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Prepositioning network design for humanitarian relief purposes under correlated demand uncertainty

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ABSTRACT

Prepositioning relief network is an effective strategy to mitigate the impact of natural disasters and public health emergencies, such as the COVID-19 pandemic. However, designing a proper network is challenging due to limited information and, more importantly, the correlated demand uncertainty that exists among affected areas. We consider a network design problem for humanitarian relief purposes, where demand correlations exist and demand information is limited, i.e., only the mean demand and covariance matrix are known. Note that the covariance matrix can explicitly capture the correlated demand among areas. We formulate this problem as a mixed-integer two-stage distributionally robust location-inventory model, which is generally NP-hard and computationally intractable. The model is further reformulated as a mixed-integer conic problem based on copositive cones, and it is tractable with positive semidefinite relaxation. To accelerate the problem-solving process, we design an interpretable branching-and-pricing heuristic with a warm start. Both semi-case study and simulation results demonstrate that explicitly modelling demand correlation can decrease unmet demand.

1. Introduction

Natural disasters and public health emergency, such as COVID-19, have always existed in the world. Losses caused by these events are immense, though our society has developed advanced response mechanisms to disasters. Every year, approximately 460 reported dramatic natural disasters, such as earthquakes, volcanoes, landslides, famines, droughts, hurricanes, etc., cause approximately 60,000 deaths worldwide, responsible for 0.1% of global deaths, and lead to 0.05% to 5% economic losses in relation to the gross domestic product by different countries (Ritchie & Roser, 2014). Besides natural disasters, global health emergencies, such as SARS, Ebola, and COVID-19, also caused millions of deaths and dramatically changed our lives. On Jan 02, 2023, the WHO COVID-19 dashboard shows that there were 651,918,402 confirmed cases of COVID-19 and 6,656,601 deaths caused by the virus.

Given the negative influence that natural disasters and public health emergencies can impose on mankind and society, experts from different areas have been striving to alleviate suffering in terms of both lives and economics through various aspects. To better understand the disaster process and harness disasters, Gupta et al. (2016) generally categorized disaster time phases into three phases: before the disaster, during the disaster, and after the disaster. However, many types of disasters only possess two phases: before (preparedness phase) and after (response phase), such as earthquakes. This is reasonable since the devastating strike of an earthquake usually lasts only minutes, during which efficient emergency service and humanitarian support cannot effectively operate. On the other hand, though some disasters, for example pandemics, may last months or years, many operational services can still be divided into preparedness and response phase, such as building makeshift hospitals or temporary PCR swab test centres. Therefore, we restrict our attention to disasters with two phases.

Relief network design is a classic but essential approach to encountering the damage. Earlier literature (Tufekci & Wallace, 1998) indicated that regarding the preparedness phase and the response phase as two separate parts would cause suboptimality. To design a reliable relief network, a two-stage location-inventory model taking into account both the preparedness phase and response phase is necessary. Specifically, decision makers are required to determine where to establish warehouses from a set of predetermined potential locations and then preposition appropriate relief materials correspondingly in the preparedness phase. The objective during the preparedness phase is to minimize the network setup cost plus expected future economic loss due to relief material shortages. In the response phase, after a disaster occurs and relief material demand realizes, the decision maker has to address a resource allocation problem, that is how to allocate the deployed relief materials to fulfil the demand as much as possible. Note that each warehouse can serve several different demand areas within its service region, and a demand area can be served by several different warehouses.

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The main challenge of designing a reliable relief network lies in the demand uncertainty posed by the disaster. Typically, demand independence among different areas is assumed to make optimization models more tractable. However, due to the inherent nature of disasters (earthquakes or pandemics), demand among areas is (possibly) positively correlated. This means that the higher the realized demand from one area is, the higher the potential demand from another area will be. A model without explicitly considering this unique feature instead of assuming spatial independence would yield an overly "optimistic" network. The meaning of optimistic is meant to emphasize that the resulting network possibly holds fewer relief materials. As a result, the shortage of relief materials would be massive after the occurrence of a disaster.

To tackle the correlated uncertainty of demand, two main approaches are commonly employed: stochastic programming and distributionally robust optimization (DRO). In stochastic programming, the correlated uncertain demand is represented as random variables with a known joint probability distribution. However, as disasters are rarely occuring events, the joint distribution of demand is hard to estimate from historical data due to the scarcity of data; thus, stochastic programming is not suitable for our scenario. Instead, we adopt a DRO technique to address the uncertainty of demand. DRO is a widely utilized technique in decision making problems under uncertainty. The aim of DRO is to minimize the worst-case expected total cost over a predetermined set of feasible demand distributions, so that decisions are robust enough to withstand extreme demand distributions. The set of feasible distributions, referred as ambiguity set, can reflect decision maker's risk attitude. And using different ambiguity sets will result in different relief networks that has better performance in various aspects.

Specifically, we adopt moments-based DRO, in which the ambiguity set is characterized by the moments of demand, e.g., the mean vector and covariance matrix. The adoption of moments-based ambiguity set with mean vector and covariance matrix is motivated by three distinguishing features of the studied problem. First of all, the demands are highly correlated in a disaster, such as an earthquake. Non-diagonal elements of a covariance matrix can capture such relationships correctly. Second, the historical data scarcity. Compared to construct a reference distribution from historical data for the discrepancy-based ambiguity set (e.g., the Wasserstein ambiguity set), estimating mean and covariance from historical data may be more reliable. Furthermore, the discrepancy-based ambiguity set usually includes some hyper-parameters, such as the radius of the Wasserstein ball in the Wasserstein ambiguity set, which is difficult to determine its value via cross-validation when historical data is limited. Third, from a tractability perspective, because of the adaptive nature of the second-stage allocation problem, it is unclear whether discrepancy-based ambiguity set leads to a tractable two-stage optimization problem without linear decision rules. In contrast, we will see later that our model with moments-based ambiguity set admits a conic reformulation, and is tractable without imposing decision rules.

Another advantage of our proposed DRO model is that the designed network is robust to demand uncertainty. The robustness to uncertainty ensures the ability to deal with various demand situations, mitigating the severity of worst-case scenarios, and maintaining an acceptable level of demand fulfilment rate. In fact, the objective to fulfil demand as much as possible follows the resilience definition in a recent review about supply chain resilience, which is quoted here "Qualitatively, we can define resilience as the ability of a supply chain to achieve a "good" match across a range of scenarios. The wider the range of scenarios under which a supply chain can continue to meet demand (fully or partially) in a timely, cost-effective, and equitable manner, the more resilient it is". Meanwhile, just as (Simchi-Levi et al., 2018) pointed out: holding inventory is a strategy to increase resilience. They noted: "we focus on two popular risk mitigation strategies: holding finished goods inventory and employing process flexibility. While it is clear that both strategies improve supply chain resilience". In our problem, the two-stage distributionally robust optimization model is devised to hold inventory to increase resilience.

In this paper, we address a prepositioning network design problem for humanitarian relief purposes under correlated demand uncertainty. First, we develop a model, explicitly considering the correlation among demand, to jointly address decisions in the preparedness and response phases. Then, we reformulate the model into a conic programming problem and obtain an exact mixed-integer formulation. Finally, we demonstrate the effectiveness of explicitly modelling correlations by conducting simulations on both synthetic datasets and a semi-case study on earthquakes. Therefore, our main contributions can be summarized as follows:

1. We introduce correlated demand uncertainties to relief network design problems by incorporating the demand covariance matrix. This problem is formulated as a mixed-integer two-stage distributionally robust optimization model (2).
2. We reformulate the two-stage model (2) as a mixed-integer conic programming problem (see Proposition 2).
3. We propose an interpretable branching method with a warm start to accelerate the branch-and-pricing solving process, thereby reducing the computational time by half (see Algorithm 1).
4. Numerical experiments based on both synthetic data and semi-case study demonstrate that modelling demand correlation is able to provide a more reliable relief network without extra costs.

Additionally, we summarize settings that we follow in developing models. Both settings are widely adopted in literature (see Ni et al. 2018).

1. Warehouses do not suffer from disruptions caused by disasters, as well as relief materials.
2. Roads through which relief materials are delivered do not become inaccessible and have unlimited transportation capacity.

The remainder of this paper is organized as follows. We first present a literature review of disaster relief network design with demand uncertainties in Section 2. We formally propose our model in Section 3, where we also propose the reformulated model and an algorithm to accelerate the solution process. In Section 4, we conduct numerous numerical studies to illustrate the advantages of the proposed models. In Section 5, several extensions are proposed to demonstrate our model's applicability to more complex situations. Finally, in Section 6, we conclude our study. All proofs are deferred to Appendix.

Notations. Throughout this paper, we use lowercase ***bold*** and UPPERCASE ***bold*** characters to denote vectors and matrices, respectively. Corresponding normal characters denote component-wise elements. Variables with tilde symbols, such as $\tilde{\boldsymbol{d}}$, represent random variables. We use $\boldsymbol{A} \succeq \boldsymbol{0}$ to indicate that matrix $\boldsymbol{A}$ is positive semidefinite. $\boldsymbol{A} \succeq_{co} \boldsymbol{0}$ and $\boldsymbol{A} \succeq_{cp} \boldsymbol{0}$ require that $\boldsymbol{A}$ belongs to a copositive or completely positive cone with proper dimensions. An n-dimensional nonnegative real space is denoted as $\mathbb{R}^n_+$, and $[n]$ is used to represent the running set $\{1, 2, 3, \ldots, n\}$. We use $\mathbb{E}_{\mathbb{P}}[\cdot]$ is the expectation with respect to distribution $\mathbb{P}$. We use $\tilde{\boldsymbol{d}} \sim \mathbb{P}$ to indicate that the random variable $\tilde{\boldsymbol{d}}$ follows distribution $\mathbb{P}$. And $\mathcal{P}(\mathbb{R}^n_+)$ defines a distribution family consisting of all distributions on an n-dimensional nonnegative real space.

2. Literature review

2.1. Disaster relief network design under demand uncertainty

The study of disaster relief network design started in 1958. It was first proposed by Baumol and Wolfe (1958), which describes a heuristic for a warehouse location model under a single-sourcing setting. Thereafter, a large number of variants that considered uncertainties were proposed to address more complex situations. Since a broad strand of literature on disaster relief network design exists and we review only related and representative works here, interested readers are encouraged to read (Behl & Dutta, 2019) for a comprehensive review.

A classic approach to model the demand uncertainty is by assuming the probability distributions or some senarios are known to the decision maker. For example, Tamura et al. (2000) constructed a probability tree to model disaster risks. Dodo et al. (2007) used a set of 47 earthquake scenarios to help make the infrastructure development decisions. Zhang et al. (2019) considered secondary disasters with the conditional probability based scenario tree. Given the computational tractability of scenario-based models, many disaster features have been embedded, such as helicopter operations (Barbarosoğlu et al., 2002), the loss of part of the prepositioned items (Pradhananga et al., 2016), the occurrence of disruptions of roads (Vitoriano et al., 2011), the disaster propagation processes (Klibi et al., 2018), the damages to disaster response facilities and population centres (Verma & Gaukler, 2015) and the impact of the risk measure (Noyan, 2012). Hoyos et al. (2015) conducted a literature survey on Operations Research (OR) models with stochastic components in disaster management, to which interested readers are referred.

All the models above can be included in a class of stochastic programming problems. However, this kind of stochastic programming problem is widely accused of assuming a known but possibly wrong distribution, especially in the field of disaster management. In contrast to stochastic models, another widely used tool is robust optimization in which only the support set information is known. For example, Ben-Tal et al. (2011) adopted a min–max criterion to dynamically assign emergency materials in time-dependent uncertainty scenarios. Najafi et al. (2013) designed a robust earthquake response mechanism by assuming that information about the transportation network is easily accessible. Ni et al. (2018) proposed a min–max robust model to jointly choose locations and inventory levels. Rezaei-Malek et al. (2016) designed a robust disaster relief network with perishable commodities. Yahyaei and Bozorgi-Amiri (2019) considered relief logistics network design under interval uncertainty and the risk of facility disruption. While a robust disaster relief network can encounter extreme cases, it is conservative and performs with lower satisfaction in terms of average metric in general.

Therefore, distributionally robust optimization (DRO) is adopted to mitigate both drawbacks. As an emerging modelling tool, DRO has been widely utilized to address humanitarian network design problems with uncertainties. For example, Liu et al. (2019) presented a two-stage model to address emergency medical service station location and sizing problems. Although they also consider the moment information, they define their ambiguity set with an ellipsoidal set proposed by Delage and Ye (2010), which simplifies the model to a conic programming. Wang and Chen (2020) considered a blood supply network with disasters, utilizing distributionally robust optimization in which the ambiguity set is characterized by moments information. They propose an approximation to transform their model into a semidefinite programming problem, but fail to give an exact reformulation. A very similar disaster relief network design problem to ours is considered in Nakao et al. (2017). Instead of involving cross-moment information, they consider only marginal moment information, i.e., mean and variance. Moreover, they use an affine approximation to approximately solve the model. Another concurrent work (Li et al., 2022) utilizes the recently developed satisficing measure to quantify demand shortages, in addition to equity consideration. However, they model the demand uncertainty with mean and absolute deviation, failing to explicitly characterize the potential demand correlation.

2.2. Moment-based distributionally robust optimization with conic programming

Moment-based distributionally robust optimization is a robust version of stochastic programming, where the objective function is formulated with respect to the worst-case expected value over the choice of a distribution that is subject to an ambiguity set constrained by a known support set and moment information. The past decade witnessed explosive growth in research on this topic after Bertsimas and Popescu (2005) connected the moment problem and semidefinite optimization (see Mishra et al., 2012 and Bertsimas et al., 2019). In practical problems, the support set should be restricted to a nonnegative orthant, i.e., demand should always be nonnegative. Natarajan et al. (2011) showed that, given the non-negative support set, mean value, and covariance matrix, computing the worst-case expected value of a problem with binary decision variables and uncertainties in the objective function can be reformulated as a conic problem based on completely positive cones. This reformulation sparked extensive applications based on a completely positive cone or its dual cone, i.e., the copositive cone. We focus on applications of both cones here, and defer the technical introduction to Appendix.

A conic programming problem where the cone is restricted to copositive cones or completely positive cones is called copositive programming or completely positive programming, respectively. Recently, copositive programming has attracted considerable attention in the optimization community as a powerful modelling technique. Many NP-hard problems have been reformulated via copositive programming or completely positive programming, although the difficulty of optimization is not reduced. In recent years, researchers have found that many operation management problems can be properly handled via copositive programming and its approximation techniques. For example, Kong et al. (2013) solved a healthcare appointment-scheduling problem, where only the mean and covariance of service duration are known. They formulated the problem as a convex copositive optimization problem with a tractable semidefinite relaxation. Li et al. (2014) studied a sequencing problem with random costs. Utilizing copositive programming, Yan et al. (2018) designed a roving team deployment plan for Singapore Changi Airport. Kong et al. (2020) further incorporated patients' no-show behaviour into a healthcare appointment scheduling problem and reformulated the problem as a copositive programme. However, we found little literature of modelling disaster relief network design problems with copositive cones or completely positive cones.

3. Humanitarian relief network design problem

3.1. The model

We consider a relief network design problem with limited demand information, i.e., only the mean $\boldsymbol{\mu}$, the second moment matrix $\boldsymbol{\Sigma}$, and the support set $\mathbb{R}_+$ are known. Suppose there is a set of potential supply nodes denoted by $\mathcal{W} = \{1, 2, \dots, m\}$ and a set of demand nodes denoted by $\mathcal{R} = \{1, 2, \dots, n\}$. The set of potential roads is known and is denoted by $\mathcal{E}$. We assume $\mathcal{E}$ defines a bipartite graph without isolated nodes to rule out absurd cases.

We model the network design problem as a two-stage problem. In the first stage, the decision maker designs the accessible network $\mathcal{E}(\boldsymbol{z})$ by selecting a subset of potential supply nodes to build warehouses and deploying an appropriate number of relief packages. Mathematically, we define the binary decision variable $\boldsymbol{z} = (z_1, z_2 \dots, z_m)$ to indicate whether location i is selected ($z_i = 1$) or not ($z_i = 0$). Building a warehouse at location i entails a fixed finite setup cost f_i, and let $\boldsymbol{f} = (f_1, f_2, \dots, f_m)$. We temporarily assume warehouses have no capacity limit, which will be relaxed in Section 5.2. We define another decision variable $\boldsymbol{y} = (y_1, y_2, \dots, y_m)$ to represent the number of deployed relief package. Here, we assume only one kind of items, i.e., the relief package. A multi-item case will be discussed in Section 5.1. Each deployed package at supply node i entails a fixed finite holding cost h_i, and we denote $\boldsymbol{h} = (h_1, h_2, \ldots, h_m)$. Therefore, the total cost in the first-stage is $\boldsymbol{f}^\top \boldsymbol{z} + \boldsymbol{h}^\top \boldsymbol{y}$.

In the second stage, after demand $\tilde{\boldsymbol{d}} = (\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n)$ is realized, the decision maker can allocate predeployed items through network $\mathcal{E}$ to satisfy as much of the demand as possible (or equivalently to minimize unmet demand). Each unit of unmet demand will trigger a penalty cost p, which is same across all demand nodes since human lives are equally important regardless of where she or he resides. For simplicity, we consider a normalized $p = 1$ in following analysis. We ignore the transshipment cost in the second stage. This is reasonable in our case. First, the penalty cost caused by unmet demand is far larger than the transshipment cost due to the importance of human lives. As long as there is a demand, transshipment should be conducted to fulfil the demand and save lives. Second, compared with the long-term prepositioning cost in the preparedness stage, the impact caused by the transshipment cost on the here-and-now decisions could be ignored. Therefore, we do not take transshipment into consideration. For any realized demand $\boldsymbol{d}$, and first-stage decisions $(\boldsymbol{z}, \boldsymbol{y})$, the second-stage cost can be obtained by solving a linear optimization problem as below,

$$g(\boldsymbol{y}, \boldsymbol{d}) = \min_{t \in \Omega(\boldsymbol{y}, \boldsymbol{d})} \sum_{j \in [n]} d_j - \sum_{(i,j) \in \mathcal{E}} t_{ij}, \tag{1}$$

where

$$\Omega(\boldsymbol{y}, \boldsymbol{d}) = \left\{ \boldsymbol{t} \in \mathbb{R}_+^{|\mathcal{E}|} | \sum_{i:(i,j) \in \mathcal{E}} t_{ij} \le d_j, \forall j \in [n]; \sum_{j:(i,j) \in \mathcal{E}} t_{ij} \le y_i, \forall i \in [m] \right\}.$$

The first constraint of the feasible region $\Omega(\boldsymbol{y})$ ensures that the total number of relief packages delivered to demand node j does not exceed the realized demand, and the second constraint is a capacity constraint for each warehouse i.

The ultimate objective is to minimize the total cost occurred in both stages. Since the decision maker has only limited information about demand, i.e., the finite first moment $\boldsymbol{\mu}$, the finite second moment $\boldsymbol{\Sigma}$, and the support set $\mathbb{R}_+^n$, we consider a distributionally robust optimization model. The model can be expressed as follow:

$$\begin{aligned} \min_{\boldsymbol{y}, \boldsymbol{z}} \ & \kappa(\boldsymbol{f}^\top \boldsymbol{z} + \boldsymbol{h}^\top \boldsymbol{y}) + \sup_{\mathbb{P} \in \mathcal{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma})} \mathbb{E}_{\mathbb{P}} \left[g(\boldsymbol{y}, \tilde{\boldsymbol{d}}) \right] \\ \text{s.t.} \ & \boldsymbol{y} \le U \boldsymbol{z}; \\ & \boldsymbol{y} \in \mathbb{R}_+^m, \boldsymbol{z} \in \{0, 1\}^m, \end{aligned} \tag{2}$$

where $\kappa \ge 0$ is a risk attitude hyperparameter that balances costs in two stages, and U is a sufficiently large positive number. Decision variables $\boldsymbol{y}$ and $\boldsymbol{z}$ are the first-stage decision variables, representing the inventory levels and indicators of building warehouse decisions, respectively. The first constraint guarantees that relief packages can be stored only at locations where a warehouse is established; the remaining constraints are standard nonnegative and binary constraints. The distribution ambiguity set $\mathcal{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ is

$$\mathcal{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \left\{ \mathbb{P} \in \mathcal{P}\left(\mathbb{R}_+^n\right) \middle| \begin{array}{l} \tilde{\boldsymbol{d}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\mathbf{d}}] = \boldsymbol{\mu} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{d}}\tilde{\boldsymbol{d}}^\top] = \boldsymbol{\Sigma} \end{array} \right\},$$

which includes all distributions that have support set $\mathbb{R}_+^n$, mean $\boldsymbol{\mu}$, and covariance matrix $\boldsymbol{\Sigma}$.

Solving Model (2) can generate the desired network. However, we remark that solving the problem is challenging.

- First, evaluating the second-stage worst-case expected penalty cost is challenging (see Proposition 1).
- Second, the problem is a mixed-integer problem, solving which generally is computationally inefficient.

Proposition 1 (*NP-hardness*)**.** *Given an inventory planning $\boldsymbol{y}$, solving the worst-case second-stage problem*

$$L(\boldsymbol{y}) := \sup_{\mathbb{P} \in \mathcal{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma})} \mathbb{E}_{\mathbb{P}} \left[g(\boldsymbol{y}, \tilde{\boldsymbol{d}}) \right] \tag{3}$$

is NP-hard.

The proof exploits the equivalence between optimization problems and separation problems (Bertsimas et al., 2010), and many existing literature have observed similar difficulty in evaluating the worst-case expected cost (Kong et al. 2013, Yan et al. 2018). Roughly speaking, the NP-hardness is due to two main factors: (1) the non-negativity of support set $\mathbb{R}_+^n$: it further imposes a constraint on the set of feasible distributions, lifting the difficulty of describing the ambiguity set. To make it clearer, we can consider to relax the support set to $\mathbb{R}^n$, then the second-stage problem can be reformulated as a problem with positive semi-definite cones, over which optimization tasks are much easier. However, such a relaxation harms the applicability of our model since demands should always be non-negative. (2) The adaptive nature of the second-stage allocation problem. Since the second stage involves allocating deployed supplies to affected areas, the allocation decisions depend on demand realization and first-stage decisions. In other words, for each demand realization, an allocation optimization problem should be solved to get the second-stage cost, making evaluating worst-case expected cost hard. Though some decision rules, such as linear decision rule, can be applied to make models computationally tractable, it is at the cost of decision sub-optimality.

3.2. A conic reformulation

To overcome preceding two difficulties, we adopt a recently developed modelling tool based on copositive cones for moments-based DRO by Natarajan et al. (2011). The set of copositive cones is a subset of semi-definite positive cones, and has been shown to be able to characterize distributions with non-negative support set. Moreover, based on copositive cones, the worst-case expected cost of the adaptive problem (3) can be expressed in the form of expected allocation amount (or equivalent expressions from max-flow min-cut theory); therefore, there is no need to trace allocation decisions for each demand realization. These insights lead to following analysis.

Before that, we provide a brief introduction to copositive cones in Appendix, and interested readers can refer to Dickinson (2013) and Li et al. (2014) for a comprehensive understanding.

As shown in the proof of Proposition 1 by max-flow min-cut theorem (Simchi-Levi & Wei, 2015), for given $\boldsymbol{y}$ and realized $\boldsymbol{d}$, the number of unmet demand is

$$g(\boldsymbol{y}, \boldsymbol{d}) = \max_{(\boldsymbol{u}, \boldsymbol{v}) \in \mathcal{L}} \boldsymbol{d}^\top \boldsymbol{v} - \boldsymbol{y}^\top \boldsymbol{u},$$

where $\mathcal{L} = \{(\boldsymbol{u}, \boldsymbol{v}) \in \{0,1\}^{m+n} | u_i \ge v_j, \forall (i,j) \in \mathcal{E}\}$. By introducing slackness variables, we can transform $\mathcal{L}$ into an equation system,

$$\mathcal{L}^\dagger = \left\{ (\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{s}^\dagger, \boldsymbol{u}^\dagger, \boldsymbol{v}^\dagger) \in \{0,1\}^N \middle| \begin{array}{l} -u_i + v_j + s_{ij}^\dagger = 0, \forall (i,j) \in \mathcal{E} \\ u_i + u_i^\dagger = 1, \forall i \in [m] \\ v_j + v_j^\dagger = 1, \forall j \in [n] \end{array} \right\}.$$

Thus, $\mathcal{L}^\dagger$ is an equation system with $N = 2\,m + 2n + |\mathcal{E}|$ variables and $M = m + n + |\mathcal{E}|$ equality constraints.

For any $k \in [M]$, let $\boldsymbol{a}_k$ denote the coefficient vector of kth equation of $\mathcal{L}^\dagger$. In addition, for any vector $\boldsymbol{\theta}$, we use $\boldsymbol{Diag}(\boldsymbol{\theta})$ to represent a diagonal matrix with the ith diagonal element equal to θ_i. And let $\boldsymbol{E}_{n \times n}$ be an n-by-n identity matrix. Then, we are ready to present our reformulation of (2), one of the main contributions of our work, as the following proposition.

Proposition 2 (*Reformulation*)**.** *Given that the moment matrix $\begin{pmatrix} 1 & \mu^\top \\ \mu & \Sigma \end{pmatrix}$ lies in the interior of a $(1+n) \times (1+n)$-dimensional completely positive cone, problem (2) can be equivalently reformulated as the following mixed-integer conic problem,*

$$(CO) L_{CO} = \min_{\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\alpha} \boldsymbol{\beta}, \boldsymbol{\theta}, \tau, \xi, \boldsymbol{\Xi}} \boldsymbol{f}^\top \boldsymbol{y} + \boldsymbol{h}^\top \boldsymbol{z} + \sum_{k \in [M]} (b_k \alpha_k + b_k^2 \beta_k) + \tau + 2\boldsymbol{\mu}^\top \xi + \boldsymbol{\Sigma} \bullet \boldsymbol{\Xi}$$

$$
\begin{aligned}
s.t.\ & \begin{pmatrix} \tau & \xi^\top & \frac{1}{2}\left(\sum_{k\in[M]} \boldsymbol{a}_i\alpha_i - \theta\right)^\top \\ \xi & \Xi & \mathbf{0} \\ \frac{1}{2}\left(\sum_{k\in[M]} \boldsymbol{a}_k\alpha_k - \theta\right) & \mathbf{0} & \sum_{i\in[M]} \boldsymbol{a}_k\boldsymbol{a}_k^\top\beta_k + \boldsymbol{Diag}(\theta) \end{pmatrix} \\
& + \frac{1}{2}\begin{pmatrix} \mathbf{0} & \mathbf{0} & \begin{bmatrix}\mathbf{0}\\ \boldsymbol{y}\\ \mathbf{0}\end{bmatrix}^\top \\ \mathbf{0} & \mathbf{0} & \begin{bmatrix}-\boldsymbol{E}^{[n]}\\ \mathbf{0}\end{bmatrix}^\top \\ \begin{bmatrix}\mathbf{0}\\ \boldsymbol{y}\\ \mathbf{0}\end{bmatrix} & \begin{bmatrix}-\boldsymbol{E}^{[n]}\\ \mathbf{0}\end{bmatrix} & \mathbf{0} \end{pmatrix} \succeq_{co} \mathbf{0}; \\
& \boldsymbol{y} \le U\boldsymbol{z}; \\
& \boldsymbol{y} \ge \mathbf{0},\ \boldsymbol{z} \in \{0,1\}^m; \\
& \boldsymbol{\alpha} \in \mathbb{R}^M,\ \boldsymbol{\beta} \in \mathbb{R}^M,\ \boldsymbol{\theta} \in \mathbb{R}^N,\ \tau \in \mathbb{R},\ \boldsymbol{\xi} \in \mathbb{R}^n,\ \boldsymbol{\Xi} \in \mathbb{R}^{n\times n},
\end{aligned}
$$

where $\boldsymbol{y}$ and $\boldsymbol{z}$ are first-stage inventory-location decisions, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\theta}$, τ, $\boldsymbol{\xi}$, and $\boldsymbol{\Xi}$ are decision variables induced from reformulation. Specifically, three decision variables $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\theta}$ together characterize the expectation of extreme points being the optimal solution in the second allocation stage. And τ, $\boldsymbol{\xi}$, and $\boldsymbol{\Xi}$ are corresponding dual variables to three constraints in the ambiguity set $\mathcal{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.

In the above model, the first constraint is derived from dualizing $L(\boldsymbol{y})$, requiring the resulting matrix lies in a copositive cone with proper dimensions. Intuitively, it captures the worst-case structure of (3). Remaining constraints are standard big-M constraint and nonnegative constraints. Please note that although we reformulate the original problem into a conic problem, the computational complexity only shifts from finding the worst-case distribution to finding a satisfied copositive cone and is therefore not reduced. We emphasize here that applying positive semi-definite relaxation to the first constraint can lead to a solvable conic model.

3.3. Acceleration of branch-and-pricing

Although we have reformulated (2) as a convex conic problem, Model (CO) still cannot be efficiently solved because of the complexity of the copositive cone and the binary decision variable $\boldsymbol{z}$. The former issue can be addressed via positive semidefinite relaxation, while the latter issue is not trivial, and brute-force search is not suitable due to exponentially many solutions. Therefore, we adopt the classic branch-and-pricing framework with depth-first search to search for optimal solutions. To accelerate the branch-and-pricing procedure, we develop a warm-start approach that exploits known information over moments and the results from Scarf (1958) to obtain an interpretable high-quality initial solution. Additionally, we propose a heuristic to quantify the value of each location in each branch-and-pricing iteration, thus accelerating the solving process by branching on a better variable. As the only integer decision variable is $\boldsymbol{z}$, the proposed branch-and-pricing technique is only applied to $\boldsymbol{z}$.

Lemma 1 (*Scarf, 1958*). *Suppose only demand mean μ and variance σ are known in advance; then a distributionally robust newsvendor problem that maximizes the expected revenue can be formulated as the follow,*

$$\max_q -hq + \inf_{\mathbb{Q}\in\mathcal{F}(\mu,\sigma)} \mathbb{E}_{\mathbb{Q}}[p\min\{\tilde{d}, q\}],$$

where h and p are the unit holding cost and selling price for each item. The optimal quantity q^* is

$$q^* = \begin{cases} \mu + \frac{\sigma}{2}\left(\sqrt{\frac{p-h}{h}} - \sqrt{\frac{h}{p-h}}\right), & if\ \frac{\mu}{\sigma} \ge \frac{h}{p-h}; \\ 0, & otherwise. \end{cases} \tag{4}$$

In our problem, we can regard each demand node $j \in \mathcal{R}$ as an independent newsvendor facing the same problem. If the relief material of demand node j is supplied by warehouse location i, then the holding cost is h_i, and the unit revenue by fulfilling demand is $p_j = 1, \forall j \in \mathcal{R}$. Therefore, if newsvendor j decides to store inventory at location i, the optimal inventory level is q_{ij}^*:

$$q_{ij}^* = \begin{cases} \mu + \frac{\sigma}{2}\left(\sqrt{\frac{p_j-h_i}{h_i}} - \sqrt{\frac{h_i}{p_j-h_i}}\right), & if\ \frac{\mu}{\sigma} \ge \frac{h_i}{p_j-h_i}; \\ 0, & otherwise, \end{cases} \tag{5}$$

where $p_j = 1, \forall j \in \mathcal{R}$.

Claim 1 (*Warm-start Policy*). *Given the optimal inventory level q_{ij}^*, we can obtain an initial solution $\boldsymbol{z}^{init}$ by solving the following assignment problem,*

$$
\begin{aligned}
\min_{\boldsymbol{z},\boldsymbol{t}}\ & \boldsymbol{f}^\top\boldsymbol{z} + \sum_{(i,j)\in\mathcal{E}} h_i q_{ij}^* t_{ij} \\
s.t.\ & \sum_{i\in[n]} t_{ij} \ge 1,\ \forall j \in [j]; \\
& t_{ij} \le z_i,\ \forall (i,j) \in \mathcal{E}; \\
& \boldsymbol{z} \in \{0,1\}^m, \boldsymbol{t} \in \{0,1\}^r.
\end{aligned} \tag{6}
$$

Furthermore, $\boldsymbol{z}^{init}$ can provide an upper bound for problem (CO), i.e., $L_{CO}(\boldsymbol{z}^{init}) \ge L_{CO}$, where $L_{CO}(\boldsymbol{z}^{init})$ is objective value of problem (CO) by fixing the variable $\boldsymbol{z} = \boldsymbol{z}^{init}$.

In this case, we manually ignore the correlated demand uncertainty between demand nodes. The decision variables are building warehouses decision $\boldsymbol{z}$ and the assignment decision $\boldsymbol{t}$. The objective is the cost of building warehouses plus the total holding cost. The first constraint requires that each demand node should be assigned to at least one warehouse. The second constraint requires that demand node j could be assigned to warehouse i only when warehouse i is established. The remaining constraints are binary requirements. Solving the problem leads to a warm-start decision $\boldsymbol{z}^{init}$, since $\boldsymbol{z}^{init}$ represents the warehouse decisions when correlated demand uncertainty is deliberately ignored. Although the quality of the warm-start solution $\boldsymbol{z}^{init}$ is not theoretically guaranteed, we will numerically demonstrate that this solution can help reduce the computational time significantly.

In our branch-and-pricing framework, in addition to determining a high-quality initial solution, another approach to accelerating the solving process is selecting a good variable to branch by appropriately pricing the value of each location variable. We apply an interpretable heuristic to produce the next branch. Let $r = |\mathcal{E}|$ be the number of roads. Note that the decision variable $\alpha_k, \forall k \in [r]$ in Model (CO) is the dual variable to the first-r road constraints of $\mathcal{L}^\dagger$. According to duality theory, economically, the solution $\alpha_k^*, \forall k \in [r]$ can be interpreted as the value of the corresponding road in the current iteration. Therefore, in each branch-and-pricing iteration, we can approximately evaluate the value of each location as the aggregated value of all roads that originate from it. Then we select a location with the highest aggregated value to branch. To describe our idea in a clearer mathematical manner, we first define an index mapping from road (i,j) to a scalar index, representing the road index in $[r]$. For example, we use subscript $_{(i,j)}, \forall (i,j) \in \mathcal{E}$ to represent road (i,j)'s index in set $[r]$. Now, we are ready to formally present our pricing policy in Claim 2.

Claim 2 (*Pricing Policy*). *During each iteration in branch-and-pricing process, after obtaining $\alpha_{(i,j)}^*, \forall (i,j) \in \mathcal{E}$ by solving positive semidefinite relaxation of Model (CO), the next warehouse location to branch is*

$$i^* = \arg\max_{i\in[m]\setminus\mathcal{I}} \sum_{j\in\Gamma(i)} \alpha_{(i,j)}^*. \tag{7}$$

where $\mathcal{I}$ is the set of warehouse locations that have been selected to branch before this iteration.

Note that $\Gamma(i)$ is the set of roads from the warehouse node i to the demand nodes. Therefore, $\sum_{j\in\Gamma(i)}\alpha^*_{(i,j)}$ can be seen as the value of the warehouse location i. Intuitively, we greedily select one warehouse with highest aggregated value that has not been visited yet.

Formally, we summarize the proposed accelerated branch-and-pricing algorithm with warm-start in Algorithm 1. We conduct further numerical experiments to illustrate the effect on reducing computational time.

Algorithm 1 Accelerated Branch-and-Pricing Algorithm

Set z^{init} by solving (6) ▷ Claim 1
Solve positive semidefinite relaxation of (CO) with z^{init} to get L^{init}
Initialize $z^* \leftarrow z^{init}$, $L^* \leftarrow L^{init}$
Initialize constraint set $\mathcal{C} := \{\mathbf{0} \le z \le \mathbf{1}\}$ and index set $\mathcal{I} =$
Run DFS($\mathcal{C}$)

function DFS($\mathcal{C}$)
  Solve linear relaxation (CO) with $\mathcal{C}$ to obtain current objective value L
  if $L < L^*$ **then**
    if z is integer **then**
      $L^* \leftarrow L$, $z^* \leftarrow z$
    else
      Select warehouse i^* according to (7) ▷ Claim 2
      $\mathcal{C}_1 \leftarrow \mathcal{C} \cup \{z_{i^*} = 1\}$
      Run DFS($\mathcal{C}_1$)
      $\mathcal{C}_0 \leftarrow \mathcal{C} \cup \{z_{i^*} = 0\}$
      Run DFS($\mathcal{C}_0$)
    end if
  end if
end function

4. Numerical study

In this section, we conduct numerous experiments to examine advantages of explicitly considering the first two moments of demand information in disaster relief network design problems. Firstly, we run our models on synthetic datasets to scrutinize the average performance and the model's robustness to initial network structures. Sensitivity analyses are conducted to examine the influences of parameters. Secondly, a semi-case study of Ya'an Earthquake that happened in 2013 is examined to demonstrate the superiority of our proposed model in practice. A stochastic model and a mean–variance model, which will be discussed in detail in the next subsection, are introduced as benchmarks. We use positive semidefinite relaxation to solve copositive programming problems. Specifically, we use $\{\boldsymbol{A}\,|\,\boldsymbol{A} = \boldsymbol{A}_1 + \boldsymbol{A}_2, \boldsymbol{A}_1 \succeq \mathbf{0}, \boldsymbol{A}_2 \ge \mathbf{0}\}$ as an inner approximation to the copositive cone. Therefore, in what follows, the statement "solve copositive programming" actually means solving copositive programming with its relaxations. We use Mosek 9.2 to solve conic problems and Gurobi 9.0.3 to solve mixed-integer problems.

4.1. Benchmarks

We introduce a two-stage stochastic model and a mean–variance distributionally robust model as benchmarks. The former model fully exploits all demand samples, while the latter considers only two statistics, mean and variance, and does not consider correlations between demand nodes. Since our model exploits only the first two moments, the amount of available information is slightly more than that of the mean–variance model but less than that of the stochastic model.

SAA with Empirical Distribution

When demand is random with an unknown distribution $\mathbb{P}'$ supported by $\mathbb{R}^n_+$, and a set of historical observed demand containing N samples is available, the network design problem can be formulated as a two-stage stochastic model as follows:

$$
\begin{aligned}
(SAA)\ \min_{\boldsymbol{y},\boldsymbol{z}}\ & \kappa(\boldsymbol{f}^\top \boldsymbol{z} + \boldsymbol{h}^\top \boldsymbol{y}) + \frac{1}{N}\sum_{i=1}^{N} g(\boldsymbol{y}, \tilde{\boldsymbol{d}}_i) \\
\text{s.t.}\ & \boldsymbol{y} \le U\boldsymbol{z}; \\
& \boldsymbol{y} \in \mathbb{R}^m_+, \boldsymbol{z} \in \{0,1\}^m,
\end{aligned}
$$

where $g(\boldsymbol{y}, \tilde{\boldsymbol{d}}_i)$ is the objective value of recourse allocation problems, which is the same as (1) but with the ith historical demand sample $\tilde{\boldsymbol{d}}_i$. The first term of the objective function is the total setup cost, including the warehouse setup cost $\boldsymbol{f}^\top\boldsymbol{z}$ and the holding cost $\boldsymbol{h}^\top\boldsymbol{y}$. The second term represents the average cost on historical demand samples when the inventory level is $\boldsymbol{y}$, which converges to $\mathbb{E}_{\mathbb{P}'}[g(\boldsymbol{y}, \tilde{\boldsymbol{d}})]$ as N goes to infinity according to the Law of Large Number. $\boldsymbol{y}$ and $\boldsymbol{z}$ are the inventory and location decision variables, respectively. The feature that distinguishes Model (SAA) from the basic model (2) is how we evaluate the expected second-stage objective value.

Mean-Variance

The other benchmark is a distributionally robust model whose ambiguity set depends only on finite mean $\boldsymbol{\mu}$ and finite variance $\boldsymbol{\sigma}^2$. In other words, the covariance is ignored. In such a case, the decision maker attempts to evaluate the expected second-stage cost under the worst-case distribution. Formally, the model is:

$$
\begin{aligned}
(MV)\ \min_{\boldsymbol{y},\boldsymbol{z}}\ & \kappa(\boldsymbol{f}^\top \boldsymbol{z} + \boldsymbol{h}^\top \boldsymbol{y}) + \sup_{\mathbb{P}\in\mathscr{F}_{mv}(\boldsymbol{\mu},\boldsymbol{\sigma}^2)} \mathbb{E}_{\mathbb{P}}\left[g(\boldsymbol{y}, \tilde{\boldsymbol{d}})\right] \\
\text{s.t.}\ & \boldsymbol{y} \le U\boldsymbol{z}; \\
& \boldsymbol{y} \in \mathbb{R}^m_+, \boldsymbol{z} \in \{0,1\}^m,
\end{aligned}
$$

where the ambiguity set $\mathscr{F}_{mv}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2)$ is

$$
\mathscr{F}_{mv}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2) = \left\{ \mathbb{P} \in \mathcal{P}\left(\mathbb{R}^n_+\right) \left| \begin{array}{l} \tilde{\boldsymbol{d}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{d}}] = \boldsymbol{\mu} \\ \mathbb{E}_{\mathbb{P}}[(\tilde{d}_j - \mu_j)^2] = \sigma_j^2,\ \forall j \in \mathcal{R} \end{array} \right. \right\},
$$

which is assumed to be non-empty. The objective function of Model (MV) is almost the same as that of Model (2) except for the slightly different ambiguity set. Although Model (MV) is still intractable in most cases, we can transform $\mathscr{F}_{mv}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2)$ into $\mathscr{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{mv})$, where $\boldsymbol{\Sigma}_{mv} = \boldsymbol{\mu}\boldsymbol{\mu}^\top + \boldsymbol{Diag}(\boldsymbol{\sigma}^2)$. Then, analysis in Proposition 2 also holds. Replacing $\boldsymbol{\Sigma}$ in Model (CO) with $\boldsymbol{\Sigma}_{mv}$ yields the resulting conic model, which is solvable via relaxations. One noteworthy feature of Model (MV) is that as $\mathscr{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \subseteq \mathscr{F}(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{mv})$, it would be more conservative than Model (CO) is, resulting in a higher expected second-stage cost under the same $\boldsymbol{y}$ and $\boldsymbol{z}$. Nevertheless, since both Model (CO) and Model (MV) balance setup costs and second-stage penalty costs, it is hard to compare the amount of inventory levels and, further, the total cost at this point.

4.2. Experiments on synthetic datasets

4.2.1. Data generation

Consider a network with m potential warehouses and n demand nodes. We randomly generate dozens or hundreds of networks, and each network is defined by a six-element tuple $(m, n, \mathcal{E}, \boldsymbol{f}, \boldsymbol{h}, \boldsymbol{\mu})$. m and n are the numbers of potential warehouse nodes and demand nodes, and $\mathcal{E}$ is a set of links. According to Ni et al. (2018), the average node degree of a typical real-world road network is approximately 2.4; to mimic reality, we therefore require $|\mathcal{E}| = 1.2(m+n)$. The following two constraints on the generated network must satisfied: (1) at least one link is connected to each supply node and (2) at least one link is connected to each demand node. These conditions help to avoid the appearance of bipartite graphs with isolated nodes. $\boldsymbol{f}$ is the fixed setup cost, which is randomly drawn from a uniform distribution U[10, 25], $\boldsymbol{h}$ is the holding cost drawn from U[0.1, 0.2], and $\boldsymbol{\mu}$ is the first moment, i.e., the mean, of demand, drawn from U[400, 600].

Table 1
Summary of the designed networks.

	total inv.	# of w.h.	# of roads	w.h. degree	d.n. degree
SAA	4903.43 (300.33)	4.06 (0.78)	11.57 (1.57)	2.91 (0.45)	1.45 (0.20)
CO	4944.17 (201.06)	3.79 (0.72)	10.96 (1.52)	2.96 (0.48)	1.37 (0.19)
MV	4638.28 (194.14)	3.77 (0.72)	11.15 (1.44)	3.02 (0.48)	1.39 (0.18)

Initially, we set the correlation coefficient between each demand node pair as $\rho = 0.3$, coefficient of variation of each demand node as $\text{cv} = 0.3$, and risk attitude coefficient $\kappa = 1$. Therefore, the standard deviation of each demand node is $\sigma_j = \mu_j \text{cv}, \forall j \in [n]$. For simplicity, we assume the correlation parameter ρ and coefficient of variation cv are applied to all demand node pairs or demand nodes, which means the covariance of any two demand nodes (i,j) is $\sigma_i\sigma_j\rho$, $\forall i \in [m], j \in [n]$. Therefore, the covariance matrix of $\tilde{\boldsymbol{d}}$ is $\boldsymbol{\Sigma}_{\rho,\text{cv}}$ whose (i,j)-position element is $\sigma_i\sigma_j\rho$. We use the subscripts $_{\rho,\text{cv}}$ to emphasize that the covariance matrix depends on ρ and cv. We randomly draw 50 demand samples from a truncated multivariate Gaussian distribution $N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\rho,\text{cv}})_+$ with domain $\mathbb{R}^n_+$, mean $\boldsymbol{\mu}$, and covariance matrix $\boldsymbol{\Sigma}_{\rho,\text{cv}}$. Based on 50 samples, we calculate the sampled mean $\hat{\boldsymbol{\mu}}$, sampled variance $\hat{\boldsymbol{\sigma}}_j$ and sampled covariance matrix $\hat{\boldsymbol{\Sigma}}_{\rho,\text{cv}}$. These sample statistics are input parameters of considered models.

After obtaining optimal solutions by solving three models, we conduct second-stage simulations on two independently generated datasets. The first dataset of 1000 demand realizations is generated from $N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\rho,\text{cv}})_+$, and the second dataset of another 1000 demand realizations is from a mixture distribution consisting of four sub-distributions, including a two-point distribution $\mathbb{P}(\tilde{\boldsymbol{d}} = \boldsymbol{\mu} - \boldsymbol{\sigma}) = \mathbb{P}(\tilde{\boldsymbol{d}} = \boldsymbol{\mu} + \boldsymbol{\sigma}) = 0.5$, element-wise independent uniform distribution $U\left[\boldsymbol{\mu} - \sqrt{3}\boldsymbol{\sigma}, \boldsymbol{\mu} + \sqrt{3}\boldsymbol{\sigma}\right]$, independent normal distribution $d_j \sim N(\mu_j, \sigma_j), \forall j \in [n]$, and log-normal distribution with mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$. The above four distributions are truncated to $\mathbb{R}^n_+$. It is easy to check that the mixture distribution ignores correlations but has a mean and standard deviation near the true $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$. We separately analyse the results for the truncated Multivariate Gaussian Distribution (MGD) and Mixture Distribution (MixD). For each network $(m, n, \mathcal{E}, \boldsymbol{f}, \boldsymbol{h}, \boldsymbol{\mu})$, we regard the average (or worst-case) performance of the two datasets as indicative of the corresponding network's performance.

4.2.2. Simulation results

Following the initial setting in the above subsection, we first restrict ourselves to the $(m,n) = (8,8)$ structure for a closer look at the model's performance. We randomly generate 200 networks with $(\rho, \text{cv}, \kappa) = (0.3, 0.3, 1.0)$. Unless stated otherwise, all analyses in this subsection rely on 200 networks: for each network, we construct a metrics based on simulation results, for example, the average unmet demand, to represent the network's performance. Then, we calculate the average over 200 networks.

First-stage Deployment

Table 1 summarizes the first-stage decisions and resulting networks. The first column, *total inv*, represents the average total inventory of the 200 networks. The numbers in parentheses show standard deviations. Clearly, Model (CO) results in the highest inventory level, while Model (MV) has the lowest inventory level. The difference results from whether the strong correlation of demand is taken into account. The second column, *# of w.h.*, reveals the number of established warehouses: on average, Model (SAA) selects almost half of potential locations to build warehouses. The third column is the number of links in designed networks: Model (CO) designs the sparsest network. The last two columns exhibit warehouse node degrees and demand node degrees in designed networks.

Table 2
Cost comparison, $(m,n) = (8,8), (\rho, \text{cv}, \kappa) = (0.3, 0.3, 1.0)$.

	MGD				MixD	
	f	**h**	**p**	total cost	**p**	total cost
SAA	67.7 (14.9)	660.2 (70.5)	64.5 (31.7)	792.5 (78.2)	70.0 (38.8)	798.0 (81.0)
CO	63.4 (13.9)	668.5 (66.5)	53.1 (12.2)	785.0 (78.5)	57.3 (15.0)	789.2 (82.0)
MV	63.3 (13.7)	636.1 (66.1)	96.1 (15.4)	795.5 (80.1)	108.7 (15.8)	808.1 (81.3)

Out-of-Sample Cost

We next investigate the cost performance, including the first-stage deployment cost and second-stage penalty due to unmet demand, which are the costs of greatest concern. Table 2 shows average costs over 200 networks under different models with MGD and MixD realizations. The numbers in parentheses represent standard deviations. The first two columns represent the setup cost $\boldsymbol{f}$ and the holding cost $\boldsymbol{h}$. And the third column $\boldsymbol{p}$ is the out-of-sample penalty cost. *Total cost* is the total cost in two stages. Clearly, Model (CO) always achieves the lowest total cost. One remarkable decrease occurs for $\boldsymbol{p}$, which decreases from approximately 64.5 (for SAA) to 53.1 (for CO), i.e., a 17.7% decrease in unmet demand. For the MixD simulation, the decrease is also considerable, from 70.0 (for SAA) to 57.3, approximately 18.4%, or from 108.7 (for MV) to 57.3, approximately 47.3%. Although the absolute decrease is marginal, the relative decrease is remarkable, which highlights the advantages of our proposed conic model: we can further substantially reduce the unmet demand even though it is already small. This managerial insight is quite attractive, especially to disaster management experts or fulfilment-oriented warehouse managers, since their priority is to reduce unmet demand without increasing expenditure.

We next scrutinize the unmet demand more deeply. Fig. 1 shows the distribution of the average unmet demand, where the horizontal axis shows the average unmet demand in simulations, and the vertical axis represents its frequency.

In the left graph, when the true demand distribution aligns with the distribution that decision makers know, Model (CO) dominates Model (MV), except in some instances of overlap at approximately 75. Although Model (CO) achieves a higher level of concentration at approximately 55, the lowest unmet demand is achieved by Model (SAA), whose performance is more scattered, entailing several worst cases at 150 levels. The right graph shows the results from the MixD simulations. Similarly, Model (CO) dominates Model (MV) most of the time. However, the most interesting change is the increased dispersion of Model (SAA). Clearly, the performance of Model (SAA) becomes unstable in different networks, while (CO) and (MV) remain concentrated near their former mean values. Furthermore, the slimmer shapes of Model (CO) in both graphs demonstrate the robustness of our proposed model to initial network graphs in terms of unmet demand.

Service Level

Another critical metric is the service level. Table 3 summarizes two kinds of service levels. The type 1 service level is an event-oriented performance criterion that measures the probability that the entire demand is completely satisfied by stock at hand. The type 2 service level is a quantity-oriented performance measure describing the proportion of fulfilled demand to total realized demand. Model (CO) performs slightly better than Model (SAA) in terms of type 2 service level. Moreover, in terms of type 1 service level, Model (CO) outperforms other models by approximately 1%.

Value-at-Risk

Furthermore, we calculate the 99.9th, 95th, 90th, 85th, and 80th quantiles of unmet demand to demonstrate the ability of the models

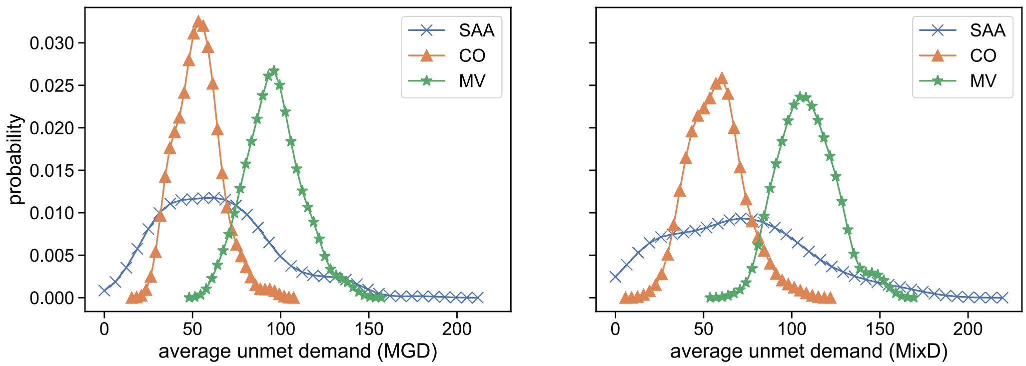
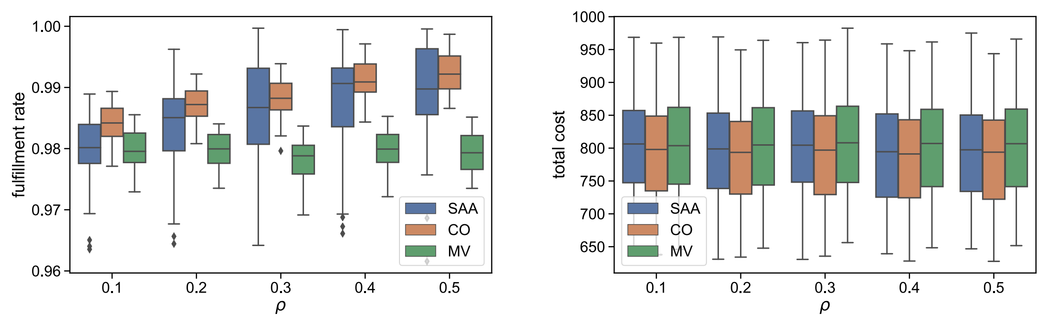
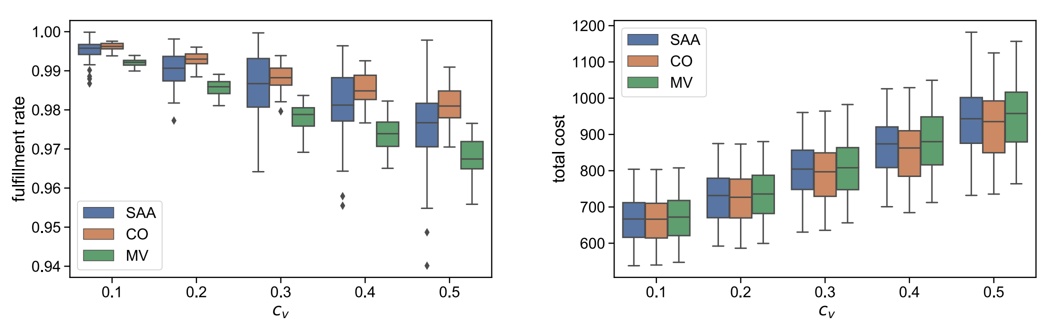


Fig. 1. Average unmet demand distribution, $(\rho, \mathrm{cv}, \kappa) = (0.3, 0.3, 1.0)$.



(a) Fulfilment Rate (b) Total Cost

Fig. 2. Sensitivity analysis of ρ.



(a) Fulfillment Rate (b) Total Cost

Fig. 3. Sensitivity analysis of cv.

Table 3
Service Level, $(m, n) = (8, 8), (\rho, \mathrm{cv}, \kappa) = (0.3, 0.3, 1.0)$.

Service level (%)	MGD		MixD	
	type1	type2	type1	type2
SAA	94.79	98.76	93.75	98.59
CO	95.61	98.99	94.32	98.85
MV	93.82	98.16	93.10	97.85

to hedge long-tail risks. When the true distribution is known to decision makers, these quantiles can be regarded as estimators of the value-at-risk (VaR) of unmet demand after implementing the corresponding first-stage solutions. Table 4 shows the VaR values under different quantiles. *MGD VaR* shows that Model (CO) dominates other models under all circumstances, which indicates that its ability to hedge long-tail risks comes from information about the true underlying distribution.

Surprisingly, columns *MixD VaR* in Table 4 further reveal that Model (CO) is sufficiently robust to the misspecification in terms of demand distributions. In terms of 95th quantile, Model (CO)'s unmet demand under MixD is roughly 20% lower than that of Model (SAA) (from 335.4 to 267.9), while the gap is only 13.6% under the MGD situation (from 406.4 to 350.9). This result verifies the advantages of Model (CO) and the distributionally robust technique, especially when historical data are limited, and misspecification in the distribution is likely.

CPU Time

Computational time is also essential in practice. Although the problem we consider is a strategy-level problem, reasonable computational time is still necessary. Therefore, we record the CPU time for solving each optimization problem, thereby highlighting the superiority of the proposed branch-and-pricing acceleration algorithm. Table 5 summarizes the time needed to solve each model. We use *(acce)* to indicate that the model is solved with the accelerated branch-and-bound technique proposed in Section 3.3. The first column, *CPU Time* (s), reports the computational time in seconds to obtain the optimal solution. The second column, *Node*, exhibits the number of visited branch-and-bound nodes before algorithm's termination. Clearly, for

Table 4
Unmet demand with various quantiles.

Unmet demand	MGD VaR					MixD VaR				
	99.9%	95%	90%	85%	80%	99.9%	95%	90%	85%	80%
SAA	1323.3	406.4	212.8	111.0	56.3	798.3	335.4	325.9	250.0	119.8
CO	1280.0	350.9	163.2	71.8	27.5	761.8	267.9	265.0	224.8	97.5
MV	1585.0	617.8	365.5	206.2	100.2	968.8	569.7	569.7	398.3	171.5

Table 5
CPU time to solve models.

	CPU time (s)	Node
SAA	0.05	–
CO	748.79	98.80
CO (acce)	337.30	44.09
MV (acce)	335.55	43.88

Model (CO), the accelerated algorithm reduces the time by more than half by visiting fewer nodes.

4.2.3. Sensitivity analysis

We modify some parameters (correlation parameter ρ, coefficient of variation cv, and risk attitude κ) to conduct sensitivity analyses. Since we have explored the metrics of interest in detail, throughout this part, we consider only 50 graphs for each parameter combination. Additionally, we focus on MixD simulations, where the true underlying distribution deviates from the distribution used in the training sample generation process. This situation is more likely to occur in reality. Results show that all the conclusions proposed earlier hold. Moreover, paired t-tests on performances between Model (CO) and other two benchmarks reject the null hypothesis with p-value ≤ 0.001, implying that the improvement of Model (CO) is statistically significant.

Correlation Parameter ρ

We first present a sensitivity analysis of the correlation parameter ρ by changing from 0.1 to 0.5 stepwisely. Fig. 2(a) shows fulfilment rates of three models under different values of parameter ρ. The fulfilment rate measures the proportion of demand that is met. Model (CO) always has a higher fulfilment rate, on average, than other models. Moreover, the perturbation of Model (CO) is much less severe than that of other models, indicating that Model (CO) is robust to the network structure. An interesting trend is that the fulfilment rate of Model (MV) keeps almost unchanged over a wide range of ρ values, while that of the other models increases. The trend is due to models' abilities to utilize correlation information. While Model (MV) does not consider any correlation between demand nodes, Model (CO) and Model (SAA) take the correlation into consideration explicitly and implicitly, respectively. Another noteworthy phenomenon is that outliers appear only in Model (SAA), reflecting its weakness in terms of hedging extreme risks. Fig. 2(b) exhibits total costs of three models under different values of ρ. Model (CO) can always achieve the lowest total cost.

Coefficient of Variation cv

Fig. 3(a) depicts fulfilment rates under a range of cv values. As cv increases, mean fulfilment rates decrease in general, and variances of fulfilment rates increase. However, among three models, Model (CO) has the slowest rate of decrease and the smallest variance, which further emphasizes the robustness of Model (CO). Fig. 3(b) shows the change in total costs in terms of cv. The total cost increases as cv increases since more warehouses and products are needed to counter the increasing uncertainty.

Risk Attitude Parameter κ

Fig. 4(a) represents the relationship between the fulfilment rate and risk attitude κ. As κ increases, i.e., the decision maker focuses more on the first-stage cost, the fulfilment rate decreases significantly. This is an expected result since a larger κ reflects a more aggressive attitude to future risk. Therefore, the corresponding prepositioning decisions become less conservative. Another noteworthy phenomenon is the diminishing performance gap between Model (CO) and Model (SAA) as κ increases. Since more attention is shifted to deployment costs, the advantage of Model (CO) in addressing demand uncertainty is gradually weakened. Therefore, Model (CO)'s superiority in terms of fulfilment rate diminishes. Fig. 4(b) expresses total costs as κ increases. Model (CO) has a slightly lower total cost, regardless of the value of κ.

In conclusion, the conic Model (CO) is demonstrated to be robust to demand uncertainty in most cases and outperforms benchmarks. Additionally, it is essential to incorporate correlation information into modelling for situations such as disaster management where demand correlation indeed exists and plays an important role. We also conduct simulations on other network structures, and leave the results and analysis to Appendix C.

4.3. Semi-case study: Ya'an earthquake

The Earthquake is one of the major natural disasters in China. After being heavily impacted by the 2008 Wenchuan Earthquake, the same area was affected by the Ya'an Earthquake, a M_s 7.0 earthquake occurred at Ya'an County in Sichuan Province, PR China, in 2013. The earthquake caused injuries of approximately 12,000 people injured and massive economic losses. The affected areas are depicted in Fig. 5(a).

In our semi-case study, we consider 12 areas that were severely attacked by the earthquake within the inner orange circle, including four cities (blue star icons) and eight towns. We assume that all 12 of the affected places are potential locations for building warehouses during our planning stage. Meanwhile, after an earthquake occurs, demand also comes from the same 12 places. After obtaining geographic information from Google Maps, we abstractly depict the affected areas as Fig. 5(b).

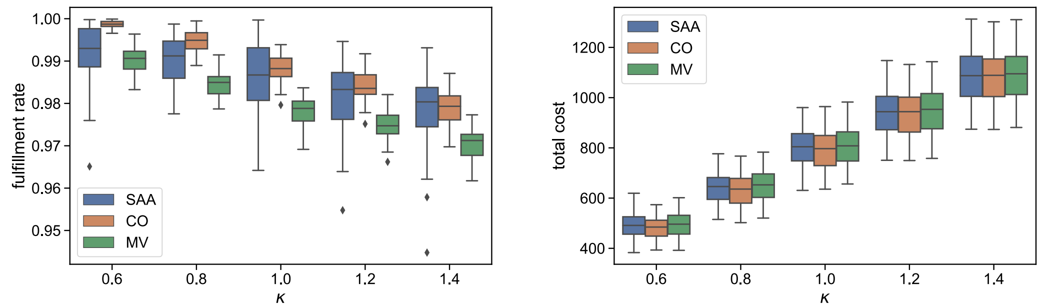
Suppose that the service radius of cities and towns are 30 kilometres and 20 kilometres, respectively. Thus, the initial network structure is

$$\mathcal{E} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

To complete the semi-case study, we collected information about considered areas from official channels and then calculated model input parameters, such as demand statistics, warehouse setup costs, and holding costs.

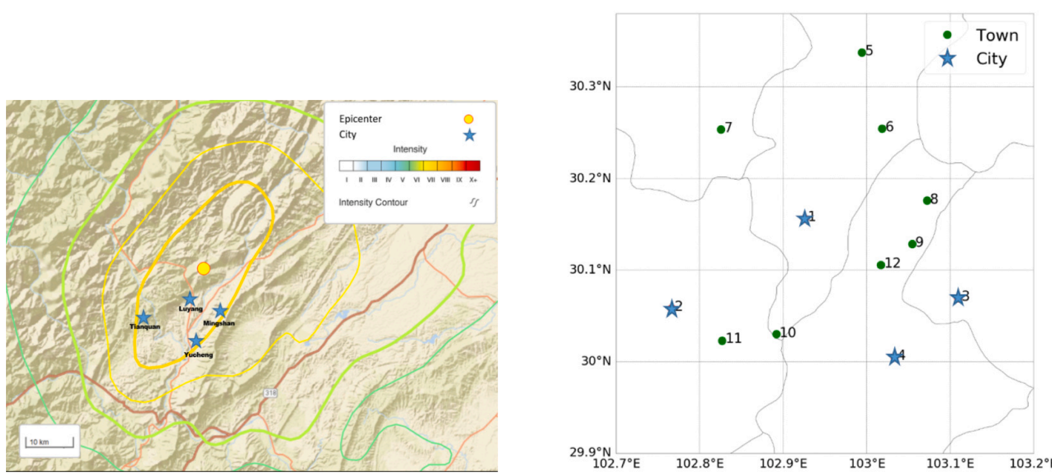
To estimate the average relief material package demand from victims, we collected the total population information from the official province-level Statistical Yearbook. Additionally, the news reported,[1]

[1] http://cpc.people.com.cn/n/2013/0508/c64387-21400852.html available on 11th, Sep, 2022.



(a) Fulfillment Rate (b) Total Cost

Fig. 4. Sensitivity analysis of κ.



(a) Areas Affected by Ya'an Earthquake (b) Areas Affected by Ya'an Earthquake

Fig. 5. Affected areas.

that, on average, 16 people lived in one relief tent. Therefore, it is reasonable to believe that each relief package can serve 16 victims. After obtaining these data, without loss of generality, we regard half of the total relief package demand as the mean demand, i.e., $\boldsymbol{\mu}$ =[980, 1231, 3992, 4068, 395, 763, 530, 394, 448, 400, 602, 326]. To estimate the setup cost, we need to consider types of building materials, price fluctuations due to earthquakes, and the extra cost of improving warehouses' resistance to earthquakes. According to the cost calculation formulation in Wang (2010), we have $f = 900 \times 120.53\% \times (1 + P) \times M$, where $P \sim U[5\%, 10\%]$ is the additional cost to improve warehouses' resistance, and $M \sim U[3000, 4500]$ is the area of a warehouse. Typically, relief warehouses are made out of brick and concrete, and the price for one square metre is 900. Furthermore, building material shortages would lead to a 20.53% rise in price. Therefore, normalized by the unit penalty cost of 100,000, the final setup cost is $\boldsymbol{f} = (51.26, 47.23, 50.47, 48.88, 42.11, 49.27, 44.48, 44.82, 42.96, 35.11, 46.76, 41.80)$. To estimate the holding cost, we first gathered a material list,[2] that recorded relief materials delivered to the affected areas within four days, as shown in Table 6.

We regard 47,000 tents as 47,000 unit packages. All prices are collected from online shopping platforms such as Taobao.com or JD.com. Thus, the holding cost for each relief package is 5418. After incorporating randomness, we obtain the normalized holding cost $\boldsymbol{h} = (0.058, 0.056, 0.056, 0.056, 0.057, 0.053, 0.056, 0.058, 0.059, 0.059, 0.057, 0.053)$. Similarly, we set coefficients of variation of demand nodes as

Table 6
Relief materials delivered to Ya'an within four days.

Materials	Quantity	Price(¥)	Shelf life(Year)
Tent	47,000	3,000	10
Quilt	199,000	399	5
Clothes	10,000	489	5
Folding Bed	10,000	2,000	10
Portable Toilet	200	3,380	5
Water-proof Cloth	700,000	3	5
Candle	100,000	1	10
Instant Noodle	70,000	48	1
Drinking Water	40,000	48	1
Emergency Lamp	900	1,350	5

$\mathrm{cv} = 0.8$, while changing correlation coefficient ρ from 0.1 to 0.9 with a step of 0.1 to examine the impact of correlation intensity. Variances of demand $\boldsymbol{\sigma}$ and covariance matrix $\boldsymbol{\Sigma}_{\rho,\mathrm{cv}}$ are calculated by the same procedure that we used in Section 4.2.1. We randomly draw 50 demand samples from a truncated multivariate Gaussian distribution $N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\rho,\mathrm{cv}})_+$ as input demand samples. Based on these samples, we solve Model (CO), Model (MV), and Model (SAA) to obtain the prepositioning network. Then 1,500 realized demand are independently drawn from a mix of distributions consisting of a multivariate Gaussian distribution $N(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\rho,\mathrm{cv}})$, a two-point distribution $\mathbb{P}(\tilde{\boldsymbol{d}} = \boldsymbol{\mu} - \boldsymbol{\sigma}) = \mathbb{P}(\tilde{\boldsymbol{d}} = \boldsymbol{\mu} + \boldsymbol{\sigma}) = 0.5$, and a truncated uniform distribution $U\left[\boldsymbol{\mu} - \sqrt{3}\boldsymbol{\sigma}, \boldsymbol{\mu} + \sqrt{3}\boldsymbol{\sigma}\right]$. We calculate their average performances and three different quantile performances, i.e., 90%, 95%, and 99%. Note that quantile performances quantify the model's robustness to hedge the worst-case situation. We exploit different data generation processes

[2] http://www.mca.gov.cn/article/zwgk/mzyw/201304/20130400448512.shtml available on May, 2020.

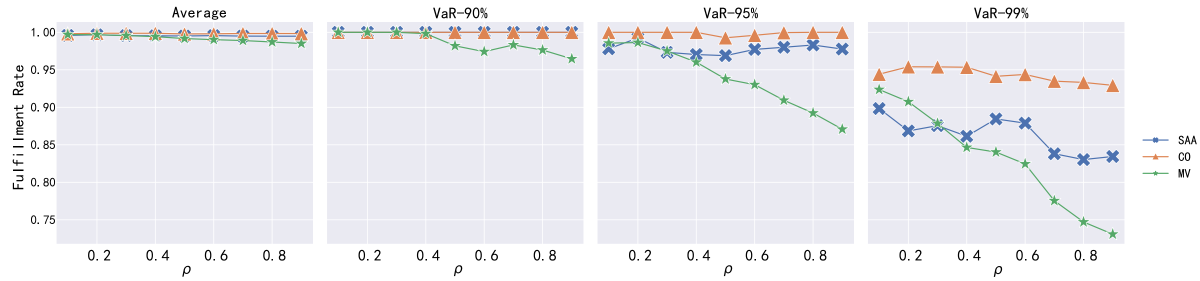


Fig. 6. Semi-case study: Fulfilment rate comparison.

Table 7
Networks designed for the semi-case study.

ρ	Model	Node	1	2	3	4	5	6	7	8	9	10	11	12	Total Inv.
	SAA		1403	2976		15321		2057				2206			23963
0.2	CO		2680	3687		18223		2701							27291
	MV		2789			17095		2111					3243		25238
	SAA		2037	2990		18006		3245							26278
0.5	CO			4265		19497		4229	1148						29139
	MV		2693	3154		17074		2031							24952
	SAA		1920	3574		18643		4053							28190
0.8	CO			4424		21157		4923	1209						31713
	MV		2828			16980		2230					3328		25366

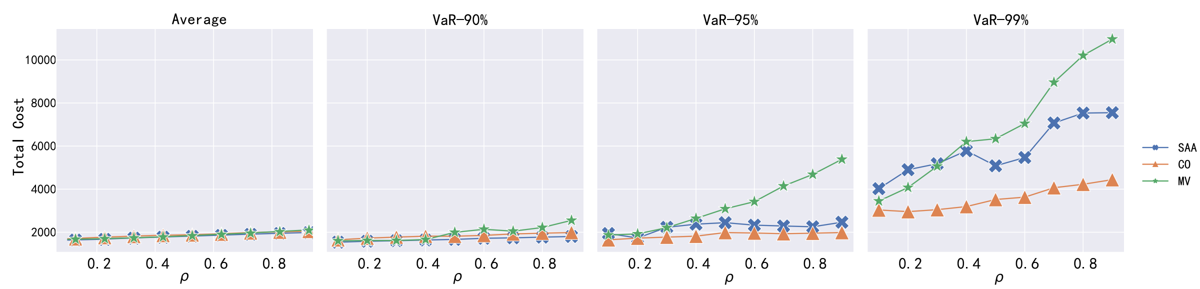


Fig. 7. Semi-case study: Total cost comparison.

when generating input samples and test samples as conducted in Ni et al. (2018) to mimic the reality that a wrong distribution assumption is a possibility.

Table 7 reveals designed prepositioning networks when $\rho = 0.2, 0.5, 0.8$. It is easy to see that in all cases, node 4 and node 6 are selected to build warehouses partially due to their high degrees. Moreover, as ρ increases, the total inventories of Model (SAA) and Model (CO) increase, but that of Model (MV) does not change substantially.

Fig. 6 displays the relationship between ρ and fulfilment rates on average and under different VaR percentiles. The first subfigure in Fig. 6 shows that, on average, Model (SAA) and Model (CO) can achieve an almost 100% fulfilment rate while that of Model (MV) is slightly lower. In cases of 90%, 95%, and 99% VaR, Model (CO) can still achieve the highest fulfilment rate, but the other two models' performances become worse, as shown in the other three subfigures. Notably, the fulfilment rate of Model (MV) decreases as ρ increases. Fig. 7 expresses total costs of the models. Surprisingly, the total costs of three models are almost the same on average. However, in case of 90%, 95%, and 99% VaR, Model (SAA) and Model (MV) have much higher total costs compared to Model (CO). It demonstrates Model (CO)'s capability of addressing extreme cases. Moreover, the increasing curve of Model (CO) always remain flat, while curves of other two models surge rapidly when ρ increases, which verifies that Model (CO) can accurately capture and correctly handle the correlated demand uncertainty while the other models cannot.

In summary, Figs. 6 and 7 show that our model can ensure a higher fulfilment rate at the lowest total cost. Furthermore, the performance of our model highlights that explicitly considering demand correlation in network design is necessary.

5. Extensions

5.1. Multiple materials

Suppose there are K different kinds of relief materials, and for each relief material $k \in [K]$, the first moment $\boldsymbol{\mu}_k$ and second moment $\boldsymbol{\Sigma}_k$ are known to decision makers. Further, assume that demands over different relief materials are independent. Under these assumptions, the second-stage problem is separable. Therefore, the network design problem can be modelled as:

$$\begin{aligned} \min_{\boldsymbol{y}_k, \boldsymbol{z}} \ & \kappa(\boldsymbol{f}^\top \boldsymbol{z} + \sum_{k\in[K]} \boldsymbol{h}_k^\top \boldsymbol{y}_k) + \sum_{k\in[K]} \sup_{\mathbb{P}\in\mathscr{F}(\boldsymbol{\mu}_k,\boldsymbol{\Sigma}_k)} \mathbb{E}_{\mathbb{P}}\left[g_k(\boldsymbol{y}_k, \tilde{\boldsymbol{d}})\right] \\ \text{s.t.} \ & \boldsymbol{y}_k \le U\boldsymbol{z}, \ \forall k \in [K]; \\ & \boldsymbol{y}_k \in \mathbb{R}_+^m, \boldsymbol{z} \in \{0,1\}^m. \end{aligned} \tag{8}$$

Let $L_k(\boldsymbol{y}) = \sup_{\mathbb{P}\in\mathscr{F}(\boldsymbol{\mu}_k,\boldsymbol{\Sigma}_k)} \mathbb{E}_{\mathbb{P}}\left[g_k(\boldsymbol{y}, \tilde{\boldsymbol{d}})\right]$. Following the analysis in Section 3, we can reformulate each $L_k(\boldsymbol{y})$ as a conic programming problem. Then the reformulated model will contain $|K|$ copositive cone constraints, which slows the computational process, but theoretically, with positive semidefinite relaxation, it does not make the model intractable.

5.2. Capacitated warehouses

Capacitated warehouses are common in reality, and the setup cost of warehouses $\boldsymbol{f}$ can be capacity-specific. Since these constraints are imposed in the first stage, they are easy to incorporate into our model. Suppose L types of warehouses are available, each of which has a limited capacity $C_l, \forall l \in [L]$ with a fixed setup cost $\boldsymbol{f}_l, \forall l \in [L]$. Now, we can model the network design problem with capacitated warehouses as:

$$\begin{aligned}
\min_{\boldsymbol{y},\boldsymbol{z}_l} \ & \kappa\Big(\sum_{l\in[L]} \boldsymbol{f}_l^\top \boldsymbol{z}_l + \boldsymbol{h}^\top \boldsymbol{y}\Big) + \sup_{\mathbb{P}\in\mathscr{F}(\boldsymbol{\mu},\boldsymbol{\Sigma})} \mathbb{E}_{\mathbb{P}}\left[g(\boldsymbol{y},\tilde{\boldsymbol{d}})\right] \quad (9)\\
\text{s.t.} \ & \boldsymbol{y} \le \sum_{l\in[L]} C_k \boldsymbol{z}_l, \ \forall l \in [L];\\
& \sum_{l\in[L]} \boldsymbol{z}_l \le \mathbf{1};\\
& \boldsymbol{y} \in \mathbb{R}_+^m, \boldsymbol{z}_l \in \{0,1\}^m, \forall l \in [L].
\end{aligned}$$

The objective function is self-evident. The first constraint requires that the number of stored products does not exceed warehouse capacity limits, while the second constraint restricts the number of warehouses at each location to be at most 1. We can still follow the same analysis as in Section 3 to reformulate the *sup* problem, and the resulting conic model is still tractable with relaxation, although it involves additional binary variables.

6. Conclusion

Motivated by the existence of correlated demand uncertainty, we studied a relief network design problem under demand uncertainty. The demand uncertainty is described as an ambiguity set characterized by the first two moments. This problem is formulated as a mixed-integer two-stage distributionally robust optimization problem, with the objective to minimize the setup cost, the holding cost, and the penalty cost due to unmet demand. We proposed an equivalent reformulation of the model as a mixed-integer conic problem. We also proposed a pricing policy with warm starts to accelerate the branch-and-pricing process. Extensive numerical studies on synthetic datasets and a semi-case study on the Ya'an Earthquake demonstrate the superiority of our proposed model and the necessity of incorporating correlated demand uncertainty when it exists. Meanwhile, sensitivity analysis shows that stronger correlations would amplify the improvement, further stressing the necessity of considering demand correlations. Several extensions have been proposed to express the compatibility of our proposed model by involving more realistic features.

It would be interesting to incorporate transportation cost into our model to capture its impact on inventory and location decisions. And in practice, warehouses and deployed relief packages are at risk of being damaged by disasters, resulting in fewer available relief packages for second-stage allocation. Taking this feature into consideration is another interesting direction. We leave them for future explorations.

CRediT authorship contribution statement

Xun Zhang: Conceptualization, Methodology, Writing – review & editing. **Du Chen:** Conceptualization, Methodology, Coding, Writing – original draft, Validation.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.cie.2023.109365.

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