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Crowdsourcing contests with entry cost

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Abstract

Today, companies can seek solutions to business problems by sponsoring cocreation activities such as crowdsourcing contests. This paper studies a crowdsourcing contest in which a sponsor seeks solutions from a number of independent contestants. Specifically, we reveal how the number of contestants affects the expected effort and maximum quality in a crowdsourcing contest. First, we show that an effort-decreasing effect exists for any number of winners when there are a number of contestants, while an effort-increasing effect may occur when there are relatively few contestants. Second, we find that the expected maximum quality at most concavely increases with the number of contestants. Finally, we extend the analysis to the case with a flexible number of contributors. Our results imply that a restricted-entry policy should be adopted if the entry cost is considered, which is not observed in the prior literature.

Keywords: crowdsourcing contests; incentive; entry policy; random shock

1. Introduction

In recent years, new business models that harness the productivity and creativity of the Internet community have proliferated. Among them, crowdsourcing is one of the most attractive models adopted by firms (sponsors) seeking solutions from a large group of Internet users (contestants) (Bockstedt et al., 2016; Wooten and Ulrich, 2017; Chen et al., 2018). There are several well-known crowdsourcing platforms worldwide, including InnoCentive and TopCoder in the United States and Taskcn and EPWK in China, and their services are in great demand. For example, up to June 2018, EPWK, which provides different services for firms, had organized 7,583,280 tasks with total awards of CNY 16,445,823,379 (approximately USD 2.4 billion) and attracted 18,833,266 registered users. On most crowdsourcing platforms, crowdsourcing contests and their variants are organized by the sponsor. These contests provide effective solutions to challenging projects. Thus, it is of great importance to explore the optimal design of crowdsourcing contests.

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To run a crowdsourcing contest, a sponsor must first announce to the contestants the following: an innovation-related project, an award scheme, and an entry policy. Typically, the award scheme specifies the number of winners and the award amount allocated to each winner. An award scheme selecting just one winner is referred to as *a single-winner award scheme*, and the winner collects the entire award (also named a winner-takes-all scheme in Ales et al., 2017b). An award scheme allowing more than one winner is called *a multiple-winner award scheme* and the sponsor fairly allocates the award among the winners. The entry policy specifies the number of contestants allowed to participate in the contest. When *a free-entry policy* (see Ales et al., 2017a) is adopted, there is no restriction on the number of participants in the contest. However, sometimes, it is worthwhile to apply *a restricted-entry policy* (see Taylor, 1995). Such a policy restricts the number of participants to provide the sponsor with higher welfare. According to the announced award scheme, the contestants decide whether to participate, and each participating contestant results in a solution of a certain quality depending on the effort expended and a random shock. The sponsor then evaluates and ranks the solutions and, finally, awards the winners according to the award scheme.

To encourage the contestants to participate, the sponsor needs to incur some advertising costs. In the evaluation process, it also incurs some evaluation costs. These constitute the entry cost for a crowdsourcing contest. As we show later in the main context, restricting the number of participants in a crowdsourcing contest can mean a more realistic entry policy if the entry cost is considered.

In this paper, we investigate the question regarding how the number of contestants affects the expected effort and maximum quality in a crowdsourcing contest, and provide the practical entry policy. The question is critical to contest mechanism design and has received some attention in the literature. Next, we review some related studies and clarify our main contributions.

The two most frequently used criteria for quantifying the expected quality are the *expected average quality* (EAQ) and the *expected maximum quality* (EMQ), both are of practical importance, for example, Kalra and Shi (2001), Moldovanu and Sela (2001), Hu and Wang (2020), and Körpeoğlu and Cho (2017). Several theoretical studies (see, e.g., Taylor, 1995; Fullerton and McAfee, 1999; Che and Gale, 2003) show that the EAQ-based model is always associated with an effort-decreasing effect in which the contestant tends to expend less effort when there are more participants. This finding implies that the restricted-entry policy is preferred for the sake of high EAQ. In addition, Boudreau et al. (2011) empirically prove their results. If the sponsor cares about EMQ, Ales et al. (2017a) show that a free-entry policy is optimal for a crowdsourcing contest when the quality uncertainty of contestants' solutions is sufficiently large or the sponsor benefits from a number of contributors whose solutions are beneficial to EMQ.

To reveal the generality of our analysis, we consider the impacts of the number of allowed participants on both EAQ and EMQ under various practical situations. In our main results, we find that the effort-decreasing effect may not hold in the EAQ-based model when there are a small number of contestants. More surprisingly, the free-entry policy is always suboptimal, rather than optimal as was claimed in Ales et al. (2017a), according to the EMQ-based model when the entry cost exists.

Based on a general contest model, we derive some new and practical results that have not been revealed in the existing crowdsourcing contest literature. Our main contributions are summarized as follows.

1. We show that an effort-decreasing effect does exist for any number of winners when the number of contestants is sufficiently large. Otherwise, more interestingly, the effort-increasing effect may

occur when the number of contestants is small. This implies that the two effects can be triggered by fierce competition and mild competition, respectively, and this was not observed previously.

2. We find that the EMQ at most concavely increases with the number of contestants, which implies that the free-entry policy is always suboptimal if the entry cost is considered. This finding complements the work of Ales et al. (2017a) who assume zero entry cost and show that the free-entry policy is optimal.
3. When the number of contributors is flexible, we find that the increasing EMQ is more likely to hold. More importantly, the EMQ also at most increases with the number of contestants, which implies that a restricted policy is also preferred if the entry cost is considered.

The remainder of this paper is organized as follows. In Section 2, we introduce and model the crowdsourcing contest. In Section 3.1, we study how the number of contestants affects the expected effort. In Section 3.2, we reveal the change of the EMQ based on the number of contestants. In Section 4, we extend our analyses to the case with a flexible number of contributors. Finally, in Section 5, we conclude our work and suggest some directions for future research.

2. The model

Consider a crowdsourcing contest where a contest sponsor (he) seeks solutions from contestants (she). The sequence of events is as follows. First, the sponsor announces the award scheme and the entry policy. Then, each contestant expends effort in finding and submitting a solution. Finally, the sponsor collects and evaluates all solutions, and awards the contestant(s) based on the announced scheme. In the following, we first discuss the model settings for the contestants and then describe the decision made by the sponsor.

Following Terwiesch and Xu (2008), we consider a homogeneous random shock contest model for contestants. In this model, the quality of a contestant's solution is determined by her effort and a random shock. Let e_i and ϵ_i be the effort and the random shock, respectively, associated with contestant i. Then, the quality of contestant i's solution, denoted by q_i, is given by

$$q_i = R(e_i) + \epsilon_i,$$

where $R(\cdot)$ is the quality function of effort e_i. Without any loss of generality, the random shock ϵ_i is assumed to be independently and identically distributed in the support $[\underline{\beta}, \overline{\beta}]$ with an expectation equal to zero, that is, $\mathbb{E}[\epsilon_i] = 0$, where $\underline{\beta} \in \mathbb{R}_- \bigcup\{-\infty\}$ and $\overline{\beta} \in \mathbb{R}_+ \bigcup\{+\infty\}$. Let F and f be the cumulative and density functions, respectively, of the probability distribution of ϵ_i. To be consistent with the existing literature, we assume that $f(\cdot)$ is either unimodal, decreasing, constant, or increasing (Ales et al., 2017a).

When there are n contestants with n realized shocks $\{\epsilon_1^n, \epsilon_2^n, \dots, \epsilon_n^n\}$ such that $\epsilon_1^n \geq \epsilon_2^n \geq \cdots \geq \epsilon_n^n$, let F_j^n and f_j^n be the cumulative function and the density function, respectively, of the probability distribution of ϵ_j^n. By noting that ϵ_j^n is the $(n-j+1)$th-order statistic among $\{\epsilon_1^n, \epsilon_2^n, \dots, \epsilon_n^n\}$, we have $f_j^n(x) = \frac{n!}{(j-1)!(n-j)!}(1-F(x))^{j-1}F(x)^{n-j}f(x)$.

For contestant i, her utility $\Phi_i = \Phi(e_i, V_i)$: $\mathbb{R}_+^2 \to \mathbb{R}$ depends on the effort e_i she exerts and the award V_i she receives. The utility takes the form $\Phi(e_i, V_i) = U(V_i) - C(e_i)$, where $U(V_i)$ is the utility that contestant i obtains if she receives the award V_i, and $C(e_i)$ is contestant i's disutility or the cost associated with her effort e_i.

In the literature, there are extant works assuming the convexity and the monotonicity of the cost function. Examples include Ales et al. (2017b), Hu and Wang (2020) and Mihm and Schlapp (2019). In addition, $R(\cdot)$ is assumed to be a concavely increasing function, as shown in Terwiesch and Xu (2008).

The sponsor evaluates the quality of each submitted solution and awards the $L \geq 1$ best-performing contestants with a total award of $\mathcal{V}$. We consider the total award $\mathcal{V}$ as a given parameter in our study. Let V_j be the award allocated to the contestant with the jth best solution. Then, the sponsor determines the number of participants n (i.e., it determines the choice of a free-entry policy or a restricted-entry policy), the number of winners $L \in \{1, 2, \ldots, n\}$, and the award vector $\mathbb{V}_L = (V_1, V_2, \ldots, V_L)$ such that $\sum_{j=1}^{L} V_j = \mathcal{V}$. We assume that $\mathbb{V}_L$ follows a constraint $V_1 \geq V_2 \geq \cdots \geq V_L > 0$ to ensure the winners' fairness in a crowdsourcing contest. For future use, we let $r_j = V_j/\mathcal{V}$, where $\sum_{j=1}^{L} r_j = 1$, and refer to r_j as the award allocation ratio (AR) for the jth award. In addition, the evaluation procedure incurs an entry cost of $n \cdot c$, where c is the unit entry cost. Despite the necessity of the evaluation procedure indicated by studies such as Terwiesch and Xu (2008) and Ales et al. (2017a), the entry cost has never been considered in the existing literature nor has its role in the design of crowdsourcing contests been explored.

Among the n submitted solutions, the sponsor is interested in the overall quality of the best K solutions, where $K(n) \in \{1, 2, \ldots, n\}$. Ales et al. (2017a, 2017b) consider a similar setting by assuming a fixed K. We, however, consider K to be an increasing function of n. Thus, the expected profit of the sponsor, denoted by $\Pi(n)$, is given by

$$\Pi(n) = \sum_{j=1}^{K(n)} \mathbb{E}\left[q_j^n\right] - (\mathcal{V} + n \cdot c), \tag{1}$$

where q_j^n is the quality of the jth best solution in $\{q_1, q_2, \ldots, q_n\}$, and $\mathcal{V} + n \cdot c$ is the cost of the sponsor. To reveal a more direct relationship between the expected profit of the sponsor $\Pi(n)$ and the efforts made by contestants e, we provide a transformation for Equation (1) as follows.

Given the award vector $\mathbb{V}_L$, we first consider a symmetric pure-strategy Nash equilibrium for the contestants under which each contestant exerts the same effort e^*. Then, the probability of contestant i providing the solution with the jth highest quality when she exerts effort e_i is (see also Ales et al., 2017b)

$$\begin{aligned}\Pr(q_i \text{ ranks order } j) = \int_{\underline{\beta}}^{\overline{\beta}} \frac{(n-1)!}{(j-1)!(n-j)!} &(1 - F(x + R(e_i) - R(e^*)))^{j-1} \\ &\times F(x + R(e_i) - R(e^*))^{n-j} f(x)dx,\end{aligned}$$

and contestant i chooses her effort e_i to maximize her expected utility by solving

$$e_i^* = \arg\max_{e_i} \left\{ \sum_{j=1}^{L} U(V_j^n) \Pr(q_i \text{ ranks order } j) - C(e_i) \right\}.$$

We characterize the symmetric effort e^* of the contestants in pure-strategy Nash equilibrium in Lemma 1.

Lemma 1. *Given n contestants, L winners, and award vector $\mathbb{V}_L$, the symmetric and optimal effort e^* of each contestant in a pure-strategy Nash equilibrium satisfies*

$$\frac{C'(e^*)}{R'(e^*)} = \left\{ \sum_{j=1}^{L-1} \left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\left[f(\epsilon_j^{n-1})\right] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n-1})\right] \right\},$$

where $\mathbb{E}[f(\epsilon_n^{n-1})] = 0$, and $C'(\cdot)$, $R'(\cdot)$ is the first-order derivative of cost function $C(\cdot)$, $R(\cdot)$ defined for the contestants.

In Lemma 1, the equilibrium effort e^* is essentially a function of the number of contestants n. Thus, we denote the equilibrium effort as $e^*(n)$ hereafter.

According to Lemma 1, we can rewrite Equation (1), the expected profit of the sponsor, as

$$\Pi(n) = K(n) \cdot e^*(n) + \sum_{j=1}^{K(n)} \mathbb{E}[\epsilon_j^n] - (\mathcal{V} + n \cdot c), \tag{2}$$

where ϵ_j^n is the jth-order statistic among n random shocks, and the term $K(n) \cdot e^*(n) + \sum_{j=1}^{K(n)} \mathbb{E}[\epsilon_j^n]$ is the expected total quality of the K best solutions. We note that when $U(\mathcal{V}) = \mathcal{V}$, $K(n) = K$, and $c = 0$, our model shown in (2) reduces to the model studied in Ales et al. (2017a).

Until now, we have completed the description and modeling of our research problem. Next, we identify the optimal entry policies by exploring how the number of contestants, n, affects the contestants' equilibrium effort $e^*(n)$ and the sponsor's expected maximum solution quality.

3. Analysis

In a crowdsourcing contest, a sponsor will receive any participant as long as his marginal value, subtracted by the unit entry cost of his solution, is positive. When the entry cost is not considered, the sponsor should always allow new participants, and this is shown in the literature by Terwiesch and Xu (2008) and Ales et al. (2017a). Next, we study whether the sponsor should restrict the number of participating contestants, that is, adopt the restricted-entry policy or the free-entry policy, when the entry cost exists.

We restate that the unit entry cost is simply c, and it is independent of n, as assumed in our basic model (2). To study the entry policy, the remainder of this section explores the impact of n on the marginal value of a new participant. Given an award allocation $\mathbb{V}_L$ (or ARs), we focus on the following two commonly applied evaluation criteria of the participants (see Terwiesch and Xu, 2008; Ales et al., 2017a):

$$\text{EAQ}(\mathbb{V}_L, n) = e^*(\mathbb{V}_L, n)$$

$$\text{and } \text{EMQ}(\mathbb{V}_L, n) = K(n)e^*(\mathbb{V}_L, n) + \sum_{j=1}^{K(n)} \mathbb{E}[\epsilon_j^n],$$

Table 1
Different conditions used in this paper

Names	Inequalities
Condition A	$\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$
Condition B	$(log(f(x)))'' \leq 0$, $lim_{x\to\overline{\beta}}((1-F(x))/f(x)) < \infty$, $lim_{x\to\underline{\beta}}(F(x)/f(x)) < \infty$

where $\text{EAQ}(\mathbb{V}_L, n)$ represents the EAQ of solutions, which is similar to the equilibrium quality (effort) defined in Lemma 1; and $\text{EMQ}(\mathbb{V}_L, n)$ represents the expected maximum solution quality that directly contributes to the sponsor's final solution. To simplify the exposition, we, respectively, use $\text{EAQ}(n)$ (or EAQ), $\text{EMQ}(n)$ (or EMQ), and $e^*(n)$ (or e^*) to denote $\text{EAQ}(\mathbb{V}_L, n)$, $\text{EMQ}(\mathbb{V}_L, n)$, and $e^*(\mathbb{V}_L, n)$ so that there is no confusion. In Section 3.2, we focus on the case where there is a fixed number of contributors, that is, $K(n) = K$ is fixed for different ns, while in Section 4, we extend to the case where there is a flexible number of contributors by treating $K(n)$ as a function of n.

Under the two evaluation criteria described above, the net profits of the sponsor are, respectively,

$$\Pi_{\text{EAQ}} = \text{EAQ} - n \cdot c - \mathcal{V} = e^*(\mathbb{V}_L, n) - n \cdot c - \mathcal{V}$$

$$\text{and} \quad \Pi_{\text{EMQ}} = \text{EMQ} - n \cdot c - \mathcal{V} = K(n)e^*(\mathbb{V}_L, n) + \sum_{j=1}^{K(n)} \mathbb{E}[\epsilon_j^n] - n \cdot c - \mathcal{V},$$

and ideally, when $\text{EAQ}(n)$ and $\text{EMQ}(n)$ are monotonic, the optimal ns for Π_{EAQ} and Π_{EMQ} are achieved at $\max\{n : \text{EAQ}(n) - \text{EAQ}(n-1) \geq c\}$ and $\max\{n : \text{EMQ}(n) - \text{EMQ}(n-1) \geq c\}$, respectively.

Practically, the above observations are supported by the contests on the EPWK platform, where the number of participants is small for a contest with high evaluation cost such as website designs and copywriter plans. As discussed above, to analyze the net profits of the sponsor Π_{EAQ} and Π_{EMQ}, we only need to focus on $\text{EAQ}(n)$ and $\text{EMQ}(n)$, respectively. Theoretically, we conjecture that the restricted-entry policy will exist whenever $\text{EAQ}(n)$ or $\text{EMQ}(n)$ is decreasing or concavely increasing with n, and this motivates the investigations of the monotonicity of $\text{EAQ}(n)$ and $\text{EMQ}(n)$ in the remainder of this subsection.

3.1. Impact of n on EAQ

Given the award scheme, we first study how $\text{EAQ}(n)$, that is, the equilibrium effort $e^*(n)$, changes with n. The main results are presented in Proposition 1. Note that the conditions of ϵ used are listed in Table 1, and we show that these are commonly satisfied in Table 2. This implies that our results hold according to the general distributions of ϵ.

Proposition 1 (Monotonicity of EAQ). *Given any ARs, along with the number of winners L, we have that* $\text{EAQ}(n)$

Table 2
The properties of some commonly used random shock distributions

Name of distribution	Condition A	Condition B	Symmetric
Normal distribution	√	√	√
Logistic distribution	√	√	√
Gumbel distribution	√	√	×
Exponential distribution	√	√	×
Laplace distribution	√	√	√
t-Distribution	√	×	√
F-distribution	√	×	×

"√" means the distribution satisfies the condition, "×" means the distribution violates the condition, and "−" means the result is unknown.

(i) *decreases with* n *for* $n \geq \overline{n}(L)$ *for some critical value* $\overline{n}(L)$ *if Condition A in Table 1 holds;*
(ii) *decreases or is unimodal in* n *for* $L = 1$ *if Condition A in Table 1 holds; and*
(iii) *decreases with* n *for* $n \geq 2L + 1$ *if* $f(\cdot)$ *is symmetric.*

Proposition 1 implies that EAQ(n) or, equivalently, the equilibrium effort $e^*(n)$ decreases in n under various situations. Accordingly, the sponsor should adopt a restricted-entry policy in the contest, and, ideally, the optimal number of participants is achieved at $\max\{n : \text{EAQ}(n) - \text{EAQ}(n-1) \geq c\}$.

Intuitively, Proposition 1(i) and (ii) suggests that when there are too many contestants, the associating fierce competition reduces the individual's winning probability; thus, the contestants would give less effort, which results in a small EAQ. Moreover, mild competition (a small n) may lead to an effort-increasing effect (an increasing EAQ), and this is new in the literature.

Proposition 1(iii) specifies that the effort-decreasing effect appears whenever $n \geq 2L + 1$ if $f(\cdot)$ is symmetric (e.g., a normal distribution and t distribution in Table 2). Numerically, when the random shock follows a normal distribution, a single-winner award scheme (i.e., $L = 1$) can trigger the effort-decreasing effect if $n \geq 3$. This is of practical value in contest design.

For illustration, we provide a numerical study for a contest with $n \in \{5, 6, \ldots, 40\}$ contestants and three awards, and record the equilibrium effort $e^*(n)$ in Fig. 1. The effort-decreasing effect appears when $n \geq 7$ with the symmetric distributions of the random shock.

We add to the results revealed by Ales et al. (2017a) that, first, the effort-decreasing effect appears in a contest with a single winner and a linear utility function $U(\mathcal{V}) = \mathcal{V}$, and, second, the effort-increasing effect appears when $f(\cdot)$ is monotonically increasing. We additionally show that, first, the effort-decreasing effect also exists in contests with multiple winners, a unimodally distributed ϵ, and a general utility function $U(\cdot)$; and, second, the effort-increasing effect may happen when n is small ($n \in \{5, 6\}$, as shown in Fig. 1). Our results are also supported by some existing works. For example, Casas-Arce and Martínez-Jerez (2009) and Garcia and Tor (2009) observe numerical evidence that the effort-decreasing effect takes place when n is larger than 50 and 20, respectively.

We conclude this subsection in Theorem 1 by presenting some sufficient conditions under which the restricted-entry policy is preferred by a sponsor to maximize Π_{EAQ}.

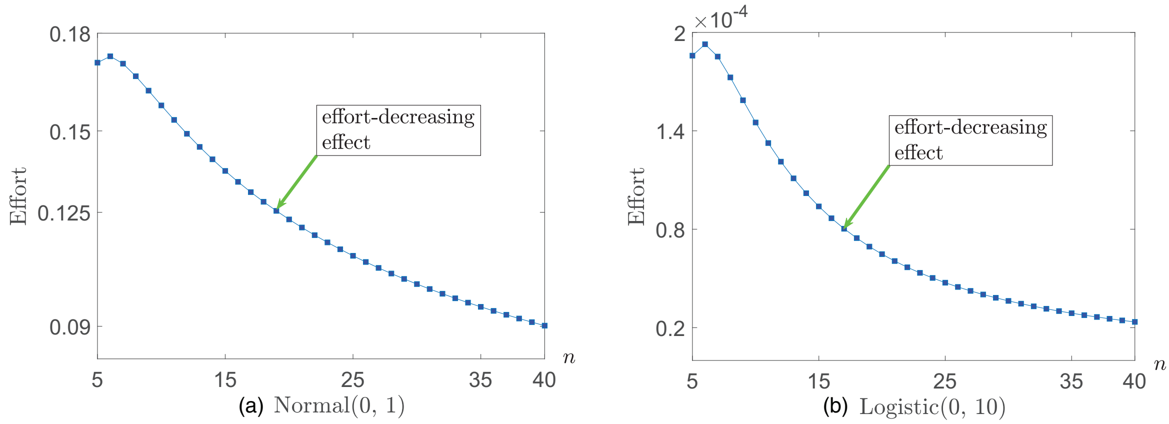

Fig. 1. The equilibrium effort $e^*(n)$ in a contest with n contestants and three awards.
Note: $K = 3$, $V_1 = 0.5$, $V_2 = 0.4$, $V_3 = 0.3$, $R(e) = e$, $C(e) = e^{1.6}$, $U(\mathcal{V}) = \mathcal{V}^{0.8}$. (a) $\epsilon \sim N(0, 1)$; (b) $\epsilon \sim Logis(0, 10)$.

Theorem 1 (Restricted-entry policy is preferred with Π_{EAQ}). *In a crowdsourcing contest, given the award scheme, when the sponsor is interested in maximizing Π_{EAQ}, the restricted-entry policy outperforms the free-entry policy if Condition A shown in Table 1 holds.*

Theorem 1 shows that the free-entry policy could be suboptimal if the sponsor cares about Π_{EAQ}. This result is also observed in the literature by Terwiesch and Xu (2008) and Ales et al. (2017a). Moreover, we extend their results for any number of winners, general utility function of contestants, and weaker conditions for the distribution of ϵ.

3.2. *Impact of n on EMQ*

Given the award scheme, we then study how $\text{EMQ}(n) = Ke^*(n) + \sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]$ changes with n, where K is a constant representing the number of contributors; and we will discuss the case with a flexible number of contributors $K(n)$ as an extension (see Section 4). Compared to $\text{EAQ}(n)$, in addition to e^*, $\text{EMQ}(n)$ is also affected by the term $\sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]$. To analyze the monotonicity of $\text{EMQ}(n)$, we follow Ales et al. (2017a) and denote the uncertainty of ϵ by $\alpha \in \mathbb{R}_+$ associated with a scale transformation $\hat{\epsilon} = \alpha\epsilon$. The main results are presented in Proposition 2 and Corollary 1.

Proposition 2 (Monotonicity of EMQ). *Given any K and ARs, along with the number of winners L, we have that* $\text{EMQ}(n)$

(i) *increases with n if $\alpha > \underline{\alpha}$ or $\mathcal{V} < \overline{\mathcal{V}}$, where $\underline{\alpha}$ and $\overline{\mathcal{V}}$ are some critical values of α and $\mathcal{V}$, respectively;*
(ii) *decreases with n for $n_1 \le n \le n_2$ if Condition A in Table 1 holds, and $\alpha < \overline{\alpha}$ or $\mathcal{V} > \underline{\mathcal{V}}$, where n_1 and n_2, $\overline{\alpha}$ and $\underline{\mathcal{V}}$ are some critical values of n, α and $\mathcal{V}$, respectively.*

Corollary 1. *The critical values $\overline{\mathcal{V}}$ and $\underline{\mathcal{V}}$ are increasing and $\overline{\alpha}$ and $\underline{\alpha}$ are decreasing with the number of contributors K if Condition B in Table 1 holds.*

As shown in Proposition 1, term $Ke^*(n)$ is generally decreasing in n while it is intuitive that the term $\sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]$ is increasing with n. Hence, the monotonicity of $\mathrm{EMQ}(n) = Ke^*(n) + \sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]$, presented in Proposition 2, is a joint effect of the above two.

Note that as the total award $\mathcal{V}$ increases, the reduction rate of $e^*(n)$ in n gets larger (see Lemma 1). However, as the uncertainty α increases, the growth rate of term $\sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]$ in n becomes larger. These two observations imply that, as shown in Proposition 2, there are some critical values of $\mathcal{V}$ and α such that if $\mathcal{V}$ is small or α is large, then $\mathrm{EMQ}(n)$ increases with n; otherwise, if $\mathcal{V}$ is large or α is small, then $\mathrm{EMQ}(n)$ may decrease in n.

Compared with the literature, we generalize the results of Ales et al. (2017a) by claiming that EMQ is increasing with n when α is sufficiently large for not only $U(\mathcal{V}) = \mathcal{V}$ but also any $U(\cdot)$. Moreover, we reveal that the monotonicity of EMQ also depends on the total award $\mathcal{V}$, and this is new to the literature.

As shown in Table 2, Condition A in Proposition 2(ii) is mild, under which the EMQ may decrease in n. Thus, the restricted-entry policy is preferred by the sponsor. As discussed before, even when the EMQ is increasing (Proposition 2(i)), the restricted-entry policy is still applicable if the EMQ is concavely increasing with n due to the existence of unit entry cost c.

Proposition 3 (Concavely increasing EMQ). *Given any ARs, along with the number of winners L, we have that* $\mathrm{EMQ}(n)$ *is concavely increasing with n if*

(i) $K = 1$ *and* $\alpha > \alpha_1$ *or* $\mathcal{V} < \mathcal{V}_1$, *where* α_1 *and* $\mathcal{V}_1$ *are some critical values of* α *and* $\mathcal{V}$, *respectively;*
(ii) $K > 1$ *when Condition B in Table 1 holds, and* $\alpha > \alpha_2$ *or* $\mathcal{V} < \mathcal{V}_2$, *where* α_2 *and* $\mathcal{V}_2$ *are some critical values of* α *and* $\mathcal{V}$, *respectively.*

Generally, Proposition 3 suggests that the EMQ is concavely increasing with n under either the single-contributor case or the multiple-contributor case when the total award is small or the uncertainty of the random shock is large. When the EMQ is shown to be concavely increasing with n, the sponsor can adopt the restricted-entry policy to maximize his Π_{EMQ}, and, ideally, the optimal number of participants is achieved at $\max\{n : \mathrm{EMQ}(n) - \mathrm{EMQ}(n-1) \geq c\}$.

To illustrate our results in Propositions 2 and 3, we conduct some numerical experiments for a contest with n contestants, ranging from 5 to 40, and the numerical results of the EMQ are shown in Fig. 2. The random shock ϵ is assumed to follow a normal distribution, under which Condition B listed in Proposition 3(ii) is valid, that is, the EMQ is concavely increasing when α is large or $\mathcal{V}$ is small.

In Fig. 2a and c, the EMQ is concavely increasing with n since the total award $\mathcal{V}$ is small and the uncertainty α is large, respectively. The optimal number of participants in the restricted-entry policy is n^*, at which the slope of the EMQ equals c. Figure 2b shows a scenario of Proposition 2(ii) under which the EMQ is first increasing and then decreasing in n. The optimal number of participants in the restricted-entry policy is n^*, at which $\mathrm{EMQ}(n^*) - \mathrm{EMQ}(n^* - 1) \geq c$ while $\mathrm{EMQ}(n^* + 1) - \mathrm{EMQ}(n^*) < c$.

We conclude this subsection in Theorem 2 by presenting a sufficient condition under which the restricted-entry policy is preferred by a sponsor to maximize Π_{EMQ}.

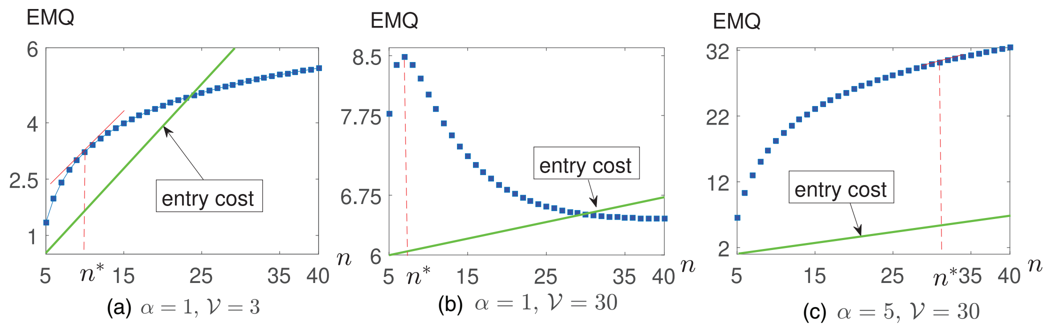

Fig. 2. The EMQ in a contest with n contestants and three awards.
Note: $K = 3$, V_1:V_2:V_3 = 5:3:2, $R(e) = e$, $C(e) = e^{1.6}$, $U(\mathcal{V}) = \mathcal{V}^{0.8}$, $\epsilon \sim N(0, 1)$. (a) $\alpha = 1$ and $\mathcal{V} = 3$, (b) $\alpha = 1$ and $\mathcal{V} = 30$, and (c) $\alpha = 5$ and $\mathcal{V} = 30$.

Theorem 2 (Restricted-entry policy is preferred with Π_{EMQ}). *In a crowdsourcing contest, given the award scheme, when the sponsor is interested in maximizing Π_{EMQ}, the restricted-entry policy outperforms the free-entry policy if Conditions A and B in Table 1 hold.*

Theorem 2 shows that the free-entry policy could be suboptimal for the sponsor when the entry cost is considered. This result is of practical importance as mentioned at the beginning, even though it is not observed in the literature by Ales et al. (2017a).

4. Extension: flexible number of contributors

We now consider a general case of Section 3.2 where the number of contributors to the EMQ is flexible rather than determined, that is, $\mathrm{EMQ}(n) = K(n)e^*(n) + \sum_{j=1}^{K(n)} \mathbb{E}[\epsilon_j^n]$ (more discussion about $K(n)$ can be obtained in Appendix C). For the purpose of conducting concrete investigations, we shift our focus to a simple concavely increasing function $\tilde{K}(n) = n - k_1$, where k_1 is a nonnegative integer. We note that $\tilde{K}(n) = n$ (i.e., $k_1 = 0$) is studied in Green and Stokey (1983) and Kalra and Shi (2001). In the following, we give the main results in Proposition 4.

Proposition 4 (Monotonicity of EMQ with flexible contributors). *Given any ARs along with the number of winners L and an example $R(e) = log(e)$ and $C(e) = e$, if Condition B in Table 1 holds, we have that*

(i) *for $k_1 = 0$, the EMQ concavely increases with n for any uncertainty α and total award $\mathcal{V}$;*
(ii) *for $k_1 > 0$, the EMQ increases with n for any uncertainty α and total award $\mathcal{V}$, and the EMQ concavely increases with n when $\alpha > \tilde{\alpha}$ or $\mathcal{V} < \tilde{\mathcal{V}}$, where $\tilde{\alpha}$ and $\tilde{\mathcal{V}}$ are some critical values of α and $\mathcal{V}$, respectively.*

Proposition 4 indicates that when $\tilde{K}(n) = n - k_1$ for $k_1 \geq 0$, the EMQ is again concavely increasing with n under certain conditions. We note that $q_i = \log(e_i) + \epsilon_i$ and $C(e) = e$ are the same as in Terwiesch and Xu (2008). In addition, Mihm and Schlapp (2019) assume that $q_i = e_i + \epsilon_i$ and $C(e) = e^2$, which results in the same optimal effort $e^*(n)$.

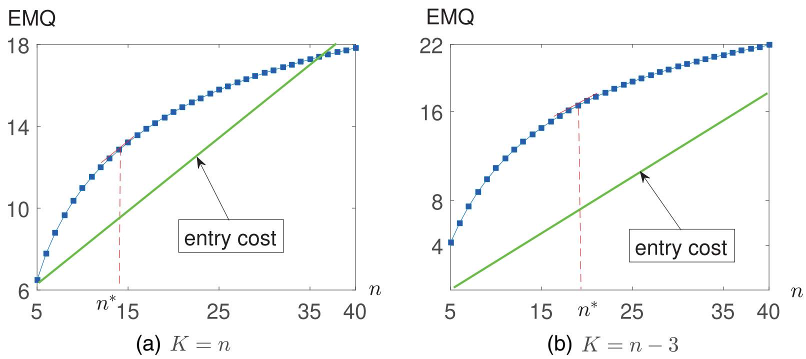

Fig. 3. The EMQ in a contest with n and $n-3$ contributors.
Note: $V_1 = 15$, $V_2 = 9$, $V_3 = 5$, $R(e) = log(e)$, $\epsilon \sim N(0, 1)$, $C(e) = e$, and $U(\mathcal{V}) = \mathcal{V}^{0.8}$. (a) $K = n$ and (b) $K = n - 3$.

Compared with the case of $K(n) = K$ (in Section 3.2), since $K(n)$ is increasing with n, it is intuitive that $\text{EMQ}(n) = K(n)e^*(n)/\alpha + \sum_{j=1}^{K(n)} \alpha\mathbb{E}[\epsilon_j^n]$ is more likely to be (concavely) increasing with n. This can be observed from the results of Propositions 2–4. Similar to the case of the EMQ with a fixed number of contributors, we also numerically illustrate the results in Proposition 4 for the case of the EMQ with a flexible number of contributors, and the results are shown in Fig. 3.

Similar to Theorem 2, we are interested in specifying when a free-entry policy is suboptimal for Π_{EMQ}. Based on Proposition 4, we obtain the following result.

Theorem 3 (Restricted-entry policy is preferred with flexibly contributed Π_{EMQ}). *Given any ARs along with the number of winners L and examples $R(e) = log(e)$ and $C(e) = e$, when the sponsor is interested in maximizing Π_{EMQ}, the restricted-entry policy outperforms the free-entry policy if Condition B in Table 1 holds.*

5. Conclusion

In this paper, we reveal the results of analyzing the monotonicity of the EAQ and the EMQ of the solutions based on the number of contestants. First, as long as the EAQ (or the EMQ) is decreasing or concavely increasing with the number of contestants due to the existence of the entry cost, the sponsor should adopt the restricted-entry policy, that is, restrict the number of participants at some level. Second, in most cases, our theoretical results indicate a preference for the restricted-entry policy for the sponsor.

There are some interesting but challenging directions for future research. First, we are among the first to observe that mild competition and fierce competition trigger the effort-increasing effect and the effort-decreasing effect, respectively. It is meaningful to empirically verify the existence of this result. Second, in this paper, we assume that the ARs are given. Thus, it is interesting to analyze the question with endogenous ARs. Third, we only analyze how the number of contestants affects the expected quality, and give the optimal number of contestants numerically. It would be interesting to analyze the properties of the optimal number of contestants, which might be very challenging.

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References

Ales, L., Cho, S.H., Körpeoğlu, E., 2017a. Innovation tournaments with multiple contributors. Working paper, Carnegie Mellon University, Pittsburgh, PA.

Ales, L., Cho, S.H., Körpeoğlu, E., 2017b. Optimal award scheme in innovation tournaments. *Operations Research* 65, 3, 693–702.

Bagnoli, M., Bergstrom, T., 2005. Log-concave probability and its applications. *Economic Theory* 26, 2, 445–469.

Bockstedt, J., Druehl, C., Mishra, A., 2016. Heterogeneous submission behavior and its implications for success in innovation contests with public submissions. *Production and Operations Management* 25, 7, 1157–1176.

Boudreau, K.J., Lacetera, N., Lakhani, K.R., 2011. Incentives and problem uncertainty in innovation contests: an empirical analysis. *Management Science* 57, 5, 843–863.

Casas-Arce, P., Martínez-Jerez, F.A., 2009. Relative performance compensation, contests, and dynamic incentives. *Management Science* 55, 8, 1306–1320.

Che, Y.K., Gale, I., 2003. Optimal design of research contests. *American Economic Review* 93, 3, 646–671.

Chen, Y.J., Dai, T., Korpeoglu, C.G., Körpeoğlu, E., Sahin, O., Tang, C.S., Xiao, S., 2018. Innovative online platforms: research opportunities. *Manufacturing Service Operations Management* 22, 430–445.

Fullerton, R.L., McAfee, R.P., 1999. Auction in entry into tournaments. *Journal of Political Economy* 107, 3, 573–605.

Garcia, S.M., Tor, A., 2009. The n-effect: more competitors, less competition. *Psychological Science* 20, 7, 871–877.

Green, J.R., Stokey, N.L., 1983. A comparison of tournaments and contracts. *Journal of Political Economy* 91, 3, 349–364.

Hu, M., Wang, L., 2020. Joint vs. separate crowdsourcing contests. *Management Science*. https://doi.org/10.2139/ssrn.2892683.

Kalra, A., Shi, M., 2001. Designing optimal sales contests: a theoretical perspective. *Marketing Science* 20, 2, 170–193.

Körpeoğlu, E., Cho, S.H., 2017. Incentives in contests with heterogeneous solvers. *Management Science* 64, 6, 2709–2715.

Mihm, J., Schlapp, J., 2019. Sourcing innovation: on feedback in contests. *Management Science* 65, 2, 559–576.

Moldovanu, B., Sela, A., 2001. The optimal allocation of prizes in contests. *American Economic Review* 91, 3, 542–558.

Taylor, C.R., 1995. Digging for golden carrots: an analysis of research tournaments. *American Economic Review* 85, 4, 872–890.

Terwiesch, C., Xu, Y., 2008. Innovation contests, open innovation, and multiagent problem solving. *Management Science* 54, 9, 1529–1543.

Wooten, J.O., Ulrich, K.T., 2017. Idea generation and the role of feedback: evidence from field experiments with innovation tournaments. *Production and Operations Management* 26, 1, 80–99.

Appendix A: Proofs

A.1. Proof of Proposition 1

Proof.

(i) Similar to (B1) in Appendix B, we can obtain

$$\mathbb{E}\Big[f(\epsilon_j^n)\Big] - \mathbb{E}\Big[f(\epsilon_j^{n-1})\Big] = \int_{\underline{\beta}}^{\overline{\beta}} \frac{(n-1)!}{(j-1)!(n-j)!}(1-F(x))^n\left(\frac{F(x)}{1-F(x)}\right)^{n-j} f'(x)dx \tag{A1}$$

for any $1 \leq j \leq L$. Next, we show that under a weak condition, (A1) is negative. Let $g_j(n) = \int_{\underline{\beta}}^{\overline{\beta}} (1 - F(x))^{j-1} F(x)^{n-j} f'(x) dx$. If $f(\cdot)$ is decreasing, obviously (A1) is negative. We next consider the case that $f(\cdot)$ is unimodal, that is, there exists $\beta_0 \in (\underline{\beta}, \overline{\beta})$ if $x \in (\underline{\beta}, \beta_0)$; then $f'(x) \geq 0$ if $x \in (\beta_0, \overline{\beta})$; and then $f'(x) \leq 0$. Let

$$G_j(n) = g_j(n+1) - g_j(n) = \int_{\underline{\beta}}^{\overline{\beta}} (1 - F(x))^{j-1} F(x)^{n-j} f'(x)(F(x) - 1) dx. \tag{A2}$$

Then, we have

$$\begin{aligned} G_j(n+1) - G_j(n) &= \int_{\underline{\beta}}^{\overline{\beta}} (1 - F(x))^{j-1} F(x)^{n-j} (F(x) - 1) f'(x)(F(x) - 1) dx \\ &> \int_{\underline{\beta}}^{\beta_0} (1 - F(x))^{j-1} F(x)^{n-j} (F(\beta_0) - 1) f'(x)(F(x) - 1) dx \\ &+ \int_{\beta_0}^{\overline{\beta}} (1 - F(x))^{j-1} F(x)^{n-j} (F(\beta_0) - 1) f'(x)(F(x) - 1) dx \\ &= (F(\beta_0) - 1) G_j(n). \end{aligned} \tag{A3}$$

Thus, $G_j(n+1) > F(\beta_0) G_j(n)$. This inequality implies that if there exists n_0 such that $G_j(n_0) \geq 0$, then for any $n > n_0$, $G_j(n) > 0$. On the other hand, according to the Dominated Convergence Theorem and integration by parts, we have $\lim_{n\to\infty} g_j(n) = 0$ for any $j \geq 1$. As a result, if $g_j(n_j) < 0$, then for any $n > n_j$, $g_j(n) < 0$, and there exists $n_{j+1} \geq n_j$ such that $g_j(n_{j+1}) < g_j(n_{j+1} + 1)$. Then,

$$g_{j+1}(n_{j+1} + 1) = g_j(n_{j+1}) - g_j(n_{j+1} + 1) < 0.$$

Similarly, we have that for any $n > n_{j+1} + 1$, $g_{j+1}(n) < 0$, and there exists $n_{j+2} \geq n_{j+1} + 1$ such that $g_{j+1}(n_{j+2}) < g_{j+1}(n_{j+2} + 1)$. To illustrate the property of $g_j(n)$, we provide Fig. A1 as follows. That is, if there exists some n_j such that $g_j(n_j) < 0$, then $g_j(n)$ (< 0) increases (Fig. A1a) or first decreases then increases (Fig. A1b) for $n \geq n_j$.

Then, if $g_1(2) < \lim_{n\to\infty} g_1(n)$, that is, $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ (Condition A in Table 1). According to the property of $g_j(n)$, it is not hard to show that there exists $\overline{n}(L)$ when $n \geq \overline{n}(L)$ and $\mathbb{E}[f(\epsilon_j^n)] < \mathbb{E}[f(\epsilon_j^{n-1})]$ for any $1 \leq j \leq L$. We note that if the inequality $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ holds, $f(\cdot)$ cannot be constant or increasing. That is, based on our assumption ($f(\cdot)$ is either unimodal, decreasing, constant, or increasing), if Condition A in Table 1 holds, $f(\cdot)$ is decreasing or unimodal.

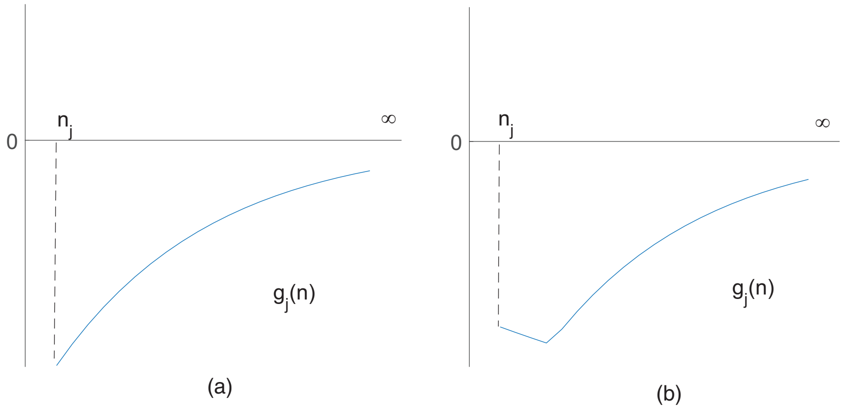

Fig. A1. The structure of $g_j(n)$.

If the ARs $r_1, r_2, \dots, r_L$ are given, by Lemma 1, $\frac{C'(e^*)}{R'(e^*)} = \sum_{j=1}^{L-1}[U(V_j) - U(V_{j+1})]\mathbb{E}[f(\epsilon_j^{n-1})] + U(V_L)\mathbb{E}[f(\epsilon_L^{n-1})]$. Then, we have

$$\begin{aligned}\frac{C'(e^*(n+1))}{R'(e^*(n+1))} &= \sum_{j=1}^{L-1}\left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\Big[f(\epsilon_j^{n})\Big] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n})\right] \\ &< \sum_{j=1}^{L-1}\left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\Big[f(\epsilon_j^{n-1})\Big] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n-1})\right] \\ &= \frac{C'(e^*(n))}{R'(e^*(n))}.\end{aligned}$$

Since the cost function $C(\cdot)$ is convex and $R(\cdot)$ is concave, then $\frac{C'(\cdot)}{R'(\cdot)}$ is increasing. As a result, if the inequality $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ holds, then $e^*(n+1) < e^*(n)$ for any $n \geq \overline{n}(L)$.

(ii) When $L = 1$, by Lemma 1, using integration by parts, we have $\frac{C'(e^*)}{R'(e^*)} = U(\mathcal{V})(f(\overline{\beta}) - g_1(n))$, where $g_1(n)$ is defined above. If the inequality $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ holds, that is, $(f(\overline{\beta}) - g_1(2)) > \lim_{n\to\infty}(f(\overline{\beta}) - g_1(n))$, from (A2) and (A3), it is straightforward to verify that $(f(\overline{\beta}) - g_1(n))$ decreases or first increases and then decreases with n. As a result, $e^*(n)$ decreases or first increases and then decreases with n if the inequality $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ holds.

(iii) If the density function of random shock $f(\cdot)$ is symmetric, that is, $\beta_0 - \underline{\beta} = \overline{\beta} - \beta_0$, $f(x) = f(2\beta_0 - x)$, $F(x) = 1 - F(2\beta_0 - x)$, $f'(x) = -f'(2\beta_0 - x)$, then we have

$$\int_{\underline{\beta}}^{\overline{\beta}} (1 - F(x))^{j-1} F(x)^{n-j} f'(x) dx$$

$$
\begin{aligned}
&= \int_{\underline{\beta}}^{\beta_0} (1-F(x))^{j-1}F(x)^{n-j}f'(x)dx + \int_{\beta_0}^{\overline{\beta}} (1-F(x))^{j-1}F(x)^{n-j}f'(x)dx \\
&= -\int_{\beta_0}^{\overline{\beta}} (1-F(x))^{n-j}F(x)^{j-1}f'(x)dx + \int_{\beta_0}^{\overline{\beta}} (1-F(x))^{j-1}F(x)^{n-j}f'(x)dx \\
&= \int_{\beta_0}^{\overline{\beta}} (1-F(x))^{j-1}F(x)^{j-1}(-(1-F(x))^{n-2j+1} + F(x)^{n-2j+1})f'(x)dx. \qquad \text{(A4)}
\end{aligned}
$$

Thus, if $n-2j+1>0$, then $-(1-F(x))^{n-2j+1}+F(x)^{n-2j+1}>0$ for $x \in (\beta_0, \overline{\beta})$, i.e., (A4) is negative. By (A1), we have $\mathbb{E}[f(\epsilon_j^n)] < \mathbb{E}[f(\epsilon_j^{n-1})]$ if $n-2j+1>0$. Hence, if $f(\cdot)$ is symmetric, $e^*(n+1) < e^*(n)$ for any $n \geq 2L+1$. □

A.2. Proof of Proposition 2

Proof.

(i) Note that

$$
\begin{aligned}
EMQ(n+1) - EMQ(n) &= \left[Ke^*(n+1) + \sum_{j=1}^{K} \mathbb{E}(\epsilon_j^{n+1})\right] - \left[Ke^*(n) + \sum_{j=1}^{K} \mathbb{E}(\epsilon_j^{n})\right] \\
&= \sum_{j=1}^{K} \Big[\mathbb{E}(\epsilon_j^{n+1}) - \mathbb{E}(\epsilon_j^{n}) + e^*(n+1) - e^*(n)\Big]. \qquad \text{(A5)}
\end{aligned}
$$

By random shock transformation, $\hat{\epsilon} = \alpha\epsilon$, we have $\mathbb{E}[f(\hat{\epsilon}_{(j)}^n)] = \mathbb{E}[f(\epsilon_j^n)]/\alpha$ and $\mathbb{E}[\hat{\epsilon}_{(j)}^n] = \alpha E[(\epsilon_j^n)]$ (see Ales et al., 2017a). If $r_1, r_2, \ldots, r_L$ are given, by Lemma 1, we have $\frac{C'(e^*(n+1))}{R'(e^*(n+1))} = \sum_{j=1}^{L-1}[U(V_j) - U(V_{j+1})]\mathbb{E}[f(\epsilon_j^n)] + U(V_L)\mathbb{E}[f(\epsilon_L^n)]$. Then,

$$
\begin{aligned}
&\mathbb{E}(\hat{\epsilon}_j^{n+1}) - \mathbb{E}(\hat{\epsilon}_j^{n}) + \hat{e}^*(n+1) - \hat{e}^*(n) \\
&= \alpha\Big[\mathbb{E}(\epsilon_j^{n+1}) - \mathbb{E}(\epsilon_j^{n})\Big] \\
&\quad + \left[\frac{C'(\cdot)}{R'(\cdot)}\right]^{-1}\left(\sum_{j=1}^{L-1}\big[U(V_j) - U(V_{j+1})\big]\mathbb{E}\Big[f(\epsilon_j^n)\Big]\Big/\alpha + U(V_L)\mathbb{E}\big[f(\epsilon_L^n)\big]\big/\alpha\right) \\
&\quad - \left[\frac{C'(\cdot)}{R'(\cdot)}\right]^{-1}\left(\sum_{j=1}^{L-1}\big[U(V_j) - U(V_{j+1})\big]\mathbb{E}\Big[f(\epsilon_j^{n-1})\Big]\Big/\alpha + U(V_L)\mathbb{E}\big[f(\epsilon_L^{n-1})\big]\big/\alpha\right). \qquad \text{(A6)}
\end{aligned}
$$

From (B1) in Appendix B, we have $\mathbb{E}[\epsilon_j^{n+1}] > \mathbb{E}[\epsilon_j^n]$ for any $j \geq 1$. From (A5) and (A6), when $\alpha \to \infty$ or $\mathcal{V} \to 0$, we can verify that $EMQ(n+1) > EMQ(n)$. Thus, there exist $\overline{\mathcal{V}}$, $\underline{\alpha}$ when $\alpha > \underline{\alpha}$ or $\mathcal{V} < \overline{\mathcal{V}}$, $EMQ(n+1) > EMQ(n)$.

(ii) If the inequality $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ holds, by Proposition 1(i), there exist n_1, n_2, $n_1 \leq n \leq n_2$, $\mathbb{E}[f(\epsilon_j^n)] < \mathbb{E}[f(\epsilon_j^{n-1})]$ for any $1 \leq j \leq L$. Similarly, there exist $\overline{\alpha}$, $\underline{\mathcal{V}}$ when $\alpha < \overline{\alpha}$ or $\mathcal{V} > \underline{\mathcal{V}}$ for $n_1 \leq n \leq n_2$, $EMQ(n+1) < EMQ(n)$. □

A.3. Proof of Corollary 1

Proof. Suppose that $EMQ(n+1)[K] - EMQ(n)[K] = Ke^*(n+1) + \sum_{j=1}^{K} \mathbb{E}[\epsilon_j^{n+1}] - (Ke^*(n) + \sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]) > 0$, where $EMQ(n+1)[K+1]$ represents the EMQ with $K+1$ contributors and $n+1$ contestants. By Lemma B1 in Appendix B, if the inequalities $((1-F(x))/f(x))' < 0$ and $lim_{x\to\underline{\beta}}(F(x)/f(x)) < \infty$ hold, we have $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] < \mathbb{E}[\epsilon_{j+1}^{n+1}] - \mathbb{E}[\epsilon_{j+1}^n]$ for any $1 \leq j \leq n-1$. If $f(x)$ is log-concave, then $((1-F(x))/f(x))' < 0$ (see Bagnoli and Bergstrom, 2005, Theorem 4). Thus, if Condition B in Table 1 holds, then $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] < \mathbb{E}[\epsilon_{j+1}^{n+1}] - \mathbb{E}[\epsilon_{j+1}^n]$ for any $1 \leq j \leq n-1$. As a result, we have

$$\begin{aligned}
&EMQ(n+1)[K+1] - EMQ(n)[K+1] \\
&= (K+1)e^*(n+1) + \sum_{j=1}^{K+1} \mathbb{E}[\epsilon_j^{n+1}] - \left((K+1)e^*(n) + \sum_{j=1}^{K+1} \mathbb{E}[\epsilon_j^n]\right) \\
&= (e^*(n+1) - e^*(n)) + \left(\mathbb{E}[\epsilon_{K+1}^{n+1}] - \mathbb{E}[\epsilon_{K+1}^n]\right) + EMQ(n+1)[K] - EMQ(n)[K] \\
&\geq \left(e^*(n+1) - e^*(n) + \frac{1}{K}\left(\sum_{j=1}^{K} \mathbb{E}[\epsilon_j^{n+1}] - \sum_{j=1}^{K} \mathbb{E}[\epsilon_j^n]\right)\right) + EMQ(n+1)[K] - EMQ(n)[K] \\
&= \left(1 + \frac{1}{K}\right)(EMQ(n+1)[K] - EMQ(n)[K]) > 0.
\end{aligned}$$

This implies that $\overline{\mathcal{V}}$ increases with K, and $\underline{\alpha}$ decreases with K. Similarly, if $EMQ(n+1)[K+1] - EMQ(n)[K+1] < 0$, we obtain $EMQ(n+1)[K] - EMQ(n)[K] < 0$. This implies that $\overline{\alpha}$ decreases with K, and $\underline{\mathcal{V}}$ increases with K. □

A.4. Proof of Proposition 3

Proof.

(i) For any distribution of ϵ, by Lemma B2 in Appendix B, we have $\mathbb{E}[\epsilon_1^{n+1}] - \mathbb{E}[\epsilon_1^n] < \mathbb{E}[\epsilon_1^n] - \mathbb{E}[\epsilon_1^{n-1}]$. Then, similar to the proof of Proposition 2, for $K = 1$, there exist α_1 and $\mathcal{V}_1$ when $\alpha > \alpha_1$ or $\mathcal{V} < \mathcal{V}_1$, $EMQ(n+1) - EMQ(n) < EMQ(n) - EMQ(n-1)$.

(ii) By Lemma B2 in Appendix B, if the inequalities $((1-F(x))/f(x))' < 0$ and $lim_{x\to\underline{\beta}}(F(x)/f(x)) < \infty$ hold, we have $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] < \mathbb{E}[\epsilon_j^n] - \mathbb{E}[\epsilon_j^{n-1}]$ for any $1 \le j \le n-1$. In addition, log-concavity implies $((1-F(x))/f(x))' < 0$. Thus, if Condition B in Table 1 holds, then $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] < \mathbb{E}[\epsilon_j^n] - \mathbb{E}[\epsilon_j^{n-1}]$ for any $1 \le j \le n-1$. Similarly, for $K > 1$, there exist α_2 and $\mathcal{V}_2$ when $\alpha > \alpha_2$ or $\mathcal{V} < \mathcal{V}_2$, $EMQ(n+1) - EMQ(n) < EMQ(n) - EMQ(n-1)$. □

A.5. Proof of Theorem 2

Proof. Noting that $EMQ(n) = Ke^*(n) + \sum_{i=1}^K \mathbb{E}[\epsilon_i^n]$, then the profit of the sponsor is given as follows:

$$
\begin{aligned}
EMQ(n) - n\cdot c - \mathcal{V} &= Ke^*(n) + \sum_{i=1}^K \mathbb{E}[\epsilon_i^n] - n\cdot c - \mathcal{V} \\
&= Ke^*(n) - n\cdot\frac{c}{2} + \sum_{i=1}^K \mathbb{E}[\epsilon_i^n] - n\cdot\frac{c}{2} - \mathcal{V}.
\end{aligned}
$$

By Proposition 1, if the inequality $\int_{\underline{\beta}}^{\overline{\beta}} f(x)^2 dx > f(\overline{\beta})$ holds, there exists $\overline{n}(L)$ when $n \ge \overline{n}(L)$, $e^*(n)$ is decreasing. To make the first term $Ke^*(n) - n\cdot\frac{c}{2}$ nonnegative, there is a bound for the number of contestants n. Log-concavity implies $((1-F(x))/f(x))' < 0$. Thus, by Lemma B2 in Appendix B, if Condition B in Table 1 holds, then $\mathbb{E}[\epsilon_i^n]$ is increasingly concave in n for any $i \ge 1$. To make the second term $\sum_{i=1}^K \mathbb{E}[\epsilon_i^n] - n\cdot\frac{c}{2}$ nonnegative, there is another bound for the number of contestants n. Hence, when the unit entry cost $c > 0$, a free-entry policy is suboptimal. □

A.6. Proof of Proposition 4

Proof. If $R(e) = log(e)$ and $C(e) = e$, by Lemma 1, we have

$$
EMQ(n) = K(n)\left(\sum_{j=1}^{L-1}\left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\left[f(\epsilon_j^{n-1})\right] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n-1})\right]\right) + \sum_{i=1}^{K(n)} \mathbb{E}[\epsilon_i^n].
$$

For $k_1 = 0$, $\sum_{i=1}^n \mathbb{E}[\epsilon_i^n] = 0$. By Lemma B3 in Appendix B, if the inequality $(log(f(x)))'' \le 0$ holds, then $(n+1)\mathbb{E}[f(\epsilon_j^n)]$ is concavely increasing with n for any $1 \le j \le L$. Thus, for any α, $\mathcal{V}$, $EMQ(n)$ is concavely increasing with n. For $k_1 > 0$, by Lemma B4 in Appendix B, if Condition B in Table 1 holds, then $\sum_{i=1}^{n-k_1} \mathbb{E}[\epsilon_i^n] = -\sum_{i=n+1-k_1}^n \mathbb{E}[\epsilon_i^n]$ is concavely increasing with n. In addition,

$$
(n-k_1)\mathbb{E}\left[f(\epsilon_j^{n-1})\right] = n\mathbb{E}\left[f(\epsilon_j^{n-1})\right]\left(1 - \frac{k_1}{n}\right).
$$

Since $(1-\frac{k_1}{n})$ and $n\mathbb{E}[f(\epsilon_j^{n-1})]$ increase with n, then for any α and $\mathcal{V}$, $EMQ(n)$ is increasing. Similar to the proof of Proposition 3, there exist $\tilde{\alpha}$ and $\tilde{\mathcal{V}}$ when $\alpha > \tilde{\alpha}$ or $\mathcal{V} < \tilde{\mathcal{V}}$, $EMQ(n)$ is concave in n. □

A.7. Proof of Theorem 3

Proof. If $R(e) = log(e)$ and $C(e) = e$, by Lemma 1, the profit of the sponsor for $k_1 = 0$ is given as follows:

$$\Pi_{EMQ} = n\left(\sum_{j=1}^{L-1}\left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\left[f(\epsilon_j^{n-1})\right] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n-1})\right]\right) - n\cdot c - \mathcal{V}.$$

By Lemma B3 in Appendix B, if Condition B in Table 1 holds, then $(n+1)\mathbb{E}[f(\epsilon_j^n)]$ is concavely increasing in n for any $1 \le j \le L$. To make the profit of the sponsor nonnegative, there is a bound for the number of contestants n.

If $R(e) = log(e)$ and $C(e) = e$, by Lemma 1, the profit of the sponsor for $k_1 > 0$ is given as follows:

$$\begin{aligned}\Pi_{EMQ} =& (n-k_1)\left(\sum_{j=1}^{L-1}\left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\left[f(\epsilon_j^{n-1})\right] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n-1})\right]\right)\\ &+\sum_{i=1}^{n-k_1}\mathbb{E}[\epsilon_i^n] - n\cdot c - \mathcal{V}\\ <& n\left(\sum_{j=1}^{L-1}\left[U(V_j) - U(V_{j+1})\right]\mathbb{E}\left[f(\epsilon_j^{n-1})\right] + U(V_L)\mathbb{E}\left[f(\epsilon_L^{n-1})\right]\right) - n\cdot\frac{c}{2}\\ &+\sum_{i=1}^{n-k_1}\mathbb{E}[\epsilon_i^n] - n\cdot\frac{c}{2} - \mathcal{V}.\end{aligned}$$

By Lemma B3 in Appendix B, if Condition B in Table 1 holds, then $(n+1)\mathbb{E}[f(\epsilon_j^n)]$ is concavely increasing in n for any $1 \le j \le L$. To make the first term $n(\sum_{j=1}^{L-1}[U(V_j) - U(V_{j+1})]\mathbb{E}[f(\epsilon_j^{n-1})] + U(V_L)\mathbb{E}[f(\epsilon_L^{n-1})]) - n\cdot\frac{c}{2}$ nonnegative, there is a bound for the number of contestants n. By Lemma B4 in Appendix B, if Condition B in Table 1 holds, then $\sum_{i=1}^{n-k_1}\mathbb{E}[\epsilon_i^n]$ is concavely increasing in n. To make the second term $\sum_{i=1}^{n-k_1}\mathbb{E}[\epsilon_i^n] - n\cdot\frac{c}{2}$ nonnegative, there is another bound for the number of contestants. Hence, when the unit entry cost $c > 0$, a free-entry policy is suboptimal. □

Appendix B: Supporting lemmas

Proof of Lemma 1. This lemma can be obtained by the first-order condition:

$$\sum_{j=1}^{L} U(V_j^n)\frac{\partial \Pr(q_i \text{ ranks order } j)}{\partial e_i} - C'(e_i) = 0.$$

Since $e_i = e^*$ in equilibrium (Terwiesch and Xu, 2008), by some calculation, we can get this lemma. □

Lemma B1. *If the inequalities* $((1-F(x))/f(x))' < 0$ *and* $lim_{x\to\underline{\beta}}(F(x)/f(x)) < \infty$ *hold, then* $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] < \mathbb{E}[\epsilon_{j+1}^{n+1}] - \mathbb{E}[\epsilon_{j+1}^n]$ *for any* $1 \le j \le L-1$.

Proof. Using integration by parts, we have

$$\begin{aligned}
\mathbb{E}[\epsilon_j^{n+1}] &= \int_{\underline{\beta}}^{\overline{\beta}} \frac{(n+1)!}{(j-1)!(n+1-j)!}(1-F(x))^{j-1}F(x)^{n+1-j}f(x)xdx \\
&= \int_{\underline{\beta}}^{\overline{\beta}} \frac{(n+1)!}{(j-1)!(n+1-j)!}(1-F(x))^{n}\left(\frac{F(x)}{1-F(x)}\right)^{n+1-j} f(x)xdx \\
&= -\frac{n!}{(j-1)!(n+1-j)!}(1-F(x))^{n+1}\left(\frac{F(x)}{1-F(x)}\right)^{n+1-j} x|_{\underline{\beta}}^{\overline{\beta}} \\
&\quad + \int_{\underline{\beta}}^{\overline{\beta}} \frac{n!}{(j-1)!(n-j)!}(1-F(x))^{n-1}\left(\frac{F(x)}{1-F(x)}\right)^{n-j} f(x)xdx \\
&\quad + \int_{\underline{\beta}}^{\overline{\beta}} \frac{n!}{(j-1)!(n+1-j)!}(1-F(x))^{n+1}\left(\frac{F(x)}{1-F(x)}\right)^{n+1-j} dx \\
&= \mathbb{E}[\epsilon_j^n] + \int_{\underline{\beta}}^{\overline{\beta}} \frac{n!}{(j-1)!(n+1-j)!}(1-F(x))^{n+1}\left(\frac{F(x)}{1-F(x)}\right)^{n+1-j} dx.
\end{aligned} \tag{B1}$$

Thus, $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] = \int_{\underline{\beta}}^{\overline{\beta}} \frac{n!}{(j-1)!(n+1-j)!}(1-F(x))^{n+1}(\frac{F(x)}{1-F(x)})^{n+1-j}dx$, and $\mathbb{E}[\epsilon_{j+1}^{n+1}] - \mathbb{E}[\epsilon_{j+1}^n] = \int_{\underline{\beta}}^{\overline{\beta}} \frac{n!}{(j)!(n-j)!}(1-F(x))^{n+1}(\frac{F(x)}{1-F(x)})^{n-j}dx$. To obtain $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] < \mathbb{E}[\epsilon_{j+1}^{n+1}] - \mathbb{E}[\epsilon_{j+1}^n]$, from (B1), we only need to show that $\frac{j}{n+1-j}\int_{\underline{\beta}}^{\overline{\beta}} (1-F(x))^j F(x)^{n+1-j}dx < \int_{\underline{\beta}}^{\overline{\beta}} (1-F(x))^{j+1}F(x)^{n-j}dx$. It is straightforward to verify that

$$\begin{aligned}
\int_{\underline{\beta}}^{\overline{\beta}} (1-F(x))^{j+1}F(x)^{n-j}dx &= \frac{1}{n-j+1}\int_{\underline{\beta}}^{\overline{\beta}} \frac{(1-F(x))}{f(x)}(1-F(x))^j dF(x)^{n-j+1} \\
&= \frac{1}{n-j+1}\frac{(1-F(x))}{f(x)}(1-F(x))^j F(x)^{n-j+1}|_{\underline{\beta}}^{\overline{\beta}}
\end{aligned}$$

$$-\frac{1}{n-j+1}\int_{\underline{\beta}}^{\overline{\beta}}((1-F(x))/f(x))'(1-F(x))^{j}F(x)^{n-j+1}dx$$

$$+\frac{j}{n-j+1}\int_{\underline{\beta}}^{\overline{\beta}}(1-F(x))^{j}F(x)^{n-j+1}dx. \tag{B2}$$

Thus, if $((1-F(x))/f(x))'<0$, and $lim_{x\to\underline{\beta}}(F(x)/f(x))<\infty$, then $\mathbb{E}[\epsilon_j^{n+1}]-\mathbb{E}[\epsilon_j^n]<\mathbb{E}[\epsilon_{j+1}^{n+1}]-\mathbb{E}[\epsilon_{j+1}^n]$. □

Lemma B2. *If the inequalities* $((1-F(x))/f(x))'<0$ *and* $lim_{x\to\underline{\beta}}(F(x)/f(x))<\infty$ *hold, then* $\mathbb{E}[\epsilon_j^{n+1}]-\mathbb{E}[\epsilon_j^n]<\mathbb{E}[\epsilon_j^n]-\mathbb{E}[\epsilon_j^{n-1}]$ *for any* $1\le j\le L$.

Proof. From (B1), it is not hard to verify that $\mathbb{E}[\epsilon_j^{n+1}]-\mathbb{E}[\epsilon_j^n]=\int_{\underline{\beta}}^{\overline{\beta}}\frac{n!}{(j-1)!(n+1-j)!}(1-F(x))^{n+1}(\frac{F(x)}{1-F(x)})^{n+1-j}dx$. Next, we show that

$$\mathbb{E}[\epsilon_j^{n+1}]-\mathbb{E}[\epsilon_j^n]<\mathbb{E}[\epsilon_j^n]-\mathbb{E}[\epsilon_j^{n-1}]. \tag{B3}$$

From (B1), (B3) can be achieved by $\int_{\underline{\beta}}^{\overline{\beta}}(1-F(x))^{j}F(x)^{n-j}dx>\int_{\underline{\beta}}^{\overline{\beta}}\frac{n}{n+1-j}(1-F(x))^{j}F(x)^{n+1-j}dx$. It is straightforward to verify that

$$\int_{\underline{\beta}}^{\overline{\beta}}(1-F(x))^{j}F(x)^{n-j}dx=\int_{\underline{\beta}}^{\overline{\beta}}(1-F(x))^{j+1}F(x)^{n-j}dx+\int_{\underline{\beta}}^{\overline{\beta}}(1-F(x))^{j}F(x)^{n+1-j}dx.$$

Thus, (B3) can be achieved by $\int_{\underline{\beta}}^{\overline{\beta}}(1-F(x))^{j+1}F(x)^{n-j}dx>\int_{\underline{\beta}}^{\overline{\beta}}\frac{j-1}{n+1-j}(1-F(x))^{j}F(x)^{n+1-j}dx$. If $((1-F(x))/f(x))'<0$ and $lim_{x\to\underline{\beta}}(F(x)/f(x))<\infty$, from (B2), we have $\mathbb{E}[\epsilon_j^{n+1}]-\mathbb{E}[\epsilon_j^n]<\mathbb{E}[\epsilon_j^n]-\mathbb{E}[\epsilon_j^{n-1}]$. □

Lemma B3. *If the inequality* $(log(f(x)))''\le 0$ *holds, then* $(n+1)\mathbb{E}[f(\epsilon_j^n)]$ *increases in n. Furthermore,* $(n+1)\mathbb{E}[f(\epsilon_j^n)]-n\mathbb{E}[f(\epsilon_j^{n-1})]<n\mathbb{E}[f(\epsilon_j^{n-1})]-(n-1)\mathbb{E}[f(\epsilon_j^{n-2})]$ *for any* $1\le j\le L$.

Proof. Similar to (B1), we have

$$(n+1)\mathbb{E}\Big[f(\epsilon_j^n)\Big]-n\mathbb{E}\Big[f(\epsilon_j^{n-1})\Big]$$

$$=\int_{\underline{\beta}}^{\overline{\beta}}\frac{n!}{(j-1)!(n-j)!}(1-F(x))^{j+1}F(x)^{n-j}(f(x)/(1-F(x)))'dx. \tag{B4}$$

Then, if $(log(f(x)))''\le 0$, which implies $(f(x)/(1-F(x)))'>0$, we have $(n+1)\mathbb{E}[f(\epsilon_j^n)]>n\mathbb{E}[f(\epsilon_j^{n-1})]$. Next, we show that

$$(n+1)\mathbb{E}\Big[f(\epsilon_j^n)\Big]-n\mathbb{E}\Big[f(\epsilon_j^{n-1})\Big]<n\mathbb{E}\Big[f(\epsilon_j^{n-1})\Big]-(n-1)\mathbb{E}\Big[f(\epsilon_j^{n-2})\Big]. \tag{B5}$$

From (B4), (B5) equals to verify that $\int_{\underline{\beta}}^{\overline{\beta}} \frac{j}{n-j}(1-F(x))^{j+1}F(x)^{n-j}(f(x)/(1-F(x)))'dx < \int_{\underline{\beta}}^{\overline{\beta}} (1-F(x))^{j+2}F(x)^{n-1-j}(f(x)/(1-F(x)))'dx$. It is straightforward to verify that

$$\begin{aligned}
&\int_{\underline{\beta}}^{\overline{\beta}} (1-F(x))^{j+2}F(x)^{n-1-j}(f(x)/(1-F(x)))'dx \\
&\quad = \frac{1}{n-j}(1-F(x))^{j+2}F(x)^{n-j}(f(x)/(1-F(x)))' \Big/ f(x)|_{\underline{\beta}}^{\overline{\beta}} \\
&\quad + \int_{\underline{\beta}}^{\overline{\beta}} \frac{j}{n-j}(1-F(x))^{j+1}F(x)^{n-j}(f(x)/(1-F(x)))'dx \\
&\quad - \int_{\underline{\beta}}^{\overline{\beta}} \frac{1}{n-j}(1-F(x))^{j}F(x)^{n-j}\Big((f(x)/(1-F(x)))'(1-F(x))^2/f(x)\Big)'dx,
\end{aligned}$$

and

$$\begin{aligned}
\Big((f(x)/(1-F(x)))'(1-F(x))^2/f(x)\Big)' &= \Big((f'(x)(1-F(x))+f(x)^2)\Big/ f(x)\Big)' \\
&= \big(f'(x)/f(x)\big)'(1-F(x)) - f'(x) + f'(x) \\
&= \big(f'(x)/f(x)\big)'(1-F(x)).
\end{aligned}$$

It is not hard to verify that $\frac{1}{n-j}(1-F(x))^{j+2}F(x)^{n-j}(f(x)/(1-F(x)))'\frac{1}{f(x)}|_{\underline{\beta}}^{\overline{\beta}} = 0$. Thus, if $(f'(x)/f(x))' < 0$, then $(n+1)\mathbb{E}[f(\epsilon_j^n)] - n\mathbb{E}[f(\epsilon_j^{n-1})] < n\mathbb{E}[f(\epsilon_j^{n-1})] - (n-1)\mathbb{E}[f(\epsilon_j^{n-2})]$. □

Lemma B4. *Letting k is a fixed positive integer, $j = n-k+1$, then $-\mathbb{E}[\epsilon_j^{n+1}]$ increases in n. Furthermore, if the inequalities $(log(f(x)))'' \leq 0$ and $lim_{x\to\overline{\beta}}((1-F(x))/f(x)) < \infty$ hold, then $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] > \mathbb{E}[\epsilon_j^n] - \mathbb{E}[\epsilon_j^{n-1}]$ for any $1 \leq j \leq L$.*

Proof. Letting $j = n-k+1$, similar to (B1), we obtain

$$\mathbb{E}[\epsilon_j^{n+1}] = \mathbb{E}[\epsilon_j^n] - \int_{\underline{\beta}}^{\overline{\beta}} \frac{n!}{(n-k)!k!}(1-F(x))^{n-k}F(x)^{k+1}dx. \tag{B6}$$

Then, we have $\mathbb{E}[\epsilon_j^{n+1}] < \mathbb{E}[\epsilon_j^n]$. Next, we show that

$$\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] > \mathbb{E}[\epsilon_j^n] - \mathbb{E}[\epsilon_j^{n-1}]. \tag{B7}$$

From (B6), (B7) equals to $\int_{\underline{\beta}}^{\overline{\beta}} F(x)^{k+1}(1-F(x))^{n-k}dx < \int_{\underline{\beta}}^{\overline{\beta}} \frac{n-k}{k}F(x)^{k+2}(1-F(x))^{n-1-k}dx$. It is straightforward to verify that

$$\begin{aligned}\int_{\underline{\beta}}^{\overline{\beta}} F(x)^{k+1}(1-F(x))^{n-k}dx = &\frac{1}{k+1}F(x)^{k+1}(1-F(x))^{n-k}(F(x)/f(x))|_{\underline{\beta}}^{\overline{\beta}}\\ &+\int_{\underline{\beta}}^{\overline{\beta}} \frac{n-k}{k+1}F(x)^{k+2}(1-F(x))^{n-1-k}dx\\ &-\int_{\underline{\beta}}^{\overline{\beta}} \frac{1}{k+1}F(x)^{k+1}(1-F(x))^{n-k}(F(x)/f(x))'dx.\end{aligned}$$

If $f(x)$ is log-concave, then $F(x)$ is log-concave (see Bagnoli and Bergstrom, 2005, Theorem 1), that is, $(F(x)/f(x))' > 0$. Thus, if $(log(f(x)))'' \leq 0$ and $\lim_{x\to\overline{\beta}}((1-F(x))/f(x)) < \infty$, then $\int_{\underline{\beta}}^{\overline{\beta}} F(x)^{n-k}(1-F(x))^{k+1}dx < \int_{\underline{\beta}}^{\overline{\beta}} \frac{n-k}{k+1}F(x)^{k+2}(1-F(x))^{n-1-k}dx < \int_{\underline{\beta}}^{\overline{\beta}} \frac{n-k}{k}F(x)^{k+2}(1-F(x))^{n-1-k}dx$. As a result, we have $\mathbb{E}[\epsilon_j^{n+1}] - \mathbb{E}[\epsilon_j^n] > \mathbb{E}[\epsilon_j^n] - \mathbb{E}[\epsilon_j^{n-1}]$. □

Appendix C: Some explanation for *K*(*n*)

Based on the assumption that $K(n)$ is a concavely increasing function and valued as an integer, $K(n)$ is more likely to be piecewise linear and constant. Since in Section 3 we consider the case that $K(n)$ is fixed, we only need to consider that the case $K(n)$ is linear in Section 4. Combining Sections 3 and 4, the EMQ in the general $K(n)$ (i.e., Fig. C1) at most concavely increases with n, which implies that a restricted-entry policy is optimal if Π_{EMQ} is preferred and the entry cost is considered.

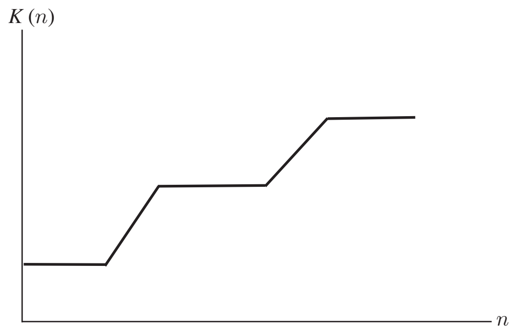

Fig. C1. The general $K(n)$.