allocation. Our work relates to two existing papers Delic and Eyers (2020) and Saavedra-Nieves (2020). Delic and Eyers (2020) examines AM order allocation by matching manufacturers with customers based on preference lists. Saavedra-Nieves (2020) investigates a joint purchasing logistics system using an economic order quantity inventory model with a λ-proportional allocation rule. The difference between theirs and our work is that we focus on cost allocation decisions within an AM integrated inventory–transportation system. In addition, we provide an in-depth analysis of cost allocation mechanisms in cooperative delivery scenarios, with an emphasis on logistics system optimisation and cost allocation strategies. We explore two coalition models for cost allocation: one involving both manufacturers and retailers and another comprising only additive manufacturing manufacturers. The former may lead to empty core scenarios, whereas the latter ensures a stable cost allocation mechanism.

Second, to mitigate the empty core issue, we introduce an optimal cost allocation problem (OCAP) model, wherein the coalition stability constraint grows exponentially with the number of manufacturers. Recognizing that real-world problem sizes often exceed the computational capabilities of solvers such as Gurobi, we propose an exact row-generation (RG) algorithm alongside a hybrid approach that integrates linear-relaxation-based (LPB) and Lagrangian-relaxation-based (LRB) methods. Through numerical experiments, we evaluate the computational efficiency and effectiveness of our algorithms. The results indicate that while the exact method is suitable for small-scale problems, the hybrid approach exhibits superior performance for large-scale cases.

Third, our numerical studies yield several key managerial insights. Cooperation between retailers and additive manufacturers reduces overall costs by enabling orders to be produced at the lowest-cost facility, with additional savings as more additive manufacturers participate. Transferring raw materials between sites further optimises inventory and lowers holding costs, while centralised procurement enhances supplier negotiations.

Moreover, the relative proportions of transportation costs are critical: when delivery costs dominate, orders should be allocated to facilities near demand points; when raw material transportation costs are higher, sites closer to suppliers are preferable. Finally, the central decision-maker exhibits a preference for additive manufacturers with greater production capacity, as such partners reduce the indivisible costs that the central decision maker must bear.

The remainder of the paper is organised as follows. Section 2 reviews the literature on AM logistics and cooperative games. Section 3 presents the logistics cost optimisation problem and formulates the game models. Section 4 details the solution method for OCAP. Section 5 reports the numerical experiments conducted to validate the proposed model and algorithms. Finally, Section 6 concludes the paper and outlines directions for future research.

2. Literature review

In this section, we present a comprehensive review of the literature focussing on two key areas: logistics cost optimisation in AM make-to-order cooperation and logistics cost allocation. First, we examine the collaborative problem developed to reduce AM logistics costs, including production planning, inventory management, and transportation optimisation. Next, we analyze cost allocation mechanisms that ensure an equitable distribution of logistics costs among stakeholders.

2.1. AM logistics cost optimisation

In the literature on AM Logistics Cost Optimization, the current research primarily focuses on inventory and transportation problems, as well as order assignment and transportation issues.

2.1.1. Inventory and transportation

Within the realm of inventory and transportation, the joint decision-making problem – which integrates inventory management with transportation across various supply chain structures – has attracted significant attention. For example, Mosca, Vidyarthi, and Satir (2019) provide a comprehensive review of inventory-transportation integration problems, categorising studies by topics, characteristics, and practical constraints. In this context, Yang et al. (2021) investigate order-related inventory holding costs, while Li, Chen, and Tang (2017) examine order delivery scheduling with time-window constraints. Similarly, Chaouch (2001) discuss vendor-managed inventory under stochastic demand, and Yokoyama (2002) explore a (t, S) inventory system. In addition, Kutanoglu and Lohiya (2008) analyze a logistics system with a stochastic base inventory and time-varying service level constraints. Although these studies effectively address the interdependence between inventory and transportation, they generally overlook the impact of distributed production on overall logistics performance.

Furthermore, the inventory routing problem explicitly incorporates elements of the travelling salesman problem and the vehicle routing problem. In this context, Coelho, Cordeau, and Laporte (2014) offer a review spanning three decades, classifying the inventory routing problem based on problem structure and customer information availability. Additional studies include Su et al. (2020), which investigates the inventory routing problem in the supply chain of gaseous products, and Fardi, Jafarzadeh Ghoushchi, and Hafezalkotob (2019), which explores robust solutions for cooperative inventory routing and revenue allocation among suppliers. Moreover, Huang and Lin (2010) develop an improved ant colony algorithm for joint multi-product replenishment under demand uncertainty, while Sofianopoulou and Mitsopoulos (2021) review the applications and efficiencies of various heuristic algorithms. A key characteristic of the inventory routing problem is its detailed consideration of transportation, rendering it highly practical yet complex. This complexity poses significant challenges for AM logistics cost optimisation, particularly regarding cost allocation among AM manufacturers. Due to the computational intractability of obtaining exact solutions, most studies rely on heuristic methods. Consequently, directly applying these approaches to the optimal cost allocation problem in AM remains impractical.

2.1.2. Order assignment and transportation

Research on order assignment and production can be broadly categorised into two directions. The first focuses on deterministic settings, where scholars examine logistics factors in order management – such as delivery batching, transportation times, and third-party logistics (Liu, He, and Max Shen 2021; Noroozi et al. 2018; Tarhan and Oğuz 2021). The second addresses uncertainties in order management, including uncertain demand, transportation capacity, and order arrivals (Aouam et al. 2018; Leng et al. 2021; Lutter and Werners 2015). Additional studies investigate capacity planning, multi-objective trade-offs, and supplier selection within order decision-making.

With the advancement of AM technology and its growing industrial applications, research on order acceptance and assignment in AM has also gained prominence. For instance, Li et al. (2019) aim to maximise average profit per unit time, while Wu et al. (2022) focus on minimising material production costs per unit volume. In another study, Darwish, El-Wakad, and Farag (2021) investigate adaptive real-time multitasking with robustness considerations. These works primarily emphasise production costs from a factory-centric perspective, aiming to enhance capacity and reduce manufacturing expenses.

Regarding product transportation, De Falco, Mastrandrea, and Rarità (2019) employ a disaggregation and localisation approach for order tasks to minimise total manufacturing and logistics costs. Likewise, Demir, Eyers, and Huang (2021) explore multi-phase production and delivery planning in urban logistics, and Gao, Yuan, and Cui (2024) examine scenarios where a mobile AM centre schedules production in transit. In our study, we assume that the delivery cost of finished products is predetermined, focussing instead on ensuring sufficient capacity to fulfil customer orders on time. Our primary concern lies in order allocation while considering the overall costs of inventory, transportation, and production within the cooperative process.

A related study by Zhong et al. (2022) shares similarities with our cooperative framework, addressing a bi-objective optimisation problem involving transportation costs and printer load balancing in distributed AM for order assignment and delivery. However, our approach differs in that we explicitly incorporate raw material transfer and procurement considerations. Moreover, while Zhong et al. (2022) employ a deep reinforcement learning approach, our optimisation problem is comparatively simpler and can be solved directly using commercial solvers.

2.2. Logistics cost allocation

Similar to the logistics cost optimisation problem, most studies on logistics cost allocation have primarily focussed on inventory or transportation games, designing allocation rules or employing classical allocation methods to address cost allocation challenges. For instance, Fiestras-Janeiro et al. (2013) propose the average value of the marginal vectors with an extreme agent first as a novel cost allocation rule for logistics costs in joint purchasing. Saavedra-Nieves (2020) investigate joint purchasing in economic order quantity inventory systems with multiple demand locations and introduce a λ-proportional sharing rule. In addition, Guajardo and Rönnqvist (2015) examine joint inventory management under demand discrepancies and propose the MIND allocation method. Özener and Ergun (2008) analyze desirable allocation properties in cooperative transportation scenarios within logistics networks and explore sharing rules based on these properties. Furthermore, Lozano et al. (2013) study horizontal transport integration and apply various allocation rules, such as the Shapley value, the nucleolus, and the core centre, while Frisk et al. (2010) investigate cooperative transportation among forestry companies using similar allocation methods. Finally, Fardi, Jafarzadeh Ghoushchi, and Hafezalkotob (2019) address cost allocation in the multi-warehouse multi-vehicle inventory routing problem by implementing classical rules such as the Shapley value, the τ-value, and the least core.

Unlike straightforward allocation rules that disregard or relax coalition stability conditions, a stream of research analyzes the structural properties of the characteristic function and develops efficient methodologies for computing optimal cost allocations. In recent years, several studies have focussed on large-scale cooperative games and have employed integer programming techniques in cost allocation models – such as the OCAP model and nucleolus-based approaches. For example, Caprara and Letchford (2010) propose an integer minimisation game and provide a comprehensive analysis of solution techniques for large-scale combinatorial optimisation games, introducing the LPB method for cost allocation. Liu, Qi, and Xu (2016) develop a framework based on the LRB method for large-scale empty-core cooperative games, demonstrating its effectiveness in facility location problems. In addition, Lu and Quadrifoglio (2019) propose a column generation-based algorithm to compute the nucleolus or approximate solutions in the ridesharing problem. In this study, we integrate the LPB and LRB methods to develop a hybrid algorithm capable of solving the large-scale OCAP problem in logistics cost optimisation.

2.3. Research gap

AM logistics cost optimisation is a complex, integrated problem that encompasses production, inventory, and transportation. While previous studies have predominantly examined the interactions between production and transportation or between inventory and production, there remains a noticeable gap in research focussing on logistics costs within the AM context – particularly those costs associated with upstream and downstream activities. With respect to cost allocation, existing research often circumvents computational challenges by prescribing allocation rules based solely on the inherent characteristics of the problem. Although these methods provide practical solutions, they frequently fail to ensure the stability of the grand coalition in large-scale settings. In the AM manufacturing sector–which is largely dominated by AM equipment manufacturers and raw material suppliers-enhancing the bargaining power of downstream partners through cooperative strategies is crucial. Therefore, addressing the issue of cooperative cost allocation between AM manufacturing plants and retailers remains an essential yet underexplored research area.

3. Logistics cost optimisation and allocation

This paper investigates the problem of cost optimisation and allocation in an integrated inventory-transportation system featuring cooperative deliveries for additive manufacturing.

In this section, we first introduce the fundamental concepts of cooperative game theory. Next, we develop a logistics cost optimisation model for additive manufacturing in a make-to-order setting. This model accounts for production costs, finished product delivery costs, transfer costs among manufacturers, and the collective replenishment cost of raw materials. Then, we integrate the optimisation model with cooperative game theory by constructing a characteristic function for the joint inventory-transportation system, establishing the foundation for cost allocation within the cooperative framework. Based on the cooperation range, we explore two coalition models: one involving both manufacturers and retailers in Section 3.3 and another comprising only additive manufacturing manufacturers in Section 3.4.

3.1. Preliminaries

Cooperative game theory investigates how to allocate costs among members of a coalition. It emphasises collective rationality in forming coalitions and achieving binding agreements through mutual coordination. Formally, if α_i denotes the share of cost allocated to member i, then the allocation vector is represented as

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_n].$$

A cooperative game is denoted by (V, c), where $V = \{1, 2, \ldots, n\}$ is the set of all players (the grand coalition) and $c : 2^V \setminus \{\emptyset\} \rightarrow \mathbb{R}$ is the characteristic function. For any nonempty coalition $s \subseteq V$, $c(s)$ represents the cooperative cost by minimising costs, thereby imposing a bound on the sum of allocations for members in s.

We now introduce the core, one of the most important solution concepts in cooperative game theory (Kuhn and William Tucker 1953). The core is defined as:

$$\mathrm{Core}(V, c) = \left\{ \boldsymbol{\alpha} \in \mathbb{R}^n : \sum_{k \in s} \alpha(k) \leq c(s), \quad \forall s \in 2^V \setminus \{\emptyset\}, \text{ and } \sum_{k \in V} \alpha(k) = c(V) \right\}. \tag{1}$$

Here, $\boldsymbol{\alpha}$ is the cost allocation vector. The constraints in (1) consist of:

- *Coalition stability constraints*: For every coalition s, the sum of the allocations does not exceed $c(s)$.
- *Budget balance constraint*: The total allocation for the grand coalition V exactly equals $c(V)$.

These constraints ensure that no sub-coalition has an incentive to break away from the grand coalition. In practice, however, the core is often empty – especially in large-scale games. To address the cost allocation problem in such settings, we relax the strict budget balance constraint and focus on the OCAP introduced by Caprara and Letchford (2010). The OCAP is formulated as:

$$\begin{aligned} \max_{\boldsymbol{\alpha}} \quad & \sum_{k \in V} \alpha(k) \\ \text{s.t.} \quad & \sum_{k \in s} \alpha(k) \leq c(s), \quad \forall s \in 2^V \setminus \{\emptyset\}. \end{aligned} \tag{2}$$

The OCAP problem, solved by a centralised decision maker, seeks to maximise the total allocated cost while ensuring coalition stability.

3.2. Optimization problem and mathematical model

In addressing the logistics cost optimisation for AM in a MTO cooperative setting, we assume the presence of a centralised decision maker acting as the coordinating headquarters. This governing entity performs four critical functions:

(1) *Demand Aggregation*: Collecting demand orders from retailers, denoted by $R = \{1, 2, \ldots, r\}$, where each retailer r has a set of orders $O_r = \{1, 2, \ldots, n_r\}$.
(2) *Production Allocation*: Optimizing production allocation among AM facilities $M = \{1, 2, \ldots, m\}$ while considering their respective production capacities C_i and production cost f_{io}.
(3) *Inventory and Transportation Planning*: The central decision-maker integrates real-time inventory data D_i with multimodal logistics cost parameters c_{ij} to design cost-efficient material transfer plans between manufacturers.

(4) *Emergency Procurement*: When transfers are insufficient to meet raw material shortages, the central decision-maker can initiate emergency centralised procurement. A total quantity z_0 is purchased based on the manufacturers' emergency demands z_i, with a unit procurement cost P and a fixed batch size Q.

The overarching objective is to minimise the total cost through synchronised decision-making.

To enhance the effectiveness of the factory-front strategy, orders collected from each retailer are redistributed by the centralised decision maker. The following general assumptions are imposed to ensure a realistic production process:

- Each order is produced by exactly one manufacturer.
- Each manufacturer may accept orders only if the total production time does not exceed its capacity.
- The raw material available at each manufacturer must meet the order demand; otherwise, the centralised decision maker will transfer raw materials from other manufacturers or procure them in an emergency.

We model the AM cooperative problem as a MILP model, denoted by $C(M \cup R)$, which aims to minimise the total cost for producing all orders from the retailers. Detailed parameters and variable notations are summarised in Table 1. The MILP formulation is given by:

$$
\begin{aligned}
C(M \cup R) = \min \quad & \sum_{i\in M}\sum_{r\in R}\sum_{o\in O_r}(f_{io} - h_i d_o)x_{io} + \sum_{i\in M} h_i \left[D_i + \sum_{i\in M}(y_{ji} - y_{ij}) + z_i\right] m_i \\
& + \sum_{i\in M}\sum_{r\in R}\sum_{o\in O_r} c_{io}x_{io} \\
& + \sum_{i\in M}\sum_{j\in M} c_{ij}y_{ij} + \sum_{i\in M} c_i z_i + P\cdot z_0 \qquad (3)\\
= \min \quad & \sum_{i\in M}\sum_{r\in R}\sum_{o\in O_r}(f_{io} + c_{io} - h_i d_o)x_{io} \\
& + \sum_{i\in M} h_i D_i m_i \\
& + \sum_{i\in M}\sum_{j\in M}(c_{ij} - h_i + h_j)y_{ij} \\
& + \sum_{i\in M}(c_i + h_i)z_i + P\cdot z_0
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{r\in R}\sum_{o\in O_r} d_o x_{io} \le \sum_{j\in M}(y_{ji} - y_{ij}) + D_i m_i + z_i, \quad \forall i \in M && (4)\\
& \sum_{r\in R}\sum_{o\in O_r} p_o x_{io} \le C_i m_i, \quad \forall i \in M && (5)\\
& \sum_{i\in M} x_{io} = 1, \quad \forall o \in O_r, r \in R && (6)\\
& y_{ij} \le \mathbf{M} m_i, \quad \forall i,j \in M && (7)\\
& y_{ij} \le \mathbf{M} m_j, \quad \forall i,j \in M && (8)\\
& z_i \le \mathbf{M} m_i, \quad \forall i \in M && (9)\\
& \sum_{i\in M} z_i \le Q z_0, \quad z_0 \in \mathbb{N}^* && (10)\\
& x_{io}, \quad m_i \in \{0,1\}, \ \forall i \in M, \ o \in O_r, \ r \in R && (11)\\
& y_{ij}, \quad z_i \ge 0, \ \forall i,j \in M. && (12)
\end{aligned}
$$

The objective function in (3) minimises the total logistics cost of the integrated inventory-production-transportation system. Specifically:

- The first term aggregates the production cost f_{io} incurred by manufacturer i for order o, while reducing the inventory cost by $h_i d_o$ when the order is accepted.
- The second term captures the holding cost at manufacturer i, which is based on its current inventory D_i and net material transfers $\sum_{j\in M}(y_{ji} - y_{ij})$, in addition to any emergency procurement z_i.
- The third, fourth, and fifth terms correspond respectively to the delivery cost for orders, the transfer cost between manufacturers, and the transportation cost from suppliers.
- Finally, the last term accounts for the fixed procurement cost P associated with emergency raw material procurement.

Constraint (4) ensures that the available raw materials at each AM site – considering net transfers and procurement – satisfy the order requirements. Constraint (5) restricts the production load at each AM facility to within its capacity C_i. Constraint (6) guarantees that each order is produced by exactly one manufacturer. Constraints (7)–(9) imply that transfer and procurement costs are incurred only when the corresponding AM site is active (i.e. $m_i = 1$). Here, the constant $\mathbf{M}$ is set sufficiently high, specifically $\mathbf{M} = \sum_{r\in R}\sum_{o\in O_r} d_o$.

Table 1. List of Notations for Parameters and Variables.

Notation	Definition
Indices	
i, j	Index for an additive manufacturer, $i, j \in M$
r	Index for a retailer, $r \in R$
o	Index for an order, $o \in O_r$
Parameters	
R	The set of retailers, $R = \{1, 2, 3, \ldots, r\}$.
M	The set of manufacturers, $M = \{1, 2, 3, \ldots, m\}$.
M_s	The sub-coalition of manufacturers, $M_s \subseteq M$.
R_s	The sub-coalition of retailers, $R_s \subseteq R$.
O_r	The set of orders from retailer r, $O_r = \{1, 2, 3, \ldots, n_r\}$.
O_R	The set of orders collected from retailers R, $O_R = \bigcup_{r \in R_S} O_r$.
d_o	The raw material consumption of orders o.
p_o	The processing time consumption of orders o.
f_{io}	The processing cost of orders o in manufacturer i.
C_i	The production capacity of manufacturer i, assumed that $\sum_{i \in M} C_i \geq \sum_{r \in R} \sum_{o \in O_r} p_o$.
D_i	The inventory amount of raw materials in manufacturer i.
h_i	The holding cost of raw material in manufacturer i.
c_{io}	The production and delivery cost of order o accepted by manufacturer i.
c_{ij}	The raw material transfer cost between manufacturers i and j.
P	The emergency raw material procurement cost.
c_i	The transportation cost from raw material supplier to manufacturer i.
Q	The amount of emergency raw material procurement batch.
Variables	
x_{io}	Binary variable, where $x_{io} = 1$ if order o is accepted by manufacturer i, and $x_{io} = 0$ otherwise.
m_i	Binary variable, where $m_i = 1$ if manufacturer i is open, and $m_i = 0$ otherwise.
y_{ij}	Continuous variable, the amount of raw material transferred from i to j.
z_i	Continuous variable, the amount of raw material procured from supplier to i.
z_0	Integer variable, the amount of raw material batch procured from the supplier.
$\gamma_i^{M_s}$	Indicator variable, where $\gamma_i^{M_s} = 1$ if $i \in M_s$, and $\gamma_i^{M_s} = 0$ otherwise.
$\gamma_r^{R_s}$	Indicator variable, where $\gamma_r^{R_s} = 1$ if $r \in R_s$, and $\gamma_r^{R_s} = 0$ otherwise.

Constraints (10) model the emergency raw material procurement process, indicating that the centralised decision maker procures raw materials in batches of size Q.

Based on the model described above, we observe that $C(M \cup R)$ is a variant of the capacitated facility location problem with an additional linear cost component. It can be reduced to the classical capacitated facility location problem using the Benders decomposition method. Numerical experiments in Section 5.2 demonstrate that commercial solvers can efficiently solve this problem, obviating the need for a specialised algorithm for $C(M \cup R)$.

3.3. Cost allocation problem for both manufacturers and retailers

Cooperation can lead to cost savings, but its foundation lies in a fair allocation scheme. Based on cooperative game theory, we aim to find a fair solution method for allocating the costs among additive manufacturers and retailers. In this subsection, we integrate cooperative game theory with the logistics cost optimisation problem $C(M \cup R)$ and analyze the existence of the core.

$$C(M_s \cup R_s) = \min\ (3)$$

$$\text{s.t. } (4) - (5),\ (7) - (12)$$

$$\sum_{i \in M} x_{io} = \gamma_r^{R_s}, \quad \forall o \in O_r, \quad r \in R \tag{13}$$

$$x_{io} \leq \gamma_i^{M_s}, \quad \forall i \in M, \quad o \in O_r, \quad r \in R \tag{14}$$

$$m_i \leq \gamma_i^{M_s}, \quad \forall i \in M \tag{15}$$

$$\gamma_r^{R_s} \in \{0, 1\}, \quad \forall r \in R, \quad s \in \mathbb{S} \tag{16}$$

$$\gamma_i^{M_s} \in \{0, 1\}, \quad \forall i \in M, \quad s \in \mathbb{S}. \tag{17}$$

To derive the characteristic function $C(M_s \cup R_s)$, we introduce indicator vectors $\gamma_i^{M_s}$ and $\gamma_r^{R_s}$ that represent whether manufacturer i or retailer r participates in coalition s, as determined by the centralised decision maker. It is evident that the cost savings from cooperation are monotonically non-decreasing as the size of the coalition increases. However, the stability of cooperation remains uncertain. If some additive manufacturers and retailers break away from the grand coalition to form smaller groups, the overall competitiveness of the grand coalition may decline. Therefore, it is necessary to verify the stability of cooperation. Our analysis reveals that the core does not always exist. This is demonstrated in the following proposition.

Proposition 3.1: *The core of $C(M_s \cup R_s)$ does not always exist.*

Proof: We illustrate that the core may be empty through an example. To facilitate intuitive understanding, we set $M = \{m_1, m_2\}$, $R = \{r_1, r_2, r_3\}$, with $|O_r| = 1$, for all $r \in R$, $P = 1, Q = 3$. Additionally, we assume $h_i = f_{io} = c_{io} = c_i = c_{ij} = d_o = 1$, and $D_i = C_i = 3$, $\forall i, j \in M$, $o \in O_r$, $r \in R$, $p_{o1} = p_{o2} = 1$ and $p_{o3} = 1.5$. Then, we obtain the following optimal values: $C(\{m_1\} \cup \{r_1, r_2\}) = C(\{m_1, m_2\} \cup \{r_1, r_3\}) = C(\{m_2\} \cup \{r_2, r_3\}) = 5$, and $C(M \cup R) = 9$.

We observe that $\sum_{i \in M \cup R} \alpha_i \leq C(\{m_1\} \cup \{r_1, r_2\}) + C(\{m_1, m_2\} \cup \{r_1, r_3\}) + C(\{m_2\} \cup \{r_2, r_3\}) = 15/2 = 7.5 < 9 = C(M \cup R)$, which contradicts the budget balance constraint. Therefore, the core is empty. ∎

To tackle this issue, we relax the budget balance constraint of the core (1) and focus on solving the OCAP problem (2). The OCAP problem, governed by a centralised decision maker, aims to maximise the total cost allocated to all players while maintaining coalition stability. However, since the number of coalition stability constraints grows exponentially with the number of players (manufacturers and retailers), an efficient solution method is essential. To overcome the computational challenges of the OCAP, we propose an exact row-generation algorithm, complemented by a hybrid approach that integrates linear and Lagrangian relaxations, as detailed in Section 4.

3.4. Cost allocation only for additive manufacturer coalition

In some cases, manufacturers can receive and produce orders independently, leading to a reduced coalition model – an AM coalition that excludes retailers. In this subsection, we analyze the scenario in which additive manufacturers collect orders on their own. Under this setting, the logistics cost problem $C(M \cup R)$ reduces to $C(M)$. Let the set of orders be defined as:

$$O_M = \bigcup_{i \in M} O_i.$$

The optimisation model for $C(M)$ (18) and the corresponding characteristic function $C(M_s)$ for any coalition s are formulated as follows.

$$\begin{aligned} C(M) = \min \quad & \sum_{i\in M}\sum_{o\in O_i}(f_{io}+c_{io}-h_i d_o)x_{io} \\ & + \sum_{i\in M} h_i D_i m_i \\ & + \sum_{i\in M}\sum_{j\in M}(c_{ij}-h_i+h_j)y_{ij} \\ & + \sum_{i\in M}(c_i+h_i)z_i + P\cdot z_0 \qquad (18) \\ \text{s.t.} \quad & (7)-(12), \\ & \sum_{o\in O_i} d_o x_{io} \le \sum_{j\in M}(y_{ji}-y_{ij}) \\ & \quad + D_i m_i + z_i, \quad \forall i \in M, \qquad (19) \\ & \sum_{o\in O_i} p_o x_{io} \le C_i m_i, \quad \forall i \in M, \qquad (20) \\ & \sum_{i\in M} x_{io} = 1, \quad \forall o \in O_i, i \in M. \end{aligned}$$

$$\begin{aligned} C(M_s) = \min \quad & \sum_{i\in M}\sum_{o\in O_i}(f_{io}+c_{io}-h_i d_o)x_{io} \\ & + \sum_{i\in M} h_i D_i m_i \\ & + \sum_{i\in M}\sum_{j\in M}(c_{ij}-h_i+h_j)y_{ij} \\ & + \sum_{i\in M}(c_i+h_i)z_i + P\cdot z_0 \\ \text{s.t.} \quad & (7)-(12),\ (19)-(20), \\ & \sum_{i\in M} x_{io} = \gamma_i^{M_s}, \quad \forall o \in O_i,\ i \in M \\ & x_{io} \le \gamma_i^{M_s}, \quad \forall i \in M,\ o \in O_i \\ & m_i \le \gamma_i^{M_s}, \quad \forall i \in M \\ & \gamma_i^{M_s} \in \{0,1\}, \quad \forall i \in M,\ s \in \mathbb{S}. \qquad (1) \end{aligned}$$

In this case, we can prove that the core of $C(M_s)$ exists, which is shown in Theorem 3.2. Our proof constructs a cost allocation rule under which both budget balance and coalition stability constraints are satisfied.

Theorem 3.2: *The core of $C(M_s)$ exists.*

Proof: We assume that $(\mathbf{m}^*, \mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$ is the optimal solution of $C(M)$. Observing the model of $C(M)$, we find that $z_0^* = \lceil \frac{\sum_{i\in M} z_i}{Q} \rceil$, $y_{ij}\cdot y_{ji} = 0$. Therefore we set $\alpha_i = \sum_{o\in O_M}(f_{io}+c_{io}-h_i d_o)x_{io}^* + h_i D_i m_i^* + \sum_{j\in M}(c_{ij}-h_i + h_j)y_{ij}^*] + c_i z_i^* + P\cdot z_0^* \cdot \frac{z_i^*}{\sum_{i\in M} z_i^*}$, $\forall i \in M$. It is obvious that $\sum_{i\in M_s}\alpha_i = C(M_s)$, $\forall s \in \mathbb{S}$. ■

Through Theorem 3.2, we demonstrate that once the optimisation problem $C(M)$ is solved, the cost allocation among manufacturers is uniquely determined. This indicates that neither raw material transfer nor joint procurement affects the final cost allocation.

4. Solution method for OCAP

To tackle the computational challenges posed by the OCAP, we develop a row generation method to obtain an exact allocation solution. Furthermore, we design the hybrid algorithm combining two approximation algorithms, a linear programming-based method and a Lagrangian relaxation-based method, for solving the large-scale OCAP problem.

4.1. Exact allocation method: row generation

In this subsection, we develop an exact row-generation method to solve the OCAP problem. Row generation is an effective technique for handling problems with a large number of constraints. The formulations of the master problem and the corresponding separation problem are presented below.

As discussed in Section 3.3, the number of coalition stability constraints grows exponentially with the number of players. Including all such constraints in the MILP model would require calculating the characteristic function for every possible coalition, leading to computational intractability as the number of players increases. To mitigate this issue, we first solve the OCAP problem with a restricted set of coalition stability constraints, which yields a solution that may be infeasible for the master problem. Next, we identify a coalition for which the allocated cost violates the coalition stability constraint and add the corresponding constraint to the model. This process is repeated iteratively until the solution satisfies the stability constraints for all coalitions.

We first consider the formulation of the master problem with a restricted coalition set denoted by $\mathbb{S}'$. The restricted master problem is formulated as:

$$\begin{aligned} \max_{\alpha} \quad & \sum_{k \in M \cup R} \alpha(k) \\ \text{s.t.} \quad & \sum_{k \in M_s \cup R_s} \alpha(k) \leq c(M_s \cup R_s), \quad \forall s \in \mathbb{S}'. \end{aligned} \tag{21}$$

The only difference between the restricted master problem (21) and the OCAP (2) lies in the use of the restricted coalition set $\mathbb{S}'$. To ensure that the solution of (21) is valid for the master problem, we then solve a separation problem to identify a coalition $s \notin \mathbb{S}'$ that violates the coalition stability constraints:

$$\begin{aligned} \xi = \min_{s \in \mathbb{S} \setminus \mathbb{S}'} \quad & C(M_s \cup R_s) - \sum_{i \in M} \gamma_i^{M_s} \bar{\alpha}(i) - \sum_{r \in R} \gamma_r^{R_s} \bar{\alpha}(r) \\ \text{s.t.} \quad & (4) - (5),\ (7) - (17). \end{aligned} \tag{22}$$

where ξ represents the optimal objective value of the separation problem, and the cost allocation $\bar{\alpha}(k)$ is obtained from the solution of the restricted master problem (21).

We summarise the procedure of the row-generation-based cost allocation in Algorithm 1. The input to the row-generation method is an initial restricted set of coalitions, $\mathbb{S}'$, which is composed of three parts:

$$\mathbb{S}' = \mathbb{S}'_1 \cup \mathbb{S}'_2 \cup \mathbb{S}'_3,$$

where:

- $\mathbb{S}'_1 = \{\{m_i, r_j\} \mid \forall i \in M, \forall j \in R\}$, in which each coalition consists of a single manufacturer and a single retailer.
- $\mathbb{S}'_2 = \{M \cup \{r_j\} \mid \forall j \in R\}$, in which each coalition comprises all manufacturers along with a single retailer.
- $\mathbb{S}'_3 = \{M \cup R\}$, the grand coalition.

This initial restricted set is designed to efficiently identify a sub-coalition s' that violates the stability constraints. If the production does not meet the orders' demand, we set $C(s) = \infty$. Given the optimal solution $\bar{\alpha}(k)$ for all $k \in M \cup R$ from the restricted master problem (21) and the current coalition set $\mathbb{S}'$, we compute the optimal objective value ξ of the separation problem. If $\xi < 0$, then there exists a sub-coalition $s' \in \mathbb{S} \setminus \mathbb{S}'$ that violates the stability constraints; in this case, we add s' to $\mathbb{S}'$ and resolve (21). This row-generation iteration continues until no violating sub-coalition is found, at which point the final solution $\bar{\alpha}(k)$ for all $k \in M \cup R$ is recorded as the optimal cost allocation α.

In Algorithm 1, we employ the commercial solver GUROBI to solve both the restricted master problem and the separation problem directly. The restricted master problem (21) is a linear programming formulation and, as such, its computational cost is relatively low. However, the separation problem (22) differs significantly from the primal problem $C(M \cup R)$ and cannot be reduced to a classical capacitated facility location problem. Consequently, for large-scale instances, we need to develop an approximate allocation method.

4.2. Approximate allocation methods

In this subsection, we design the hybrid algorithm combining LPB and LRB methods for solving the large-scale OCAP problem. LPB method is proposed by Caprara and Letchford (2010), which is simple and direct. However, the gap between the linear programming (LP) relaxation and the integer programming (IP) solution is significant, leading to a suboptimal cost allocation that fails to appropriately distribute costs among players. LRB framework is proposed by Liu, Qi, and Xu (2016), which yields a solution closer to the exact allocation but requires additional computational time to determine the optimal Lagrangian multiplier λ^*. We then combine them to develop a hybrid algorithm that leverages the simplicity of the LPB method and the accuracy of the LRB framework.

4.2.1. Linear-relaxation-based method

The key component of the LPB method is the concept of assignable constraints. In this subsection, we formally introduce the assignable constraint in Definition 4.1, which represents a special type of inequality. Then we prove that the logistics cost allocation problem satisfies the assignable constraint property in Lemma 4.2, and we establish that the LPB method can approximate the core in Theorem 4.3.

Definition 4.1 (Caprara and Letchford 2010): An inequality $ax \geq \eta$ which is valid for Conv $\{x \in \mathbb{Z}^q : Ax \geq B\mathbf{1} + E\}$ is said to be assignable if there exists an inequality $ax \geq by$, which is valid for Conv Q^{xy} such

Algorithm 1 Row Generation Method for OCAP

Input: Initial restricted coalition set $\mathbb{S}'$.
Output: Optimal cost allocation solution α.

1: Solve the initial restricted master problem (21) and obtain the current cost allocation $\bar{\alpha}(k)$, $\forall k \in M \cup R$.
2: Solve the separation problem (22) and obtain the optimal value ξ and sub-coalition s'. The optimal coalition refers to the coalition with the smallest value.
3: **while** $\xi < 0$ **do**
4: Add s' to the restricted coalition set $\mathbb{S}'$.
5: Resolve the restricted master problem (21) and update the cost allocation $\bar{\alpha}(k)$, $\forall k \in M \cup R$.
6: Solve the separation problem (22) and obtain the optimal value ξ and sub-coalition s'.
7: **end while**
8: Record the optimal cost allocation as $\alpha(k) = \bar{\alpha}(k)$, $\forall k \in M \cup R$.

that $b\mathbf{1} = \eta$, where $Q_{xy} := \{(x, y) : Ax \geq By + E, y = y(s) \text{ for some } s \in \mathbb{S},\ x \in \mathbb{Z}^q,\ y \in \{0, 1\}^n\}$.

Lemma 4.2: *The constraints of $c(M_s \cup R_s)$ is assignable.*

Proof: According to the Definition 4.1, it is evident that if we set $\gamma_i^{M_s} = \gamma_r^{R_s} = 1$, $\forall i \in M$, $r \in R$. ■

Theorem 4.3: *Core of $c_{LP}(M_s \cup R_s)$ exists.*

Proof: Constraints of $c(M_s \cup R_s)(Dx \geq f)$ is assignable $(Dx \geq Ef)$. And based on the Lemma 2 of Caprara and Letchford (2010), the product of dual solution π^* of $c_{LP}(M_s \cup R_s)$ and E lies in the core. ■

The assignable constraints were first introduced by Caprara and Letchford (2010), where the linear relaxation of an integer minimisation game with assignable constraints guarantees a non-empty core via linear duality. In the context of this study, the linear relaxation proportionally allocates the costs associated with production and transportation activities among the players, and any discrepancies arising from the relaxation of integer constraints are absorbed by the central decision-maker.

4.2.2. Lagrangian-relaxation-based method

In this subsection, we design the LRB method for solving the OCAP problem in the logistics cost allocation framework, following the LRB approach proposed by Liu, Qi, and Xu (2016). The key idea is to relax the 'hard' constraints into the objective function, allowing the remaining problem to be solved more efficiently.

In the Lagrangian relaxation (LR) process, we decompose constraint (13) into two parts: $\{\sum_{i \in M} x_{io} \geq \gamma_r^{R_s}, \forall o \in O_r,\ r \in R\}$ and the constraint (23). We then relax the constraints $\{\sum_{r \in R} \sum_{o \in O_r} x_{io} \geq \gamma_r^{R_s},\ \forall r \in R\}$ by incorporating them into the objective function using the Lagrangian multipliers $\lambda = \{\lambda_{or} : o \in O_r,\ r \in R\}$. The remaining constraints from (23) are retained in the LR model, denoted as $C_{LR}(M_s \cup R_s)$.

$$
\begin{aligned}
C_{LR}(M_s \cup R_s) = \min \quad & \sum_{i \in M} \sum_{r \in R} \sum_{o \in O_r} (f_{io} + c_{io} - h_i d_o \\
& - \lambda_{or}) x_{io} + \sum_{i \in M} h_i D_i m_i \\
& + \sum_{r \in R} \sum_{o \in O_r} \lambda_{or} \gamma_r^{R_s} \\
& + \sum_{i \in M} \sum_{j \in M} (c_{ij} - h_i + h_j) y_{ij} \\
& + \sum_{i \in M} (c_i + h_i) z_i + P \cdot z_0 \quad (2) \\
& \text{s.t. } (4) - (5),\ (7) - (12), \\
& (14) - (17), \\
& \sum_{i \in M} x_{io} \leq \gamma_r^{R_s}, \\
& \forall o \in O_r, \quad r \in R. \qquad (23)
\end{aligned}
$$

According to Lagrangian dual theory, the Lagrangian dual problem can be formulated as:

$$
C_{d_{LR}}(M_s \cup R_s) = \max_{\lambda \geq 0} C_{LR}(M_s \cup R_s).
$$

We apply the subgradient method (Held, Wolfe, and Crowder 1974) to determine the optimal Lagrangian multiplier, λ^*, and obtain the LR lower bound for $C(M_s \cup R_s)$. The Lagrange multipliers act as penalty factors, quantifying the cost of violating the constraints $\sum_{r \in R} \sum_{o \in O_r} x_{io} \geq \gamma_r^{R_s}$, $\forall r \in R$. In essence, they represent the marginal penalty incurred when an order is unfulfilled. In the subsequent OCAP problem, these fixed Lagrange multipliers directly influence cost allocation by determining the penalty associated with unfulfilled orders. It is crucial to emphasise that the effectiveness of the LRB method is highly dependent on the quality of the LR process. If the LR process fails to generate a lower bound that closely approximates the exact IP solution within a reasonable computational time, the resulting LRB solution may not be satisfactory.

Next, we decompose the primal game into two subgames: subgame 1 and subgame 2. The characteristic function of subgame 1, denoted as $C_{LR_1}(M_s \cup R_s)$, is given by:

$$C_{LR_1}(M_s \cup R_s) = \sum_{r \in R} \sum_{o \in O_r} \lambda_{or} \gamma_r^{R_s}. \tag{24}$$

According to Theorem 1 and Lemma 1 in Liu, Qi, and Xu (2016), we can set $\alpha^{\lambda}_{LR_1}(i) = 0$ for all $i \in M$ and $\alpha^{\lambda}_{LR_1}(r) = \sum_{o \in O_r} \lambda_{or}$ for all $r \in R$ in subgame 1, ensuring that it has a non-empty core.

For subgame 2, the characteristic function is formulated as:

$$\begin{aligned} C_{LR_2}(M_s \cup R_s) = \min \quad & \sum_{i \in M} \sum_{r \in R} \sum_{o \in O_r} (f_{io} + c_{io} - h_i d_o \\ & - \lambda_{or}) x_{io} + \sum_{i \in M} h_i D_i m_i \\ & + \sum_{i \in M} \sum_{j \in M} (c_{ij} - h_i + h_j) y_{ij} \\ & + \sum_{i \in M} (c_i + h_i) z_i + P \cdot z_0 \\ & \text{s.t. } (4) - (5),\ (7) - (12), \\ & (14) - (17),\ (23). \end{aligned} \tag{25}$$

Subgame 2 is a variant of the fixed-charge multiple knapsack problem, incorporating an additional cost for each knapsack. It can be transformed into a fixed-charge multiple knapsack problem using the Benders decomposition method. Additionally, based on numerical experiments, we find that commercial solvers can efficiently solve $C_{LR_2}(M_s \cup R_s)$, eliminating the need for a specialised algorithm.

To compute the optimal cost allocation $\alpha^{\lambda}_{LR_2}$ for $C_{LR_2}(M_s \cup R_s)$, we apply the row generation algorithm. Specifically, the separation problem involves identifying a sub-coalition s' with the smallest reduced cost ξ. The separation problem is formulated as:

$$\begin{aligned} \xi = \min_{s \in \mathbb{S} \setminus \mathbb{S}'} \quad & C_{LR_2}(M_s \cup R_s) - \sum_{i \in M} \gamma_i^{M_s} \bar{\alpha}^{\lambda}_{LR_2}(i) \\ & - \sum_{r \in R} \gamma_r^{R_s} \bar{\alpha}^{\lambda}_{LR_2}(r) \\ \text{s.t.} \quad & (4) - (5),\ (7) - (12),\ (14) - (17),\ (23). \end{aligned} \tag{26}$$

with $\bar{\alpha}^{\lambda}_{LR_2}$ represents the optimal dual solution from the corresponding master problem of the OCAP problem.

The row generation algorithm follows the same procedure as described in Section 4.1. Although solving the separation problem in polynomial time is challenging, we introduce Lemma 4.4 to reduce the problem size when solving (26).

Lemma 4.4: *For the separation problem (26), certain variables can be sequentially fixed to zero without altering the optimal objective value, using the following steps:*

(1) *For each $i \in M, o \in O_r, r \in R$, if $f_{io} + c_{io} - h_i d_o - \lambda_{or} > 0$, then set $x_{io} = 0$ and update $f_{io} + c_{io} - h_i d_o - \lambda_{or} = \infty$.*
(2) *For each $r \in R$, if*

$$\sum_{i \in M} \min \left\{ \sum_{o \in O_r} (f_{io} + c_{io} - h_i d_o - \lambda_{or}), 0 \right\} - \bar{\alpha}^{\lambda}_{LR_2}(r) > 0,$$

then set $\gamma_r^{R_s} = 0$ and $x_{io} = 0$, $\forall i \in M,\ o \in O_r$.

Proof: First, suppose there exists a pair of indices (i, o) such that $x_{io} = 1$ and $f_{io} + c_{io} - h_i d_o - \lambda_{or} > 0$ in a feasible solution to the separation problem (26). In this case, setting $x_{io} = 0$ yields another feasible solution, where the objective function value decreases by at least $f_{io} + c_{io} - h_i d_o - \lambda_{or}$.

Second, consider a retailer r such that $\gamma_r^{R_s} = 1$ and

$$\sum_{i \in M} \min \left\{ \sum_{o \in O_r} (f_{io} + c_{io} - h_i d_o - \lambda_{or}), 0 \right\} - \bar{\alpha}^{\lambda}_{LR_2}(r) > 0$$

holds in a feasible solution to (26). Setting $\gamma_r^{R_s} = 0$ and $x_{io} = 0$ for all $i \in M, o \in O_r$ produces another feasible solution, reducing the objective function value by at least

$$\sum_{i \in M} \min \left\{ \sum_{o \in O_r} (f_{io} + c_{io} - h_i d_o - \lambda_{or}), 0 \right\} - \bar{\alpha}^{\lambda}_{LR_2}(r).$$

This completes the proof. ■

4.2.3. Hybrid algorithm

Based on the above analysis, we integrate the LPB and LRB methods and summarise the process in Algorithm 2. The key idea is to first use the LPB method to obtain a feasible cost allocation solution quickly, and then refine the solution using the LRB method to improve its quality.

The detailed process of the approximate allocation algorithm is as follows. First, we solve the linear relaxation of $C(M_s \cup R_s)$ to obtain the LPB solution α_{LP}.

Algorithm 2 Hybrid Approximate Allocation Algorithm for OCAP

Input: Basic parameters: $R, M, O, d_o, p_o, C_i, D_i, h_i, f_{io}, c_{io}, c_{ij}, P, c_i, Q$
Output: Optimal cost allocation solution α

1: Solve the linear relaxation of $C(M_s \cup R_s)$ to obtain the LPB solution α_{LP}.
2: Solve the Lagrangian relaxation of $C(M_s \cup R_s)$ to obtain the Lagrangian relaxation lower bound $C_{LR}(M_s \cup R_s)$.
3: **if** $C_{LR}(M_s \cup R_s) \le C_{LP}(M_s \cup R_s)$ **then**
4: Record $\alpha = \alpha_{LP}$ and terminate the algorithm.
5: **else**
6: Decompose $C_{LR}(M_s \cup R_s)$ into subgame 1 and subgame 2.
7: For subgame 1, set $\alpha^{\lambda}_{LR_1}(i) = 0$ for all $i \in M$ and $\alpha^{\lambda}_{LR_1}(r) = \sum_{o \in O_r} \lambda_{or}$ for all $r \in R$.
8: For subgame 2, apply Lemma 4.4 to preprocess the problem and compute the optimal cost allocation $\alpha^{\lambda}_{LR_2}$ using the row generation method.
9: **end if**
10: Compare α_{LP} and $\alpha^{\lambda}_{LR} = \alpha^{\lambda}_{LR_1} + \alpha^{\lambda}_{LR_2}$, and record the better one as α.

Next, we solve the Lagrangian relaxation of $C(M_s \cup R_s)$ and compute the LR lower bound $C_{LR}(M_s \cup R_s)$. If LR lower bound is smaller than that of the linear relaxation, we terminate the algorithm immediately. Otherwise, we decompose $C_{LR}(M_s \cup R_s)$ into subgame 1 and subgame 2. For subgame 1, we set $\alpha^{\lambda}_{LR_1}(i) = 0$ for all $i \in M$ and $\alpha^{\lambda}_{LR_1}(r) = \sum_{o \in O_r} \lambda_{or}$ for all $r \in R$. For subgame 2, we preprocess using Lemma 4.4 and compute the optimal cost allocation solution $\alpha^{\lambda}_{LR_2}$ using the row generation method. Finally, we compare α_{LP} and $\alpha^{\lambda}_{LR} = \alpha^{\lambda}_{LR_1} + \alpha^{\lambda}_{LR_2}$, and record α as the better solution.

5. Numerical experiment

Our computational investigations are undertaken with several objectives in mind. Firstly, we establish a non-cooperative model as a benchmark to assess the advantages of cooperation across varying problem scales. Secondly, we showcase the computational efficiency and efficacy of our proposed algorithms. Thirdly, we conduct sensitivity analyses across diverse scenarios to extract pivotal managerial insights.

5.1. Experiment description

Within this subsection, we execute numerical experiments utilising Gurobi 11.0.0 as the solver. The solution methodologies are implemented in MATLAB R2023a and run on a Windows 11 personal computer with an Intel Core i7-8700K CPU and 32 GB of RAM. The temporal constraints for different processes are delineated as follows: 300 seconds for the LR process and 3600 seconds for the row generation process. If the computation exceeds the allocated time, the result is denoted as '-'. The instances used in our computational experiments are generated according to the following procedure. We summarise the parameter configurations in Table 2. Coefficients $\eta = [\eta_1, \eta_2, \eta_3]$ and $\theta = [\theta_1, \theta_2, \theta_3]$ are introduced to modulate the proportional relationships among different cost components. The coordinates of the manufacturers and demand locations are randomly generated from a planar region $U[0, 1000] \times U[0, 1000]$, wherein the distance E between two nodes is quantified by Euclidean distance.

Table 2. Setting of key parameters.

Constraints related		Objective related	
Parameters	Range	Parameters	Range
$\lvert O_r \rvert$	$U[1, 3]$	f_{io}	$U[20, 100]$
d_o	$U[1, 5]$	h_i	$U[20, 100] \times \eta_1$
p_o	$U[10, 30]$	P	$200 \times \eta_2$
C_i	$U[50, 100]$	c_i	$U[100, 700] \times \eta_3 \times \theta_1$
D_i	$U[10, 30]$	c_{io}	$E_{io} \times \theta_2$
Q	1000	c_{ij}	$E_{ij} \times \theta_3$

5.2. Benefits of cooperation

In this section, we demonstrate that the benefits of cooperation by comparing the model with and without centralised procurement and inter-facility material redistribution settings. In the cases of raw material shortages, all AM manufacturers procure directly from suppliers in the non-cooperative scenario, as shown in the Appendix. Each problem instance is repeated 20 times, and the logistics cost savings achieved through cooperative delivery are documented in Tables 3 and 4.

The first two columns in Tables 3 and 4 outline the problem size for $M = 5$ and $M = 10$, where (M, R) denotes the number of manufacturers and retailers, and $\lvert O_R \rvert$ represents the total number of orders. The columns labelled C/C_{no-co} show the ratio of cooperative cost to non-cooperative cost, while those labelled *Time* document the computational time required by the commercial

Table 3. Comparison of Cooperative and Non-Cooperative Costs with $M = 5$.

	$\lvert O_R \rvert$			C/C_{no-co}/%			Time /s		
(M,R)	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.
(5,5)	27	36	16	89.56	100.00	65.17	0.01	0.04	0.00
(5,10)	55	69	39	67.29	75.72	55.23	0.01	0.02	0.00
(5,15)	82	105	61	66.04	76.02	39.15	0.01	0.02	0.00
(5,20)	105	129	86	64.85	76.40	50.48	0.01	0.02	0.00
(5,25)	143	171	122	63.33	73.23	47.40	0.01	0.03	0.01
(5,30)	168	190	142	65.82	74.31	51.93	0.02	0.05	0.01
(5,35)	188	217	160	64.86	72.68	47.65	0.02	0.03	0.01
(5,40)	217	246	190	64.25	77.17	47.76	0.02	0.06	0.01
(5,45)	245	285	210	64.79	71.78	50.39	0.03	0.05	0.01
(5,50)	268	319	233	65.51	76.63	52.18	0.03	0.07	0.02

Table 4. Comparison of Cooperative and Non-Cooperative Costs with $M = 10$.

	$\lvert O_R \rvert$			C/C_{no-co} /%			Time /s		
(M,R)	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.
(10,5)	29	38	17	99.51	100.00	94.08	0.04	0.13	0.01
(10,10)	55	69	40	89.44	100.00	57.85	0.05	0.13	0.01
(10,15)	82	104	54	71.97	99.54	52.38	0.04	0.08	0.02
(10,20)	110	140	75	63.76	73.35	29.46	0.04	0.11	0.02
(10,25)	138	178	118	61.26	70.20	48.03	0.05	0.12	0.02
(10,30)	165	206	127	60.55	73.11	44.59	0.03	0.07	0.01
(10,35)	188	225	153	58.78	72.22	43.70	0.05	0.15	0.03
(10,40)	214	252	172	59.58	69.51	40.56	0.04	0.12	0.02
(10,45)	243	301	203	57.16	66.84	43.85	0.05	0.13	0.02
(10,50)	273	298	212	58.00	66.02	48.09	0.06	0.12	0.02

solver. As evidenced in Tables 3 and 4, the solver operates efficiently, achieving optimality in all instances within 0.15 seconds.

To better visualise the trend, we present Figure 1. The horizontal axis represents the number of retailers R, while the vertical axis depicts the cost ratio C/C_{no-co}. The blue line corresponds to $M = 5$, and the red line corresponds to $M = 10$. Figure 1 demonstrates that the total cost decreases sharply when R is small and then stabilises around 60% as R increases. This trend occurs because a larger number of additive manufacturing sites leads to higher inventory costs, which form a significant portion of the total cost. When R is small, inter-facility material redistribution effectively reduces costs. However, as R increases, the influence of inventory costs diminishes, and procurement costs become the dominant factor. While centralised procurement alleviates a portion of the initial order cost, when the number of orders becomes sufficiently large, each additive manufacturer tends to procure raw materials independently.

Notably, we observe that for $R \leq 15$, the cost reduction achieved with $M = 5$ surpasses that with $M = 10$. Conversely, when $R > 15$, the situation reverses. This phenomenon arises because a greater number of additive manufacturing sites leads to higher inventory costs, which constitute a larger proportion of the total cost. However, when these sites receive a sufficiently large number of orders, their proximity to retailers reduces production delivery costs and enables a more responsive supply chain.

Overall, the most pronounced effects of cooperation occur in order scheduling and inventory coordination. An efficient central decision-maker can significantly reduce overall costs. Moreover, broader collaboration leads to further cost reductions, with a larger alliance of manufacturers and retailers yielding even greater savings.

5.3. Computational efficiency of algorithms

In this subsection, we evaluate the computational efficiency and effectiveness of our algorithms for problems of varying scales. Unlike the previous subsection, we conduct two sets of experiments. The first set, presented in Tables 5 and 6, focuses on small-scale cases ($R \leq 25$). The second set, shown in Tables 7 and 8, addresses large-scale cases ($R > 25$). We then summarise the results based on the optimal cost allocation ratio $\left(\frac{\sum_{k\in\{M\cup R\}}\alpha(k)}{C(M\cup R)}\right)$ and the computing time.

In Tables 5 and 7, we denote the optimal cost of model (18) computed by Gurobi as C. We use $C_{IPB}, C_{LPB}, C_{LRB}$ to represent the exact solution obtained by the row generation algorithm, the approximate optimal cost allocation solution obtained by the LPB and LRB methods, respectively, along with their average, maximum, and minimum values. Note that instances with

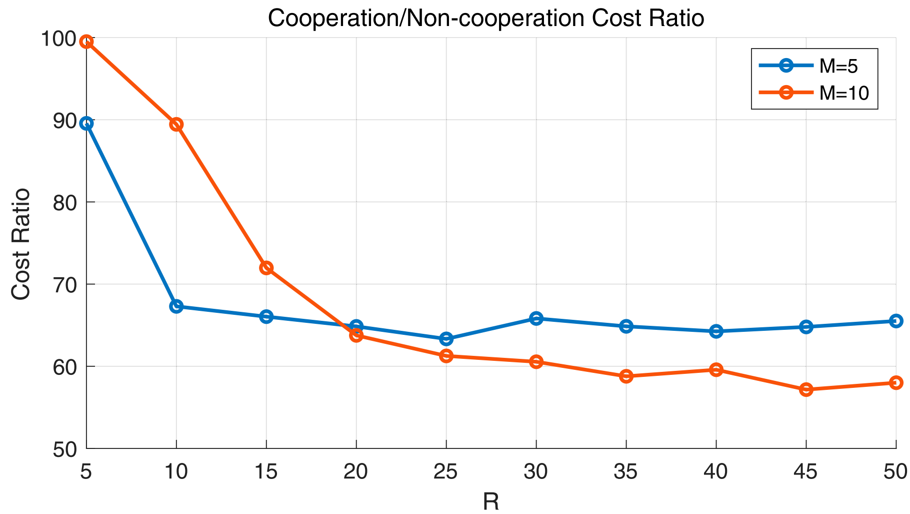


Figure 1. Cost Ratio of Cooperation and Non-Cooperation with $M = 5$ and $M = 10$.

Table 5. Cost Allocation Comparison of Proposed Algorithms for Small-Scale Cases.

	$\vert O_R\vert$			C_{IPB}/C /%			C_{LPB}/C /%			C_{LRB}/C /%		
(M,R)	*Avg.*	*Max.*	*Min.*	*Avg.*	Max.	*Min.*	Avg.	Max.	Min.	*Avg.*	Max.	Min.
(5,5)	27	40	7	100.00	100.00	100.00	93.09	99.30	70.84	99.10	100.00	94.62
(5,10)	56	73	40	100.00	100.00	100.00	99.46	99.84	98.53	99.98	100.00	99.72
(5,15)	83	96	68	100.00	100.00	100.00	99.82	99.93	99.43	99.96	100.00	99.63
(5,20)	109	132	87	100.00	100.00	100.00	99.87	99.96	99.46	99.97	100.00	99.52
(5,25)	140	163	118	100.00	100.00	100.00	99.94	99.98	99.82	100.00	100.00	99.97
(10,5)	26	40	15	99.85	100.00	98.23	83.56	98.35	35.20	95.57	100.00	88.36
(10,10)	57	72	35	99.94	100.00	99.39	97.67	99.40	93.84	98.79	100.00	95.25
(10,15)	85	104	61	100.00	100.00	100.00	99.05	99.73	97.86	99.83	100.00	98.20
(10,20)	108	126	87	100.00	100.00	100.00	99.59	99.89	99.02	99.88	100.00	98.83
(10,25)	135	163	113	–	–	–	99.75	99.91	99.38	99.98	100.00	99.90

Table 6. Computation Time Comparison of Proposed Algorithms for Small-Scale Cases.

	t_{IPB} /s			t_{LPB} /s			t_{LRB_LR} /s			t_{LRB_RG} /s		
(M,R)	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.
(5,5)	104.93*	–	1.38	0.02	0.05	0.00	11.63	32.23	0.13	0.51	0.91	0.11
(5,10)	106.19*	–	47.55	0.02	0.06	0.00	8.83	32.53	0.69	1.52	2.78	0.67
(5,15)	216.69*	–	30.17	0.04	0.08	0.00	11.59	33.36	0.70	2.57	3.28	1.66
(5,20)	451.63*	–	53.42	0.04	0.16	0.00	7.39	24.83	0.48	3.56	5.02	2.59
(5,25)	1076.90*	–	137.53	0.05	0.16	0.02	11.33	36.94	0.94	5.51	6.92	4.38
(10,5)	54.37*	–	11.23	0.02	0.08	0.00	17.10	40.52	3.33	0.63	1.23	0.25
(10,10)	1039.66	2413.97	258.81	0.04	0.16	0.00	37.80	70.55	7.38	4.19	5.83	1.42
(10,15)	1861.69*	–	423.97	0.08	0.19	0.02	22.26	53.13	3.11	7.15	9.50	4.88
(10,20)	1864.31*	–	1452.89	0.09	0.20	0.03	28.55	84.78	3.09	11.00	13.02	9.03
(10,25)	–	–	–	0.12	0.20	0.03	39.11	81.98	5.50	20.12	26.83	15.92

Table 7. Cost Allocation Comparison of Proposed Algorithms for Large-Scale Cases.

	$\vert O_R\vert$			C_{IPB}/C /%			C_{LPB}/C /%			C_{LRB}/C /%		
(M,R)	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.
(5,30)	168	198	150	–	–	–	99.96	99.98	99.86	99.99	100.00	99.93
(5,35)	195	224	153	–	–	–	99.96	99.99	99.90	99.99	100.00	99.96
(5,40)	225	266	196	–	–	–	99.98	99.99	99.96	100.00	100.00	99.96
(5,45)	239	272	186	–	–	–	99.98	99.99	99.95	99.96	100.00	99.44
(5,50)	274	308	239	–	–	–	99.98	99.99	99.94	99.96	100.00	99.33
(10,30)	158	192	122	–	–	–	99.78	99.94	99.44	99.94	100.00	99.65
(10,35)	193	223	162	–	–	–	99.89	99.95	99.80	99.98	100.00	99.92
(10,40)	218	253	174	–	–	–	99.91	99.97	99.82	99.99	100.00	99.93
(10,45)	243	270	209	–	–	–	99.94	99.98	99.84	99.99	100.00	99.94
(10,50)	266	318	232	–	–	–	99.96	99.98	99.93	100.00	100.00	99.97

Table 8. Computation Time Comparison of Proposed Algorithms for Large-Scale Cases.

	t_{IPB} /s			t_{LPB} /s			t_{LRB_LR} /s			t_{LRB_RG} /s		
(M, R)	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.
(5,30)	–	–	–	0.06	0.17	0.00	13.06	48.88	1.59	7.80	10.63	5.69
(5,35)	–	–	–	0.07	0.19	0.03	16.84	83.53	2.13	11.29	14.25	9.09
(5,40)	–	–	–	0.09	0.19	0.03	18.37	74.48	1.92	14.47	17.59	13.34
(5,45)	–	–	–	0.09	0.20	0.03	21.43	71.44	1.52	16.47	19.11	12.89
(5,50)	–	–	–	0.11	0.22	0.05	31.40	80.31	5.09	20.90	25.11	18.11
(10,30)	–	–	–	0.13	0.20	0.05	27.77	79.80	2.69	27.54	34.20	21.33
(10,35)	–	–	–	0.20	0.25	0.09	34.91	101.00	2.86	39.73	49.31	34.27
(10,40)	–	–	–	0.23	0.41	0.09	44.55	124.05	5.00	53.91	70.34	38.31
(10,45)	–	–	–	0.27	0.41	0.20	40.53	111.66	5.91	69.56	80.16	54.48
(10,50)	–	–	–	0.33	0.47	0.25	46.49	93.13	6.73	87.77	116.70	69.95

computation times exceeding 3600 seconds are excluded from the average calculation, as indicated by $*$. Similarly, in Tables 6 and 8, we use $t_{IPB}, t_{LPB}, t_{LRB_{LR}}, t_{LRB_{RG}}$ to denote the CPU time of the row generation algorithm, the approximate optimal cost allocation solution obtained by the LPB and LRB methods, respectively, along with their average, maximum, and minimum values. Specifically, we divide the LRB computation time into the RG process and the LR process, as the LR process plays a critical role in the LRB method. Obtaining a better lower bound from the LR process is essential for the overall performance of LRB.

From Tables 5 and 6, we observe that for small-scale problems, the cost allocation obtained by the LRB method outperforms that of the LPB method. In the best case, the LRB method achieves results identical to the exact row-generation method, with its average cost allocation exceeding 95%. In contrast, the cost allocation produced by the LPB method is less stable, with the worst case achieving only 35.20%. Furthermore, Table 6 reveals that the exact row-generation method requires significantly more time to solve the OCAP problem compared to the LPB and LRB methods. For instance, in the case of $(M, R) = (10, 25)$, the exact row-generation method fails to obtain an optimal solution within the allotted time. While the LPB method requires minimal time (less than 0.2 seconds), the LRB method, leveraging the strengths of both the row-generation and LPB methods, can obtain an approximate optimal solution in an average of less than 60 seconds. However, it is worth noting that the effectiveness of the LRB method depends on the LR process; if the LR process fails to produce a lower bound close to the IP exact solution within the limited time, the LRB solution may not be satisfactory.

For the large-scale problems presented in Tables 7 and 8, we observe that the exact RG method struggles to obtain an optimal solution within the limited time. However, both the LPB and LRB methods continue to perform effectively for large-scale problems. Notably, compared to small-scale cases, the performance of the LPB method improves significantly – the worst-case ratio of the approximate optimal solution increases from 35.20% to 99.44%. In some instances, the LPB solution even surpasses the LRB method. The LRB method excels in large-scale scenarios, with the average ratio of the approximate optimal solution exceeding 99.94%. However, the running time of the LRB method is notably higher than that observed for small-scale problems.

Overall, each of the three proposed algorithms has its own strengths and limitations. From a runtime perspective, the approximate algorithms complete significantly faster than the exact approach – particularly, the LPB method computes more quickly than the LRB method in large-scale problems. In terms of accuracy, although the row-generation algorithm is exact, it sometimes fails to yield an optimal solution within the preset time for small-scale problems. Conversely, the LPB algorithm exhibits larger errors on small-scale problems, but its results gradually converge to those of the LRB algorithm as the problem scale increases. Notably, the LRB algorithm strikes a better balance between computational time and accuracy. However, to better leverage the strengths of both the LPB and LRB methods, we employ a hybrid algorithm to solve the OCAP problems in the sensitivity analysis cases.

5.4. *Sensitivity analysis and management insights*

We further conduct sensitivity analyses across three distinct scenarios:

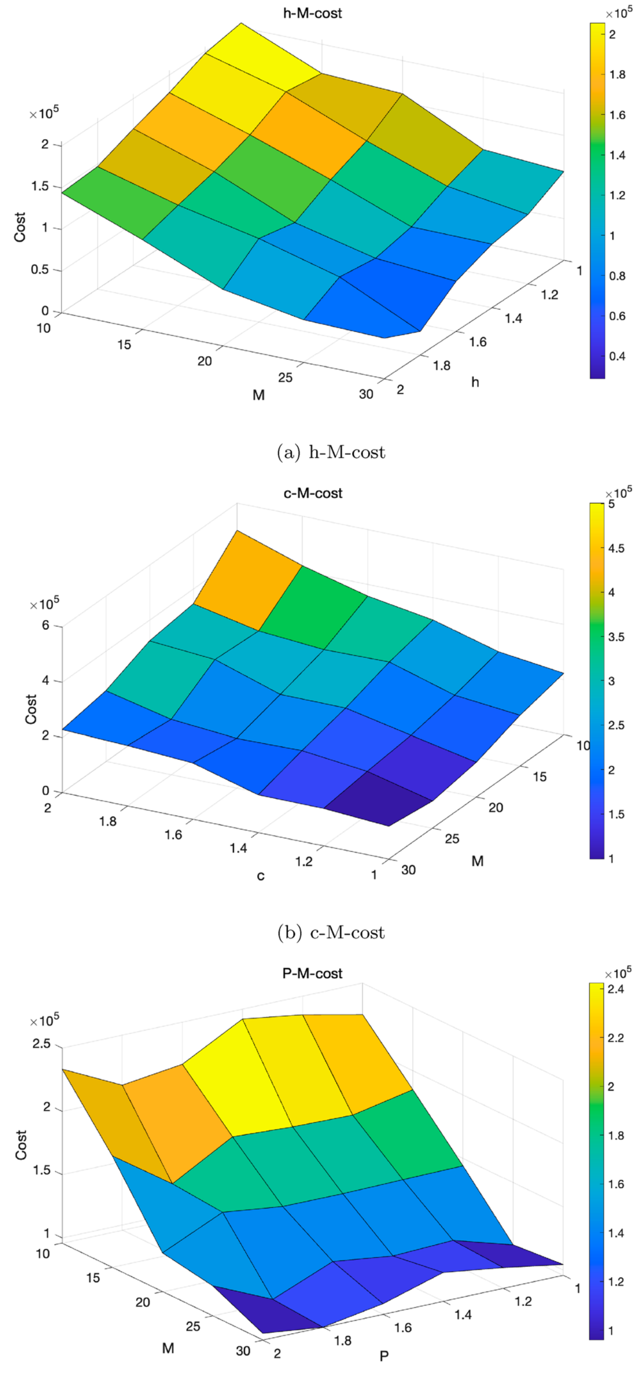
- Cost Component Analysis: Examination of inventory cost h, emergency raw material procurement cost P, and transportation cost (c_{io}, c_{ij}, c_i);
- Transportation Cost Analysis: Evaluation of finished product delivery cost c_{io}, raw material transfer cost c_{ij}, and transportation cost from supplier c_i;
- Cost Allocation Gap Analysis: Investigation of cost allocation gaps across varying process times and capacities.

5.4.1. Inventory-transportation-procurement cost comparison

We systematically analyze the impact of varying cost components by fixing certain parameters. Specifically, we set the number of retailers $R = 50$ and investigate the effects of changing the number of additive manufacturers $M \in \{10, 15, 20, 25, 30\}$. The coefficients η_1, η_2 and η_3 are varied from 1 to 2 (for example, $\eta_1 = \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$). Additionally, we examine the sensitivity by fixing $M = 20$ and varying $R \in \{10, 20, 30, 40, 50, 60, 70, 80, 100\}$ while keeping η_1, η_2 and η_3 within the range of 1 to 2. The results of these experiments are illustrated in Figures 2 and 3.

We observe that when the number of retailers is fixed (i.e. the demand is fixed), the total cost varies with changes in h, c, P and M as shown in Figures 2 and 3. Specifically, the total cost decreases as M increases (see Figure 2(a–c)), indicating that cooperation among additive manufacturers can reduce total costs associated with inventory, procurement, and transportation. However, each cost component exhibits a different level of sensitivity. Figure 2(a) shows that with a fixed number of retailers, the total cost decreases as the inventory cost coefficient increases. This reduction is due to inter-facility transfers, where raw materials are moved from sites with high inventory costs to those with lower costs. In contrast, Figure 3(a) indicates that when the number of additive manufacturers is fixed, the inventory cost coefficient has little influence on the total cost if the number of retailers is small; its influence becomes more pronounced as the number of retailers increases. Figures 2(b) and 3(b) illustrate that as the transportation coefficient increases, the total cost also increases. This effect is especially noticeable when the number of additive manufacturers is small or the number of retailers is large, potentially causing raw material shortages and a higher proportion of transportation costs. Finally, Figures 2(c) and 3(c) show that the procurement cost coefficient has only a minor effect on the total cost when the demand for raw materials is fixed. However, we observe that as R increases, there is a zone where the total cost increases slowly, which may be attributed to raw material factors. In this zone, the transfer of raw materials between AM sites decreases to zero, and all sites procure raw materials directly from the supplier.

Overall, our findings indicate that cooperation between retailers and additive manufacturers offers several advantages. First, orders can be produced at the cost-minimising factory, and the cost reduction increases as the number of additive manufacturers grows. Second, transferring raw materials between additive manufacturing sites helps optimise inventory utilisation and reduce



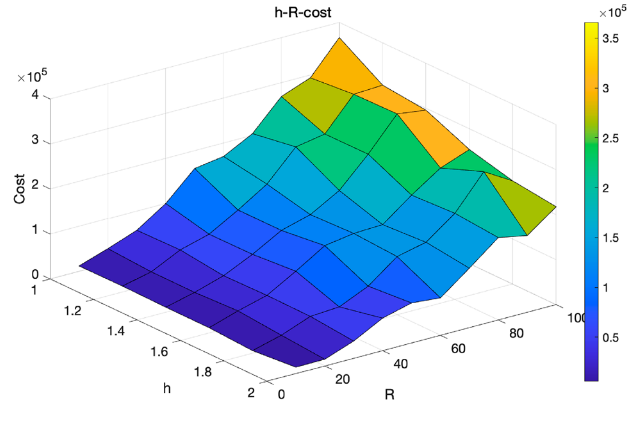
(a) h-M-cost

(b) c-M-cost

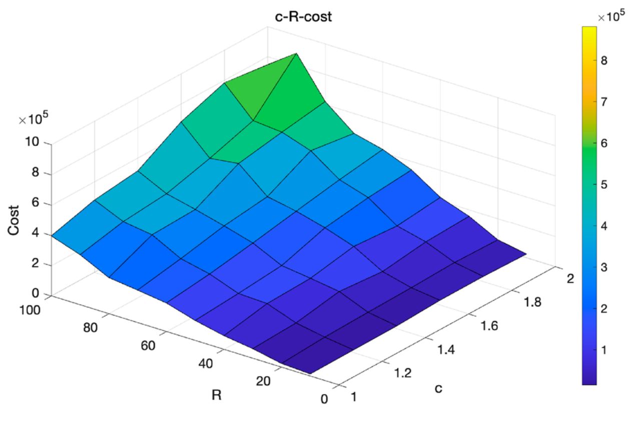
(c) P-M-cost

Figure 2. Cost Comparison with Varying Number of Manufacturers (M), Inventory Cost Coefficient (h), Transportation Cost Coefficient (c), Procurement Cost Coefficient (P), and Fixed Retailers Number ($R = 50$). (a) h-M-cost. (b) c-M-cost. (c) P-M-cost.

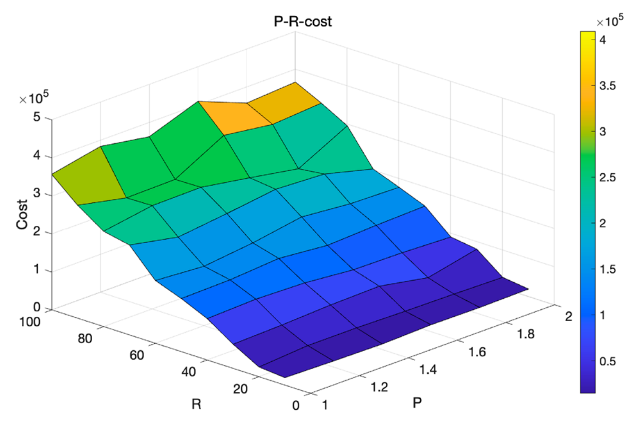
holding costs, particularly when differences in inventory costs are substantial. Finally, centralised procurement coordinated by the headquarters enables downstream manufacturers and retailers to negotiate with raw material suppliers at better terms.



(a) h-R-cost



(b) c-R-cost



(c) P-R-cost

Figure 3. Cost Comparison with Varying Number of Retailers (R), Inventory Cost Coefficient (h), Transportation Cost Coefficient (c), Procurement Cost Coefficient (P), and Fixed Manufacturers Number ($M = 20$). (a) h-R-cost. (b) c-R-cost. (c) P-R-cost.

5.4.2. Cost comparison among different transportation links

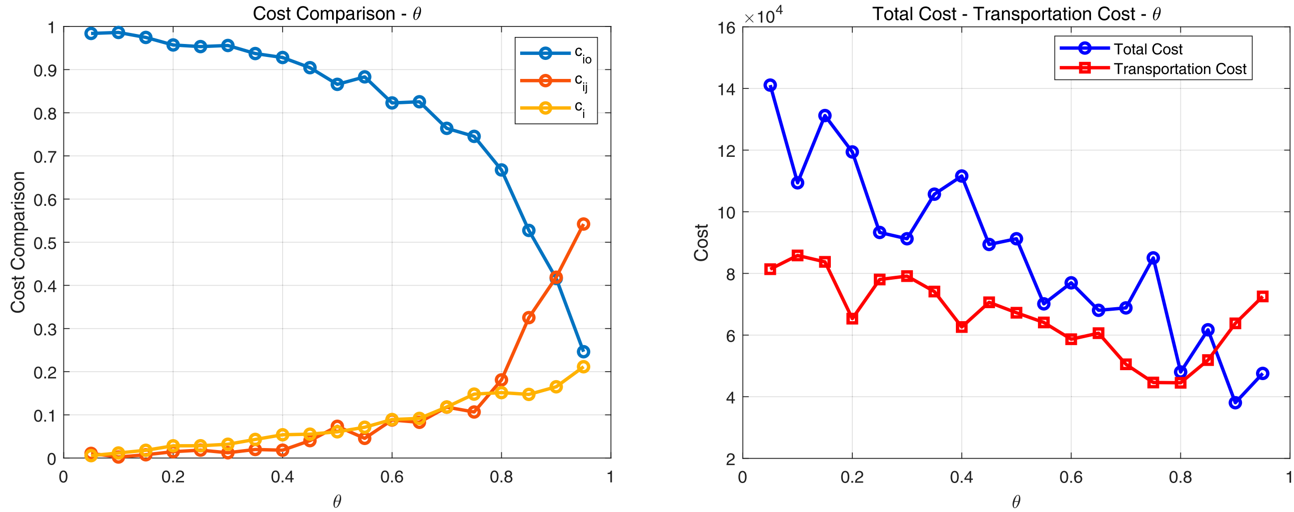
We further conduct sensitivity analyses across three distinct transportation cost components: delivery cost c_{io}, raw material transfer cost c_{ij}, and transportation cost from raw material supplier c_i. We systematically analyze the impact of varying transportation cost components by fixing certain parameters. Specifically, we set the number of retailers $R = 50$ and the number of additive manufacturers $M = 20$. Since the transportation cost per kilometre is assumed to be the same for c_{ij} and c_i, we set $\theta_2 = \theta_3$. To facilitate comparative experiments, we further define $\theta_2 = \theta_3 = 1 - \theta_1$. Then, we vary the coefficient θ_1 from 0.05 to 1(for example, $\theta_1 = \{0.05, 0.10, 0.15, \dots, 0.90, 0.95\}$). The results are presented in Figure 4. The left panel, Figure 4(a), shows the cost ratios for delivery cost c_{io}, raw material transfer cost c_{ij}, and transportation cost from raw material supplier c_i. The right panel, Figure 4(b), displays the total cost along with the overall transportation cost.

Based on the setting of θ, when θ_1 is small, it indicates that the finished products delivery cost coefficient c_{io} is lower than the raw transportation cost coefficient c_{ij} and c_i. Conversely, when θ_1 is large, the transportation cost coefficient for finished products is higher than that for raw materials.

Upon analyzing Figure 4(a), our analysis reveals that as θ_1 increases, the proportion of finished product transportation cost decreases, while raw material transportation cost increases. This trend occurs because, as the cost of finished product transportation rises, production orders are preferentially assigned to additive manufacturer sites closer to the order demand points, thereby minimising total system cost. This observation is further supported by Figure 4(b), where the blue line, representing total cost, shows a general downward trend as θ_1 increases.

Notably, when $\theta_1 > 0.8$, the raw material transfer costs from AM sites surpass the transportation cost from the raw material supplier, as shown in Figure 4(a). Additionally, in Figure 4(b), the total cost falls below the sum of transportation costs. This phenomenon arises because as the raw material transportation cost coefficient decreases, the transfer cost becomes lower than the inventory cost difference between two AM sites. Consequently, inventory within the manufacturer coalition is redistributed to reduce costs. This result aligns with the findings in Figure 2(a).

Overall, our analysis indicates that the relative proportions of transportation cost components play a pivotal role in logistics cost optimisation. When the finished product delivery cost constitutes a significant portion of the total cost, it is more cost-effective to allocate orders to AM sites closer to demand points. Conversely, when the raw material transportation cost dominates the total cost, assigning orders to sites closer to raw material suppliers is preferable. Furthermore, inventory costs and raw



(a) $c_{io} - c_{ij} - c_i$ comparison (b) Total cost-Transportation cost

Figure 4. Comparison of Transportation Component Costs and Total Costs. (a) $c_{io} - c_{ij} - c_i$ comparison. (b) Total cost-Transportation cost.

material transportation costs jointly determine the value of inter-facility material transfers.

5.4.3. Cost allocation gap exploration

In this subsection, we investigate the impact of varying process times p_o and capacities C_i on the cost allocation gap, defined as $\left(1 - \frac{\sum_{k\in\{M\cup R\}} \alpha(k)}{C(M\cup R)}\right) \times 100\%$. Specifically, we examine three distinct groups: Group 1 $(M,R) = (10,10)$ with $C_i = 50$, $\forall i \in M$, Group 2 $(M,R) = (10,10)$ with $C_i = 150$, $\forall i \in M$, and Group 3 $(M,R) = (10,30)$ with $C_i = 150$, $\forall i \in M$. The process times p_o are selected from the set $\{7,11,13,17,19,23\}$, which serve as the horizontal axis in our analysis. For each group, we conduct 50 trials to analyze their characteristics.

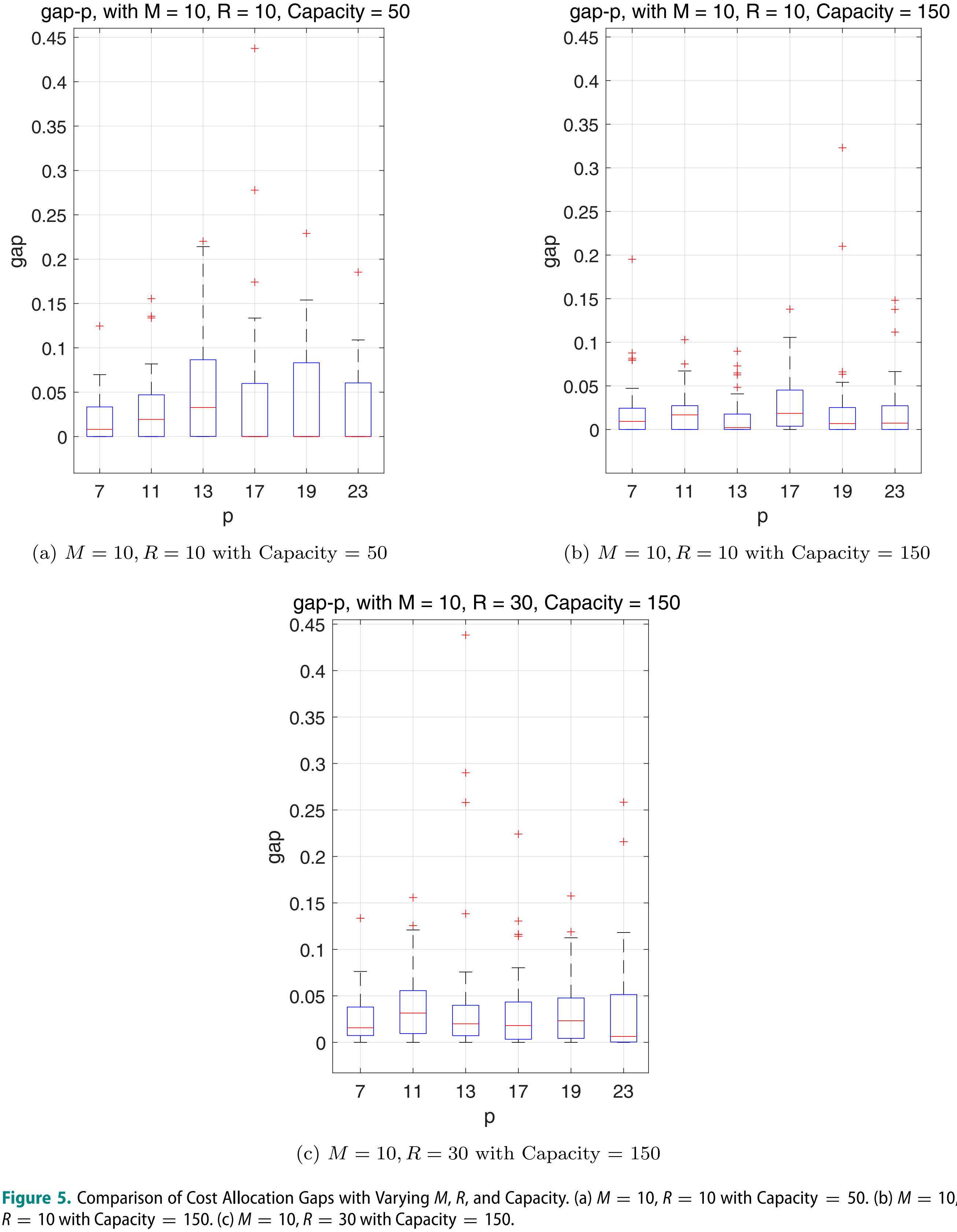
The results are presented in Figure 5. Our findings reveal that the capacity of factories and the number of orders (retailers) significantly influence the degree of dispersion in the cost allocation gap. When comparing Figure 5(a,b), Group 2, which possesses a higher capacity, exhibits a narrower range of gap variation compared to Group 1. Similarly, Figure 5(b,c) demonstrate that the gap variation range for Group 3, with more orders (retailers), is larger than that of Group 2. Notably, the impact of process time on the cost allocation gap is less predictable. However, in Figure 5(a), we observe that the median initially increases as the process time rises but drops to zero when the process time exceeds 15. This phenomenon may be attributed to capacity constraints, as a single factory can only serve one retailer when the process time is large.

Overall, our analysis indicates, the central decision-maker exhibits a preference for AM providers with greater production capacity, as such partners reduce the indivisible costs that the central decision maker must bear.

6. Conclusion

In this paper, we address logistics cost optimisation and allocation for additive manufacturing make-to-order cooperation by developing a mixed-integer linear programming model that integrates production, inventory, and transportation costs. Our numerical experiments confirm that cooperation among additive manufacturers can lead to significant cost reductions, with transportation cost ratios playing a critical role in cost allocation. We analyze two distinct cost allocation models – one involving both manufacturers and retailers and another comprising only additive manufacturing manufacturers. The former may result in an empty core scenario, whereas the latter ensures a stable cost allocation mechanism. To address the empty core issue, we propose an exact row-generation algorithm and a hybrid approximation method. Experimental results demonstrate that the exact algorithm is effective for small-scale problems, whereas the hybrid method excels in large-scale cases.

Our study also provides several managerial insights: centralised cooperation enables production at the lowest-cost facility, optimises raw material transfers for inventory efficiency, and enhances centralised procurement for better supplier negotiations. Additionally, the allocation



(a) $M = 10, R = 10$ with Capacity $= 50$

(b) $M = 10, R = 10$ with Capacity $= 150$

(c) $M = 10, R = 30$ with Capacity $= 150$

Figure 5. Comparison of Cost Allocation Gaps with Varying M, R, and Capacity. (a) $M = 10$, $R = 10$ with Capacity $= 50$. (b) $M = 10$, $R = 10$ with Capacity $= 150$. (c) $M = 10$, $R = 30$ with Capacity $= 150$.

strategy should prioritise the dominant cost driver – whether it is finished product delivery or raw material transportation. Furthermore, the central decision-maker exhibits a preference for AM providers with greater production capacity, as such partners reduce the indivisible costs that the central decision maker must bear.

Despite these contributions, our work also identifies several limitations. The additive manufacturing production process, which involves multiple operational stages, is not fully captured in our model. Expanding the framework to incorporate non-linear cost functions would enhance its practical applicability, though it would also introduce additional computational challenges. Furthermore, the paper does not explore real-world production constraints, which remain largely unexamined. Investigating these constraints and their impact on cost allocation represents a valuable direction for future research.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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Data availability

The data that support the findings of the study are available from the corresponding author, Y. Zhu, upon reasonable request.

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Appendix. Baseline model of benefit comparison

We simplify the model (18) by removing centralised procurement and inter-facility material redistribution, resulting in a baseline model (A1).

$$
\begin{aligned}
C_{no\text{-}co}(M \cup R) = \min \quad & \sum_{i\in M}\sum_{r\in R}\sum_{o\in O_r}(f_{io} + c_{io} - h_i d_o)x_{io} \\
& + \sum_{i\in M}((c_i + h_i)z_i + h_i D_i m_i + P\cdot z_i') \\
\text{s.t.} \quad & \sum_{r\in R}\sum_{o\in O_r} d_o x_{io} \le D_i m_i + z_i, \\
& \forall i \in M \\
& \sum_{r\in R}\sum_{o\in O_r} p_{io}x_{io} \le C_i m_i, \quad \forall i \in M \\
& z_i \le z_i', \quad \forall i \in M \\
& z_i' \le Q m_i, \quad \forall i \in M \\
& \sum_{i\in M} x_{io} = 1, \quad \forall o \in O_r, \ \ r \in M \\
& x_{io}, m_i \in \{0,1\}, \quad \forall i \in M, \\
& o \in O_r, \ \ r \in M \\
& z_i \ge 0, \quad z_i' \in \mathbb{N}, \ \ \forall i \in M. \qquad \text{(A1)}
\end{aligned}
$$

This adjustment brings the partnership model closer to real-world practices. For orders with small profit margins, manufacturers typically coordinate production through standard transactions without engaging in more complex cooperative agreements involving raw material sharing or joint procurement alliances.