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Supply chain with random yield and financing☆

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ABSTRACT

In this paper, we consider a supply chain consisting of a well-capitalized manufacturer, a well-capitalized and reliable supplier, and a capital-constrained and unreliable supplier with random yield. The interactive relationship among them is modeled as a Stackelberg game with the manufacturer as the leader and the suppliers as the followers. The optimal production quantity of the suppliers and ordering quantity of the manufacturer are derived. We investigate the manufacturer's optimal sourcing strategy considering a fixed interest rate. The analytical model shows that when external financing is available, the optimal sourcing strategy depends on the interest rate threshold. Specifically, if the interest rate is lower than the threshold, the unreliable channel dominates; otherwise, the reliable channel dominates. When financing is not available and the supply price of the reliable supplier is high, dual-channel sourcing is the optimal choice. Computational studies are performed to explore the impacts of the initial capital and the interest rate on the optimal decisions and profits. It shows capital constraint does not always decrease the attraction of the unreliable supplier for the manufacturer. Actually the attraction of using an unreliable supplier can increase if the interest rate is low or the initial capital of the unreliable supplier is small. Several extensions are considered in this paper. We show the attraction of the unreliable supplier for the manufacturer will increase if the sourcing price from the unreliable supplier is endogenously determined by the manufacturer or the demand is random.

1. Introduction

Due to economic globalization, many manufacturers in North America have begun to source from unreliable (small but cheap) suppliers in developing countries such as China and India [16]. Manufacturers can certainly benefit from global sourcing by reducing their sourcing costs. However, at the same time, the overall supply risk increases because of the yield uncertainty of the unreliable suppliers. In practice, yield uncertainty is a common phenomenon in many industries. It refers to the situation where the output quantity may differ from the input quantity. For example, in the agriculture industry, the crop output is uncertain due to many uncontrollable factors, such as weather condition, temperature or natural disasters [10]. In the semiconductor industry, the yield of the manufacturer's production system may differ from the initial production quantity because of production process risks or the failure of a replenishment batch to satisfy quality standards [20,39]. In reality, supply uncertainty can lead to huge losses for firms. For example, Ericsson suffered a $400 million in lost sales due to supply disruption [33]. Hence, many firms such as Nokia and Wal-Mart adopt dual sourcing strategy to reduce the supply risk [15,33]. To study the sourcing strategies of the manufacturer, in the basic model, we focus on the random yield risk and consider a determined demand setting that has practical implications for production planning in some industries [3,14].

Usually, the manufacturer can simply choose to enhance the order quantity from the unreliable supplier to ease the negative effects of random yield. However, when taking the capital shortage of the unreliable supplier into consideration, the scenarios may be different, because the unreliable supplier's production decision is affected by the financing situation (e.g., the bank's interest rate), thus influencing the manufacturer's ordering behavior. In fact, capital shortage and financing are not rare in developing countries. For example, China has more than 40 million small and medium enterprises (SMEs), most of which face capital shortages and urgently need financing. But these financial needs are hardly satisfied, as banks (the main financing access) usually prefer large firms to SMEs, and sometimes, the financing interest rates are high.

It is clear that the capital shortage and the financing pressure (i.e., the financing interest rate) significantly affect the whole

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supply chain. From the perspective of the unreliable supplier, due to capital shortage, the production plan depends on not only the manufacturer's order quantity but also on the supplier's initial capital level and the financing interest rate. From the perspective of the manufacturer, when facing reliable and unreliable suppliers, he could choose a sole-reliable-sourcing channel, a sole-unreliable-sourcing channel, or a mixture of these two channels to balance the trade-off between cost savings and supply risks. In addition to the channel choice, the manufacturer must decide the allocation of order quantities between the two different types of suppliers, according to factors such as the initial capital level of the unreliable supplier and the bank's financing interest rate.

To address the concerns raised above, in this paper, we consider a simple supply chain consisting of a well-capitalized manufacturer and two suppliers. One supplier is well capitalized and reliable in production, while the other supplier is short of capital and has random yield. To overcome capital shortage, the unreliable supplier may borrow money from the bank. As the interest rate in some countries such as China is controlled by external parties, in the basic model, we assume the interest rate of bank loan is exogenous. For example, the Bank of Zhengzhou launches a financing product “Weimiaodai” for small and micro enterprises. The bank sets unified interest rate (the daily interest rate is 0.045%) for all small and micro enterprises. The interactive relationship among the manufacturer and the suppliers is modeled as a Stackelberg game where the manufacturer is the leader and the suppliers are the followers. At the beginning, a determined demand is realized to the manufacturer. By observing the reliable and unreliable suppliers as well as the bank, the manufacturer can decide the optimal order quantities from each supplier to maximize his own profit. After receiving sourcing orders from the manufacturer, the suppliers start production. For the reliable supplier, the optimal production quantity is simply equal to the manufacturer's order quantity, while for the unreliable suppler, due to the capital shortage and random yield, the optimal production quantity would rely heavily on the supplier's initial capital level and the bank's interest rate. After the order quantities are produced, the manufacturer is able to collect the deliveries and satisfy demand.

According to the model built above, we can find some straightforward results. For example, the unreliable supplier would choose to over-produce due to random yield; when the interest rate is high, the manufacturer would prefer the reliable supplier to avoid high supply risks. Certainly, there are many other interesting questions, which are hard to answer directly. Our goal in this paper is to study the joint effects of random yield and financing on the whole supply chain.

The remainder of this paper is organized as follows: In Section 2, we review some relevant literature on mitigation strategies for supply risk and supply chain finance. In Section 3, we describe the model and introduce the notations. In Section 4, we analyze the optimization problem of the follower (the unreliable supplier) under different initial capital scenarios; and in Section 5, we investigate the optimization problem of the leader (the manufacturer) and try to provide the manufacturer with appropriate sourcing strategies in different situations. In Section 6, we carry out computational experiments that not only demonstrate the results derived in Sections 4 and 5, but also provide numerical insights into problems that are hard to study analytically. In Section 7, we provide extensions and in Section 8, we conclude the paper.

2. Literature review

As an attempt to unify the supply risk and financing in a supply chain, we review the literature in three streams, i.e., random yield, sourcing strategy and supply chain finance.

Random yield has drawn extensive attentions from many researchers in the community of operations management. Henig and Gerchak [18] made a comprehensive analysis of a general periodic review production and inventory model in the random yield setting. Yano and Lee [47] reviewed many models determining the lot size of a production or inventory system with random yield risk. Federgruen and Yang [13] developed an algorithm to find the optimal sourcing strategy that includes a set of suppliers with random yield, and their order shares. Dong et al. [10] examined the effect of random yield on the sourcing decision and supply diversification under two pricing schemes. In addition, Wang et al. [40] summarized uncertain supply models as three types—random capacity, random yield, and random disruption, making the intuitive distinction between random disruption and random yield—assuming that the random disruption results in either 0% or 100% of the target yield. Thus, supply disruption can be regarded as a particular case, and the literature concerning random yield cases is also suitable for supply disruption cases.

To overcome the drawbacks of random yield, decision makers always choose to order products from different suppliers. Dada et al. [7] examined the procurement problem of a newsvendor that sourced from multiple reliable or unreliable suppliers and surprisingly found that supply reliability is not a necessary condition of a given supplier being chosen, but the supply price is. Federgruen and Yang [11,12] developed procedure to help a newsvendor select the optimal set of suppliers with random yield in the service constraint model (SCM) and the total cost model (TCM), respectively. Wu and Zhang [42] characterized the equilibrium outcome of efficient sourcing with responsive sourcing and found efficient sourcing had an advantage. Gurnani et al. [16] considered the dual sourcing problem. However, they assumed that two suppliers had different delivery reliabilities, i.e., one suppliler had reliable delivery, and the other faced random yield risk and disruption risk. He et al. [17] adopted a dual sourcing strategy to mitigate disruption risk through a real-option approach. Niu et al. [35] studied the dual sourcing strategy of an original equipment manufacturer (OEM) faced with a competitive supplier and a non-competitive supplier whose yield was uncertain. Hsieh and Lai [19] investigated the manufacturer's sourcing strategy in the presence of downward substitution and yield uncertainty. In general, a common assumption in current research on channel selection is that suppliers always have adequate initial capital. This makes our paper distinct, because we unify random yield and capital shortage (financing) in a supply chain

Supply chain finance has been one of the most popular directions in operations management since Buzacott and Zhang [4] and Xu and Birge [44] first modeled the joint operations and finance decisions of a newsvendor. In the study of supply chain finance, works mainly explore trade credit (e.g., [8,28,32,38,41,46,48]), bank credit (e.g., [1,4,6,25]) and their comparisons (e.g., [5,21,23]) under demand uncertainty. These works mainly assumed the retailer faces capital constraint, however, some works also explored the problem in different settings where the supplier is capital constrained. For example, Lai et al. [27] considered the inventory risk sharing problem in a capital-constrained supply chain and explored the implications of financial constraints for the supplier and the supply chain by comparing the values of three types of supply modes. Deng et al. [9] explored the problem of financing multiple heterogeneous financial-constrained suppliers in an assembly system supply chain with demand uncertainty. Li et al. [34] compared bank lending with buyer lending in a bilateral supply chain composed of a retailer and a capital-constrained supplier subject to spectral risk measures. Jin et al. [22] compared three financing strategies, i.e., a non-collaborative strategy (bank financing separately), and two collaborative strategies (bank financing with trade credit and bank financing with the supplier's guarantee) in a two-echelon supply chain where both the supplier and retailer are capital constrained. Lee and Rhee [30] and Kouvelis and Zhao [24] designed contracts to coordinate the supply chain where the supplier and the retailer confronted capital constraint. Xiao and Zhang [43] considered advance selling as a way to mitigate the supplier's financial distress and derived the supplier's optimal strategy. Similarly, Zhao and Huchzermeier [49] considered two financing methods, advance payment discount (APD) and buyer-backed purchase order financing (BPOF), to alleviate the supplier's capital restriction. The authors showed under what conditions the retailer preferred APD to BPOF. Instead of addressing the problem of the supplier, retailer or supply chain, Raghavan and Mishra [36] mainly considered the lender's decision in a capital-constrained supply chain. These works on addressing the problem of the capital-constrained supplier assumed that the supply from the supplier is reliable, and that the sole uncertainty faced by the supply chain exists on the demand side.

In summary, although many works have studied random yield and financing in a supply chain, few integrated random yield and financing in a unified framework and explored their joint effect. Notable exceptions include Babich et al. [2] and Tang et al. [37], which are the most relevant works to our research. Babich et al. [2] explored the optimal choice of a number of homogeneous suppliers that face stochastic proportional yield and offer funds to the capital-constrained newsvendor. Their results showed that firms operating in developing economies should not contract with more suppliers than firms operating in developed economies when the supply is uncertain and there is a high fixed cost of an extra supplier. Different from Babich et al. [2], we consider the scenario where two suppliers are available for the manufacturer and one of them faces capital constraint and stochastic yield. Tang et al. [37] examined the value of buyer financing versus bank financing, and the impact of the manufacturer's information advantage on buyer financing dominance in the scenario where an unreliable supplier is solely available and the stochastic delivery probability is endogenously determined. Our work is different from Tang et al. [37] in two ways. First, our work focuses on the channel preference of the manufacturer sourcing from two channels, one of which is reliable and the other unreliable. Second, we extend their endogenously determined delivery binomial probability for the unreliable supplier to an exogenously determined general continuous distribution.

3. Model description

We consider a two-echelon supply chain consisting of a well-capitalized manufacturer and two suppliers. Both suppliers can provide the manufacturer with identical products. In particular, one of the suppliers is reliable, whose initial capital level is adequate, and yield is non-random; the other supplier is unreliable, whose initial capital level may be limited and yield is random. We assume all the supply chain players are risk-neutral and there is no information asymmetry in the supply chain.

The determined demand faced by the manufacturer is D, and the unit retail price is p. The unit sourcing prices from the reliable and unreliable suppliers are w_h and w_l, respectively. The unit production costs of the reliable and unreliable suppliers are c_h and c_l, respectively. In the basic model, we assume that the initial capital level of the unreliable supplier, denoted by y, is zero (e.g., [21,31]). In this case, the unreliable supplier has to finance production and borrow money from a bank at a fixed interest rate of r. When the unreliable supplier plans to produce Q units of products, he will first borrow cQ from the bank and then yield XQ units of products, where X is a random proportional variable distributed between 0 and 1 with cumulative distribution function (CDF) $F(x)$ and probability density function (PDF) $f(x)$ [2]. Define the complementary cumulative distribution function as $\bar{F}(x) = 1 - F(x)$ and generalized failure rate as $H(x) = \frac{xf(x)}{\bar{F}(x)}$, where $H(x)$ increases in x [29]. Without loss of generalization, we assume the unreliable supplier has limited liability and will go bankrupt if the sales value cannot repay the loan amount [23–26].

To avoid trivial cases, we assume that $p > w_h > w_l$, $w_h > c_h$, and $w_l > \max\{c_l/E[X], c_l(1+r)\}$, where $w_h > w_l$ reflects the difference in the reliabilities of the two suppliers, $w_l > c_l/E[X]$ ensures that the unreliable supplier has the incentive to produce, and $w_l > c_l(1+r)$ ensures that the unreliable supplier has the incentive to borrow. Furthermore, for simplicity, we do not consider several traditional concepts, such as the salvage value of unsold products and the opportunity cost of capital, because they don't have a significant impact on the main conclusions of this paper [21]. For convenience, we summarize the key notations in Table 1.

To facilitate discussions, we model the manufacturer and the suppliers as players in a Stackelberg game, where the manufacturer is the leader and the suppliers are the followers. The sequence of events and decisions is shown in Fig. 1.

To be specific, when D, p, w_h, w_l, c_h, c_l, r, and the distribution of X are revealed, the manufacturer first decides the respective order quantities, denoted as q_h and q_l, from the reliable and unreliable suppliers. Then, the suppliers start production, where the reliable supplier's production quantity is exactly q_h, while the unreliable supplier's production quantity Q is decided by q_l, w_l, c_l and r. When production is completed, the manufacturer receives q_h and $\min\{XQ, q_l\}$ units of goods from the reliable and unreliable suppliers, respectively, and then pays the sourcing costs. Note that the

Table 1
Summary of notations.

Notation	Definition
p	Unit retail price
w_h	Unit sourcing price from the reliable supplier
w_l	Unit sourcing price from the unreliable supplier
D	Deterministic market demand
c_h	Unit production cost of the reliable supplier
c_l	Unit production cost of the unreliable supplier
X	Random yield rate, where the probability density function is $f(x)$ and the cumulative distribution function is $F(x)$
q_h	Order quantity from the reliable supplier
q_l	Order quantity from the unreliable supplier
Q	Production quantity of the unreliable supplier
y	Initial capital of the unreliable supplier
r	Interest rate
π	Profit of the unreliable supplier
Π	Profit of the manufacturer

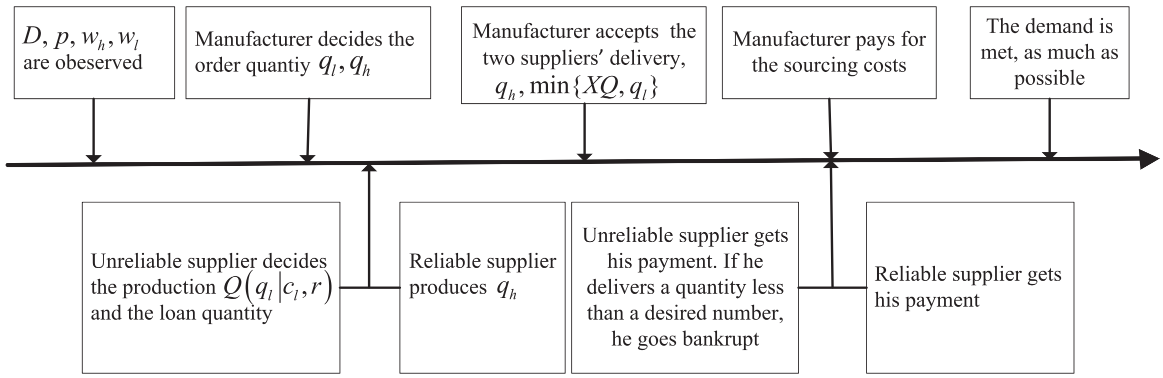

Fig. 1. Sequence of events and decisions.

unreliable supplier may go bankrupt if his yields cannot meet the desired order quantity.

4. Optimization problem of the unreliable supplier

We first study the optimization problems of the followers (suppliers) in the Stackelberg game. Remind that the optimal production quantity of the reliable supplier is always q_h, because the reliable supplier's yields are determined, and the initial capital is adequate. Therefore, we focus on the optimization problem of the unreliable supplier. The investigation includes three scenarios where the initial capital is adequate, limited, and zero, respectively.

4.1. Unreliable supplier with adequate capital

When the initial capital is adequate for the unreliable supplier, the supplier does not need to finance from the bank, and the profit under a given production quantity Q is simply $w_l \min(XQ, q_l) - c_l Q$. Therefore, the expected profit of the unreliable supplier can be expressed as:

$$\pi_a = w_l Q \int_0^{q_l/Q} \overline{F}(x)dx - c_l Q. \tag{1}$$

By solving Eq. (1), we can obtain the optimal production quantity for an unreliable supplier with adequate initial capital. The main results are shown in Lemma 4.1.

Lemma 4.1. *For a given order quantity q_l, we have that:*

(i) *the optimal production quantity Q_A^* of the unreliable supplier with adequate capital satisfies $w_l \int_0^{q_l/Q_A^*} x f(x)dx - c_l = 0$.*
(ii) *Q_A^* is larger than and increases in the order quantity q_l;*
(iii) *Q_A^* increases in the unit sourcing price w_l.*

4.2. Unreliable supplier with capital shortage

We now consider the case where the unreliable supplier has limited initial capital, i.e., $0 < y < c_l Q_A^*$. Then, the expected profit of the unreliable supplier can be expressed as:

$$\pi_l = E[w_l \min(XQ, q_l) - (c_l Q - y)(1+r)]^+ - y. \tag{2}$$

Intuitively, there exists a bankruptcy threshold on the realization of X, denoted by δ_l, below which the unreliable supplier will go bankrupt, that is, the unreliable supplier cannot repay the loan to the bank after selling the produced order to the manufacturer. By letting $w_l \delta_l Q = (c_l Q - y)(1+r)$, we can obtain that $\delta_l = (c_l Q - y)(1+r)/(w_l Q)$.

According to the definition of δ_l, we can rewrite Eq. (2) as:

$$\pi_l = w_l Q \int_{\delta_l}^{q_l/Q} \overline{F}(x)dx - y. \tag{3}$$

When π_l is a concave function in production quantity Q, we can find the optimal production quantity for an unreliable supplier with limited initial capital by solving Eq. (3). The main results are shown in Proposition 4.1.

Proposition 4.1. *For an unreliable supplier with limited initial capital ($0 < y < c_l Q_A^*$), if $\frac{q_l^2}{Q_l^2} f(\frac{q_l}{Q_l}) - (\frac{c_l(1+r)}{w_l} - \delta_l)^2 f(\delta_l) > 0$, where Q_l satisfies $\int_{\delta_l}^{q_l/Q_l} x f(x)dx = \frac{c_l(1+r)}{w_l} \overline{F}(\delta_l)$, we have that:*

(i) *the optimal production quantity of the unreliable supplier is*
$$Q_L^* = \begin{cases} Q_l & \text{if } \int_0^{\min\{1, c_l q_l/y\}} x f(x)dx > (c_l(1+r)/w_l), \\ y/c_l & \text{otherwise}. \end{cases}$$
(ii) *Q_l is larger than and increases in the order quantity q_l.*

By Proposition 4.1, we can see that when the interest rate is low, the unreliable supplier will borrow from the bank during the production process to meet the manufacturer's order; otherwise, the unreliable supplier will not borrow money from the bank and spend all the initial capital in the production. Once the supplier decides to borrow from the bank, the resulting production quantity Q_l is always larger than order quantity q_l to ease the negative effects of random yield.

Note that Proposition 4.1 relies on the condition that $\frac{q_l^2}{Q_l^2} f(\frac{q_l}{Q_l}) - (\frac{c_l(1+r)}{w_l} - \delta_l)^2 f(\delta_l) > 0$. However, if X follows a uniform distribution or a normal distribution, it is easy to show the prerequisite is satisfied, which implies that the results of Proposition 4.1 are general. In addition, if this prerequisite is not satisfied, the unreliable supplier's optimal production Q_L^* is either at stationary point or $Q_L^* = y/c$.

4.3. Unreliable supplier with zero initial capital

In the zero-capital case, for a given order quantity q_l, the unreliable supplier has to borrow money from the bank at an interest rate of r. Thus, the profit of the unreliable supplier can be expressed as:

$$\pi_z = E[w_l \min(XQ, q_l) - c_l Q(1+r)]^+. \tag{4}$$

Similar to the previous cases, there also exists a bankruptcy threshold on the realization of X, which is denoted by δ_z and equal to $c_l(1+r)/w_l$.

According to the above analyses, we can rewrite π_z as:

$$\pi_z = w_l Q \int_{\delta_z}^{q_l/Q} \overline{F}(x)dx. \tag{5}$$

By solving Eq. (5), we can find the optimal production quantity for an unreliable supplier with zero initial capital. The main results are shown in Proposition 4.2.

Proposition 4.2. *For a given order quantity q_l, we have that:*

(i) *the optimal production quantity Q_Z^* of the unreliable supplier with zero initial capital satisfies $\int_{\delta_z}^{q_l/Q_Z^*} xf(x)dx = \delta_z \overline{F}(\delta_z)$;*
(ii) *Q_Z^* is larger than and increases in the order quantity q_l;*
(iii) *Q_Z^* increases in the unit sourcing price, w_l; and decreases in the interest rate r.*

The results of Proposition 4.2 coincide with those of Proposition 4.1. In addition, to balance the trade-off between marginal profit and interest, it is intuitive that Q_Z^* is increasing in w_l and decreasing in r.

5. Optimization problem of the manufacturer

Based on the results derived in Section 4, we are ready to study the optimization problem of the leader (manufacturer). As the leader of the Stackelberg game, the manufacturer has multiple choices of sourcing channels by appropriately allocating order quantities from each supplier, including sourcing solely from a reliable supplier, sourcing solely from an unreliable supplier, and sourcing from both suppliers. Note that in the first case, the result is straightforward where the manufacturer orders D units of goods from the reliable supplier. In the remainder of this section, we investigate the latter two cases.

5.1. Sole-unreliable-sourcing (SUS) channel

When sourcing solely from the unreliable channel, the manufacturer places an order with quantity $q_{l,s}$ to the supplier. Due to the random yield, the realized delivery from the unreliable supplier is $\min(XQ, q_{l,s})$. Then, the realized sales to customers can be denoted as $\min(XQ, q_{l,s}, D)$. Therefore, the expected profit of the manufacturer is

$$\Pi_s = E\Big[p \min(XQ, q_{l,s}, D) - w_l \min(XQ, q_{l,s})\Big]. \tag{6}$$

Lemma 5.1. *In the SUS case, the optimal order quantity for the manufacturer is larger than or equal to the market demand,* i.e., *$q_{l,s}^* \geq D$.*

According to Eq. (6), since $q_{l,s}^* > D$, we rewrite the expected profit of the manufacturer as

$$\Pi_s = pQ\int_0^{D/Q} \overline{F}(x)dx - w_l Q\int_0^{q_{l,s}/Q} \overline{F}(x)dx. \tag{7}$$

By solving Eq. (7), we can obtain the optimal order quantity $q_{l,s}^*$ of the manufacturer from the unreliable supplier shown in Proposition 5.1.

Proposition 5.1. *In the SUS case, the optimal order quantity $q_{l,s}^*$ of the manufacturer solves $p\int_0^{D/Q^*} xf(x)dx = w_l \int_0^{q_{l,s}^*/Q^*} \overline{F}(x)dx$.*

Note that Q^* in Proposition 5.1 denotes the optimal production quantity of the unreliable supplier when the order quantity from the manufacturer is revealed. The value of Q^* can be substituted by either Q_Z^* or Q_A^* to indicate the case where the initial capital level of the unreliable supplier is zero or adequate. Let $q_{l,s}^{z*}$ and $q_{l,s}^{a*}$ be the manufacturer's optimal order quantities when facing an unreliable supplier with zero and adequate capital in the SUS case, respectively. Consequently, denote the resulting production quantities and expected profits of the supplier as $Q_Z^*(q_{l,s}^{z*})$, $\Pi_s(q_{l,s}^{z*})$ and $Q_A^*(q_{l,s}^{a*})$, $\Pi_s(q_{l,s}^{a*})$, respectively. Then we have Corollary 5.1 revealing some of their properties.

Corollary 5.1. *(i) In the SUS case, $q_{l,s}^{z*}$ increases in r, while $Q_Z^*(q_{l,s}^{z*})$ decreases in r;*

(ii) If $r < \tilde{r}$, we have $Q_Z^*(q_{l,s}^{z*}) > Q_A^*(q_{l,s}^{a*})$ and $\Pi_s(q_{l,s}^{z*}) > \Pi_s(q_{l,s}^{a*})$; otherwise, we have $Q_Z^*(q_{l,s}^{z*}) \leq Q_A^*(q_{l,s}^{a*})$ and $\Pi_s(q_{l,s}^{z*}) \leq \Pi_s(q_{l,s}^{a*})$, where $\tilde{r}$ solves $c_l/w_l = \int_0^{\delta_z(\tilde{r})} \overline{F}(x)dx$.

Corollary 5.1(i) shows that when the interest rate increases, the manufacturer's order quantity increases while the supplier's production quantity decreases. To be simple, under a given order quantity, it is clear that the supplier will reduce production to ease the negative effects from the increase in the interest rate. In this case, the manufacturer has to increase his order quantity to get enough deliveries in the end.

Corollary 5.1(ii) shows that, when the interest rate is low, the manufacturer benefits in the case where the supplier is zero-capitalized; otherwise, the manufacturer prefers an unreliable supplier with adequate capital. The intuition is that, in the low interest case, a zero-capitalized supplier tends to produce more to avoid bankruptcy compared with an adequately capitalized supplier. Therefore, the expected delivery to the manufacturer is higher, and this, in turn, benefits the manufacturer by reducing the order quantity at the beginning.

We now compare the SUS case with the sole-reliable-sourcing (SRS) case. As stated in the beginning of this section, the optimal order and production quantities in the SRS case are both D. Therefore, the resulting profit of the manufacturer is simply $(p - w_h)D$. Denote Δ as the gap of the expected profits between the SUS and SRS cases, we have that

$$\Delta = pQ_Z^*\left(q_{l,s}^{z*}\right)\int_0^{D/Q_Z^*\left(q_{l,s}^{z*}\right)} \overline{F}(x)dx - w_l Q_Z^*\left(q_{l,s}^{z*}\right)\int_0^{q_{l,s}^{z*}/Q_Z^*\left(q_{l,s}^{z*}\right)} \overline{F}(x)dx \\ -(p - w_h)D.$$

According to Proposition 5.1, by substituting $w_l \int_0^{q_{l,s}^{z*}/Q_Z^*(q_{l,s}^{z*})} \overline{F}(x)dx$ with $p\int_0^{D/Q_Z^*(q_{l,s}^{z*})} xf(x)dx$, we can rewrite Δ as:

$$\Delta = pD\Big(w_h/p - F\Big(D/Q_Z^*\Big(q_{l,s}^{z*}\Big)\Big)\Big). \tag{8}$$

By letting $\Delta = 0$, we can find an indifferent threshold of r, denoted by $\widehat{r}$, below which $\Delta > 0$ (SUS is preferred), and $\Delta < 0$ (SRS is preferred), otherwise. The detailed results are shown in Theorem 5.1.

Theorem 5.1. *For any given interest rate r, the manufacturer: (i) prefers SUS, if $r < \widehat{r}$; (ii) prefers SRS, if $r > \widehat{r}$; (iii) is indifferent between SUS and SRS, where $\widehat{r}$ ensures*

$$\begin{cases} F\Big(D/Q_Z^*\Big(q_{l,s}^{z*}\Big)\Big) = w_h/p, \\ \int_{\delta_z(\widehat{r})}^{q_{l,s}^{z*}/Q_Z^*(q_{l,s}^{z*})} xf(x)dx = \delta_z(\widehat{r})\overline{F}(\delta_z(\widehat{r})), \\ p\int_0^{D/Q_Z^*(q_{l,s}^{z*})} xf(x)dx = w_l \int_0^{q_{l,s}^{z*}/Q_Z^*(q_{l,s}^{z*})} \overline{F}(x)dx. \end{cases}$$

Note that if $w_h \leq pF(D/Q_Z^*(q_{l,s}^{z*}))|_{r=0}$, i.e., the unit sourcing price from the reliable supplier is relatively low, the manufacturer will always prefer SRS even though the interest rate is zero. Based on Theorem 5.1, we can further analyze the dependences of $\widehat{r}$ on the parameters of the supply chain (w_h, w_l, p, and D). The results are shown in Theorem 5.2.

Theorem 5.2. *The indifferent threshold $\widehat{r}$ of SUS and SRS: (i) increases in w_h; (ii) decreases in w_l and p; and (iii) is independent on D.*

As an additional cost of the unreliable supplier, it is intuitive that $\widehat{r}$ increases in w_h and decreases in w_l. When the retail price p increases, due to a higher marginal profit, the manufacturer tends to order more from the reliable supplier to reduce the risk of demand loss. Therefore, $\widehat{r}$ would be smaller to indicate the decline of the unreliable supplier's competitiveness. The independence between $\widehat{r}$ and D implies that the market demand is just a scaling effect and does not influence the manufacturer's channel choice.

5.2. Mixed reliable and unreliable sourcing channel

When the manufacturer chooses to source from reliable and unreliable channels, that is, sources from a mixed-sourcing channel, the realized deliveries are $q_{h,\,d}$ and $min(XQ,\,q_{l,\,d})$, respectively. Therefore, the expected profit of the manufacturer is

$$\Pi_d = E\Big[p\min\big(\min(XQ, q_{l,d}) + q_{h,d}, D\big) - w_l \min(XQ, q_{l,d}) - w_h q_{h,d}\Big]. \tag{9}$$

We need to point out that it is hard to derive analytical results for Eq. (9) when the effects of limited initial capital $0 < y < c_l Q_A^*$ and interest rate r are considered. Thus, we carry out numerical experiments in Section 6 to show how they jointly affect the manufacturer's channel decisions.

Instead, in this section, we study two special cases of Eq. (9) where either the initial capital is zero ($y = 0$) or there is no access to financing.

5.2.1. Facing an unreliable supplier with zero capital

In the mixed-sourcing case, it is easy to show that the manufacturer's total order quantities from both suppliers are not lower than the total market demand, i.e., $q_{l,d}^{z*} + q_{h,d}^{z*} \geq D$. When the initial capital level of the unreliable supplier is zero, the expected profit of the manufacturer can be expressed as

$$\Pi_d = pD - p(D - q_{h,d})F\left(\frac{D - q_{h,d}}{Q_Z^*}\right) + pQ_Z^* \int_0^{\frac{D - q_{h,d}}{Q_Z^*}} xf(x)dx$$
$$-w_l Q_Z^* \int_0^{\frac{q_{l,d}}{Q_Z^*}} \overline{F}(x)dx - w_h q_{h,d}. \tag{10}$$

By solving Eq. (10), we can obtain the necessary condition under which the manufacturer can choose the mixed-sourcing channel.

Theorem 5.3. *When $y = 0$, the mixed-sourcing channel can be optimal for the manufacturer if $p\int_0^{F^{-1}(\frac{w_h}{p})} xf(x)dx - w_l \int_0^{t_z^*} \overline{F}(x)dx = 0$, where t_z^* satisfies $\int_{\delta_z}^{t_z^*} xf(x)dx = \delta_z \overline{F}(\delta_z)$.*

Theorem 5.3 gives a necessary condition for a manufacturer choosing the mixed-sourcing channel. For example, if X is uniformly distributed over interval [0, 1], $y = 0$, $r = 0.03$, $c_l = 0.2$, $w_l = 0.5$, $w_h = 0.6208$, $p = 0.8$, then the manufacturer is indifferent between the two channels and can choose the mixed-sourcing channel. However, it is easy to see the condition is quite strict, which indicates that the sole-sourcing channel is more likely to be optimal for the manufacturer when the manufacturer faces an unreliable supplier with zero capital. The results agree with those of Dong et al. [10] and Gurnani et al. [16], who showed that the sole-sourcing channel is always optimal to the manufacturer.

5.2.2. Facing an unreliable supplier with no access to financing

When there is no access to financing, or equivalently, the interest rate is quite high, we have Proposition 5.2 that specifies the manufacturer's sourcing strategy by solving Eq. (9).

Proposition 5.2. *When $0 < y < c_l Q_A^*(q_{l,s}^{a*})$ and there is no access to financing, the optimal sourcing strategy of the manufacturer depends on the sourcing price w_h, specifically, if $w_h > \overline{w_h}$, we have $q_{h,d}^{y*} = \max\{D - yF^{-1}(w_h/p)/c_l, 0\}$ and $q_{l,d}^{y*} = q_0$, otherwise we have $q_{h,d}^{y*} = D$ and $q_{l,d}^{y*} = 0$, where q_0 solves $y = c_l Q_A^*(q_0)$ and $\overline{w_h} = pF(D/Q_A^*(q_{l,s}^{a*}))$.*

Proposition 5.2 shows when financing is not available, there exists an effective wholesale price $\overline{w_h}$ separating the dominance of the two channels. If $w_h < \overline{w_h}$, the manufacturer will choose the reliable supplier as the sole-sourcing channel; Otherwise, when $w_h > \overline{w_h}$, the manufacturer prefers the unreliable-sourcing channel and orders as many products as possible from the unreliable supplier so that the unreliable supplier uses up initial capital.

6. Numerical experiments

In this section, we conduct some numerical experiments to demonstrate the results derived in the previous sections. Furthermore, the analyses of these numerical examples can provide practical insights to the case when the manufacturer choose the mixed-sourcing channel (with limited interest rate and initial capital), which is hard to mathematically analyze as stated in Section 5.2. For initialization, we let $y = 0.5$, $r = 0.05$, $c_l = 0.2$, $w_l = 0.5$, $w_h = 0.625$, $D = 10$, and $p = 0.8$, and X is uniformly distributed over interval [0, 1].

6.1. Impacts of interest rate and initial capital on channel preference

By substituting different pairs of values of r and y in Eq. (9), we can analyze the manufacturer's channel preference shown in Table 2. For simplification, we denote the channel preferences of sole-reliable-sourcing, sole-unreliable-sourcing, and mixed-sourcing as R, U and M, respectively.

In Table 2, the dashed line represents the adequate boundary of the unreliable supplier below which the supplier is equipped with adequate initial capital. This table shows that when the interest rate and the initial capital level are high, the sole-reliable-sourcing channel is the optimal channel choice for the manufacturer. When they are low, the sole-unreliable-sourcing is optimal. The explanations are as follows: (1) For any given y, when r increases, the unreliable supplier tends to reduce production to save costs. Thus, the manufacturer has to increase his order quantity to overcome the effect of random yield, until he finds that it is more profitable to order from the reliable supplier; (2) For any given r, when y increases, the unreliable supplier is more sensitive to bankruptcy and will reduce production. Similarly, the manufacturer will be less attracted by the unreliable channel.

To further analyze the manufacturer's channel preference, we change the value of w_h slightly from 0.625 to 0.725, while keeping the remaining parameters unchanged. The results are shown in Table 3.

As the sourcing price from the reliable supplier is larger, the manufacturer prefers the unreliable supplier more. From Table 3, it can be observed when the unreliable supplier has sufficient capital ($y \geq 2.55$), the optimal choice is the unreliable channel. However,

Table 2
Channel preference of the manufacturer with different y and r ($w_h = 0.625$).

y \ r	0.05	0.10	0.15	0.20	0.25	0.30
0.05	U	U	U	R	R	R
0.30	U	U	U	R	R	R
0.55	U	U	R	R	R	R
0.80	U	U	R	R	R	R
1.05	U	R	R	R	R	R
1.30	U	R	R	R	R	R
1.55	R	R	R	R	R	R
1.80	R	R	R	R	R	R
2.05	R	R	R	R	R	R
2.30	R	R	R	R	R	R
2.55	R	R	R	R	R	R
2.80	R	R	R	R	R	R

Table 3
Channel preference of the manufacturer with different y and r ($w_h = 0.725$).

y \ r	0.05	0.1	0.15	0.2	0.25	0.3
0.05	U	U	U	U	M	M
0.3	U	U	U	U	M	M
0.55	U	U	U	U	M	M
0.8	U	U	U	U	M	M
1.05	U	U	U	U	M	M
1.3	U	U	U	U	M	M
1.55	U	U	U	U	M	M
1.8	U	U	U	U	M	M
2.05	U	U	U	U	M	M
2.3	U	U	U	U	U	U
2.55	U	U	U	U	U	U
2.8	U	U	U	U	U	U

when the unreliable supplier faces capital constraint ($y \le 2.3$), for a given$y$($<$ 2.3), if the interest rate is relatively low, the unreliable channel is still the unique optimal channel; otherwise, dual-channel sourcing is optimal. That is because while the supplier faces capital constraint, the relatively low interest rate spurs the unreliable supplier to borrow and produce more, which makes the unreliable channel more attractive. However, when the interest rate is relatively high, the unreliable channel has no incentive to borrow, and consequently, the production quantity is low, making the risk for the manufacturer high if it uses only the unreliable channel. In such a case, the unreliable supplier would not choose a bank loan and the manufacturer would order first from the unreliable supplier and then from the reliable supplier (i.e., use dual sourcing). For a given y ($= 2.3$), the unreliable channel is still the unique optimal channel even if the unreliable supplier faces capital constraint. The reason for this exception is that when the supplier's initial capital is large, the production is large, too, which makes it possible for the manufacturer to cover the market demand with the delivery of the unreliable channel if the achieved value of the stochastic yield factor is big. In that scenario, if the manufacturer also sources from the reliable channel, then the new incurred overstock risk and high sourcing cost will decrease the manufacturer's expected profits.

The consequences shown in Tables 2 and 3 can be summarized as follows: Given an initial capital level, there exists an interest rate threshold to separate the channel preference, which is consistent with Theorem 5.1 and Proposition 5.2. Proposition 5.2 hints that the single channel sourcing is not always the optimal strategy, and the manufacturer should take the interest rate and the capital state of its suppliers into consideration before making a sourcing channel decision.

6.2. Impacts of interest rate and initial capital on the order and production quantities

By changing the values of r or y in Eq. (9), we can compute the resulting order and production quantities of the manufacturer and the suppliers, respectively. The results are shown in Figs. 2 and 3.

From Figs. 2 and 3, as the interest rate r or the initial capital level y increases, we have the following observations: (1) the manufacturer's order quantity from the reliable supplier increases; (2) the manufacturer's order quantity from the unreliable supplier increases first and then decreases to zero; (3) the unreliable supplier's production quantity is always larger than the corresponding order quantity.

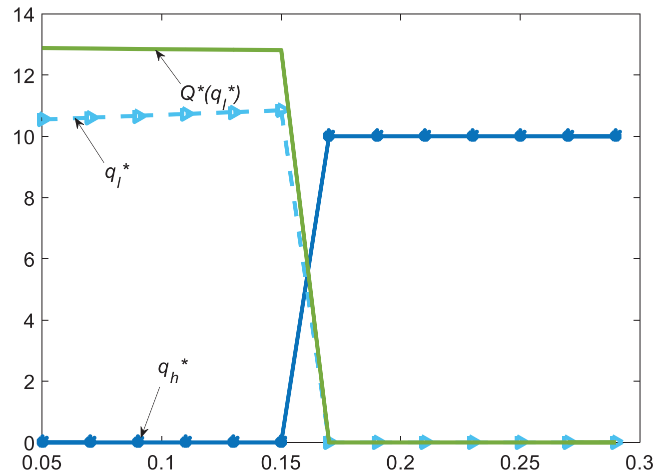

Fig. 2. Impact of interest rate on the order and production quantities.

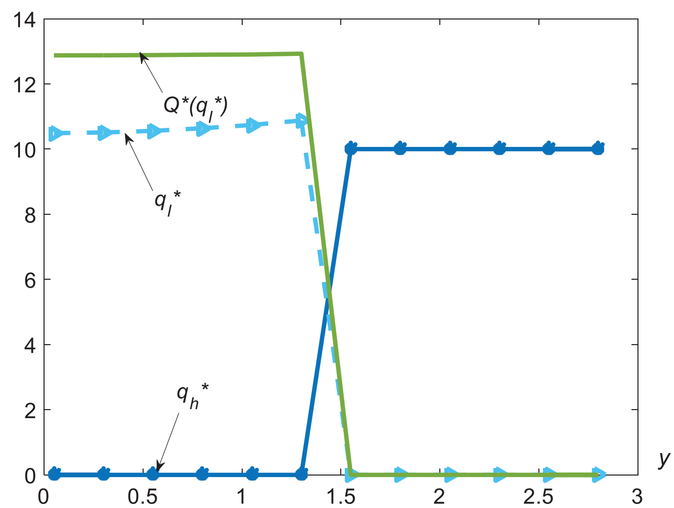

Fig. 3. Impact of initial capital level on the order and production quantities.

The equilibrium order quantity for the unreliable supplier increases in the initial capital level and the interest rate, but the equilibrium production of the unreliable supplier varies inversely with the two parameters. The intuitional explanation for this is that as r or y increases, the incentive of limited liability for the production quantity weakens so that the production of the unreliable supplier will decrease for a given order, which induces the manufacturer to increase the order quantity to stop the production from decreasing. Nevertheless, a high interest rate does not increase the real ability of the unreliable supplier to help cover the negative effect of the interest rate, but a high initial capital level does.

6.3. Impacts of interest rate and initial capital on profits allocation

We now check how the net profits of the manufacturer and the suppliers change according to r or y. Note that the net profit of the reliable supplier is simply $q_h^*(w_h - c_h)$, therefore, we omit the associated explanations in the following analyses. Figs. 4 and 5 illustrate the impacts of y and r on the expected profits of the unreliable supplier and the manufacturer. These figures illustrate that the unreliable channel is the dominant channel if the interest rate or the initial capital quantity is small, whereas the reliable channel is the dominant one otherwise. In addition, the expected profits of the manufacturer and the unreliable supplier decrease in y and r.

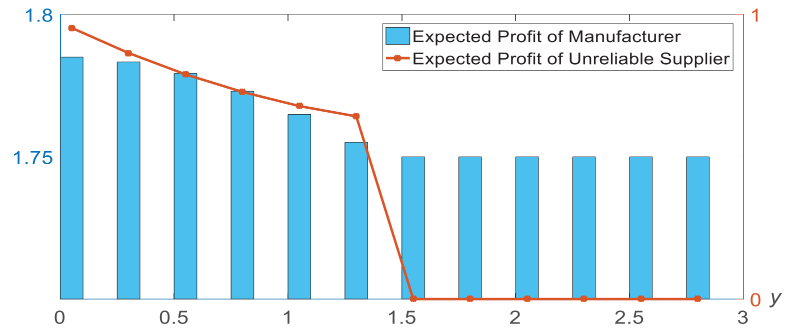

Fig. 4. Impact of initial capital on profits allocation.

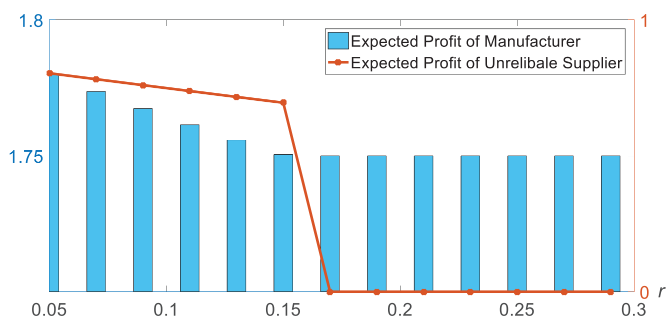

Fig. 5. Impact of interest rate on profits allocation.

The observations above imply that capital constraint and financing details can change the manufacturer's channel preference. The preference changes due to the interest rate, and the capital-constrained unreliable supplier and manufacturer prefer a small initial capital quantity and a low interest rate. This scenario occurs because the equilibrium production of the unreliable supplier versus the order quantity is magnified more critically when he has a low initial capital and a low interest rate due to the unreliable supplier's limited liability. The improved production not only benefits the supplier but also gives the manufacturer higher delivery reliability at a lower cost.

7. Extensions

In this section, we provide extensions to derive interesting results considering an endogenous interest rate, an endogenous sourcing price from the unreliable supplier, and random demand.

7.1. Endogenous interest rate

In the basic model, we do not consider the bank's decision and assume the interest rate is exogenous. If the interest rate is endogenously determined by the bank that requires a break-even condition (i.e., the bank loan is competitively priced [26,45]), we have

$$\min\{w_l \min(XQ, q_l), (c_l Q - y)(1+r)\} = c_l Q - y. \tag{11}$$

By substituting Eq. (11) into Eq. (2), and after simplification, the expected profit of the unreliable supplier with capital shortage can be rewritten as

$$\pi_l = w_l E \min(XQ, q_l) - c_l Q,$$

which is the same as that without capital constraint. Accordingly, the optimal production quantity is the same as that without capital constraint. Therefore, under a competitively priced bank loan, when the manufacturer chooses to source from reliable and unreliable channels, the expected profit of the manufacturer can be written as

$$\begin{aligned}\Pi_d &= E\Big[p \min\big(\min(XQ_A^*, q_{l,d}) + q_{h,d}, D\big) \\ &\quad - w_l \min(XQ_A^*, q_{l,d}) - w_h q_{h,d}\Big] \\ &= pD - pQ_A^* \int_0^{\frac{D-q_{h,d}}{Q_A^*}} F(x)dx - w_l Q_A^* \int_0^{\frac{q_{l,d}}{Q_A^*}} \overline{F}(x)dx - w_h q_{h,d}.\end{aligned} \tag{12}$$

By solving Eq. (12), we can obtain the manufacturer's optimal sourcing strategy.

Theorem 7.1. *If the bank loan is competitively priced or the initial capital level of the unreliable supplier is adequate, the mixed-sourcing channel can be optimal for the manufacturer if* $p\int_0^{F^{-1}(\frac{w_h}{p})} x f(x)dx = w_l \int_0^{t^*} \overline{F}(x)dx$*, where* t^* *satisfies* $\int_0^{t^*} x f(x)dx = \frac{c_l}{w_l}$.

Similar to Theorem 5.3, Theorem 7.1 gives a necessary condition for a manufacturer choosing the mixed-sourcing channel under a competitively priced bank loan or with an unreliable supplier with adequate capital, which is quite strict. Furthermore, similar to Proposition 5.2, it is easy to get that if $w_h > \overline{w_h}$, the manufacturer prefers the unreliable channel; if $w_h < \overline{w_h}$, the manufacturer prefers the reliable channel.

7.2. Endogenous sourcing price from the unreliable supplier

In the basic model, we assume w_l is exogenous. However, as a Stackelberg leader, the manufacturer may determine w_l to maximize the expected profit. Therefore, in this section, we provide a numerical example to examine whether the channel preference changes if w_l is endogenously determined by the manufacturer. The initialization values of the parameters are the same as those in Section 6. The results are shown in Table 4.

Table 4 shows when the unreliable supplier has adequate capital, the manufacturer prefers the unreliable channel. As the manufacturer can determine w_l, he will set a low w_l and increase the order quantity from the unreliable supplier to earn the largest profit, making the unreliable supplier more attractive. Furthermore, the table shows that when the interest rate and the initial capital level are low, the manufacturer prefers the unreliable channel, which is the same as that in Table 2. However, given any interest rate r (initial capital y), when y (r) increases, the manufacturer may prefer the mixed-sourcing channel. The reason is that as r or y increases, the unreliable supplier tends to reduce production, making the unreliable channel less attractive. When r or y is relatively high, the manufacturer chooses to set a low w_l and order first from the unreliable supplier (who will use all the initial capital) and then from the reliable supplier.

Table 4
Channel preference of the manufacturer under endogenously w_l.

y \ r	0.05	0.10	0.15	0.20	0.25	0.30
0.05	U	U	U	U	M	M
0.30	U	U	U	U	M	M
0.55	U	U	U	M	M	M
0.80	U	U	U	M	M	M
1.05	U	U	M	M	M	M
1.30	U	U	M	M	M	M
1.55	U	M	M	M	M	M
1.80	U	M	M	M	M	M
2.05	U	M	M	M	M	M
2.30	M	M	M	M	M	M
2.55	M	M	M	M	M	M
2.80	U	U	U	U	U	U

Table 5
Channel preference of the manufacturer under random demand.

y \ r	0.05	0.10	0.15	0.20	0.25	0.30
0.05	U	U	U	U	M	M
0.30	U	U	U	U	M	M
0.55	U	U	U	U	M	M
0.80	U	U	U	U	M	M
1.05	U	U	U	U	M	M
1.30	U	U	U	U	M	M
1.55	U	U	U	U	M	M
1.80	U	U	U	U	M	M
2.05	U	U	U	U	M	M
2.30	U	U	U	U	U	U
2.55	U	U	U	U	U	U
2.80	U	U	U	U	U	U

7.3. Uncertain demand

To investigate the random yield risk, for simplicity, we assume the demand is determined in the basic model. If the demand is uncertain, we let S denote stochastic demand. The CDF and PDF of S are denoted by $G(s)$ and $g(s)$, respectively. Accordingly, Lemma 4.1 and Propositions 4.1 and 4.2 also hold under demand uncertainty.

When sourcing solely from the unreliable channel, the manufacturer's expected profit is

$$\begin{aligned}\Pi_s &= E\left[p\min(XQ, q_{l,s}, S) - w_l\min(XQ, q_{l,s})\right] \\ &= \int_0^{q_{l,s}} \left[p\bar{G}(s) - w_l\right]\bar{F}\left(\frac{s}{Q}\right)ds. \end{aligned} \tag{13}$$

By solving Eq. (13), we can get the manufacturer's optimal order quantity.

Proposition 7.1. *If the demand is random, in the SUS case, the optimal order quantity $q^*_{l,s}$ of the manufacturer capital satisfies*

$$\bar{F}\left(\frac{q^*_{l,s}}{Q^*}\right)\left[p\bar{G}(q^*_{l,s}) - w_l\right] + \frac{Q^*}{q^*_{l,s}}\int_0^{\frac{q^*_{l,s}}{Q^*}}\left[p\bar{G}(Q^*x) - w_l\right]xf(x)dx = 0.$$

Similar to Proposition 5.1, the value of Q^* can be substituted by either Q^*_Z or Q^*_A to indicate the case where the unreliable supplier's initial capital level is zero or adequate.

When the manufacturer sources solely from the reliable channel, it is easy to get the manufacturer's optimal order quantity, which is $G^{-1}(\frac{p-w_h}{p})$. And the manufacturer's optimal expected profit is $(p-w_h)G^{-1}(\frac{p-w_h}{p}) - p\int_0^{G^{-1}(\frac{p-w_h}{p})} G(s)ds$.

When sourcing from the two channels, the manufacturer's expected profit is

$$\begin{aligned}\Pi_d &= E\left[p\min\left(\min(XQ, q_{l,d})+q_{h,d}, S\right) - w_l\min(XQ, q_{l,d}) - w_h q_{h,d}\right] \\ &= p\left[\int_0^{q_{l,d}+q_{h,d}}\bar{G}(s)ds - \int_{q_{h,d}}^{q_{l,d}+q_{h,d}} F\left(\frac{s-q_{h,d}}{Q}\right)\bar{G}(s)ds\right] \\ &\quad - w_l\int_0^{q_{l,d}}\bar{F}\left(\frac{s}{Q}\right)ds - w_h q_{h,d}. \end{aligned} \tag{14}$$

Similarly, as it is difficult to derive analytical results for Eq. (14), we analyze it in a numerical experiment. We assume the demand follows a uniform distribution, that is, $S \sim U[0, 20]$. The initialization values of the parameters are the same as those in Section 6. Then the channel preference of the manufacturer is in shown in Table 5.

Compared to Table 2, we can see the manufacturer would be more likely to choose the unreliable channel when the demand is random. Furthermore, if the unreliable supplier's initial capital is adequate, then the manufacturer will always choose the unreliable channel. Although uncertain demand makes both suppliers less attractive to the manufacturer, it has more impact on the reliable supplier, making the unreliable supplier more attractive.

8. Conclusion

In this paper, we aim to explore the implication of capital constraint for the supplier and the manufacturer. Specifically, we examine the impact of capital constraint and financing details on the supplier's production and on the manufacturer's channel preference of the manufacturer. Although there is substantial literature on supply risk and supply chain finance, few works integrate the supply risk and capital constraint into a framework to explore the implication of capital constraint for the supplier and the manufacturer. Our work mainly focuses on such a question.

To address the question, we consider a two-echelon supply chain model, in which the manufacturer can order from two channels: one that is reliable and one that faces capital constraint and stochastic yield. Consequently, we find that a single sourcing channel is almost always the optimal sourcing choice unless the two channels are identical or the dominant unreliable channel faces capital constraint and an exorbitant interest rate. There exists an interest rate threshold separating the dominance of the two channels: If the interest rate is lower than the threshold, the unreliable channel dominates; otherwise the reliable channel dominates. The interest rate threshold is impacted by the supply price and the retail price, but is independent of the absolute value of the demand. Apart from interest rate considerations, dual-sourcing can be the optimal decision if the supply price of the reliable channel is high and the unreliable channel has no access to financing. Surprisingly, given a low interest rate, the manufacturer obtains more profit with a capital-constrained supplier than in the scenario where the unreliable supplier has adequate initial capital. In addition, our numerical example also indicates that if the capital level or the interest rate is low, the capital constraint can make the unreliable supplier become dominant even though it would not be dominant if it had adequate initial capital. In addition, in the capital constraint scenario, the equilibrium expected profits of the manufacturer and supplier decrease in the initial capital level of the unreliable channel and decrease in the interest rate.

Our research shows that in some situations, single sourcing is not optimal. Capital constraint and lack of financing options for the unreliable supplier can make dual sourcing be the optimal manufacturer's decision. In the situation of an unreliable supplier with capital constraint, a low interest rate or a small initial capital level can turn a dominated unreliable supplier into the dominant one, and even have benefits for the supplier and the manufacturer, although the capital constraint and financing carry bankruptcy risk and higher cost for the unreliable supplier. It is interesting that capital constraint does not always decrease the attraction of the unreliable supplier for the manufacturer; capital constraint can actually increase the attraction if the interest rate is relatively low.

We also extend our model to consider an endogenous interest rate. We find the manufacturer's optimal decision in a competitively priced bank loan is the same as that with an unreliable supplier with adequate capital. We further show if the sourcing price from the unreliable supplier is endogenously determined by the manufacturer or the demand is uncertain, the attraction of the unreliable supplier for the manufacturer will increase.

Further research to expand our work might consider the following directions. First, in our research, we do not consider the risk attitude of the supply chain players. The results may be different if the manufacturer or suppliers are risk averse. Second, this research assumes all information is common knowledge among all parties. In reality, information asymmetry may exist between supply chain members. In addition, taking into account the supplier's development over multiple periods of time may be an interesting future research direction.

CRediT authorship contribution statement

Xiaoyong Yuan: Conceptualization, Methodology, Software, Formal analysis, Validation, Visualization, Writing - original draft, Writing - review & editing. **Gongbing Bi:** Conceptualization, Methodology, Formal analysis, Visualization, Writing - review & editing, Supervision, Project administration. **Yalei Fei:** Methodology, Visualization, Validation, Writing - original draft, Writing - review & editing. **Lindong Liu:** Methodology, Writing - review & editing.

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Appendix

Proof of Lemma 4.1

From Eq. (1), the first and second order condition of π_a with respect to Q are

$$d\pi_a/dQ = w_l\int_0^{q_l/Q} xf(x)dx - c_l, \text{ and } d^2\pi_a/dQ^2 = -w_l(q_l^2/Q^3)f(q_l/Q) < 0.$$

Hence, $\pi_a$is concave in Q. As $\lim_{Q\to 0} d\pi_a/dQ = w_l E[X] - c_l > 0$, the optimal quantity Q_A^* satisfies the first order condition.

Now, we prove that Q_A^* increases in q_l and w_l, and that is more than q_l. From the first order condition, $w_l\int_0^{q_l/Q} xf(x)dx = c_l$, it can be found that $dQ_A^*/dq_l = Q_A^*/q_l > 0$, that is, Q_A^* increases in q_l. Similarly, it can be got that $dQ_A^*/dw_l = \frac{c_l Q_A^{*3}}{w_l^2 q_l^2 f(q_l/Q_A^*)} > 0$, that is, Q_A^* increases in w_l. Assume $Q_A^* \le q_l$, then the first order condition can be simplified as $w_l\int_0^1 xf(x)dx = c_l$, which is contradicted with $c_l/w_l < E[X]$. Therefore, the assumption $Q_A^* \le q_l$ does not hold, and $Q_A^* > q_l$ holds.

Proof of Proposition 4.1

From Eq. (3), the first order condition (FOC) of π_l with respect to Q is as follows. $d\pi_l/dQ = w_l[\int_{\delta_l}^{q_l/Q} xf(x)dx - (c_l(1+r)/w_l)\bar{F}(\delta_l)] = 0$, with Q_l solving

$$\int_{\delta_l}^{q_l/Q_l} xf(x)dx = (c_l(1+r)/w_l)\bar{F}(\delta_l).$$

Convert FOC to $\int_{\delta_l}^{q_l/Q_l}\bar{F}(x)dx = (q_l/Q_l)\bar{F}(q_l/Q_l) + (c_l(1+r)/w_l - \delta_l)\bar{F}(\delta_l)$, and then get the following second order condition at stationary point.

$$\left.\frac{d^2\pi_l}{dQ^2}\right|_{Q=Q_l} = w_l\begin{bmatrix} -\frac{q_l}{Q_l^2}\bar{F}\left(\frac{q_l}{Q_l}\right) - \frac{c_l(1+r)/w_l-\delta_l}{Q_l}\bar{F}(\delta_l) + \frac{q_l}{Q_l^2}\bar{F}\left(\frac{q_l}{Q_l}\right) \\ -\frac{q_l}{Q_l}f\left(\frac{q_l}{Q_l}\right)\frac{q_l}{Q_l^2} - (-1)\frac{c_l(1+r)/w_l-\delta_l}{Q_l}\bar{F}(\delta_l) \\ -(c_l(1+r)/w_l-\delta_l)(-1)f(\delta_l)\frac{c_l(1+r)/w_l-\delta_l}{Q_l} \end{bmatrix} = -\frac{w_l}{Q_l}\left[\frac{q_l^2}{Q_l^2}f\left(\frac{q_l}{Q_l}\right) - \left(\frac{c_l(1+r)}{w_l}-\delta_l\right)^2 f(\delta_l)\right].$$

If $d^2\pi_l/dQ^2|_{Q=Q_l} < 0$, i.e., $\frac{q_l^2}{Q_l^2}f(\frac{q_l}{Q_l}) - (\frac{c_l(1+r)}{w_l} - \delta_l)^2 f(\delta_l) > 0$, the unique optimal solution must satisfy the FOC under financing. Next, we derive the condition that the supplier prefers to borrow from financial institutions under this condition. When borrowing is beneficial for the supplier facing capital constraint, we have $\lim_{Q\to y/c_l} d\pi_l/dQ > 0$, i.e., $\int_0^{\min\{1,c_l q_l/y\}} xf(x)dx > c_l(1+r)/w_l$. Otherwise, borrowing is unprofitable and the supplier will just spend its own initial capital on production. To summarize, the optimal production Q_L^* is, (i) the solution to$\int_{\delta_l}^{q_l/Q_L^*} xf(x)dx = (c_l(1+r)/w_l)\bar{F}(\delta_l)$, if $\int_0^{\min\{1,c_l q_l/y\}} xf(x)dx > c_l(1+r)/w_l$; (ii) y/c_l, otherwise.

Assume $q_l/Q_l > 1$, then it can be found a $\underline{q_l}$($\underline{q_l} < q_l$ and $\underline{q_l}/Q_l \le 1$) to meet the FOC. From the Eq. (3), the unreliable supplier can be found the same amount of profit at order quantity $\underline{q_l}$ and q_l. Thus, Q_l is not the optimal response of q_l, and the assumption $q_l/Q_l > 1$ does not hold, which means that $q_l/Q_l \le 1$.

Taking the derivative of $\int_{\delta_l}^{q_l/Q_l} xf(x)dx = (c_l(1+r)/w_l)\bar{F}(\delta_l)$ with respect to q_l, we get

$$\frac{dQ_l}{dq_l} = \frac{(q_l/Q_l)f(q_l/Q_l)}{(q_l/Q_l)^2 f(q_l/Q_l) - (c_l(1+r)/w_l - \delta_l)^2 f(\delta_l)} > 0.$$

Proof of Proposition 4.2

From Eq. (5), the first and second order condition of π_z with respect to Q are

$$\frac{d\pi_z}{dQ} = w_l\left[\int_{\delta_z}^{q_l/Q} xf(x)dx - \delta_z\bar{F}(\delta_z)\right], \text{ and}$$
$$\frac{d^2\pi_z}{dQ^2} = -\frac{w_l q_l^2}{Q^3}f\left(\frac{q_l}{Q}\right) < 0.$$

Therefore, π_z is concave in Q. As $\lim_{Q\to 0}\frac{d\pi_z}{dQ} = w_l\int_{\delta_z}^1 \bar{F}(x)dx > 0$, the optimal production satisfies the first order condition.

From the first order condition, $\int_{\delta_z}^{q_l/Q} xf(x)dx = \delta_z\bar{F}(\delta_z)$, it can be found that q_l/Q_Z^* (and Q_Z^*) is a one-variable function of δ_z, and $\frac{dQ_Z^*}{d\delta_z} = \frac{\overline{F}(\delta_z)}{(q_l^2/(Q_Z^*)^3)f(q_l/Q_Z^*)} < 0$. As $\delta_z = c_l(1+r)/w_l$, it is clear that $d\delta_z/dw_l = -c_l(1+r)/w_l^2 < 0$, $d\delta_z/dr = c_l/w_l > 0$ so $dQ_Z^*/dr < 0$ and $dQ_Z^*/dw_l > 0$. Similarly, we can get $dQ_Z^*/dq_l = Q_Z^*/q_l > 0$.

Proof of Lemma 5.1

When the unreliable channel is the only one available, if $q_l \le D$, the expected profit of the manufacturer is $\Pi_s = (p-w)Q\int_0^{q_{l,s}/Q}\overline{F}(x)dx$.

The first derivative of Π_s with respect to $q_{l,s}$ follows:

$$\frac{d\Pi_s}{dq_{l,s}} = (p-w)\left[\frac{dQ}{dq_{l,s}}\int_0^{q_{l,s}/Q}\overline{F}(x)dx + Q\overline{F}\left(\frac{q_{l,s}}{Q}\right)\times\frac{1}{Q^2}\left(Q - q_{l,s}\frac{dQ}{dq_{l,s}}\right)\right] \quad \text{for} \quad Q = \begin{cases} Q_A^*, \\ Q_Z^*. \end{cases}$$
$$= (p-w)\left[\int_0^{q_{l,s}/Q} xf(x)dx\frac{dQ}{dq_{l,s}} + \overline{F}\left(\frac{q_{l,s}}{Q}\right)\right] > 0,$$

This means it is always profitable to increase $q_{l,s}$ if $q_{l,s} \le D$, and so the lemma holds.

Proof of Proposition 5.1

Lemma 5.1 implies $q_{l,s}^* \ge D$, under which condition, from Eq. (7), it is clear that,

$\frac{d\Pi_s}{dq_{l,s}} = 0 \Rightarrow q_{l,s}^*$ solves $\int_0^{D/Q^*} xf(x)dx = \frac{w_l}{p}\int_0^{q_{l,s}^*/Q^*}\overline{F}(x)dx$.

$$\frac{d^2\Pi_s}{dq_{l,s}^2}|_{Q=Q_Z^*} = \frac{pQ_Z^*}{q_{l,s}}\left[\frac{D}{Q_Z^*}f\left(\frac{D}{Q_Z^*}\right)(-1)\frac{D}{\left(Q_Z^*\right)^2}\frac{dQ_Z^*}{dq_{l,s}} - \frac{w_l}{p}\overline{F}\left(\frac{q_{l,s}}{Q_Z^*}\right)\frac{Q_Z^* - q_{l,s}dQ_Z^*/dq_{l,s}}{\left(Q_Z^*\right)^2}\right]$$
$$= -p\left(\frac{Q_Z^*}{q_{l,s}}\right)^2\frac{D^2}{\left(Q_Z^*\right)^3}f\left(\frac{D}{Q_Z^*}\right) < 0.$$

Similarly, it is easy to get $\frac{d^2\Pi_s}{dq_{l,s}^2}|_{Q=Q_A^*} < 0$.

Hence, Π_s is concave in $q_{l,s}$ and the optimal sourcing quantity $q_{l,s}^*$ from the unreliable channel satisfies the first order condition.

Proof of Corollary 5.1

Combining Propositions 4.2 and 5.1, property (i) of Corollary 5.1 holds obviously and the proof of it is omitted here. Next, we prove property (ii). When the unreliable channel without capital constraint is the unique available channel, it is easy to determine the expected profit of the manufacturer, $\Pi_s = pQ_A^*\int_0^{D/Q_A^*}\overline{F}(x)dx - w_lQ_A^*\int_0^{q_{l,s}/Q_A^*}\overline{F}(x)dx$ and the equilibrium order of the manufacturer satisfying $\int_0^{D/Q_A^*} xf(x)dx = \frac{w_l}{p}\int_0^{q_{l,s}^*/Q_A^*}\overline{F}(x)dx$. Substitute the equilibrium order condition into the expected profit of the manufacturer to get $\Pi_s(q_{l,s}^{a*}) = pD\overline{F}(D/Q_A^*(q_{l,s}^{a*}))$. Similarly, when the unreliable supplier faces capital constraint, it is easy to get a similar formulation of the expected profit of the manufacturer, $\Pi_s(q_{l,s}^{z*}) = pD\overline{F}(D/Q_Z^*(q_{l,s}^{z*}))$. From the profit formulations of the manufacturer in these two scenarios, it can be found that the expected profit of the manufacturer depends just on the equilibrium production. Combining the first order condition of the unreliable supplier and that of the manufacturer, it can be found that the two equilibrium production quantities can be expressed in a unified function type $Q_{opt} = G(.)$, which is an increasing function. Further, combining the Lemma 4.1 with Proposition 4.2, it is obvious that both Q_A^* and Q_Z^* are more than $q_{l,s}$ with Q_A^* solving $\int_0^{q_{l,s}/Q_A^*} xf(x)dx = c_l/w_l$, and Q_Z^* solving $\int_0^{q_{l,s}/Q_Z^*} xf(x)dx = \int_0^{\delta_z}\overline{F}(x)dx$. For $r = 0$, we have $\int_0^{\delta_z}\overline{F}(x)dx < c_l/w_l$; and for $r = w_l/c_l - 1$, we have $\int_0^{\delta_z}\overline{F}(x)dx = E[X] > c_l/w_l$ and $\int_0^{\delta_z}\overline{F}(x)dx (< c_l/w_l)$ is an increasing function of r, so there must be a $\widetilde{r} \in (0, w_l/c_l - 1)$ such that if $r < \widetilde{r}$, then $\int_0^{\delta_z}\overline{F}(x)dx < c_l/w_l$; otherwise, $\int_0^{\delta_z}\overline{F}(x)dx \ge c_l/w_l$, then it is easy to see that if $r < \widetilde{r}$, then $Q_Z^*(q_{l,s}^{z*}) > Q_A^*(q_{l,s}^{a*})$; otherwise, $Q_Z^*(q_{l,s}^{z*}) \le Q_A^*(q_{l,s}^{a*})$. Further, substitute the equilibrium production into the expected profit to get that if $Q_Z^*(q_{l,s}^{z*}) > Q_A^*(q_{l,s}^{a*})$, then $\Pi_s(q_{l,s}^{z*}) > \Pi_s(q_{l,s}^{a*})$; and if $Q_Z^*(q_{l,s}^{z*}) \le Q_A^*(q_{l,s}^{a*})$, then $\Pi_s(q_{l,s}^{z*}) \le \Pi_s(q_{l,s}^{a*})$. Hence, Corollary 5.1(ii) holds.

Proof of Theorem 5.1

Define $t^* = q_{l,s}^{z*}/Q_Z^*(q_{l,s}^{z*})$ and $\alpha^* = D/Q_Z^*(q_{l,s}^{z*})$. From Propositions 4.2 and 5.1, it is clear that t^* and α^* meet the conditions: $\int_{\delta_z}^{t^*} xf(x)dx = \delta_z\overline{F}(\delta_z)$ and $\int_0^{\alpha^*} xf(x)dx = \frac{w_l}{p}\int_0^{t^*}\overline{F}(x)dx$.

From these equations, it is easy to determine that $d\alpha^*/dr > 0$, i.e., $\alpha^* = D/Q_Z^*(q_{l,s}^{z*})$ increases in r. Based on the assumption, we know $r < w_l/c_l - 1$. If $r = w_l/c_l - 1$, the unreliable supplier will not borrow and the manufacturer will always prefer SRS. Therefore, there is a unique $\widehat{r} < w_l/c_l - 1$ ensuring $F(D/Q_Z^*(q_{l,s}^{z*})) = w_h/p$, and it can be found that (i) if $r < \widehat{r}$ then $\Delta = pD(w_h/p - F(D/Q_Z^*(q_{l,s}^{z*}))) > 0$; (ii) if $r = \widehat{r}$ then $\Delta = 0$; and (iii) if $r > \widehat{r}$ then $\Delta < 0$. Note that if $w_h \le pF(D/Q_Z^*(q_{l,s}^{z*}))|_{r=0}$, $\widehat{r}$ will be negative. As $r > 0$, in this case, the manufacturer will always prefer SRS.

Proof of Theorem 5.2

Firstly, we prove the $\widehat{r}$ to be increasing in w_h. The proof of Theorem 5.1 shows that α^* increases in r, i.e., $d\alpha^*/dr \ge 0$, which also means $dr/d\alpha^* \ge 0$. In the equation, $F(D/Q_Z^*(q_{l,s}^{z*})) = w_h/p$, since w_h/p increases in w_h, $F(\alpha^*)|_{r=\widehat{r}}$ increases in w_h, too. Also in consideration of α^* being increasing in r, it is easy to get that $\widehat{r}$ increases in w_h.

Secondly, we prove the correlation of r and w_l. From the equations in the proof of Theorem 5.1, the following hold:

$$\frac{d\alpha^*}{dw_l} = \frac{t^*f(t^*)\int_0^{t^*}\overline{F}(x)dx - \delta_z\overline{F}(\delta_z)\overline{F}(t^*)}{p\alpha^*f(\alpha^*)t^*f(t^*)}$$
$$= \frac{t^*f(t^*)\int_0^{t^*}\overline{F}(x)dx - \overline{F}(t^*)\int_0^{t^*}xf(x)dx + \overline{F}(t^*)\int_0^{\delta_z}xf(x)dx}{p\alpha^*f(\alpha^*)t^*f(t^*)}$$
$$= \frac{\int_0^{t^*}\overline{F}(x)\overline{F}(t^*)[H(t^*) - H(x)]dx + \overline{F}(t^*)\int_0^{\delta_z}xf(x)dx}{p\alpha^*f(\alpha^*)t^*f(t^*)} > 0.$$

That is, α^* increases in w_l. Thus, when w_l increases, α^* increases too (if r is a constant value) and there should be a lower $\widehat{r}$ to decrease α^* so as to keep $F(\alpha^*) = w_h/p$, which means that $\widehat{r}$ decreases in w_l. Similarly, it can be proved that $pF(\alpha^*) - w_h$ increases in p, therefore, $\widehat{r}$ decreases in p.

Since $\widehat{r}$ solves $F(\alpha^*) = w_h/p$ and α^* does not depend on D, it is obvious that D has no effect on $\widehat{r}$.

Proof of Theorem 5.3

From Eq. (10), the two first-order derivatives of Π_d on q_l and q_h can be expressed as the following equations:

$$\begin{cases} \dfrac{\partial\Pi_d}{\partial q_{l,d}} = \frac{p}{t_z^*}\int_0^{\beta^*} xf(x)dx - \frac{w_l}{t_z^*}\int_0^{t_z^*}\overline{F}(x)dx, \\ \dfrac{\partial\Pi_d}{\partial q_{h,d}} = pF(\beta^*) - w_h. \end{cases}$$

We note $t_z^* = q_{l,d}/Q_Z^*$, $\beta^* = (D - q_{h,d})/Q_Z^*$. If Π_d has stationary point, then we can get $p\int_0^{F^{-1}(\frac{w_h}{p})} xf(x)dx - w_l\int_0^{t_z^*}\overline{F}(x)dx = 0$, where t_z^* satisfies $\int_{\delta_z}^{t_z^*} xf(x)dx = \delta_z\overline{F}(\delta_z)$. In this case, the optimal choice can be single channel or dual channel sourcing due to there being no difference between the two channels. Otherwise, if this condition cannot be satisfied, the optimal choice of sourcing channel is single channel sourcing.

Proof of Proposition 5.2

According to Dong et al. [10] and Gurnani et al. [16], the sole sourcing is the optimal decision of the manufacturer in the case where the unreliable supplier has no capital constraint. In that case, similar to Eq. (8), we can get the threshold of supply price of the reliable channel, $\overline{w_h}$ solving $pD(\overline{w_h}/p - F(D/Q_A^*(q_{l,s}^{a*}))) = 0$. If $w_h > \overline{w_h}$, the unreliable supplier is chosen; otherwise, the reliable channel is chosen.

Note$y = c_l Q_A^*(q_0)$, so according to Lemma 4.1, we have that if $q_{l,\,d} < q_0$, then $Q_A^*(q_{l,d}) < Q_A^*(q_0)$ and $c_l Q_A^*(q_{l,d}) < y$, i.e., the unreliable supplier is sufficiently capitalized. Similarly, if $q_{l,\,d} \ge q_0$, the unreliable supplier faces capital constraint.

If $w_h < \overline{w_h}$, reliable channel is chosen, and the sourcing decisions of the manufacturer are obviously $q_{h,d} = D, q_{l,d} = 0$. However, if $w_h > \overline{w_h}$, the case is different. Next, we mainly consider the two scenarios in the case.

We firstly consider the case $(q_{l,\,d} \ge q_0)$ where the unreliable supplier produces y/c_l units of product. Substitute $Q_Z^* = y/c_l$ into Eq. (10) to get

$$\Pi_d = pD - p(D - q_{h,d})F\left(\frac{c_l(D - q_{h,d})}{y}\right) + \frac{py}{c_l}\int_0^{\frac{c_l(D-q_{h,d})}{y}} xf(x)dx - \frac{w_l y}{c_l}\int_0^{\frac{c_l q_{l,d}}{y}} \overline{F}(x)dx - w_h q_{h,d}.$$

For a given $q_{h,\,d}$, $\partial\Pi_d/\partial q_{l,d} = -w_l\overline{F}(c_l q_{l,d}/y) < 0$. Thus, for any given $q_{h,\,d}$, $q_{l,d}^{y*} = q_0$ will always be the optimal decision, and we can get $q_{h,d}^{y*} = \max\{D - yF^{-1}(w_h/p)/c_l, 0\}$.

In the scenario $q_{l,\,d} < q_0$, the production of the supplier is $Q_A^*(q_{l,d})$ and the expected profit of the manufacturer is

$$\Pi_d = pD - p(D - q_{h,d})F\left(\frac{D - q_{h,d}}{Q_A^*}\right) + pQ_A^*\int_0^{\frac{D-q_{h,d}}{Q_A^*}} xf(x)dx - w_l Q_A^*\int_0^{\frac{q_{l,d}}{Q_A^*}} \overline{F}(x)dx - w_h q_{h,d}.$$

Note $q_{l,d}/Q_A^* = t_y^*$, which can be found to be a function of c_l/w_l from Lemma 4.1. Similarly, note $\beta_y^* = \frac{D-q_{h,d}}{Q_A^*}$, which can be converted to $\beta_y^* = \frac{t_y^*(D-q_{h,d})}{q_l}$. Substitute $t_y^* = q_{l,d}/Q_A^*$ and $\beta_y^* = \frac{t_y^*(D-q_{h,d})}{q_{l,d}}$ into Π_d to get

$$\Pi_d = pD - p(D - q_{h,d})F(\beta_y) + \frac{pq_{l,d}}{t_y^*}\int_0^{\beta_y} xf(x)dx - \frac{w_l q_{l,d}}{t_y^*}\int_0^{t_y^*} \overline{F}(x)dx - w_h q_{h,d}.$$

Similar to the scenario with zero capital and financing access, it is easy to find that the single sourcing decision is always optimal and that the optimal channel choice is the unreliable channel and $q_{l,d}^{y*} > q_0$. Thus, this scenario does not hold.

In summary, in the scenario where the unreliable supplier does not have adequate initial capital and adopt external financing, the optimal decision of the manufacturer is dual-channel sourcing and the orders from two channels are $q_{h,d}^{y*} = \max\{D - yF^{-1}(w_h/p)/c_l, 0\}$ and $q_{l,d}^{y*} = q_0$, respectively if $w_h > \overline{w_h}$.

Proof of Theorem 7.1

The proof is similar to the proof of Theorem 5.3 and is omitted here.

Proof of Proposition 7.1

From Eq. (13), the first and second order condition of Π_s with respect to $q_{l,\,s}$ are

$$\frac{d\Pi_s}{dq_{l,s}} = \overline{F}\left(\frac{q_{l,s}}{Q}\right)\left[p\overline{G}(q_{l,s}) - w_l\right] + \frac{1}{q_{l,s}Q}\int_0^{q_{l,s}} \left[p\overline{G}(s) - w_l\right]sf\left(\frac{s}{Q}\right)ds$$
$$= \overline{F}\left(\frac{q_{l,s}}{Q}\right)\left[p\overline{G}(q_{l,s}) - w_l\right] + \frac{Q}{q_{l,s}}\int_0^{\frac{q_{l,s}}{Q}} \left[p\overline{G}(Qx) - w_l\right]xf(x)dx.$$

$$\frac{d^2\Pi_s}{dq_{l,s}^2} = -p\left[\overline{F}\left(\frac{q_{l,s}}{Q}\right)g(q_{l,s}) + \left(\frac{Q}{q_{l,s}}\right)^2\int_0^{\frac{q_{l,s}}{Q}} g(Qx)x^2 f(x)dx\right] < 0.$$

Hence, Π_s is concave in $q_{l,\,s}$ and the optimal sourcing quantity $q_{l,s}^*$ from the unreliable channel satisfies the first order condition.

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