Chapter Five

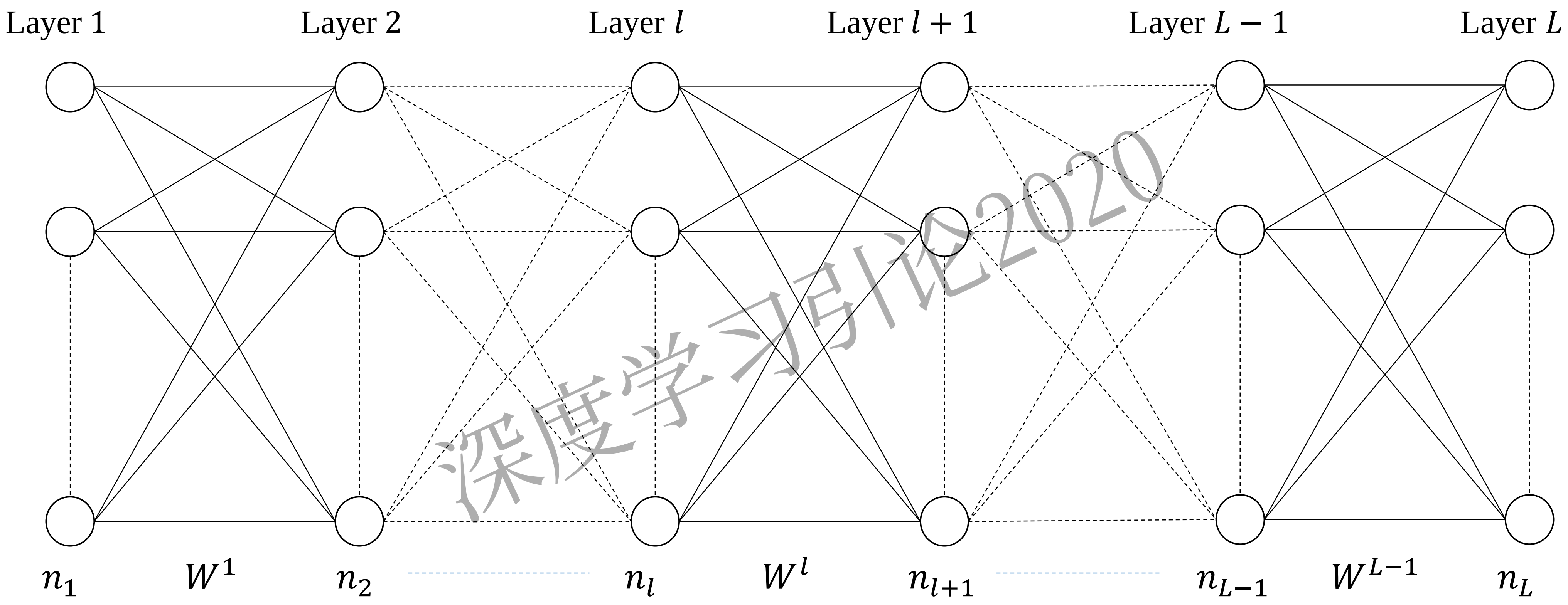
On Some Problems of BP

Zhang Yi, *IEEE Fellow*
Autumn 2020

Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Learning Rule
  - On the Target Output
  - On the Network Prediction
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

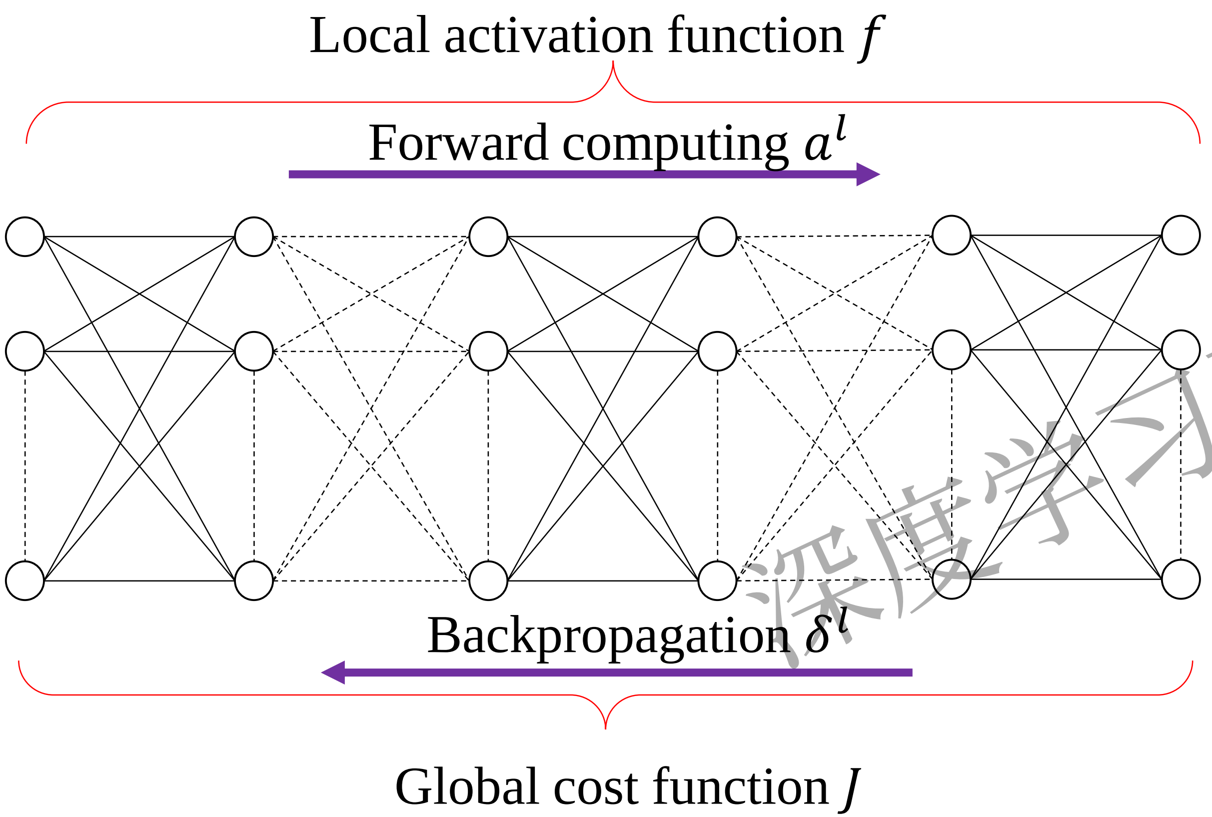
Network Structure

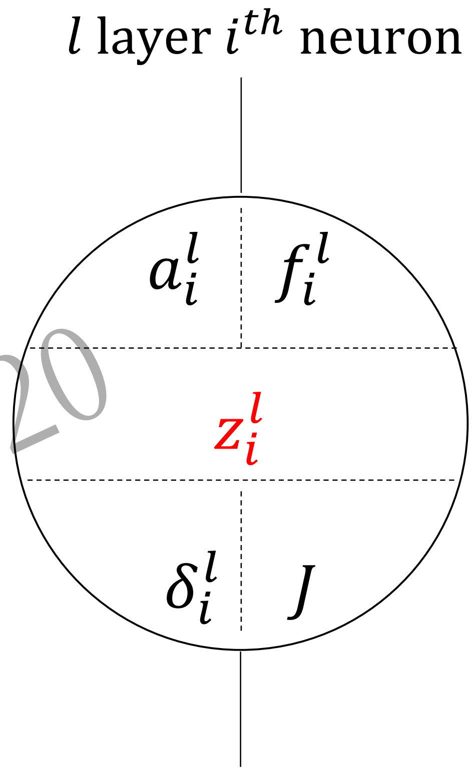


Two important characters:

- No any connection in any layer
- No any connection across any layer

Network Concepts



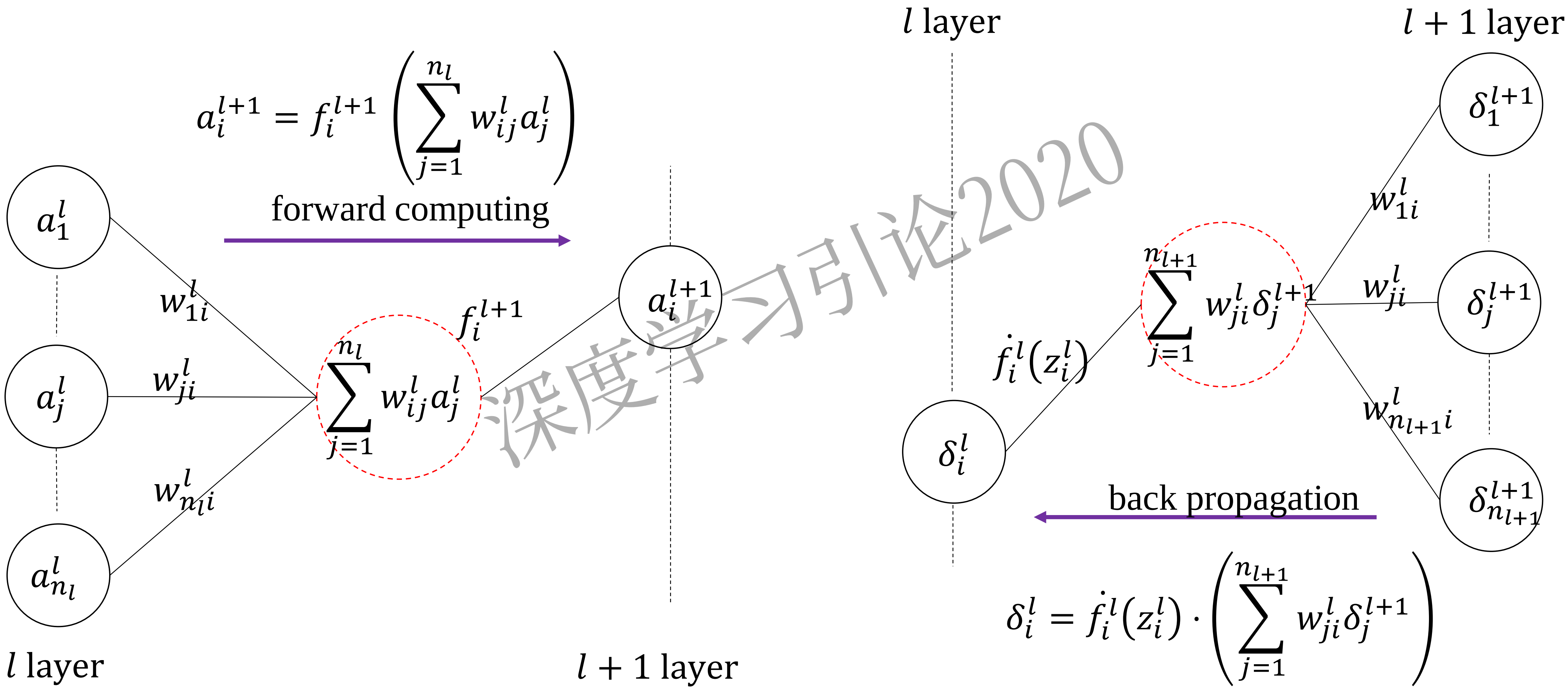


Global J Local f_i^l

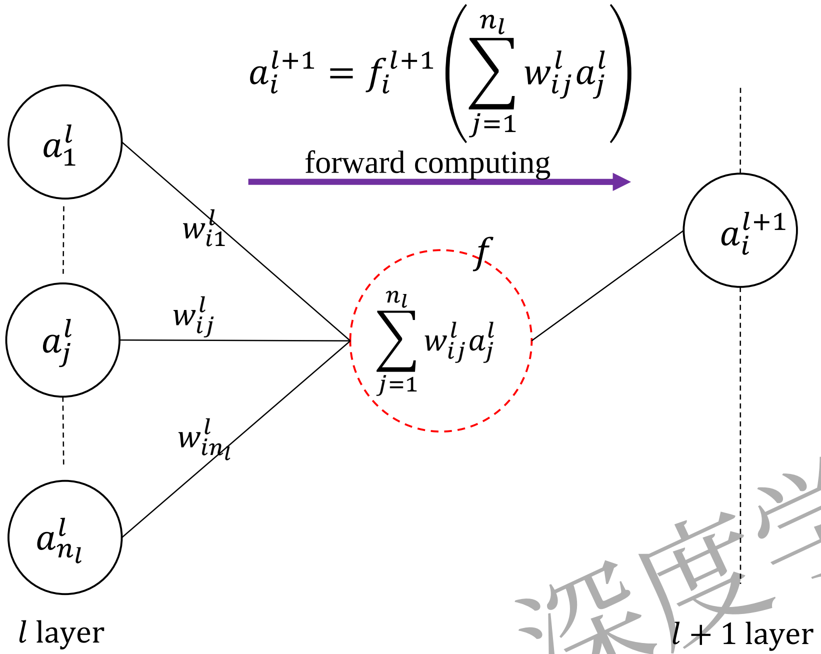
$$\frac{\partial J}{\partial z_i^l} = \delta_i^l \longleftarrow z_i^l \longrightarrow a_i^l = f_i^l(z_i^l)$$

Bridge

Network Operations



Network Functions



function $bc\left(w^l, \delta^{l+1}\right)$

$for\ i = 1{:}\,n_l$

$$\delta_i^l = \dot{f}_i^l\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$

end

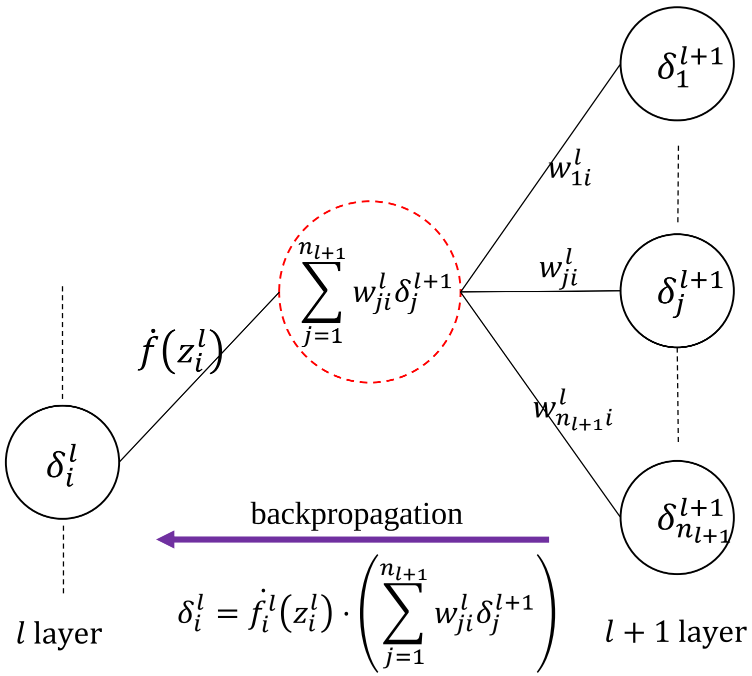
function $fc\left(w^l, a^l\right)$

$for\ i = 1{:}\,n_{l+1}$

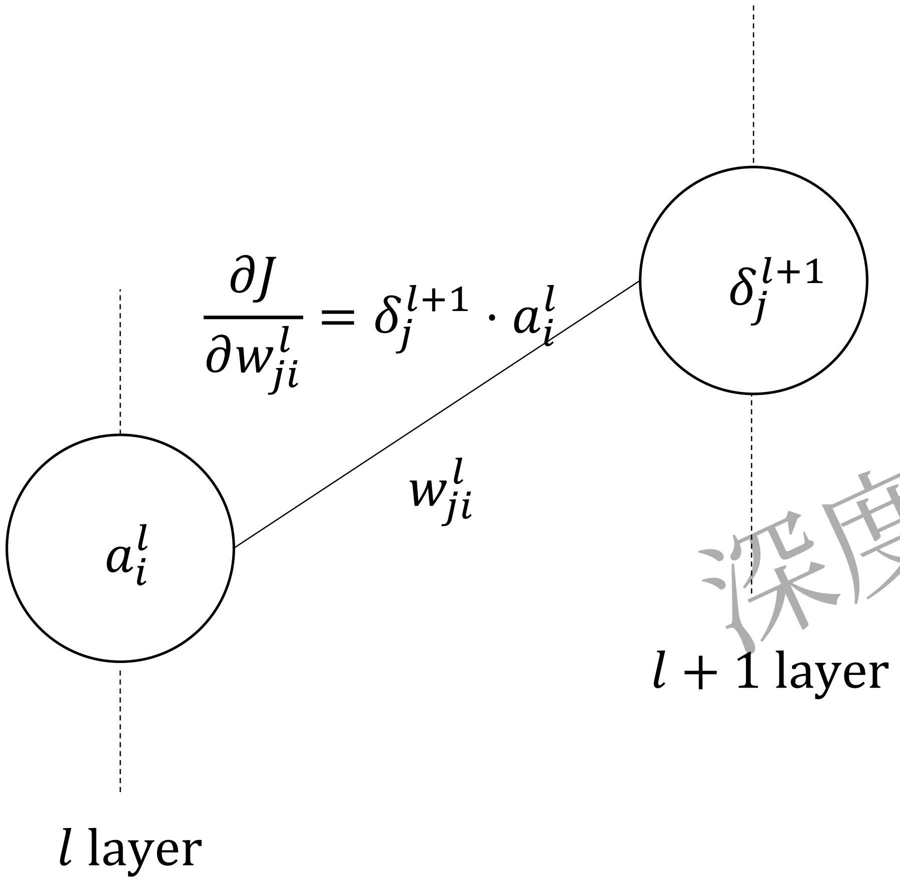
$$z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$$

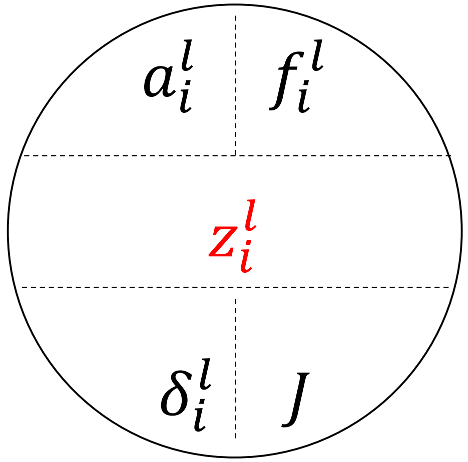
$$a_i^{l+1} = f_i^{l+1}\left(z_i^{l+1}\right)$$

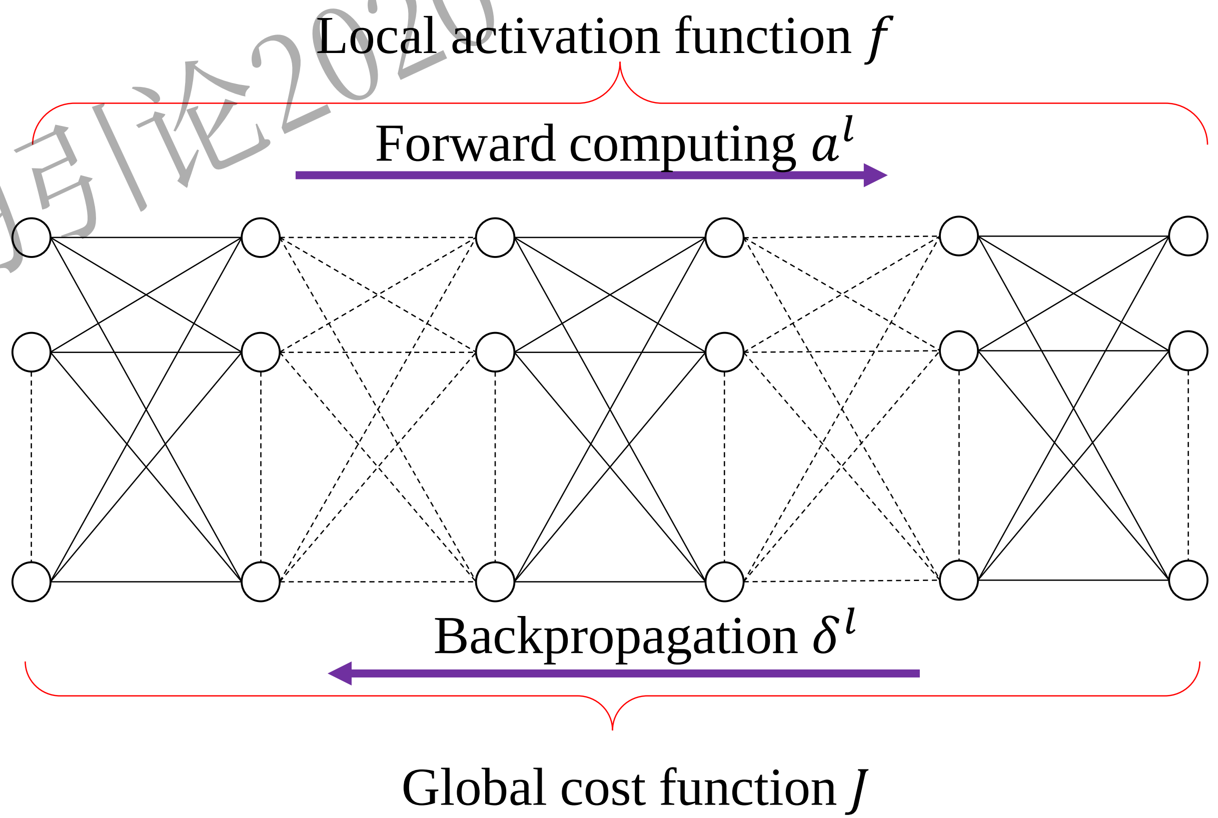
end



Network Relationship





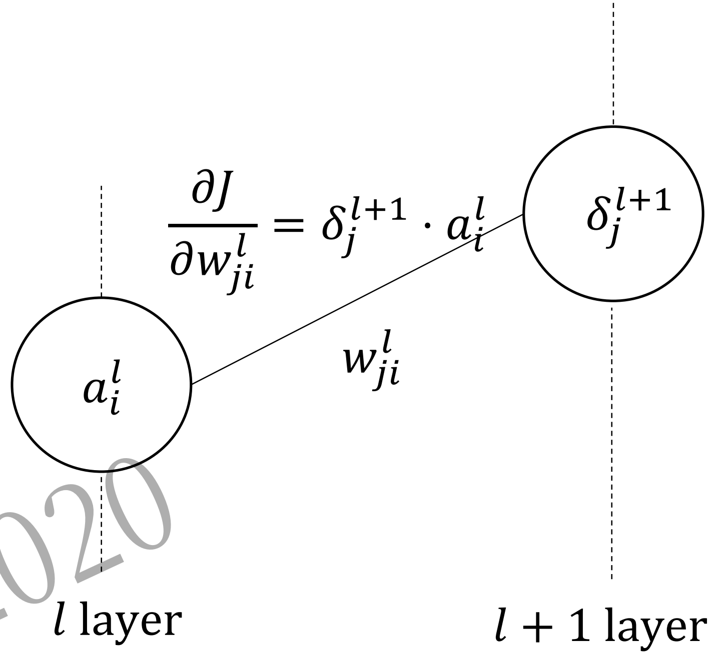


Network Learning Rule

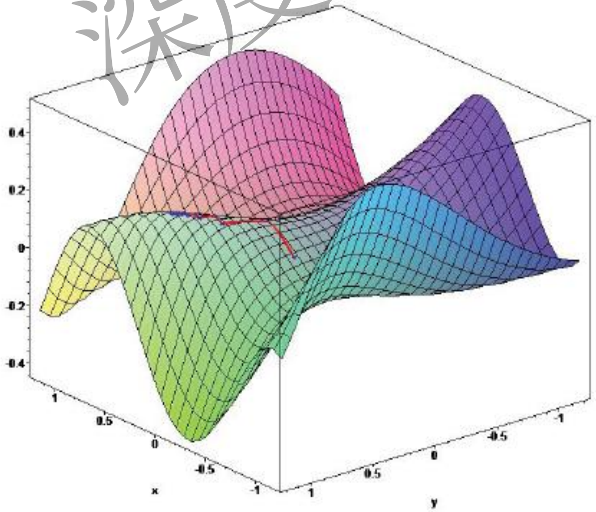
Learning rule

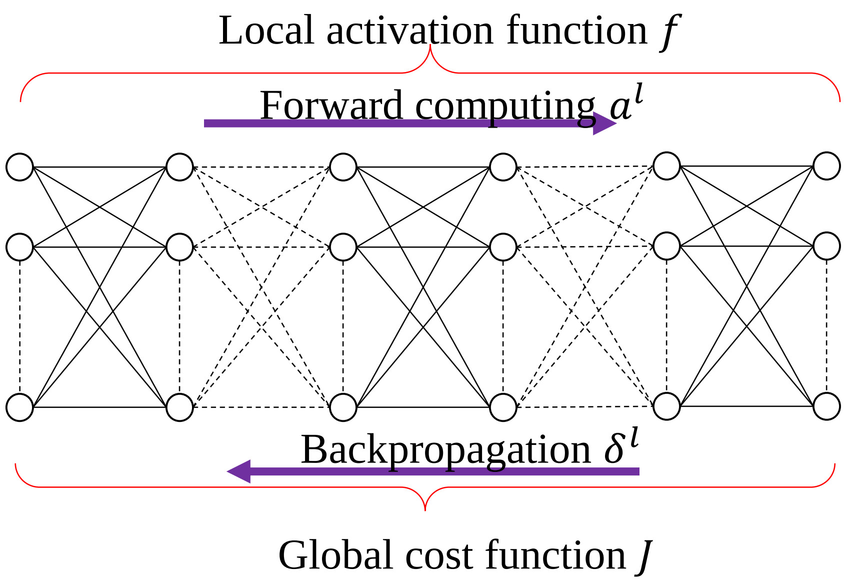
$$w_{ji}^{l} \leftarrow w_{ji}^{l} - \alpha \cdot \frac{\partial J}{\partial w_{ji}^{l}}$$

$$w_{ji}^{l} \leftarrow w_{ji}^{l} - \alpha \cdot \left(\delta_j^{l+1} \cdot a_i^{l}\right)$$



$$J = J(\cdots, w_{ji}^{l}, \cdots)$$





The BP Algorithm

Step 1. Input the training data set $D = \{(x, y)\}$

Step 2. Initialize each w_{ij}^{l}, and choose a learning rate α.

Step 3. for each mini-batch sample $D_m \subseteq D$

for each $x \in D_m$

$a^1 \leftarrow x \in D_m;$

for $l = 2{:}L$

$a^{l+1} \leftarrow fc(w^l, a^l);$

end

$$\delta^L = \frac{\partial J(x)}{\partial z^L};$$

for $l = L-1{:}2$

$\delta^l \leftarrow bc(w^l, \delta^{l+1});$

end

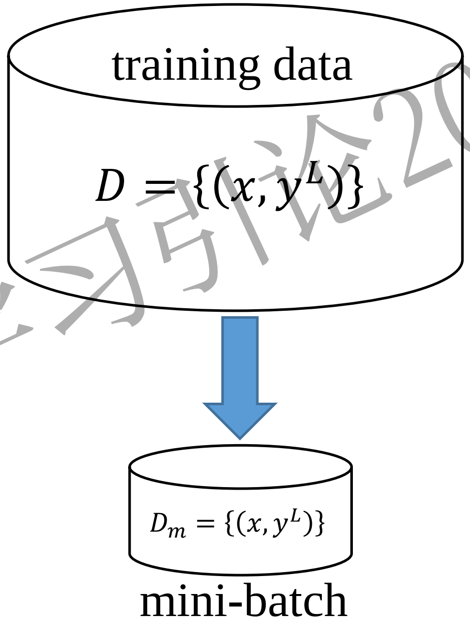
$$\frac{\partial J}{\partial w_{ji}^l} \leftarrow \frac{\partial J}{\partial w_{ji}^l} + \delta_j^{l+1} \cdot a_i^l;$$

end

$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l};$$

end

Step 4. Return to Step 3 until each w^lconverge.



function $fc(w^l, a^l)$

$for\ i = 1{:}n_{l+1}$

$$z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$$

$$a_i^{l+1} = f_i^{l+1}(z_i^{l+1})$$

end

Relationship:

$$\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$$

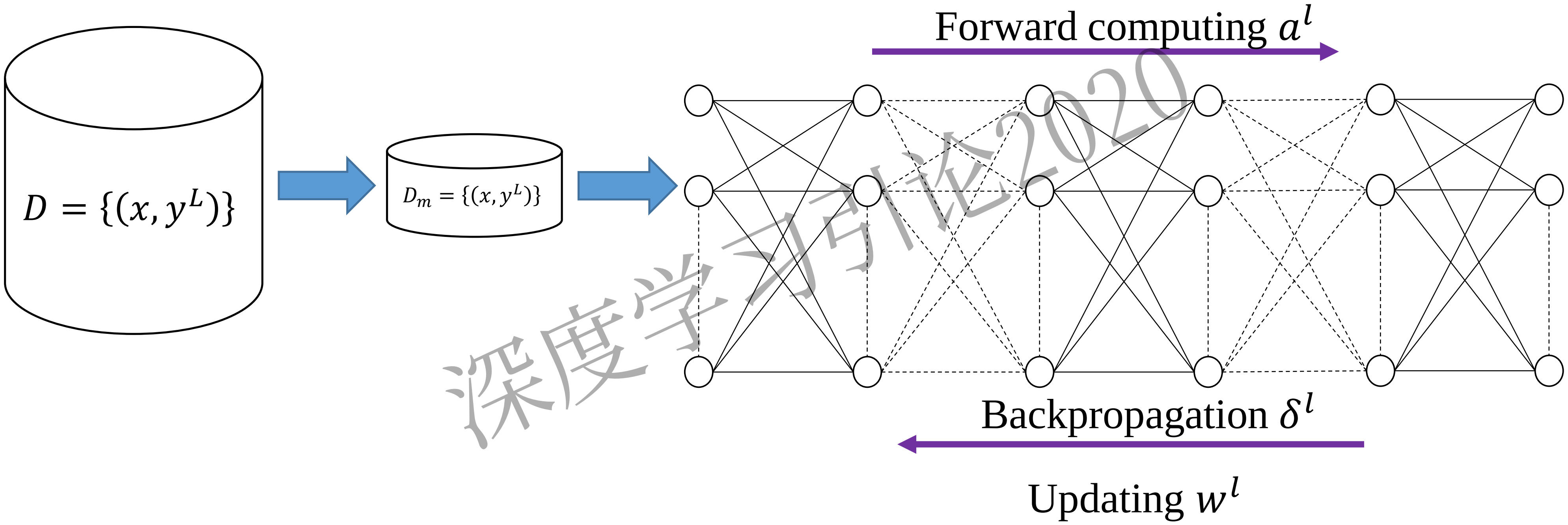
function $bc(w^l, \delta^{l+1})$

$for\ i = 1{:}n_l$

$$\delta_i^l = \dot{f}_i^l(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$

end

Network Training



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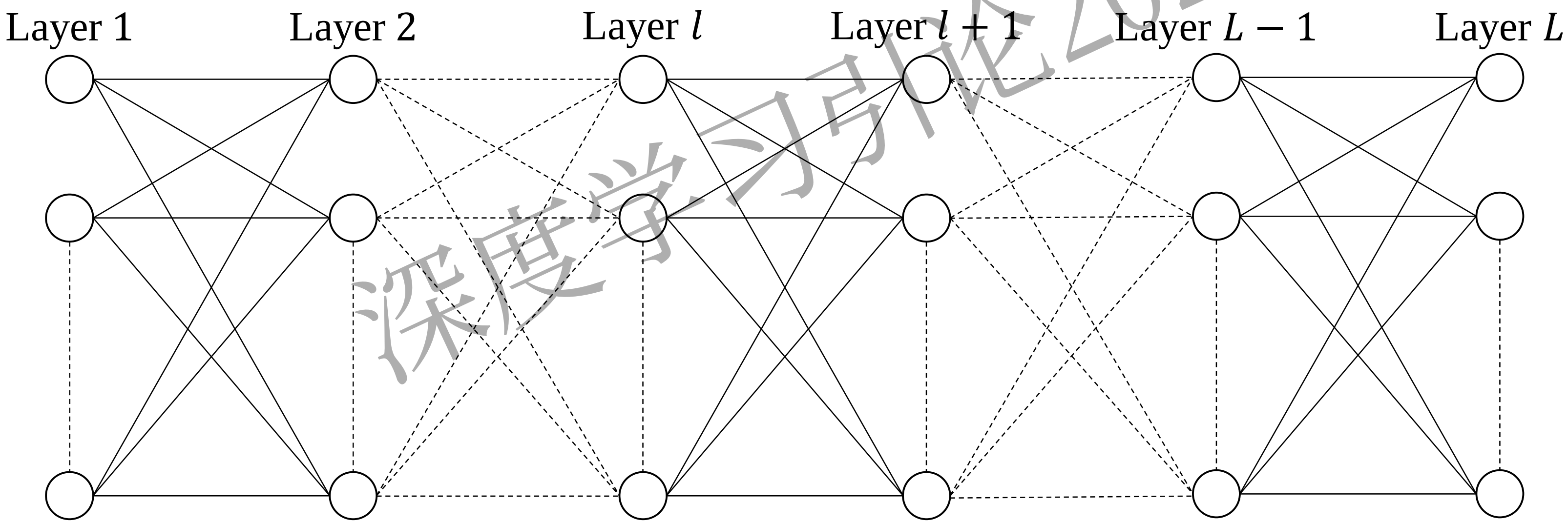
On the Network Structure

Two important characters:

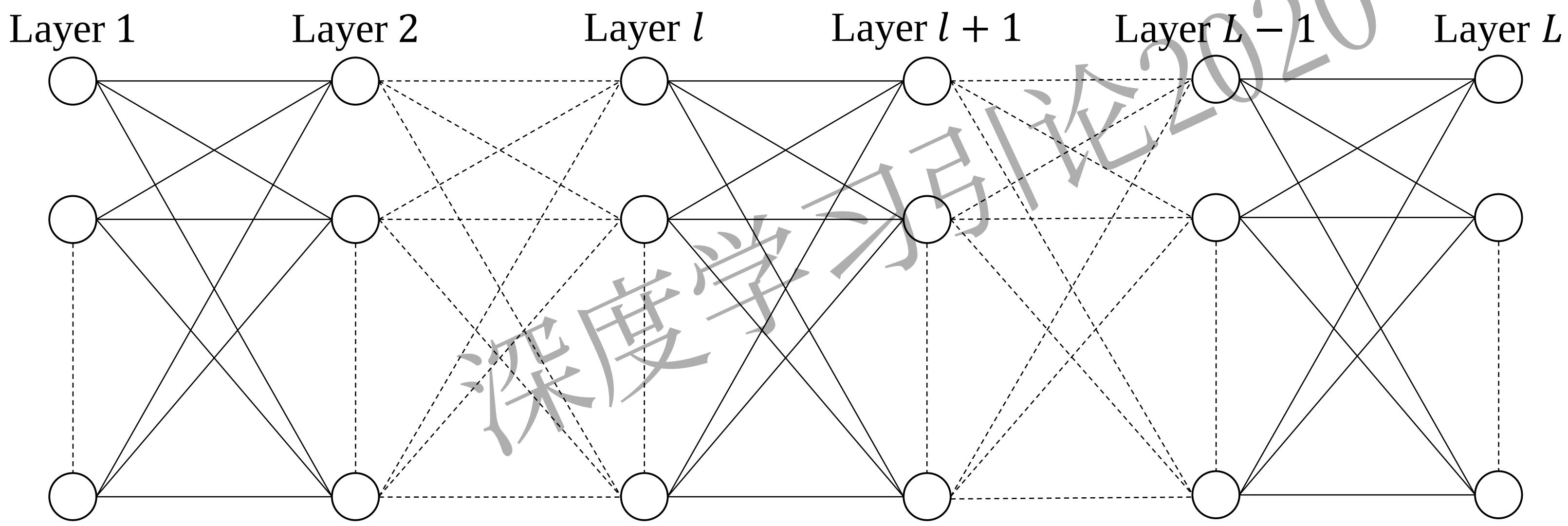
- No any connection in any layer
- No any connection across any layer



neurons	n_1		n_2		n_l		n_{l+1}		n_{L-1}	n_L
weights		W^1		------		W^l		------	W^{L-1}	
predictions	a^1		a^2		a^l		a^{l+1}		a^{L-1}	a^L

On the Network Structure

Recurrent Neural Networks



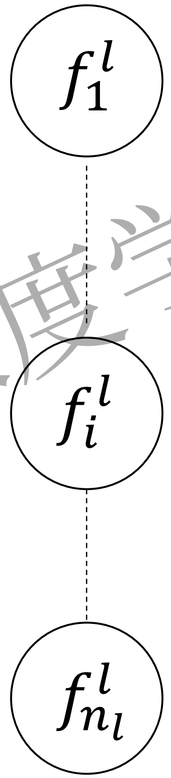
neurons	n		n		n		n		n		n
weights		W		......		W		......		W	
predictions	a^1		a^2		a^l		a^{l+1}		a^{L-1}		a^L

$$a^{l+1} = f\left(Wa^l\right)$$

On the Network Structure

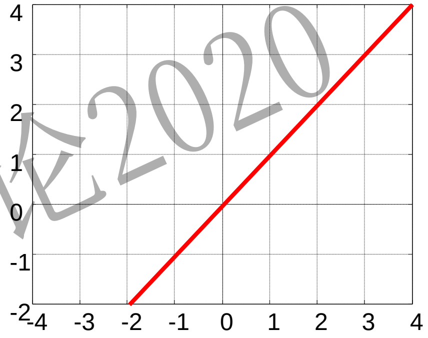
Activation functions of each neuron can be different

Layer l



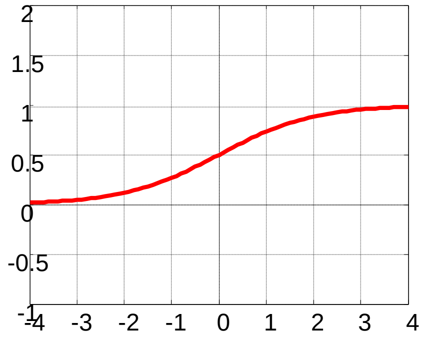
Linear function

$$f(z) = z$$



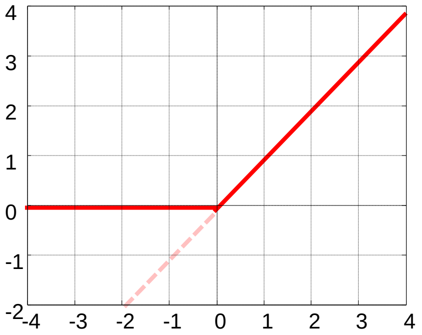
Sigmoid function

$$f(z) = \frac{1}{1 + e^{-z}}$$



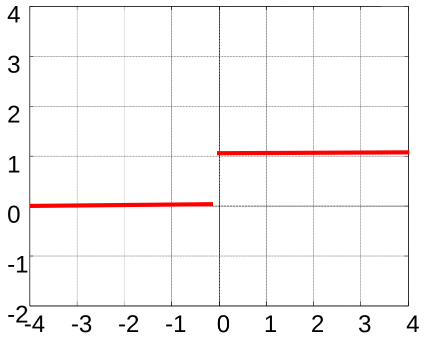
Rectifier function

$$f(z) = \begin{cases} z, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

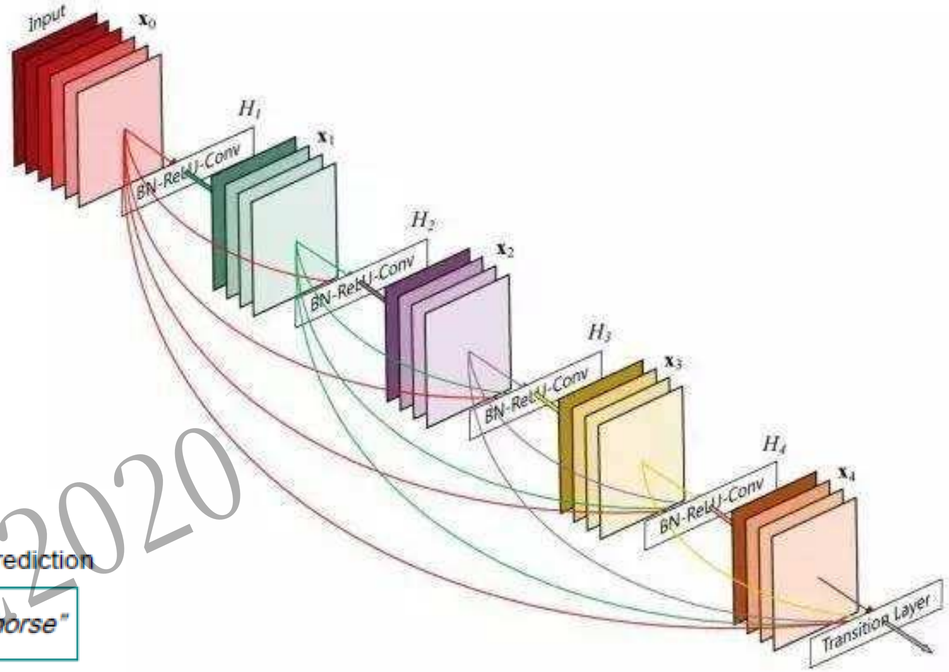


Hard-limit function

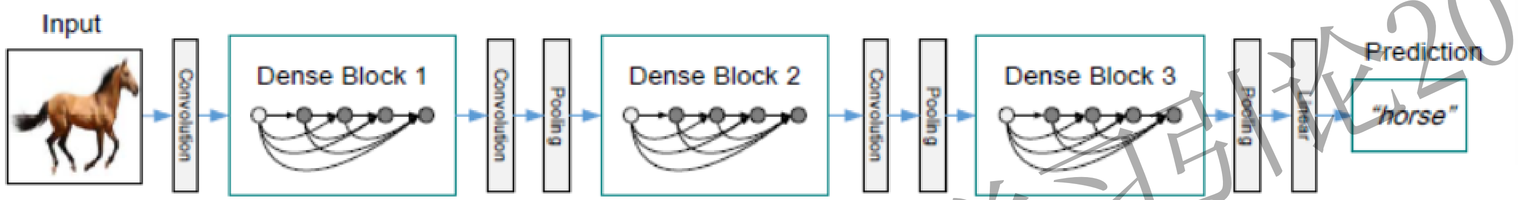
$$f(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

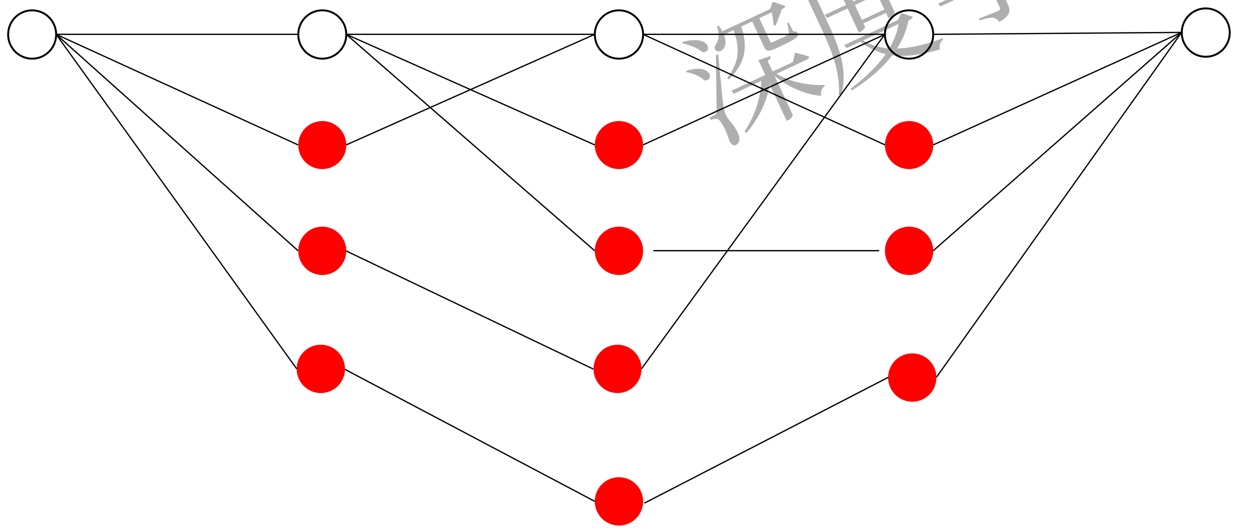


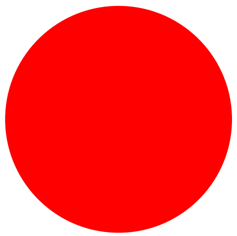
On the Network Structure



DenseNets





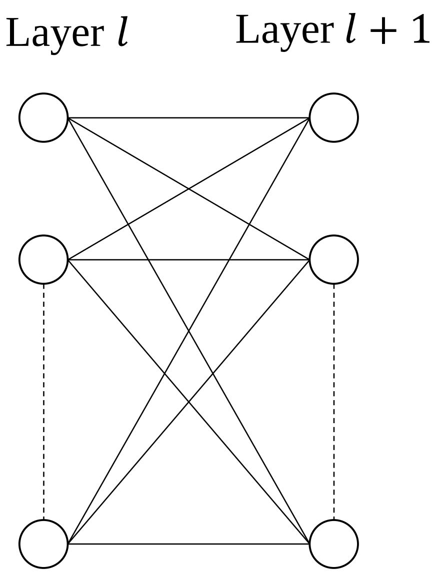


Linear neuron

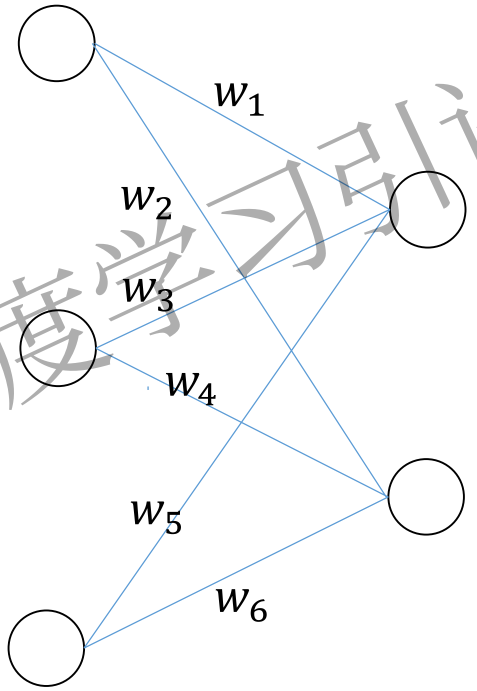
$f(s) = s$

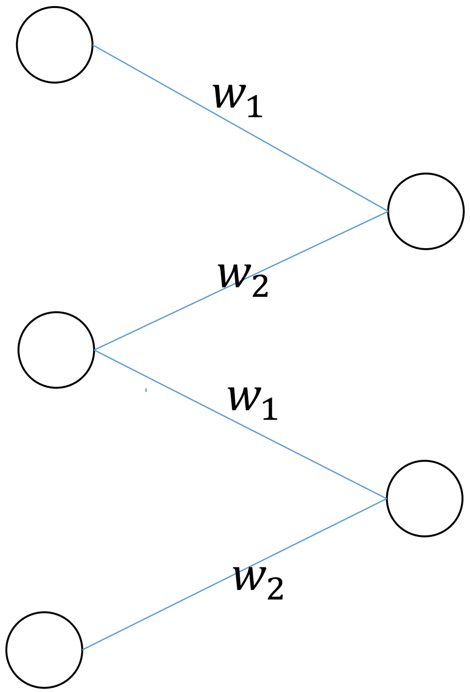
On the Network Structure

Connection weights between two layers can share some weights



$$W^l = \left(w_{ij}^l\right)_{n_{l+1}\times n_l}$$

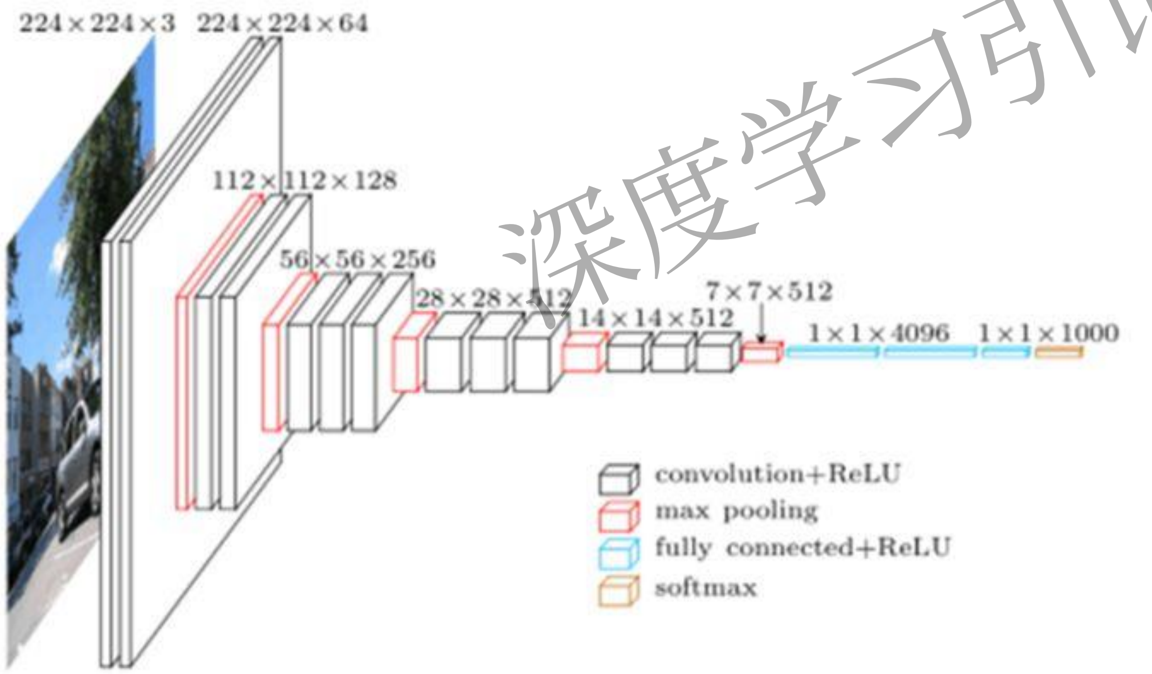


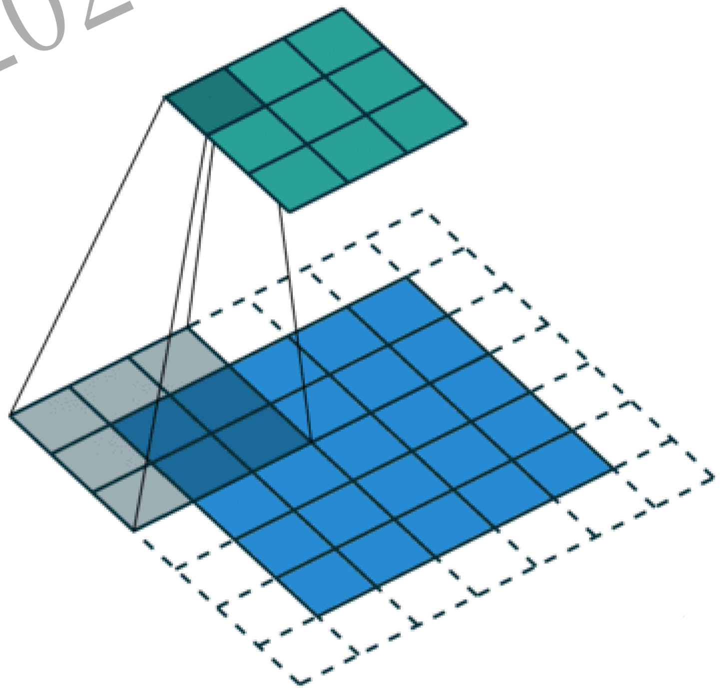


On the Network Structure

CNNs

Sharing of connection weights between two layers





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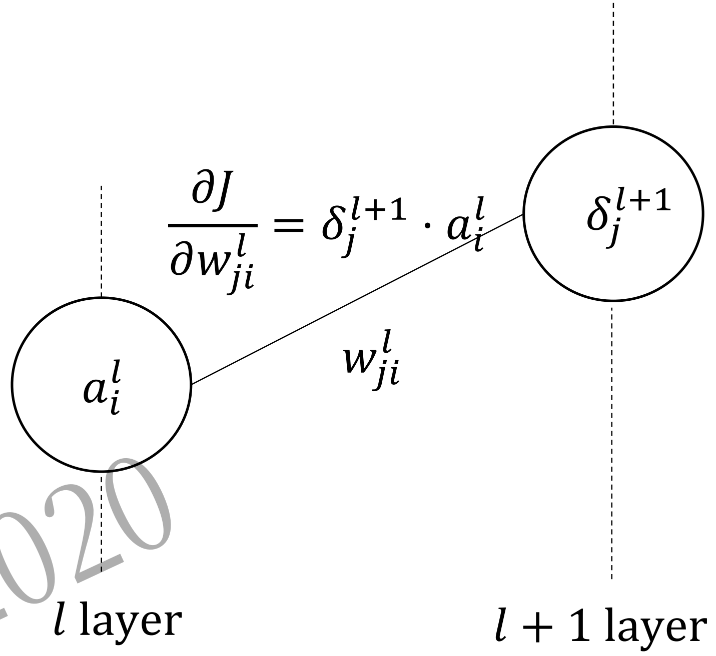
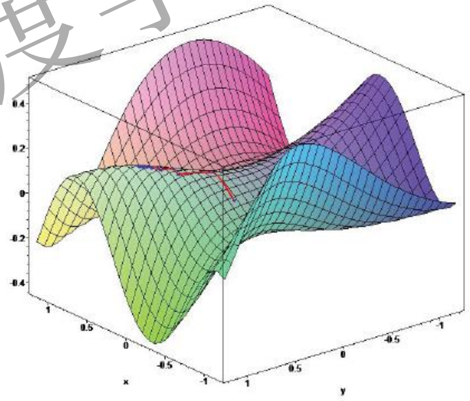
$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \left(\delta_j^{l+1} \cdot a_i^l\right)$$

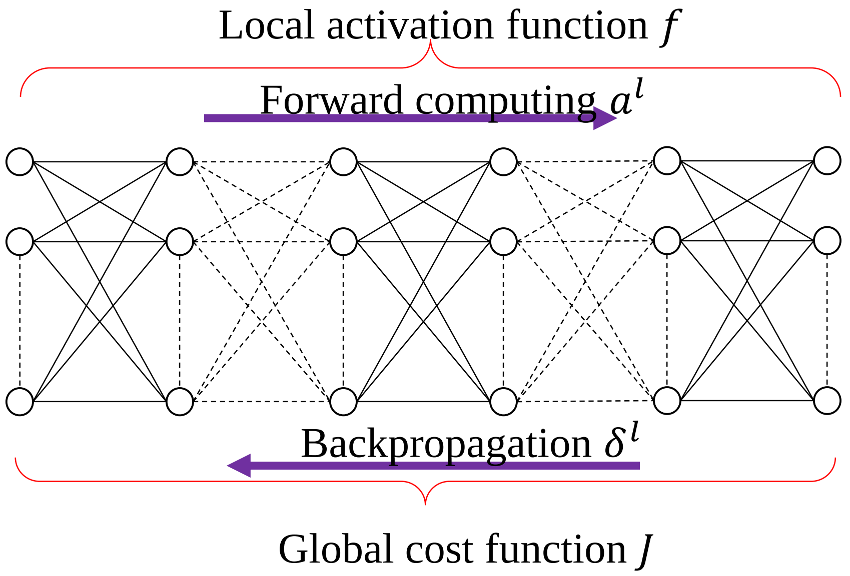
$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \left(\frac{\partial J}{\partial z_j^{l+1}}\right) \cdot f\left(z_i^l\right)$$



$$\delta_j^{l+1} = \frac{\partial J}{\partial z_j^{l+1}}$$



$$J = J(\cdots, w_{ji}^l, \cdots)$$

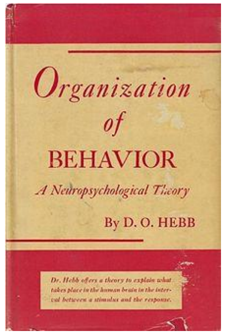


On the Network Learning Rule

Hebb’s Postulate

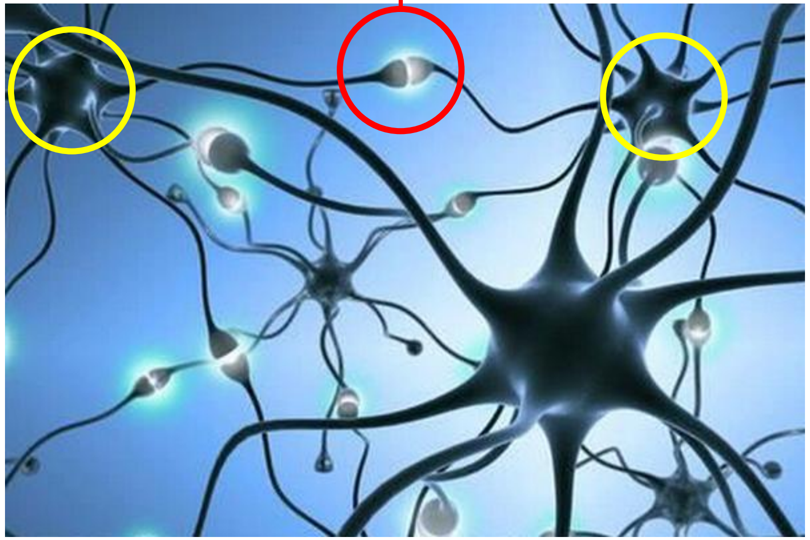
When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased.

——D.O Hebb， 1949



D. O. Hebb
Father of Cognitive Psychobiology
1904-1985

Synapse



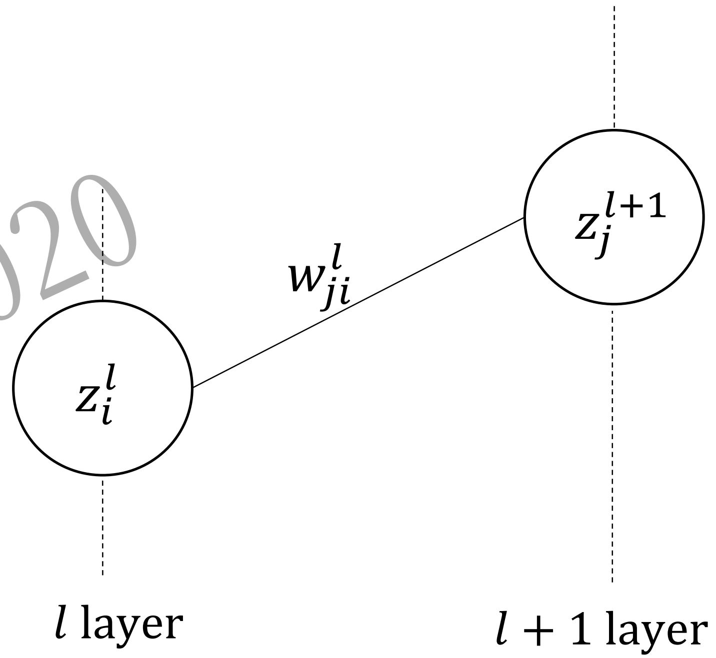
On the Network Learning Rule

When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased.

If two neurons of either side of a synapse are activated simultaneously, the strength of the synapse will increase.

Hebbian Learning

$$w_{ji}^{l} \leftarrow w_{ji}^{l} + F\left(z_j^{l+1}, z_i^{l}\right)$$



On the Network Learning Rule

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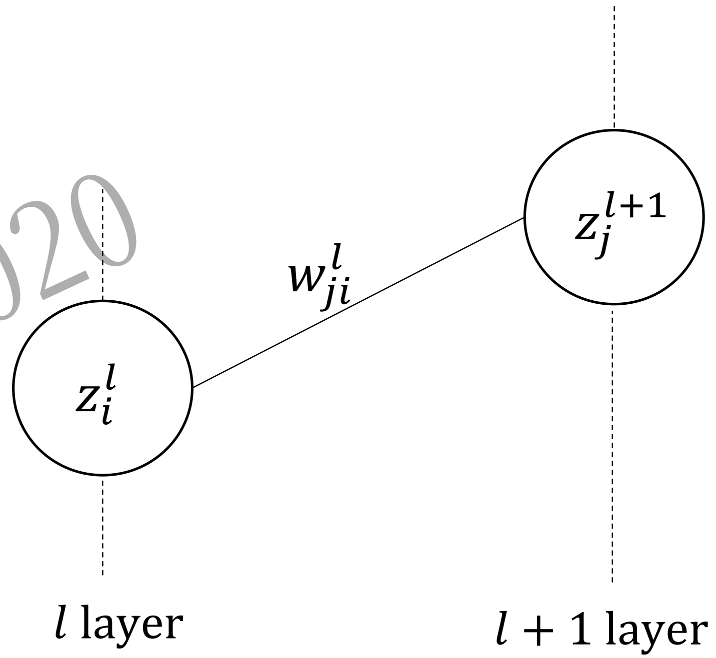
Hebbian Learning

$$w_{ji}^l \leftarrow w_{ji}^l + F\left(z_j^{l+1}, z_i^l\right)$$

$$w_{ji}^l \leftarrow w_{ji}^l + \alpha \cdot f_j^{l+1}\left(z_j^{l+1}\right) \cdot f_i^l\left(z_i^l\right)$$

$$w_{ji}^l \leftarrow w_{ji}^l + \alpha \cdot a_j^{l+1} \cdot a_i^l$$



On the Network Learning Rule

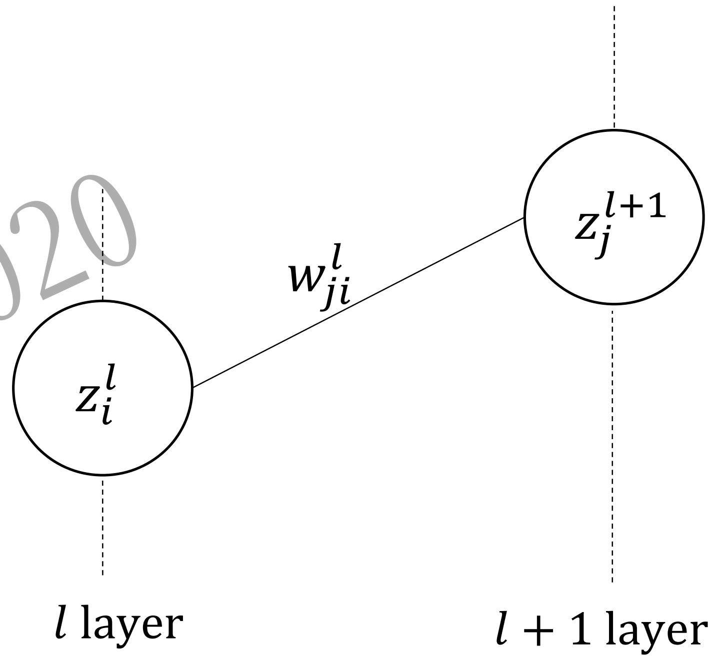
If two neurons of either side of a synapse are activated simultaneously, the strength of the synapse will increase.

Hebbian Learning

$$w_{ji}^{l} \leftarrow w_{ji}^{l} + F\left(z_j^{l+1}, z_i^{l}\right)$$

BP Learning

$$w_{ji}^{l} \leftarrow w_{ji}^{l} - \alpha \cdot \left(\frac{\partial J}{\partial z_j^{l+1}}\right) \cdot f\left(z_i^{l}\right)$$



$$F\left(z_j^{l+1}, z_i^{l}\right) = -\alpha \cdot \left(\frac{\partial J}{\partial z_j^{l+1}}\right) \cdot f\left(z_i^{l}\right)$$

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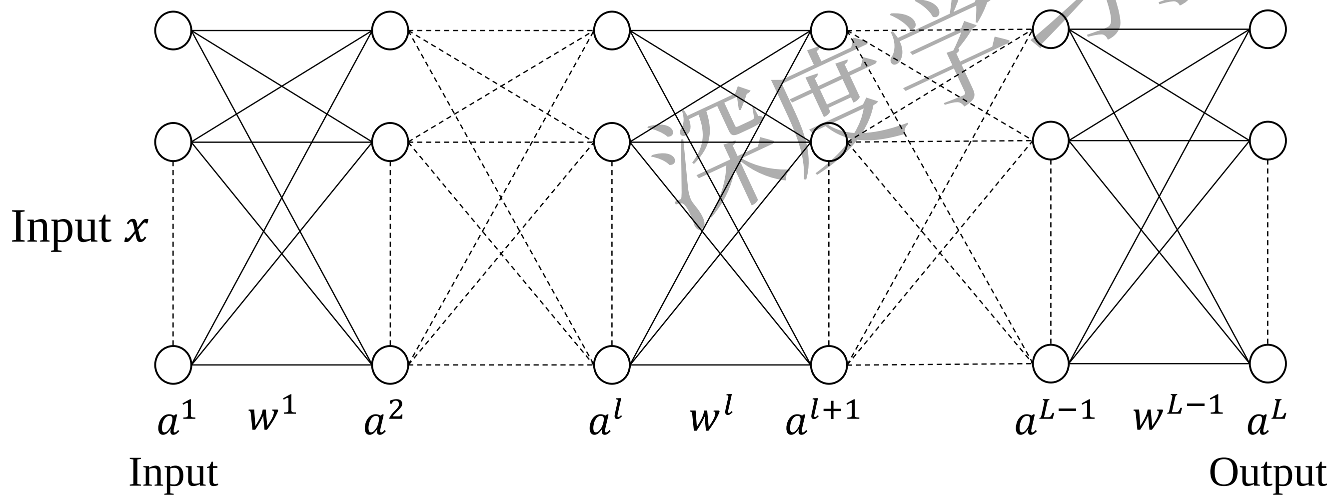
On the Target Output

Problem: How to define target output?

In principle, it can be defined in any way by users. However, it must fit the meaning of applications. Thus, it is application originated. A target output must correspond to its associated input.



Defined on the last layer
Target Output

$$\underbrace{y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix} \Leftrightarrow \text{Input } x}_{\text{A training sample } (x,\, y^L)}$$

$$\dim(a^L) = \dim(y^L)$$

On the Target Output

Classification Problem

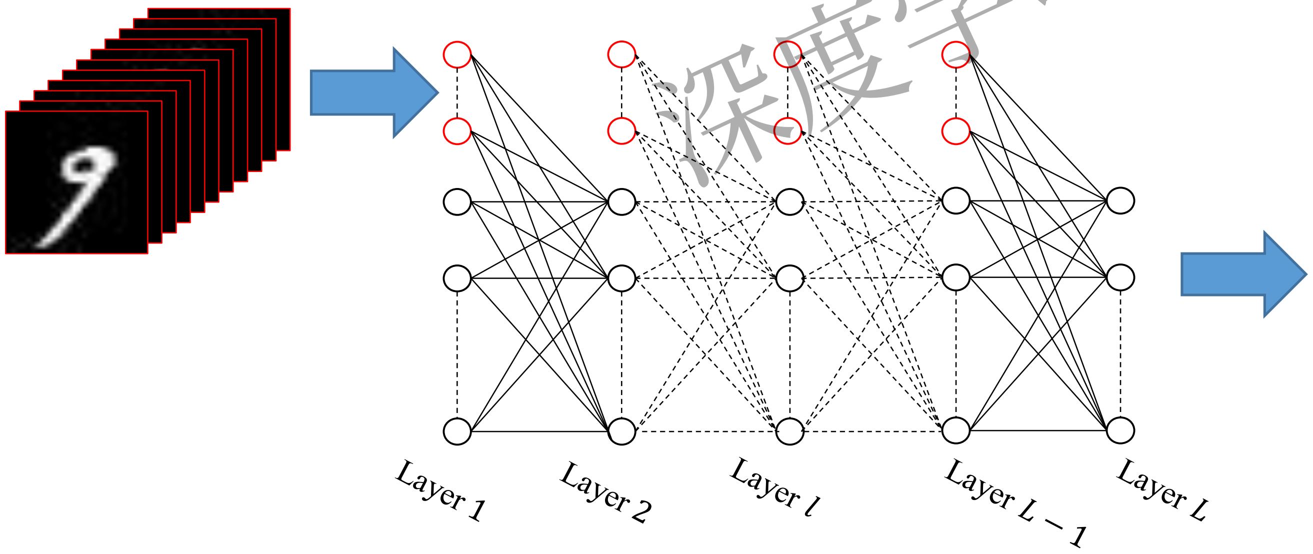
The target is to assign each input data sample to its class label. Thus, the target output can be defined by the representation of the label.

Tip:
The number of output neurons equals to the number of classes.

Representation of the label associated with the input.

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

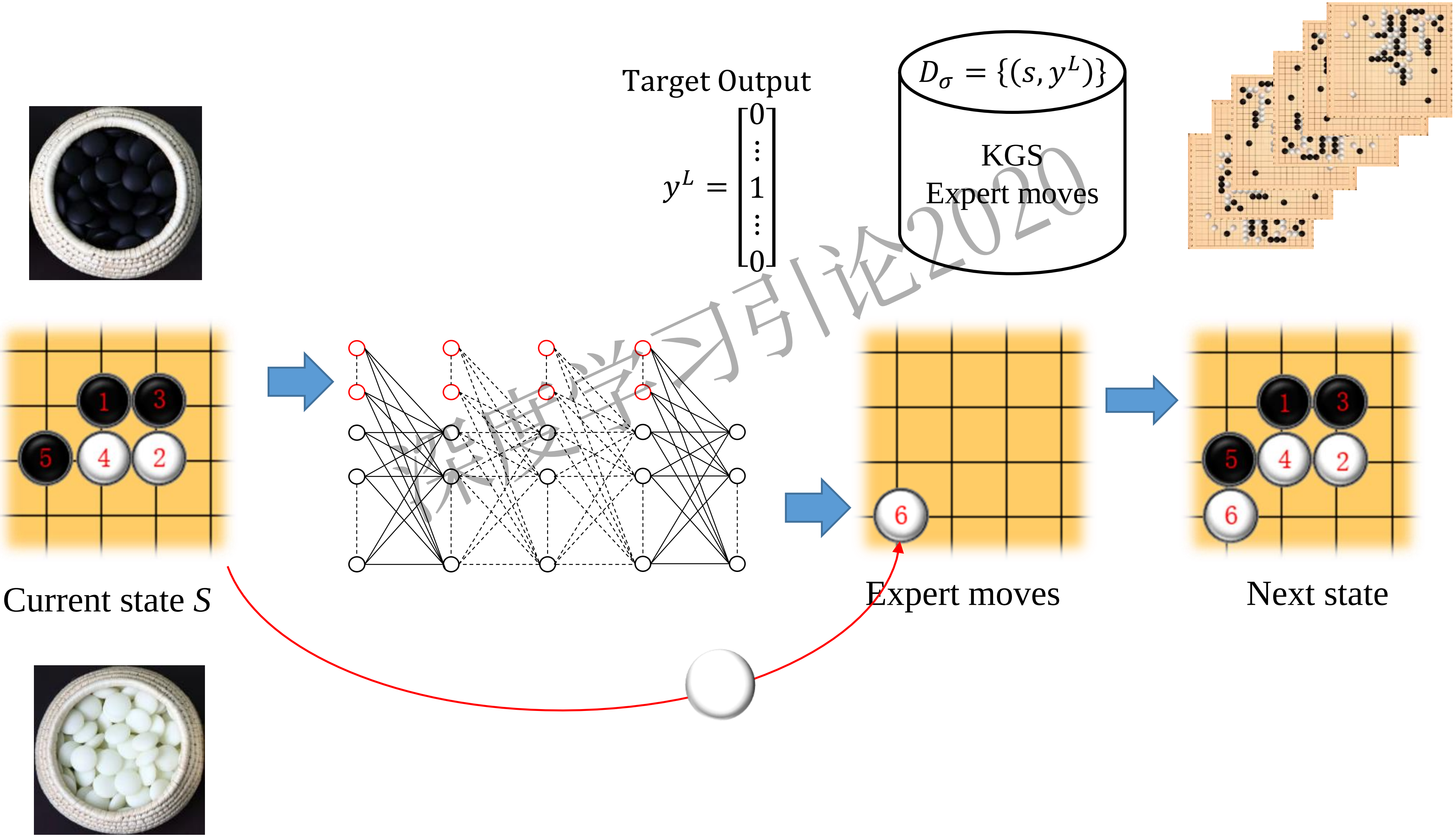


Classes Label

0	1	2	3	4	5	6	7	8	9
$\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\1\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\1\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\0\\1\\0\\0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\0\\0\\1\\0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\0\\0\\0\\1\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\1\\0\end{bmatrix}$

Representation

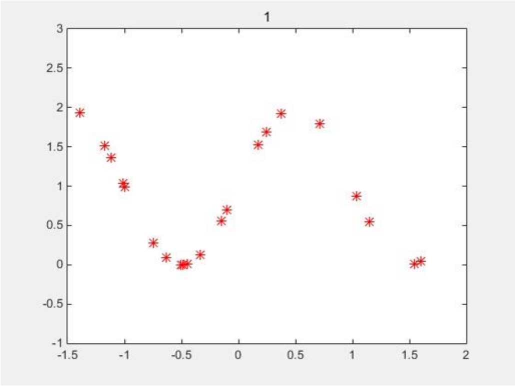
On the Target Output



On the Target Output

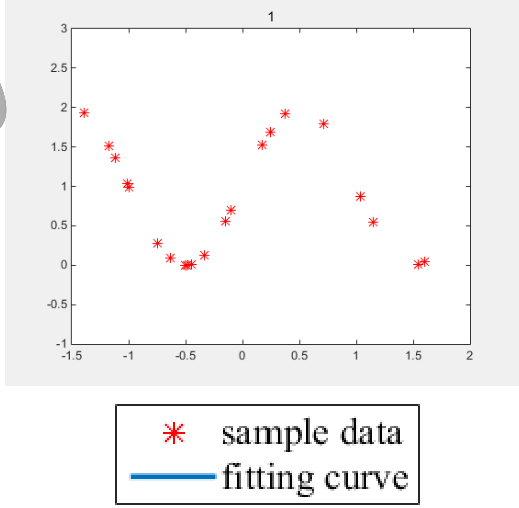
Curve Fitting Problem

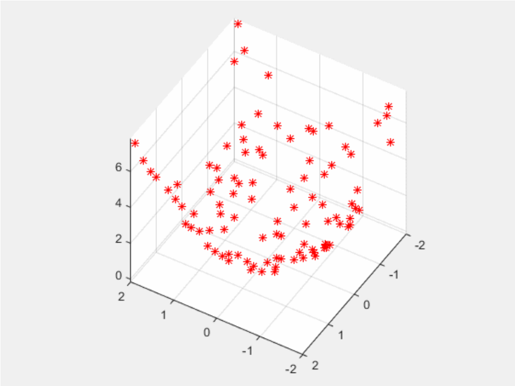
Given a set of sample data, estimates a curve that go through the samples.



Sample data

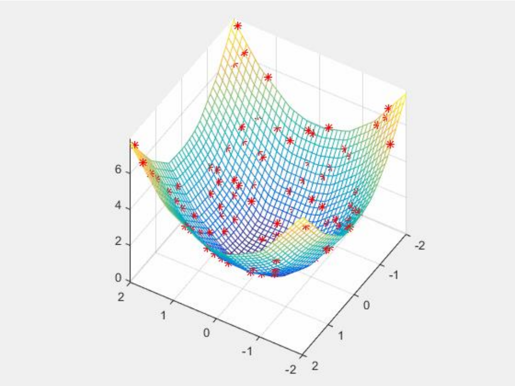
	1	2	3	4	5	6
x	-0.5000	0.1740	0.7100	-0.9980	-0.6340	1.0400
y	0	1.5198	1.7902	0.9937	0.0873	0.8747



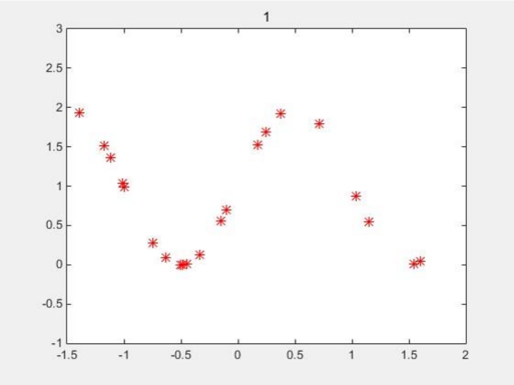


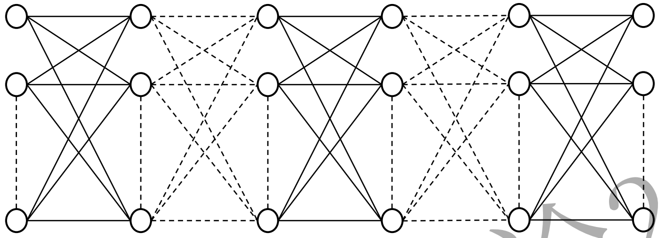
Sample data

	1	2	3	4	5	6
	-0.2000	-1.9000	1.9000	0.4000	-1.9000	0.8000
x	1.4000	-1.9000	-1.5000	-0.5000	0.3000	-0.1000
y	2.0000	7.2200	5.8600	0.4100	3.7000	0.6500

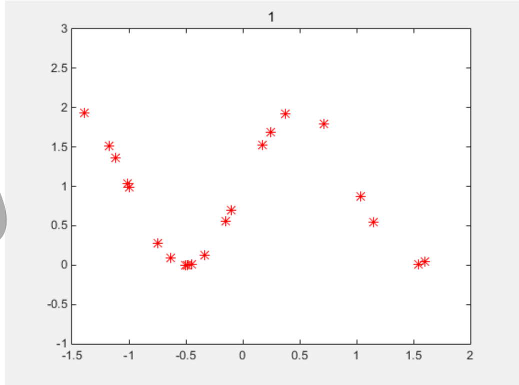


On the Target Output

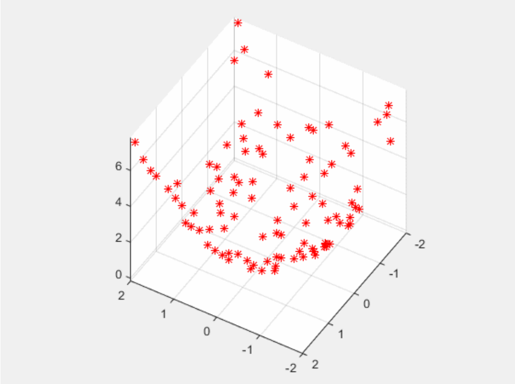


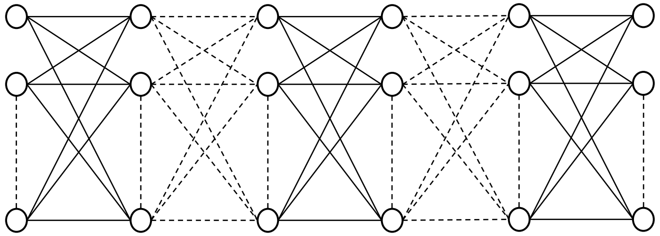


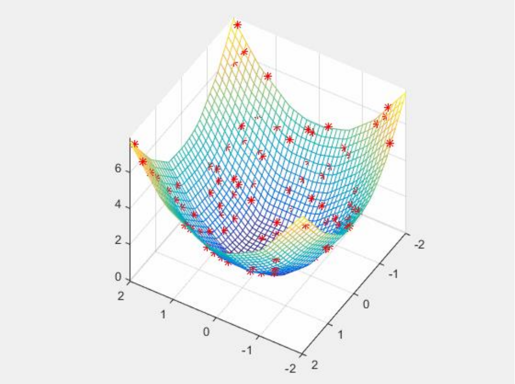
Training sample(x, y)



The target output is the value of y corresponding to x of each sample.







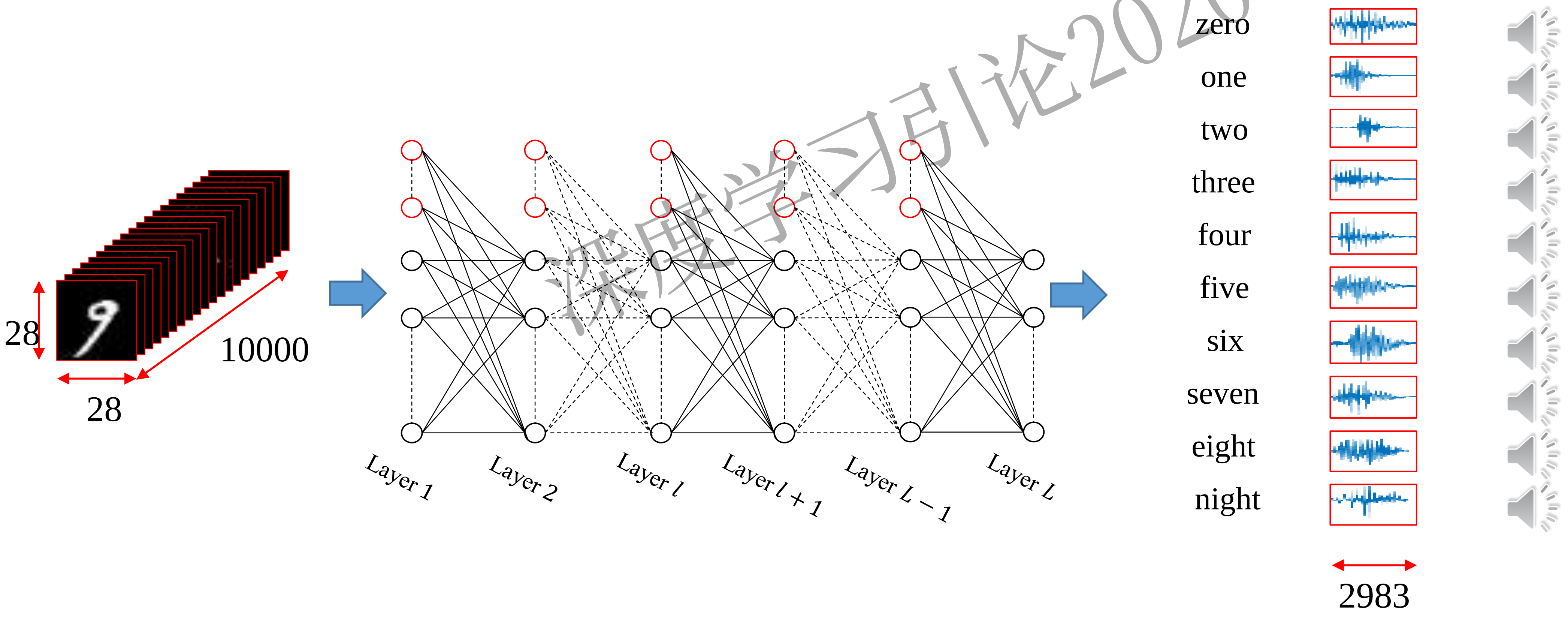
Target Output

$$y^L = y$$

On the Target Output

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ y_2^L \\ \vdots \\ y_{2983}^L \end{bmatrix}$$



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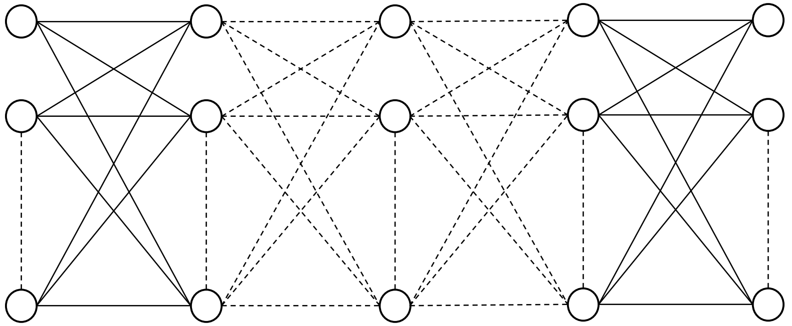
On the Network Prediction



a^1 w^1 a^2 w^l a^l a^{L-1} w^{L-1} a^L

Input Output

Network Prediction

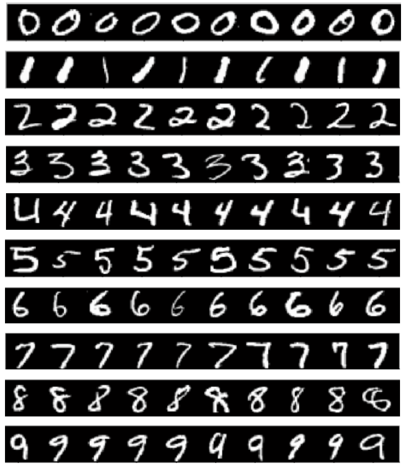
$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Define the last layer activation function f^L so that the network output a^L can match the target output y^L. Note that f^L should be differentiable.

$$a_i^L = f_i^L(z_i^L)$$

Target

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

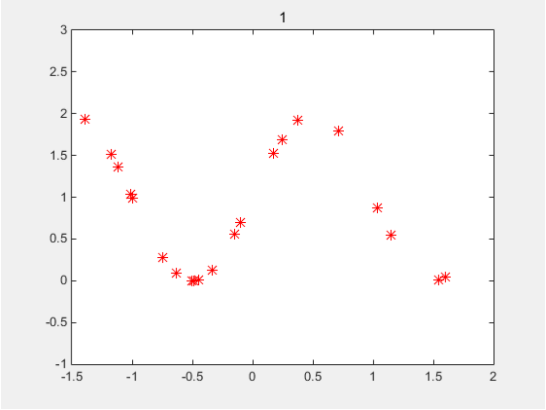


Sigmoid function

$$f(s) = \frac{1}{1+e^{-s}} \in (0,1)$$

$$\begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix} \xleftrightarrow{\text{Threshold } \theta} \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

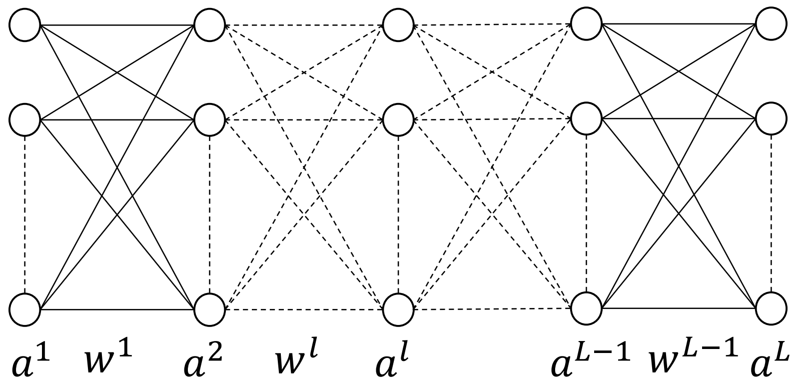
$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Linear function

$$f(s) = s$$

On the Network Prediction



Target

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

$$0 \le y_i^L \le 1$$

$$\sum_{i=1}^{n_L} y_i^L = 1$$

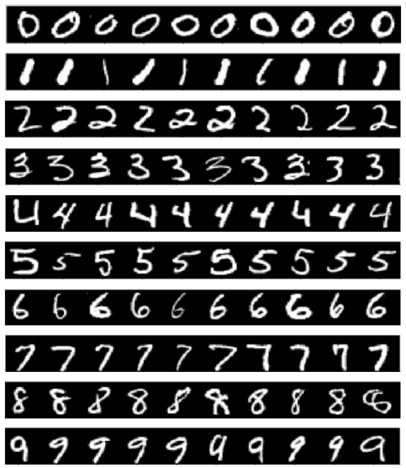
$$a_i^L = \frac{e^{z_i^L}}{e^{z_1^L} + \cdots + e^{z_{n_L}^L}}$$

Softmax function

Network Prediction

$$0 < a_i^L < 1$$

$$\sum_{i=1}^{n_L} a_i^L = 1$$



0	1	2	3	4	5	6	7	8	9
0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix} \qquad 0 \le y_i^L \le 1, \sum_{i=1}^{n_L} y_i^L = 1$$

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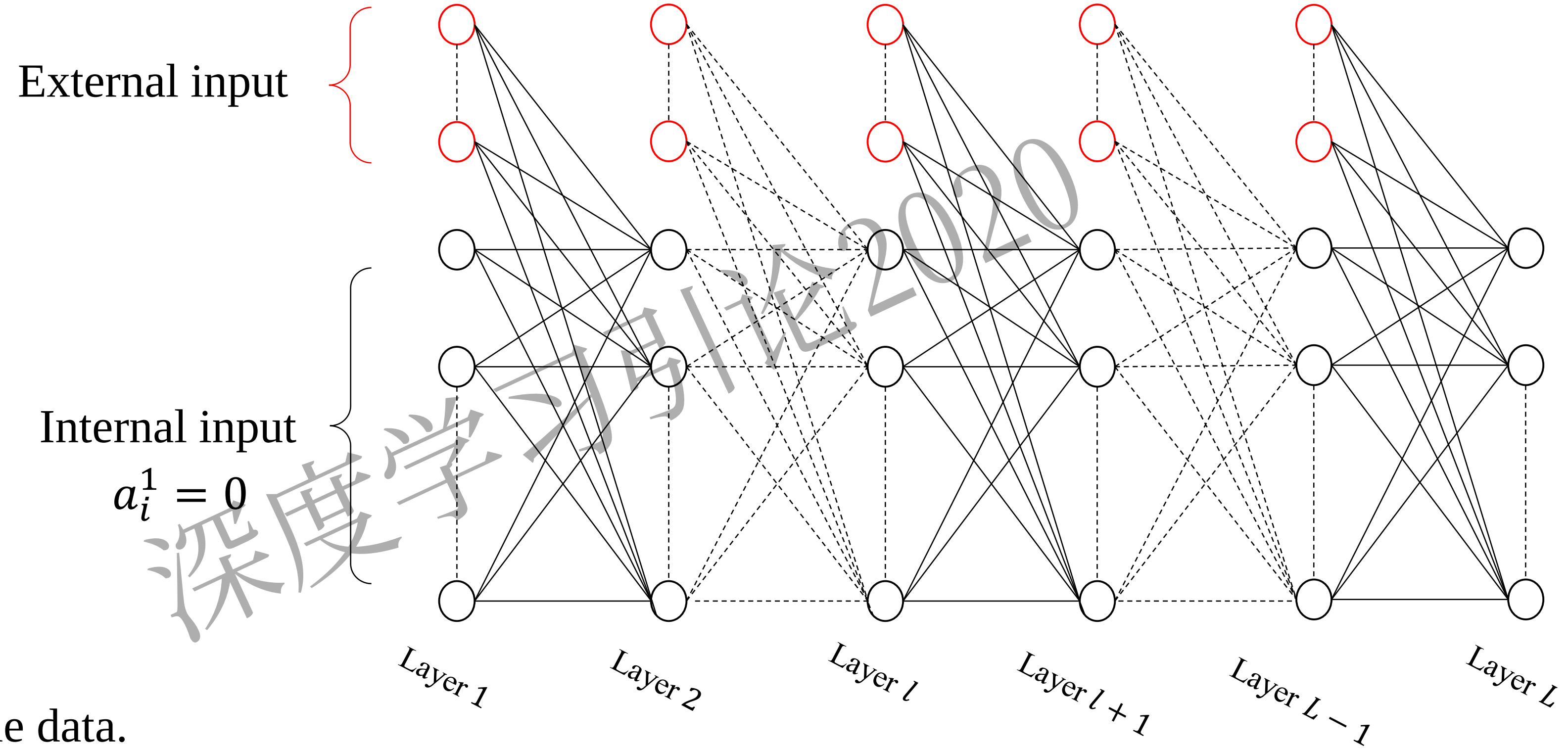
On the Network Input



External input:

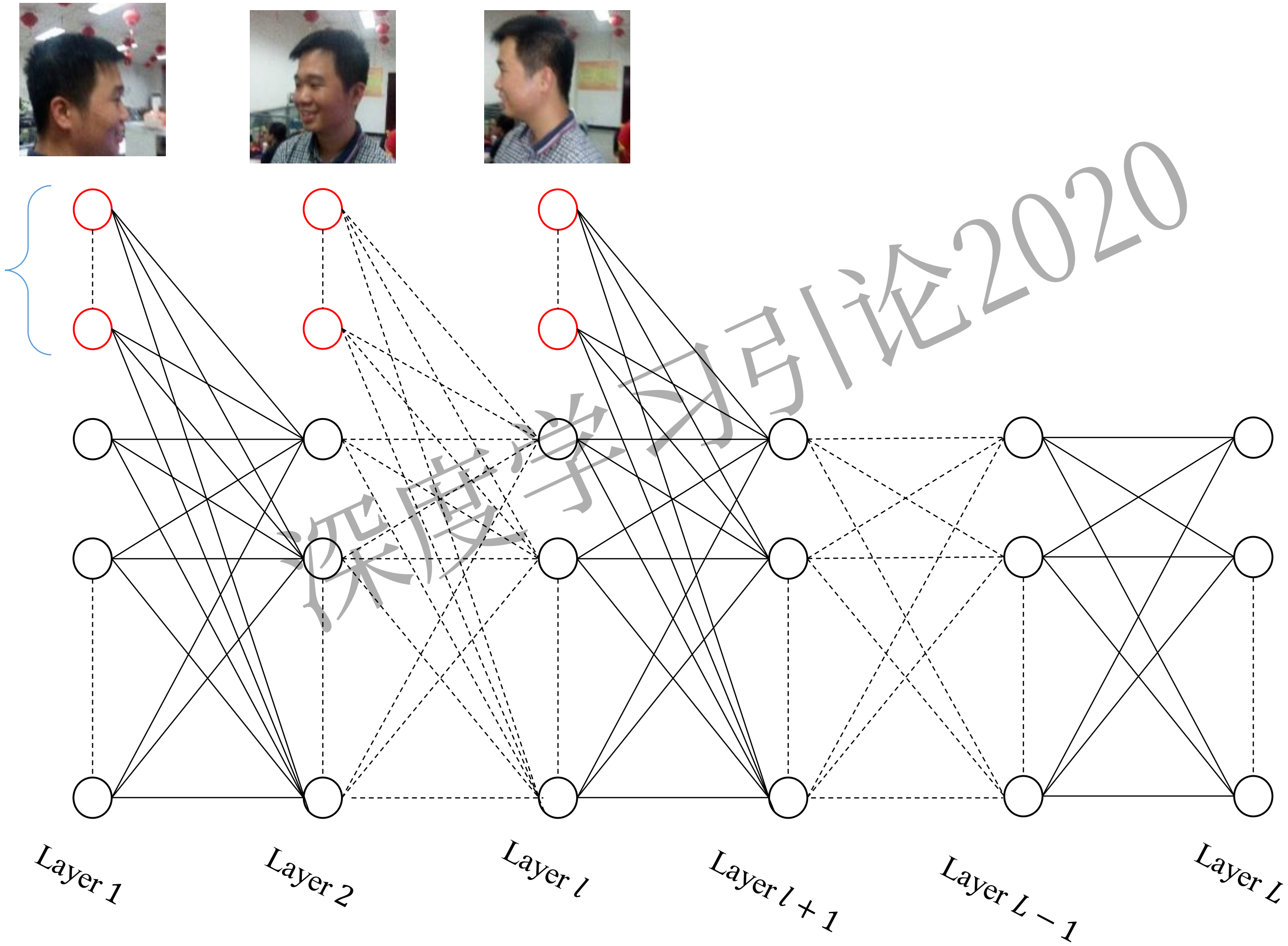
- Directly from sample data.

Internal input:

- Generated by former layer
- Maintain a working memory for the neural network
- The first layer internal input is generated by user

On the Input

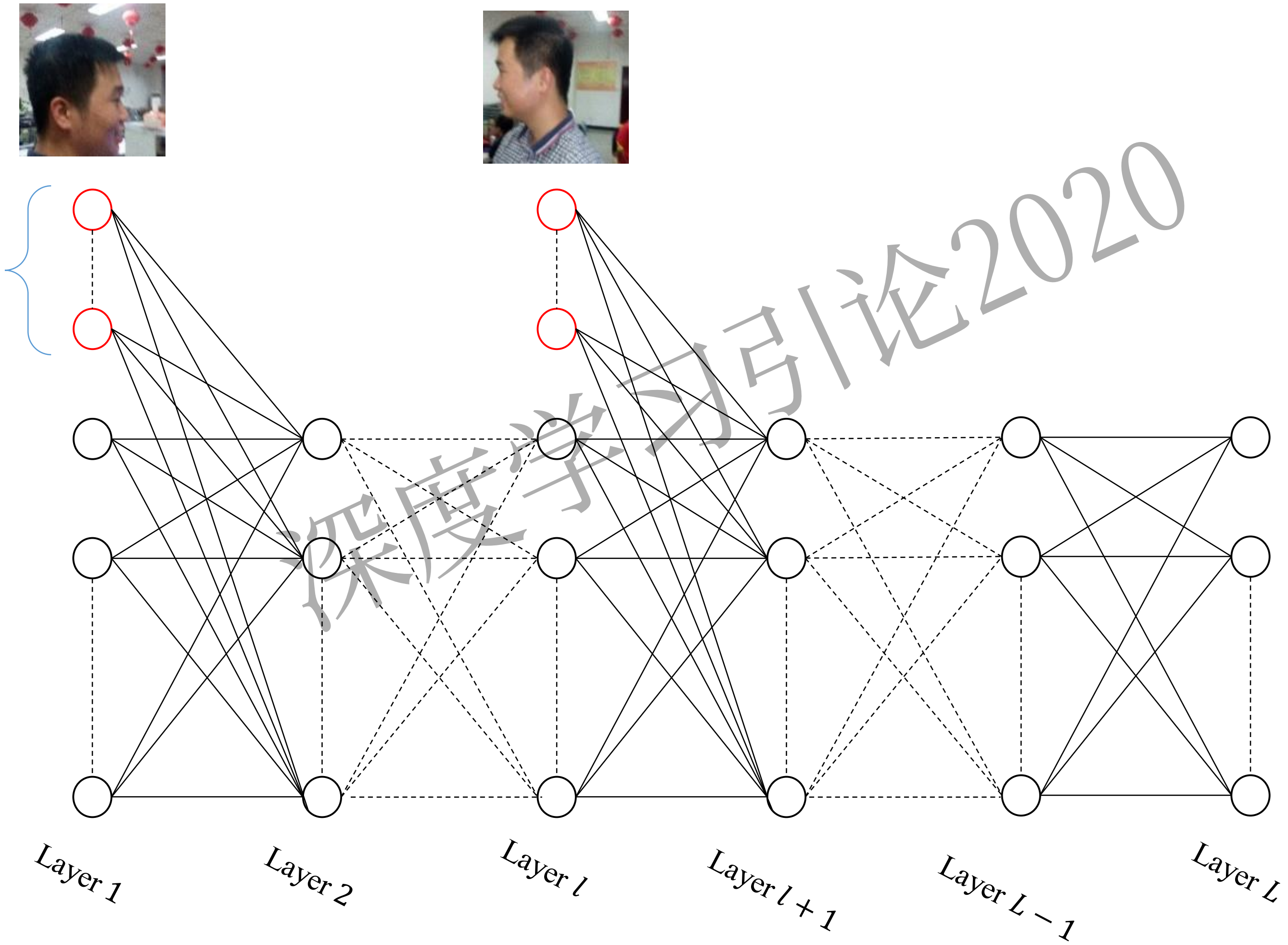
Sequence Input



Recognize
the identity

On the Input

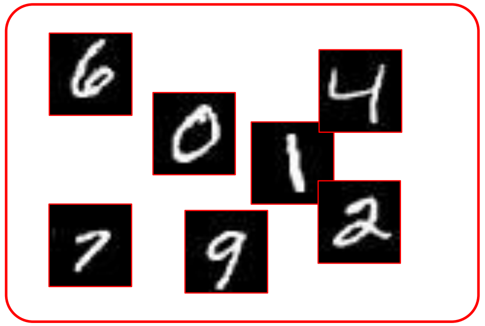
Sequence Input

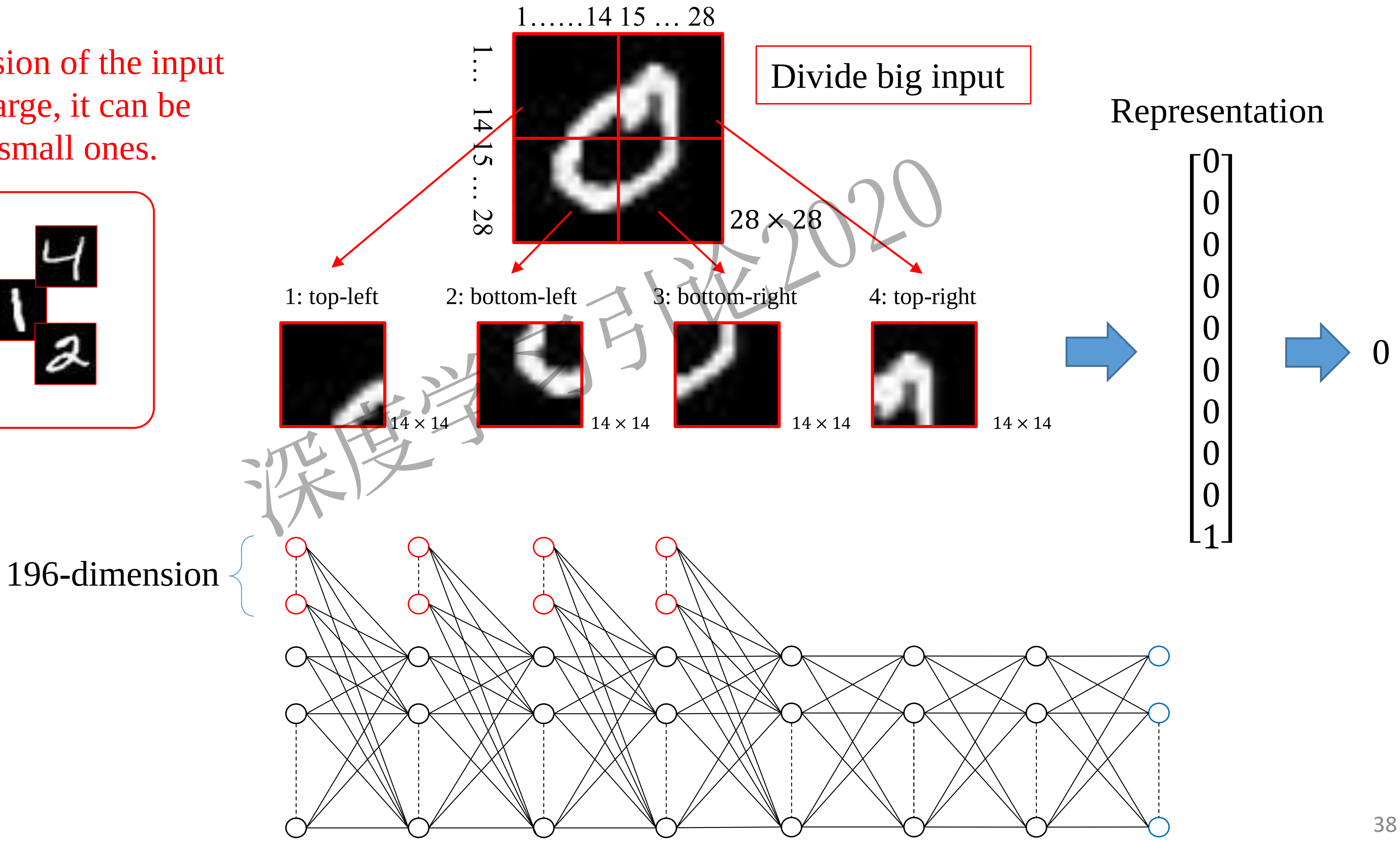


Recognize
the identity

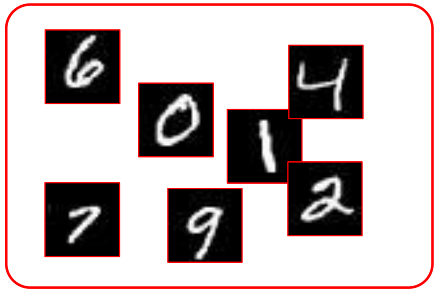
On the Input

If the dimension of the input data is too large, it can be divided into small ones.

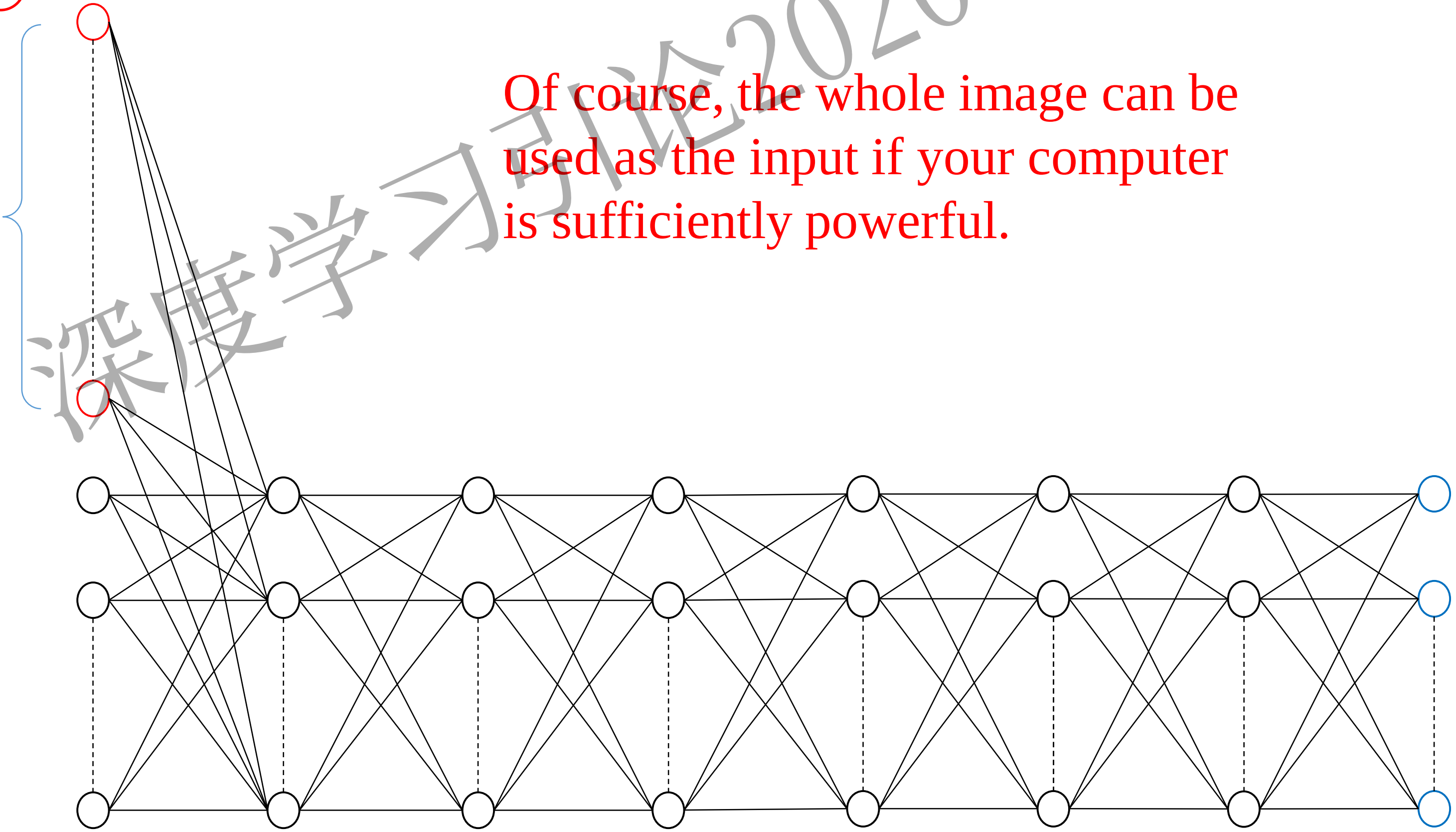




On the Input

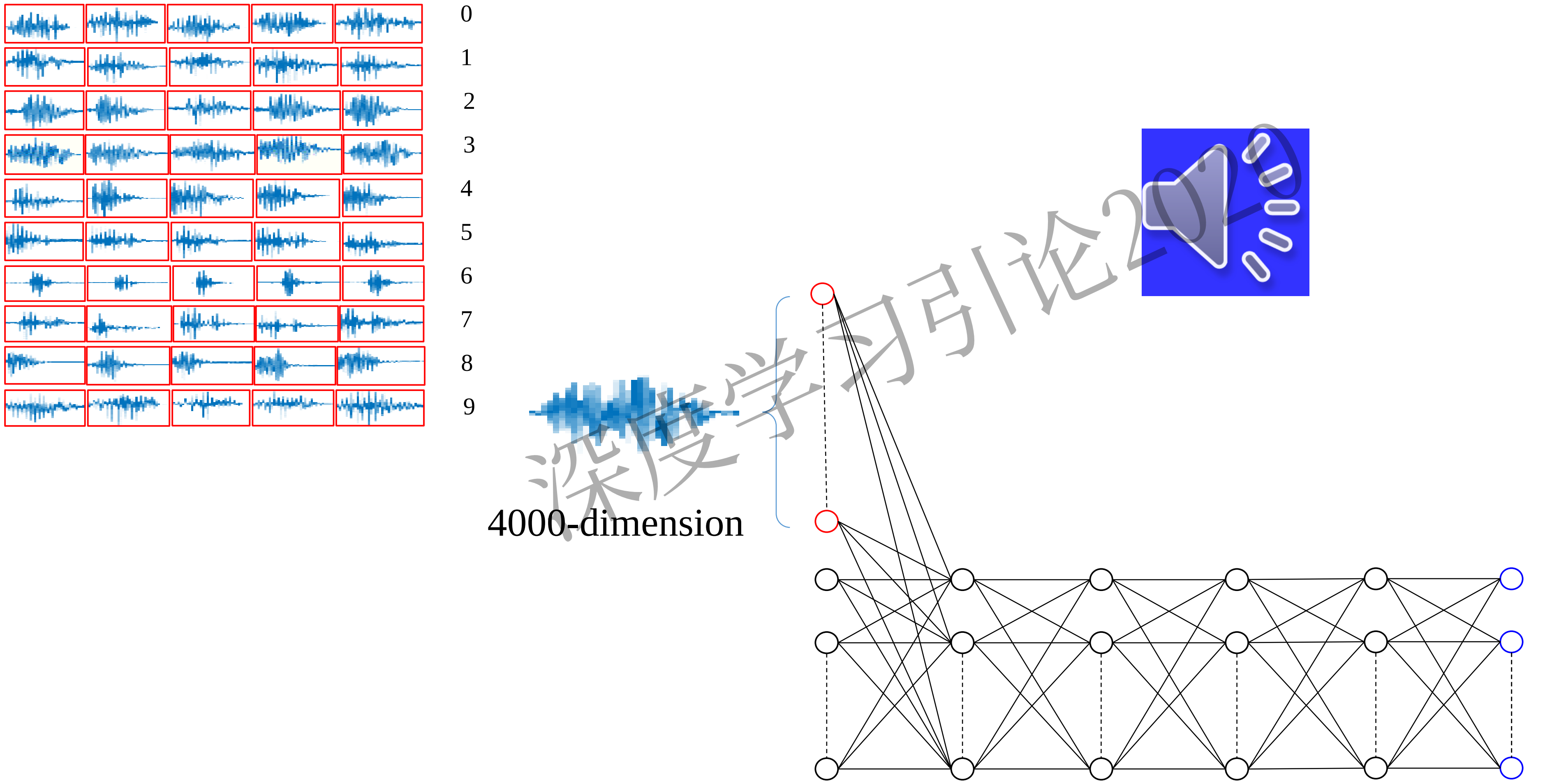


784-dimension



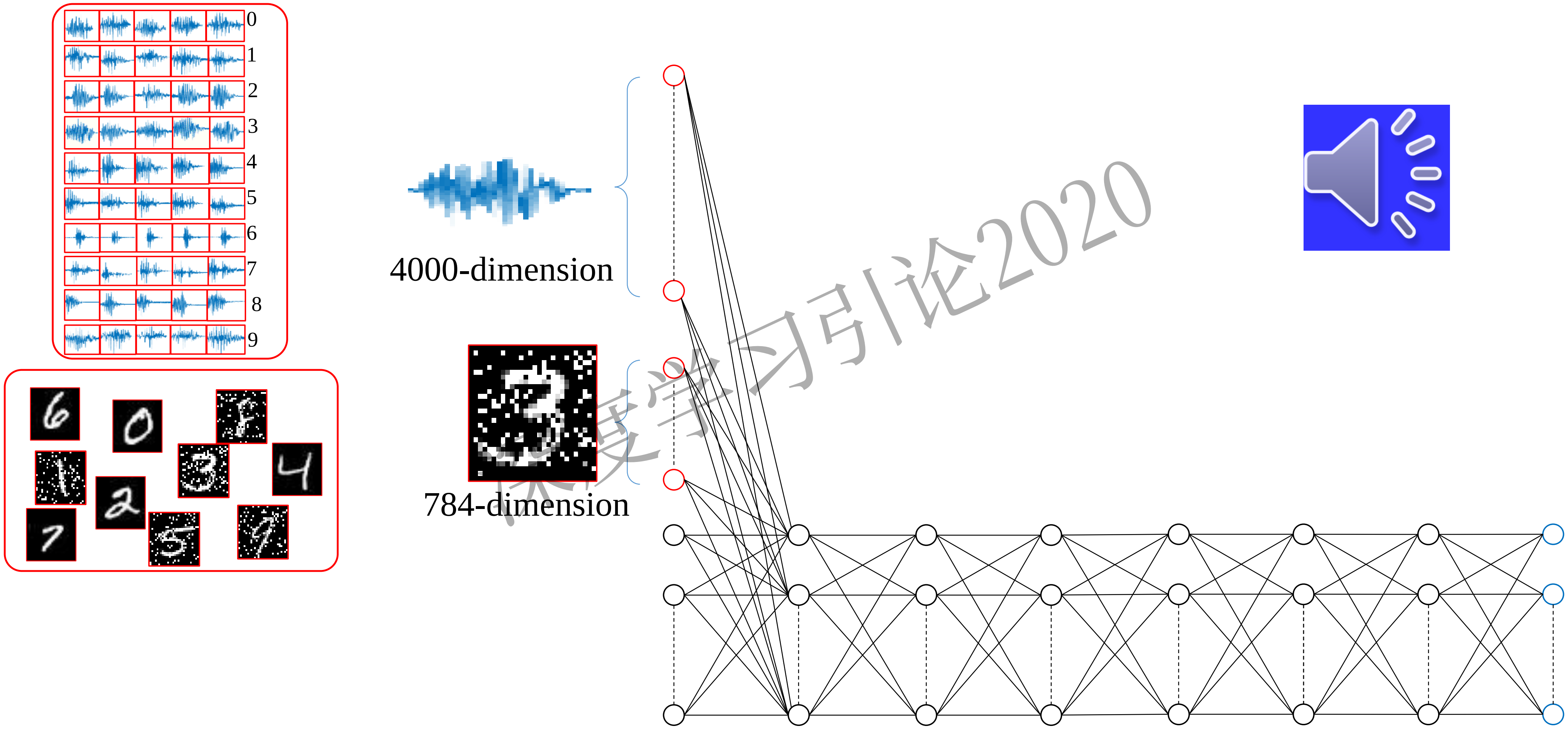
Of course, the whole image can be used as the input if your computer is sufficiently powerful.

On the input

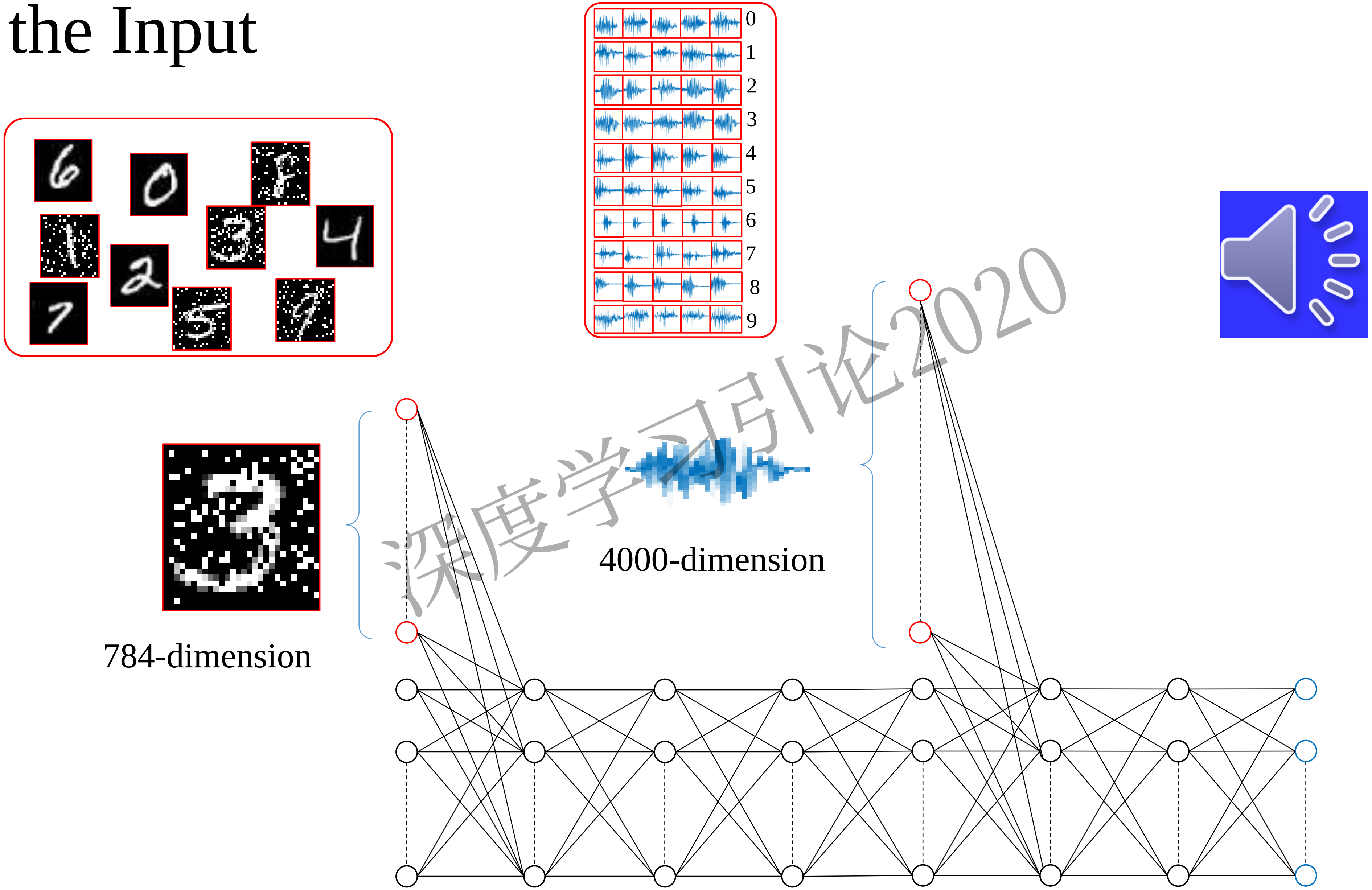


$$\begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

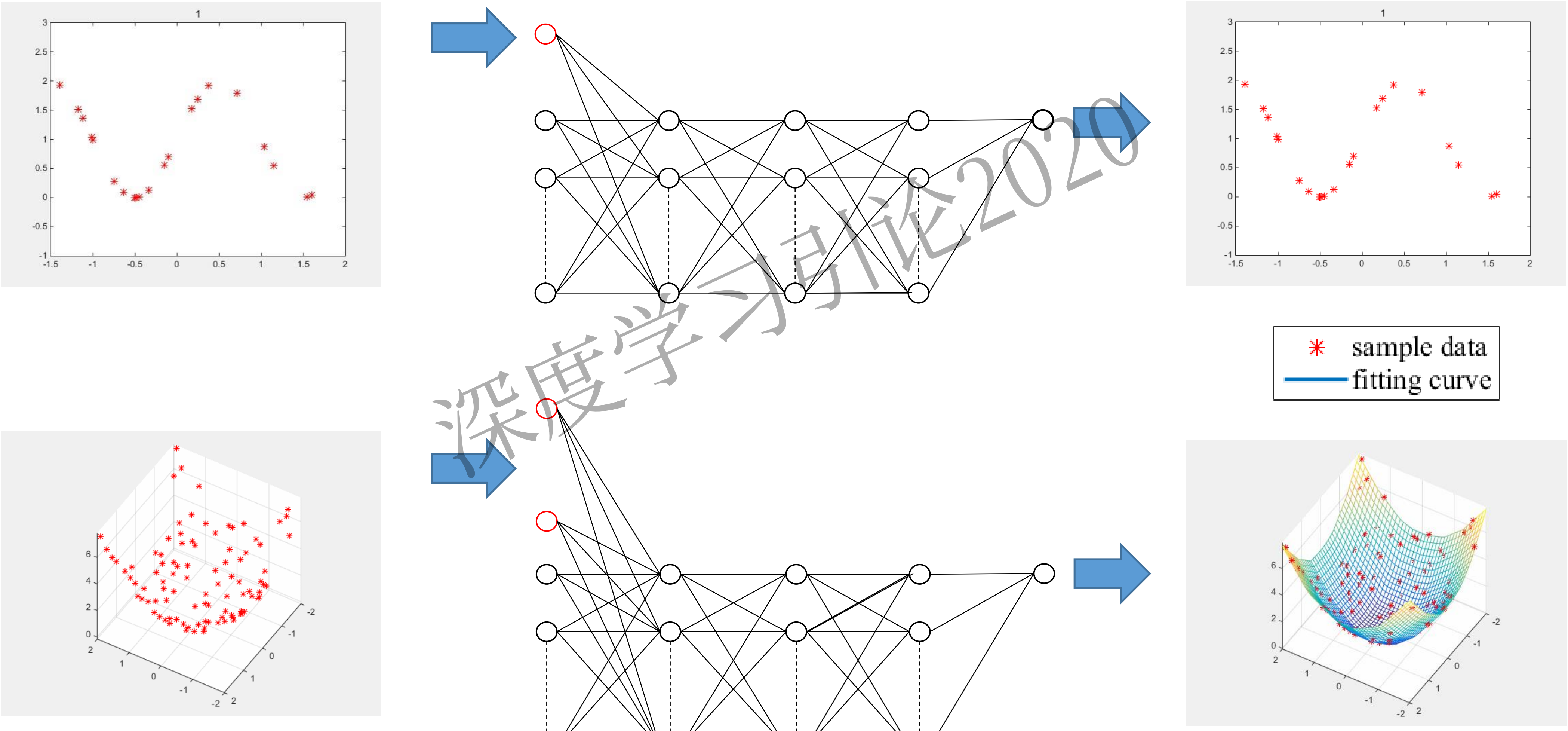
On the input



On the Input



On the input



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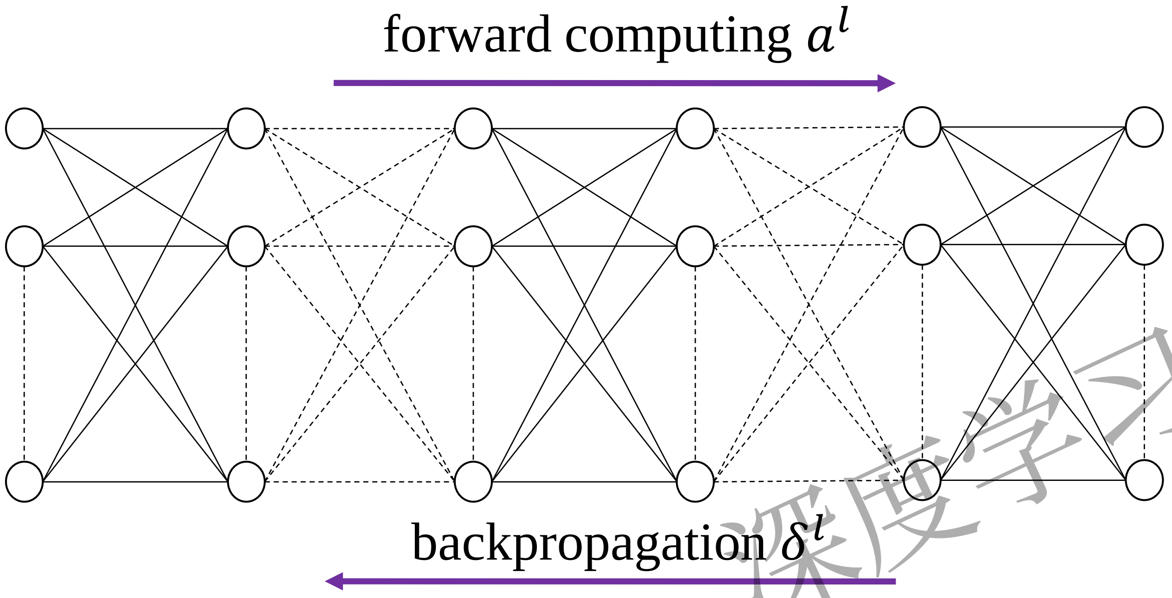
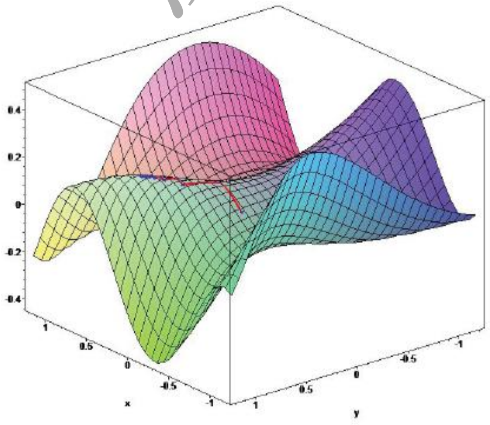
On the Cost Function





Network Output

$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

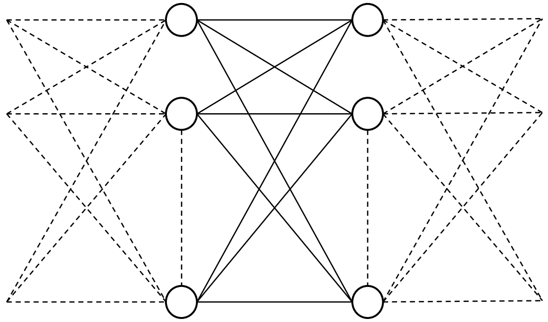
$$J(a^L, y^L)$$

Cost function $J(a^L, y^L)$ is used to describe the closeness between a^L and y^L, $J(a^L, y)$ is indeed a function of $(w^1, \cdots, w^{L-1})$, i.e.,

$$J = J(w^1, \cdots, w^{L-1}).$$

On the Cost Function

forward computing a^l



backpropagation δ^l

Square Error

$$\begin{cases} J = \frac{1}{2}\sum_{j=1}^{n^L}\left(a_j^L - y_j^L\right)^2 \\ \delta_i^L = \frac{\partial J}{\partial z_i^l} = \left(a_i^L - y_i^L\right)\cdot \dot{f}\left(z_i^L\right) \end{cases}$$

$0 \le y_i^L \le 1 \ (i = 1, \cdots, n_L)$

$$a_i^L = f(z_i^L) = \frac{1}{1+e^{-z_i^L}}$$

Sigmoid function

Network Output

$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

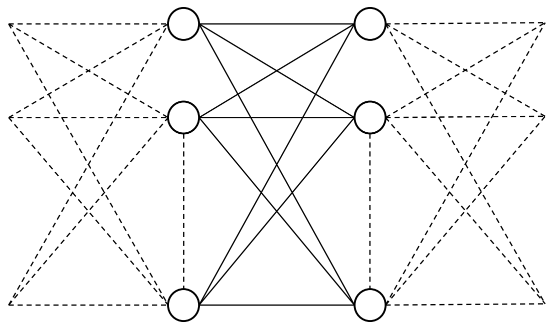
$J(a^L, y^L)$

Cost function $J(a^L, y^L)$ is used to describe the closeness between a^L and y^L, $J(a^L, y)$ is indeed a function of $(w^1, \cdots, w^{L-1})$, i.e.,

$$J = J(w^1, \cdots, w^{L-1}).$$

On the Cost Function

forward computing a^l



backpropagation δ^l

Cross Entropy

$$J = -\sum_{j=1}^{n_L} y_j^L \cdot \log(a_j^L) + \lambda \cdot \sum (w_{ij}^l)^2$$

$$a_j^L = \frac{e^{z_j^L}}{\sum_{i=1}^{n_L} e^{z_i^L}}$$

$$\delta_i^L = a_i^L - y_i^L$$

$$\sum_{j=1}^{n_L} y_i^L = 1$$

$$a_j^L = \frac{e^{z_j^L}}{e^{z_1^L} + \cdots + e^{z_{n_L}^L}}$$

Softmax function

Network Output

$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

$J(a^L, y^L)$

Cost function $J(a^L, y^L)$ is used to describe the closeness between a^L and y^L, $J(a^L, y)$ is indeed a function of $(w^1, \cdots, w^{L-1})$, i.e.,

$$J = J(w^1, \cdots, w^{L-1}).$$

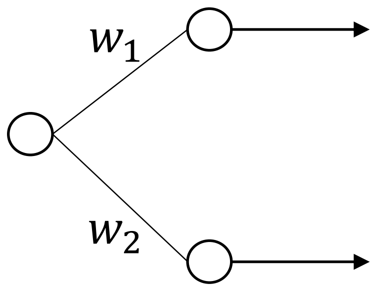
On the Cost Function

An example

Sample data

	1	2
x	0.8000	0.2000
y	0	1
	1	0

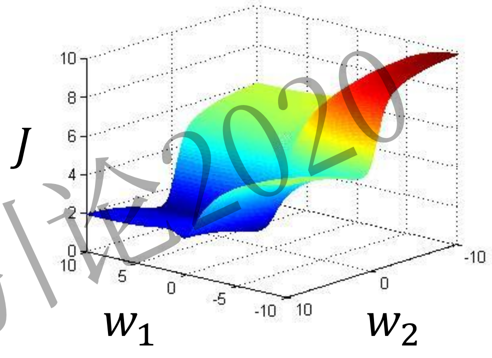
Network

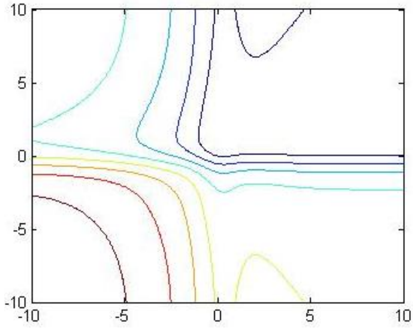


Layer 1 Layer 2

Square Error

$$\begin{cases} J = \frac{1}{2}\sum_{j=1}^{2}(a_j - y_j)^2 \\ a_j = \frac{1}{1+\exp(-z_j)} \\ z_j = w_j \cdot x \end{cases}$$

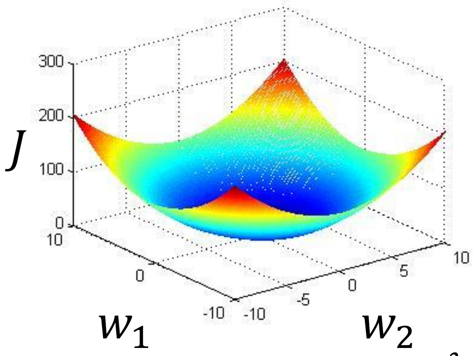




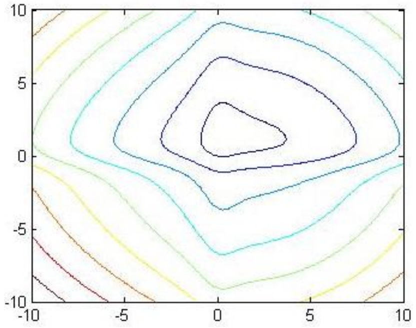
contour

Cross Entropy

$$\begin{cases} J = -\sum_{j=1}^{2} y_j \cdot \log(a_j) + \lambda(w_1^2 + w_2^2) \\ a_j = \frac{e^{z_j}}{\sum_{i=1}^{2} e^{z_j}} \\ z_j = w_j \cdot x \end{cases}$$



$\lambda = 0.05$



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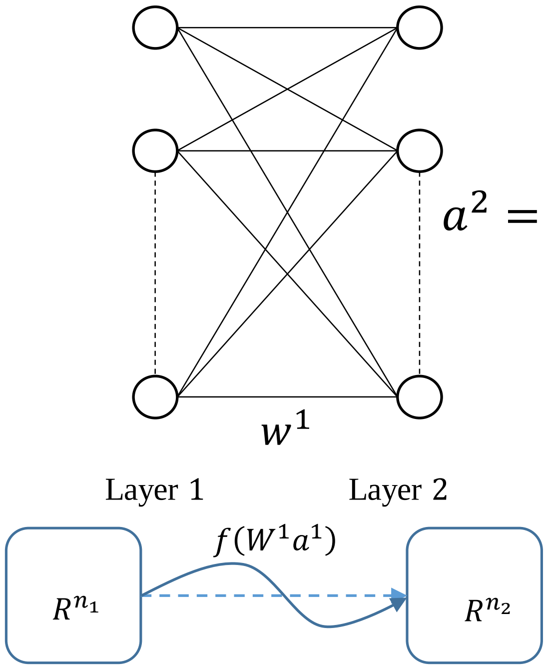
On the Depth of the Networks

Shallow neural network

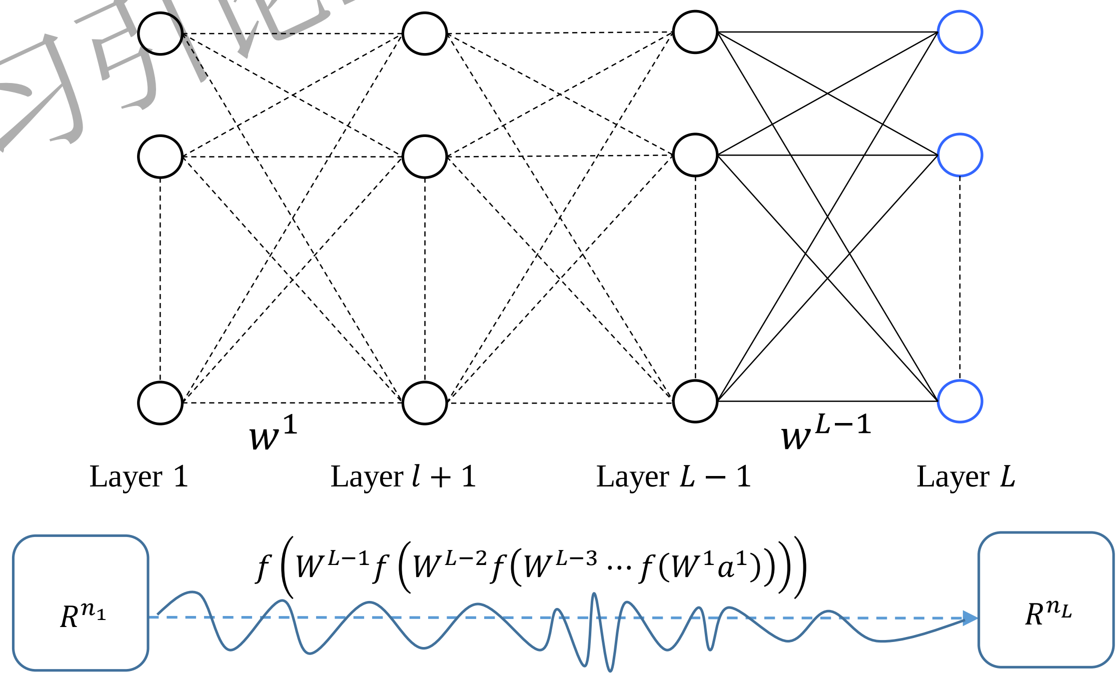
- $L = 2$
- too shallow to learn complex mappings

Deep neural network

- $L > 2$
- can approximate any nonlinear mappings in any precise provided sufficient neurons in the networks

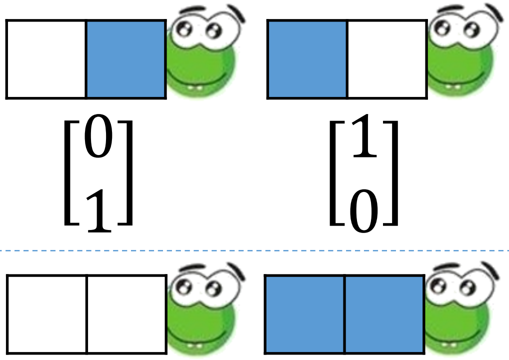


$a^2 = f(w^1 x)$



An example: XOR problem

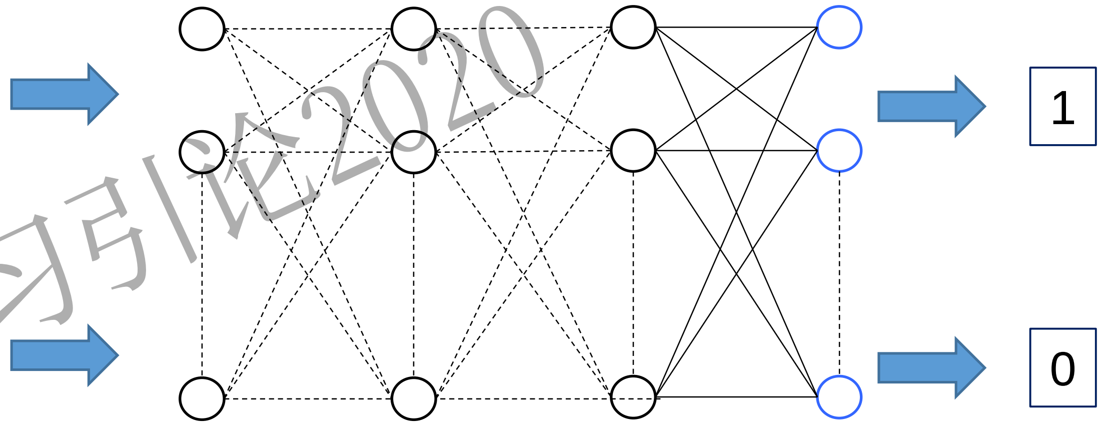
Doted worms

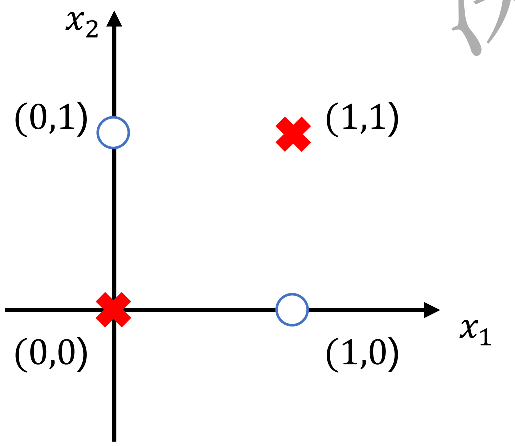


$\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1\\0\end{bmatrix}$

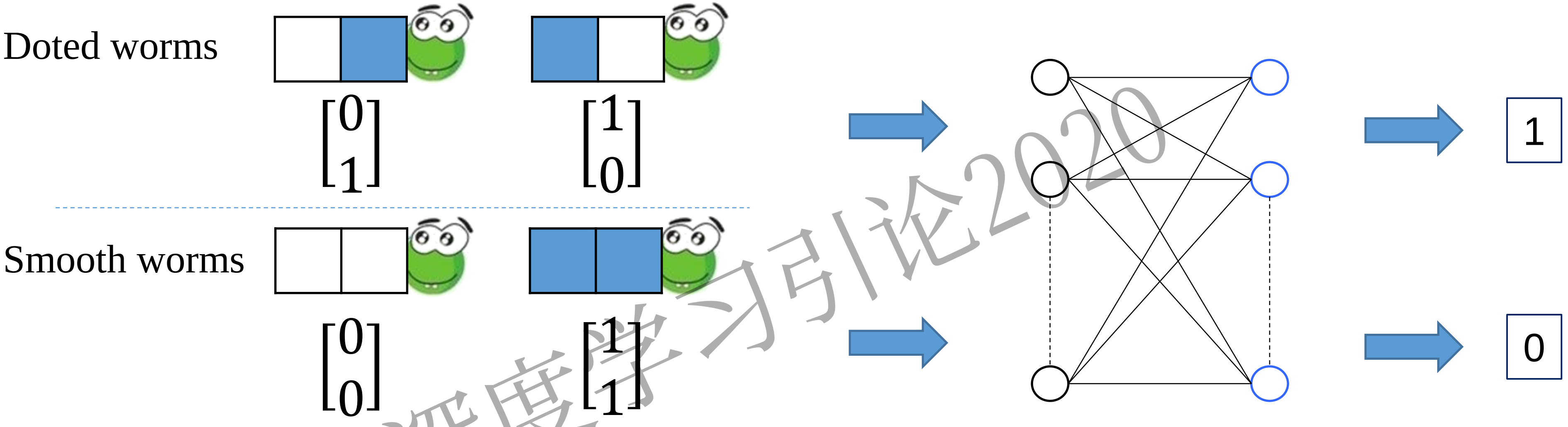
Smooth worms

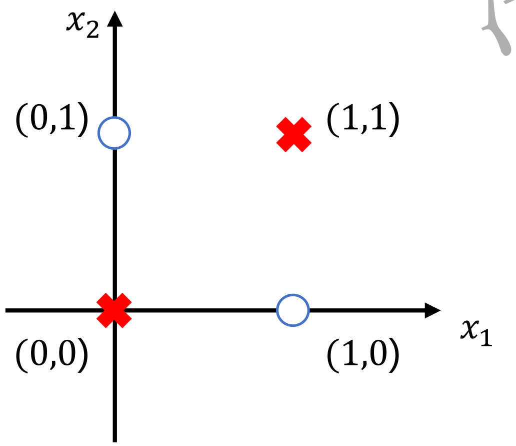
$\begin{bmatrix}0\\0\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}$





An example: XOR problem



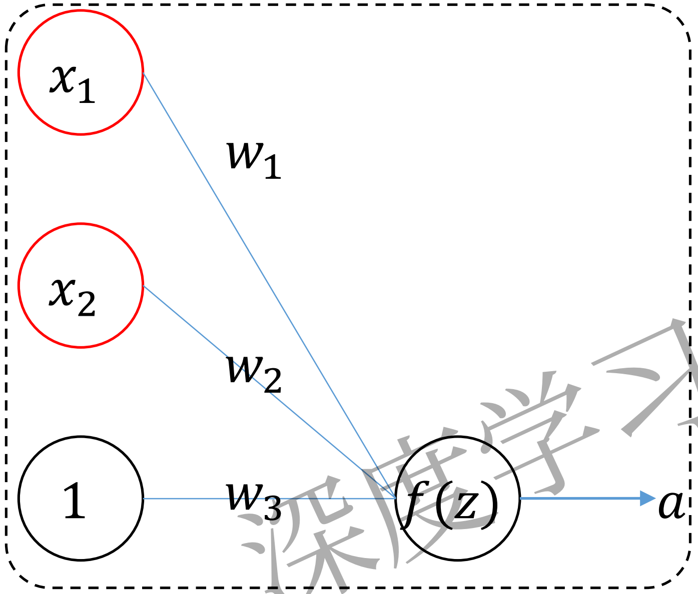


The classification task CANNOT be completed by using two layers network.

An example: XOR problem

$\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1\\0\end{bmatrix}$

$\begin{bmatrix}0\\0\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}$



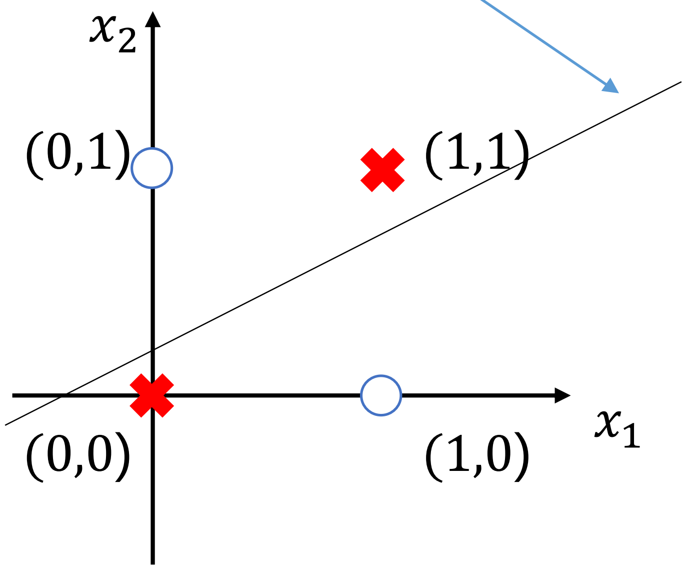
Equivalent

$$a = f(z)$$

$$z = w_1 x_1 + w_2 x_2 + w_3$$

Decision line

$$w_1 x_1 + w_2 x_2 + w_3 = 0$$



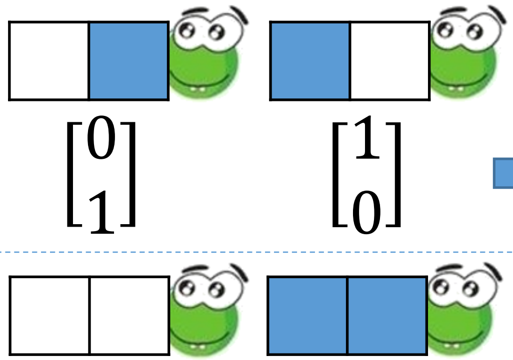
CANNOT use a line to separate the two classes!

An example: XOR problem

At least three layers are required for XOR problem.

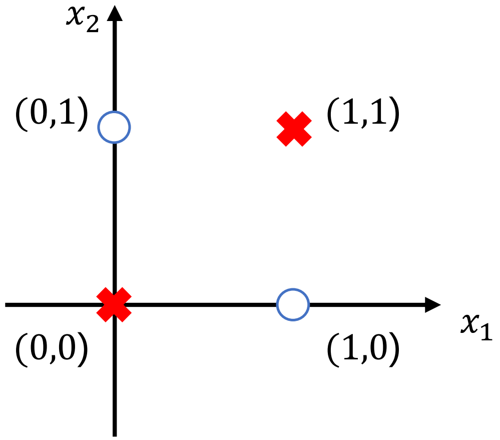
Doted worms

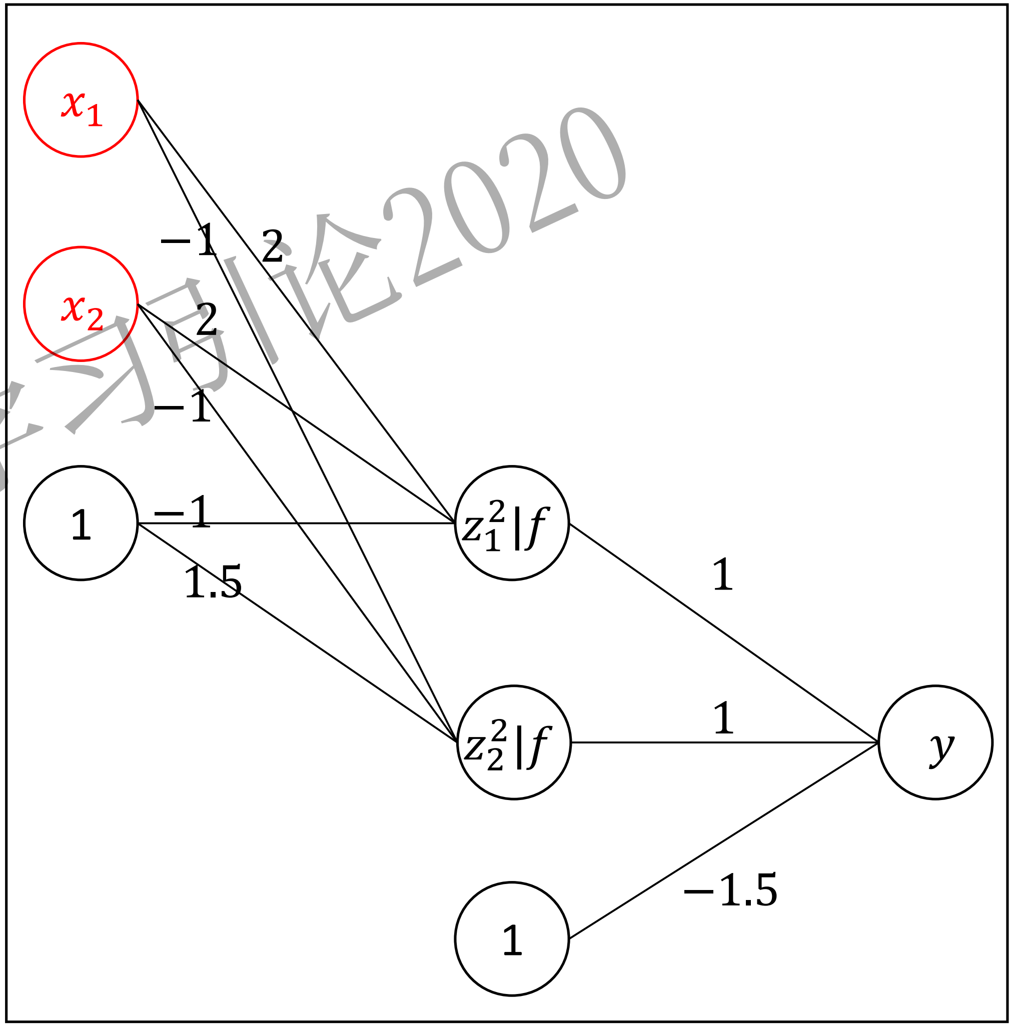
Smooth worms



Doted worms: $\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1\\0\end{bmatrix}$ → 1

Smooth worms: $\begin{bmatrix}0\\0\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}$ → 0





1

0

On the Depth of the Networks

Gradient Vanishing Problem

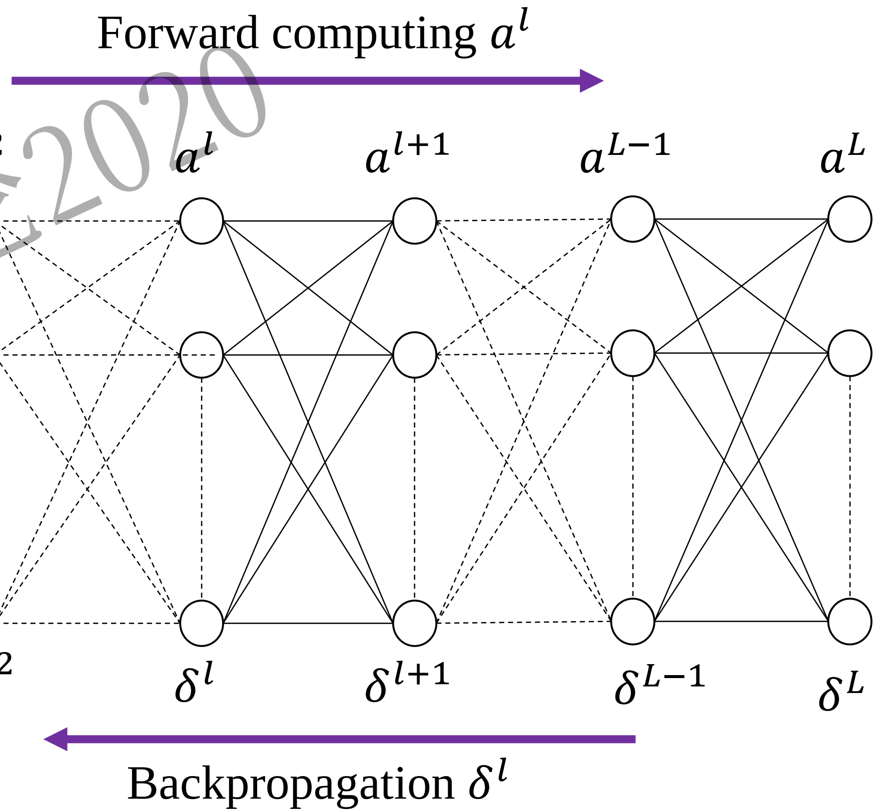
Cost function: $J(w^1, \cdots, w^{L-1})$

Updating rule: $w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$

Relationship: $\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$

key:

$$\delta_i^l = \dot{f}(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1} \right)$$



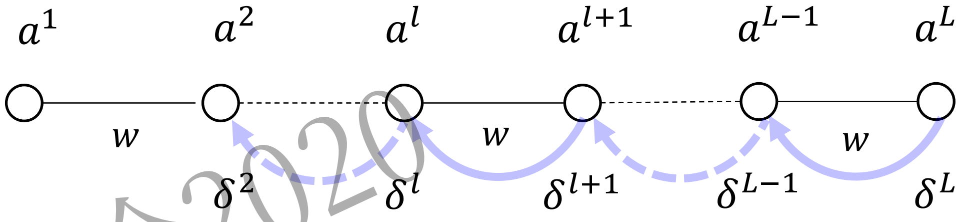
On the Depth of the Networks

Gradient Vanishing Problem

a simple example

$$w = w^{\,l}$$

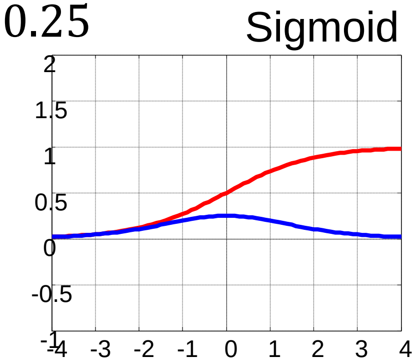
$$\delta^l = \dot{f}(z^l) \cdot w \cdot \delta^{l+1}$$



$$\begin{aligned}
\delta^l &= \dot{f}(z^l) \cdot w \cdot \delta^{l+1} \\
&= \dot{f}(z^l) \cdot w \cdot \dot{f}(z^{l+1}) \cdot w \cdot \delta^{l+2} \\
&= w \cdot \dot{f}(z^l) \cdot w \cdot \dot{f}(z^{l+1}) \cdots w \cdot \dot{f}(z^{L-1}) \cdot \delta^L \\
&= \prod_{m=L-1}^{l} \left(w \cdot \dot{f}(z^m)\right) \cdot \delta^L
\end{aligned}$$

$$\left|\frac{\partial \delta^l}{\partial \delta^L}\right| = \prod_{m=L-1}^{l} \left|w \cdot \dot{f}(z^m)\right| \leq |w|^{L-l+1} \cdot (0.25)^{L-l+1}$$

$$\dot{f}(z^m) \leq 0.25$$

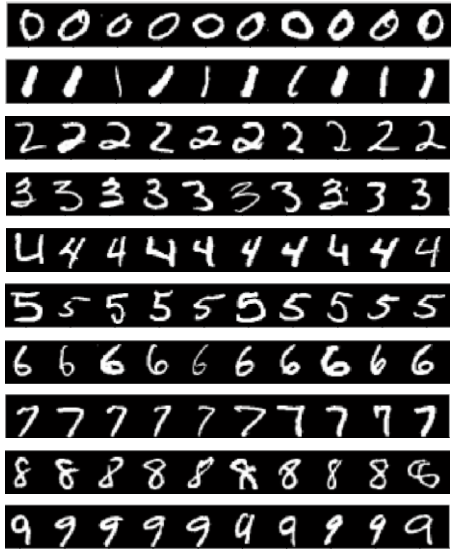


Notes：

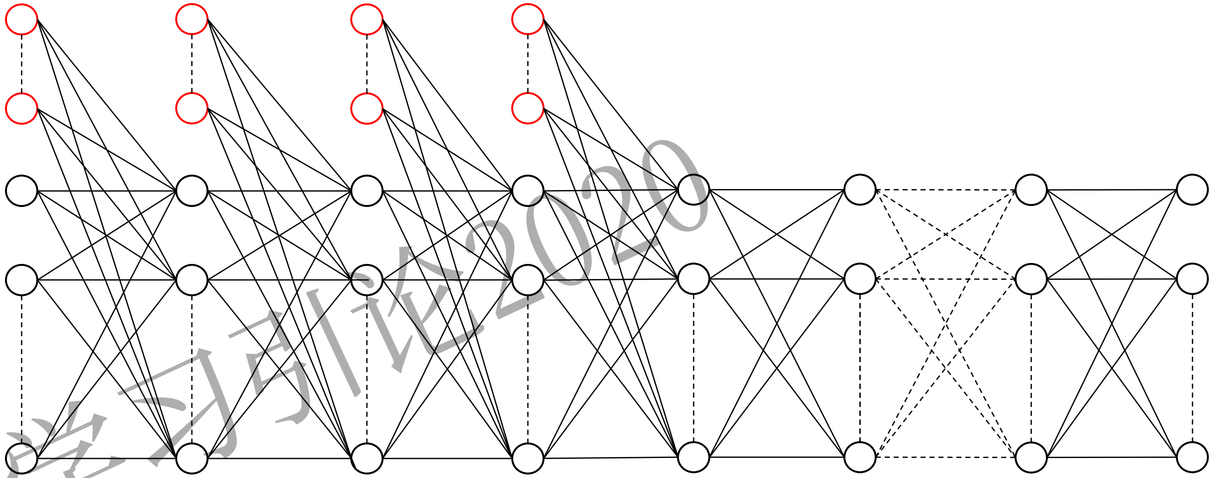
The exponential descent of δ^l causes the gradient vanish problem.

On the Depth of the Networks

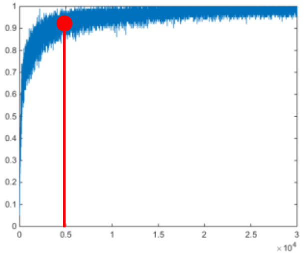
The depth of the network is correlated to the problem.



Handwritten digits recognition problem

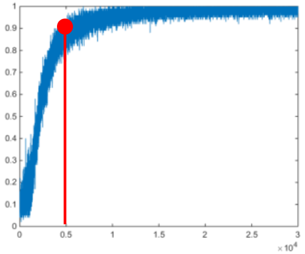


5 layers



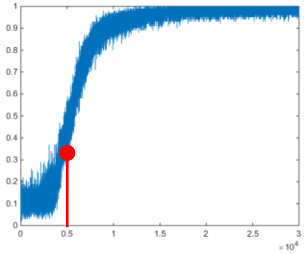
Accuracy
- Training=97.55%
- Testing=95.25%

8 layers



Accuracy
Training=98.65%
Testing=95.10%

9 layers



Accuracy
- Training=98.45%
- Testing=93.20%

Outline

- Brief Review of Backpropagation Algorithm
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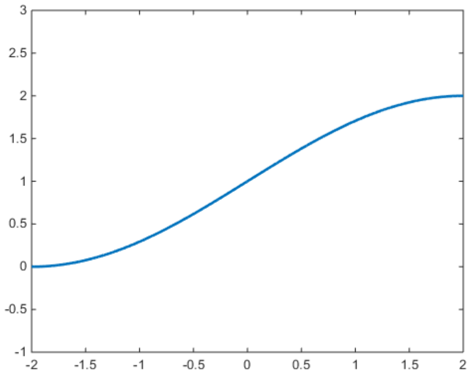
  - On the Depth of the Network
  - On the Training Data
- Assignment

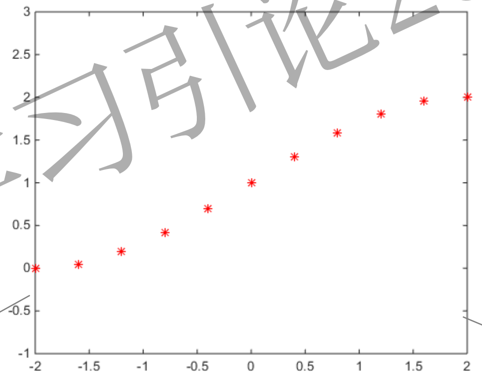
On the Training Data

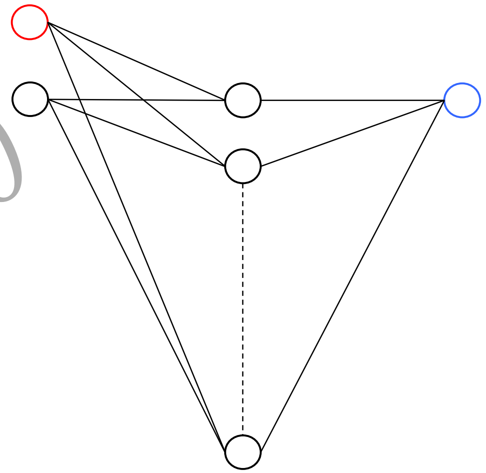
Using a 2-9-1 network to fit a partial sin curve

$$y = g(x) = 1 + \sin\left(\frac{\pi}{4}x\right), x \in [-2,2]$$



Samples





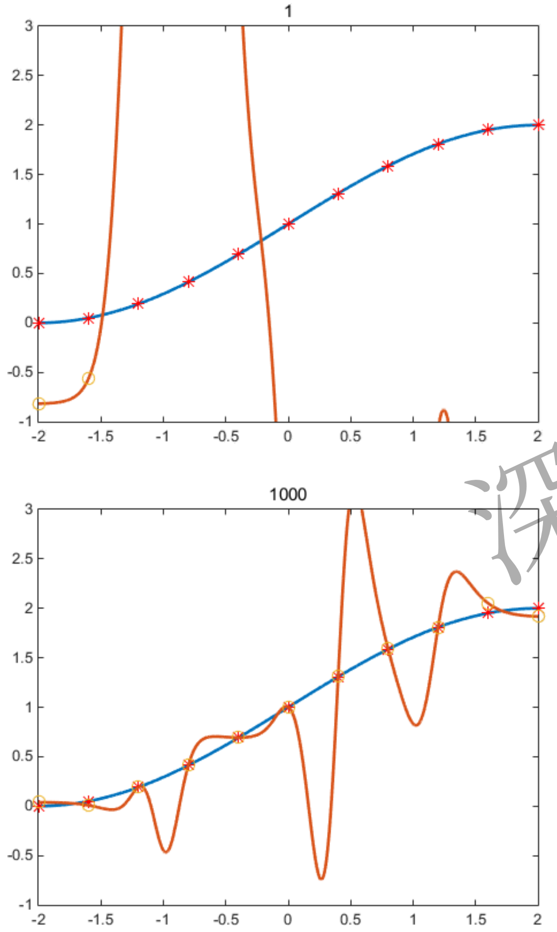
(1,1) → (0,9) → (0,1)

	1	2	3	4	5	6	7	8	9	10	11
x	-2	-1.6000	-1.2000	-0.8000	-0.4000	0	0.4000	0.8000	1.2000	1.6000	2
y	0	0.0489	0.1910	0.4122	0.6910	1	1.3090	1.5878	1.8090	1.9511	2

11 samples

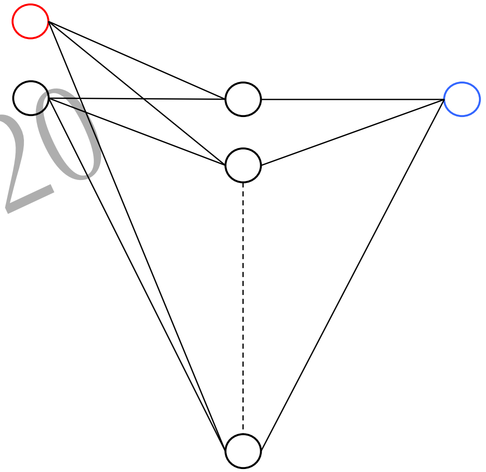
On the Training Data

Overfitting



11 data samples

Using a 2-9-1 network to fit a partial sin curve

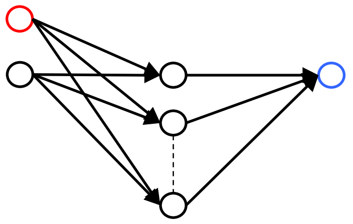


2-9-1 network has 27 weights to be tuned.
In general, we need more samples than the number of unknown parameters in a system.

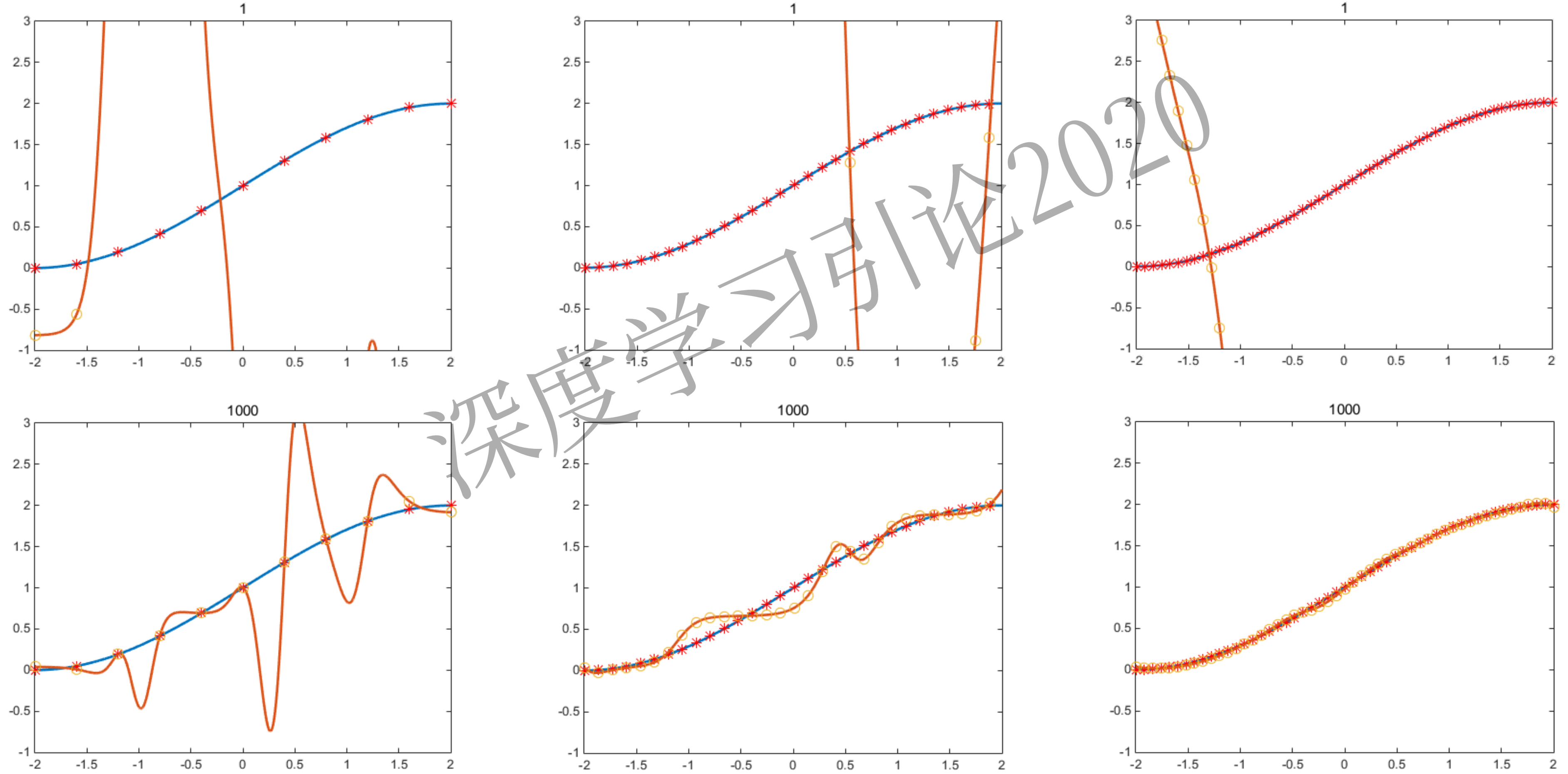
The network fit the data sample properly, but nowhere else on the curve! Overfitting!

- Fit training data well
- Cannot fit testing data
- We need **MORE** data!

On the Training Data

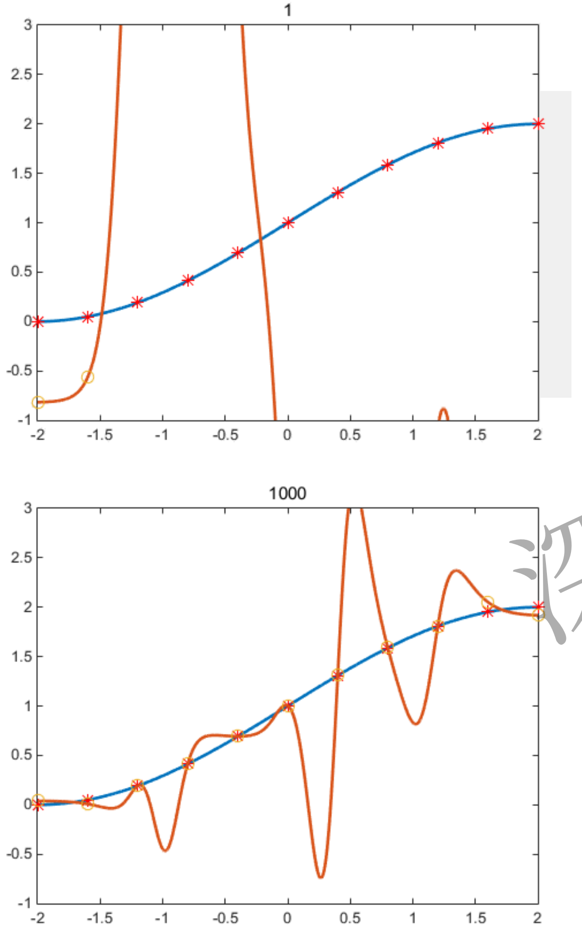


(1,1) → (0,9) → (0,1)

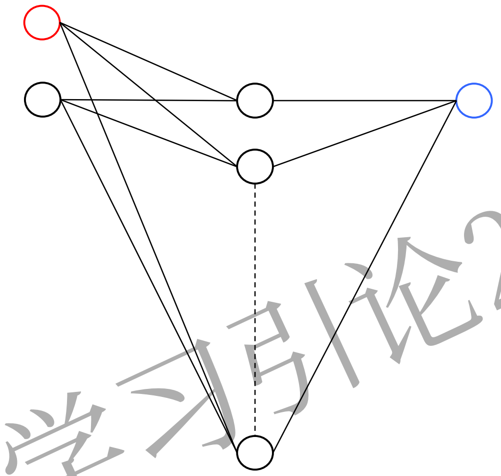


11 data samples | 23 data samples | 51 data samples

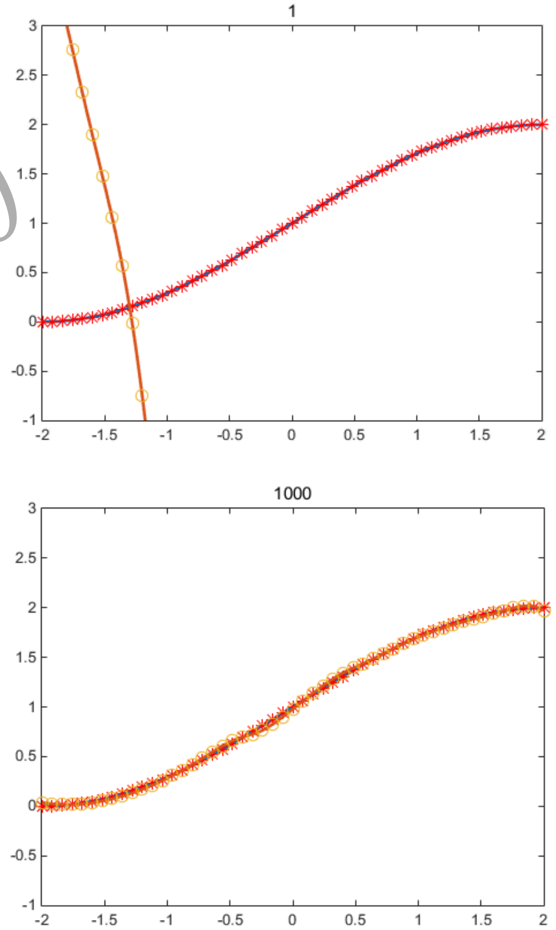
On the Training Data



11 data samples



For a network to be able to generalize, it should have fewer parameters than there are data points in the training set.



51 data samples

On the Training Data

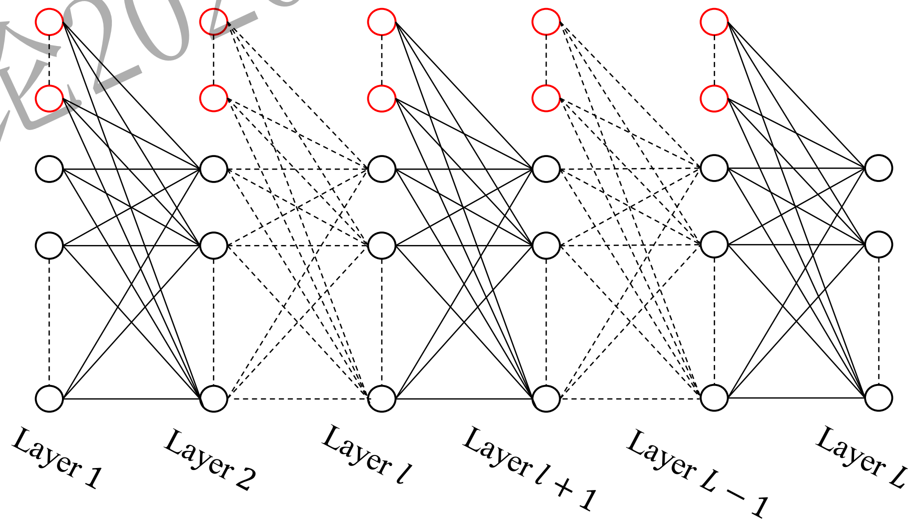
Big data

Complex patterns in **big data** need complex model to deal with.

Abundant data sample for training model (samples)

Highly nonlinear, flexible, and trainable model (complexity)

DNN



Huge number of parameters in **DNN** models need to be determined.

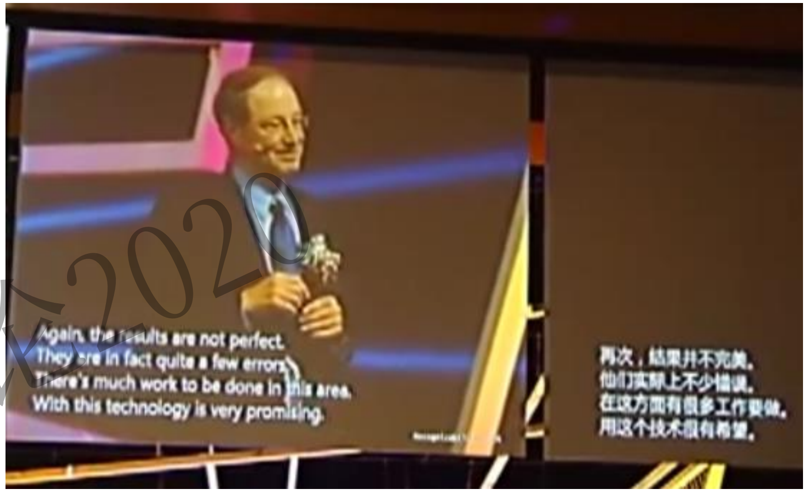
Big data + DNN Example

Speech Recognition

1950s Wave of speech + pattern recognition = few words

1970s Gaussian Mixture Model + Hidden Markov Model = ~80% recognition rate

2011 Deep neural network for modeling speech = awesome real-time recognition!



Real-time speech translation

English speech → English text → Chinese text → Chinese speech

English speech → Voice Feature → Chinese speech

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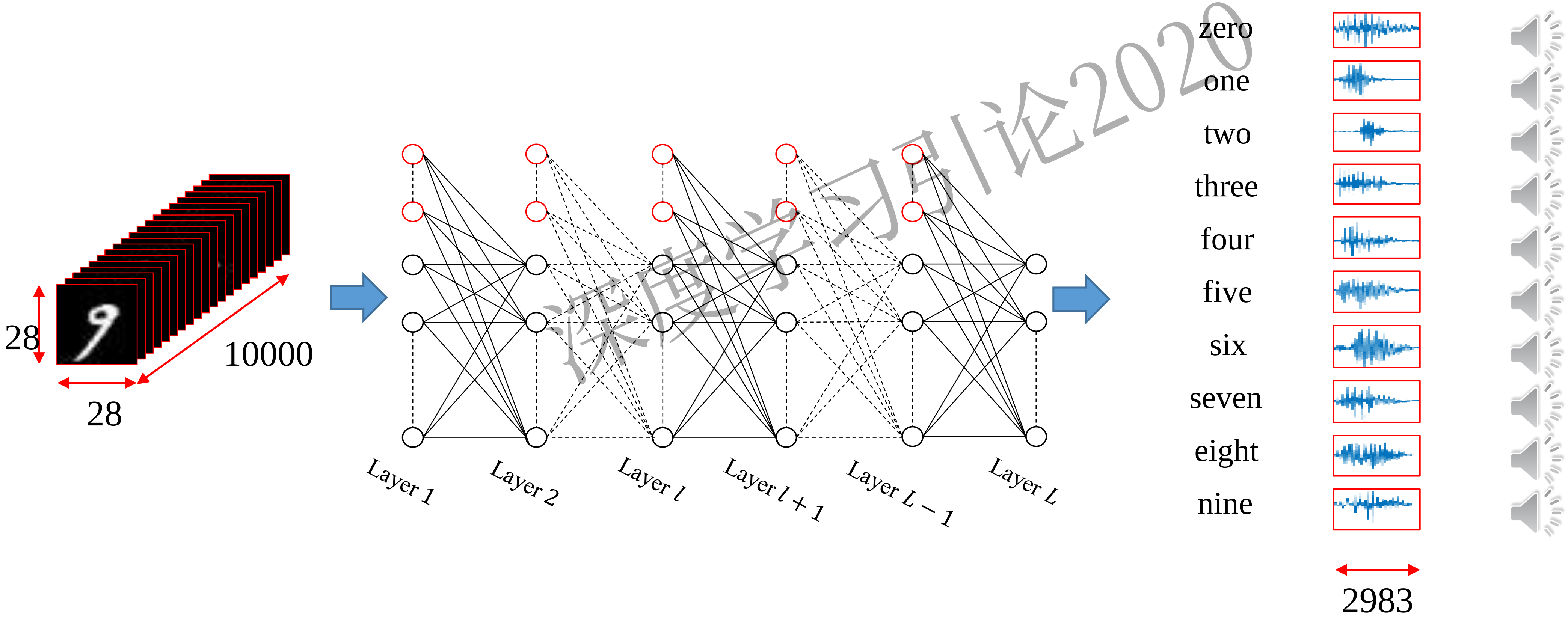
Assignment

Implement the handwritten digits to speech convertor by MATLAB.

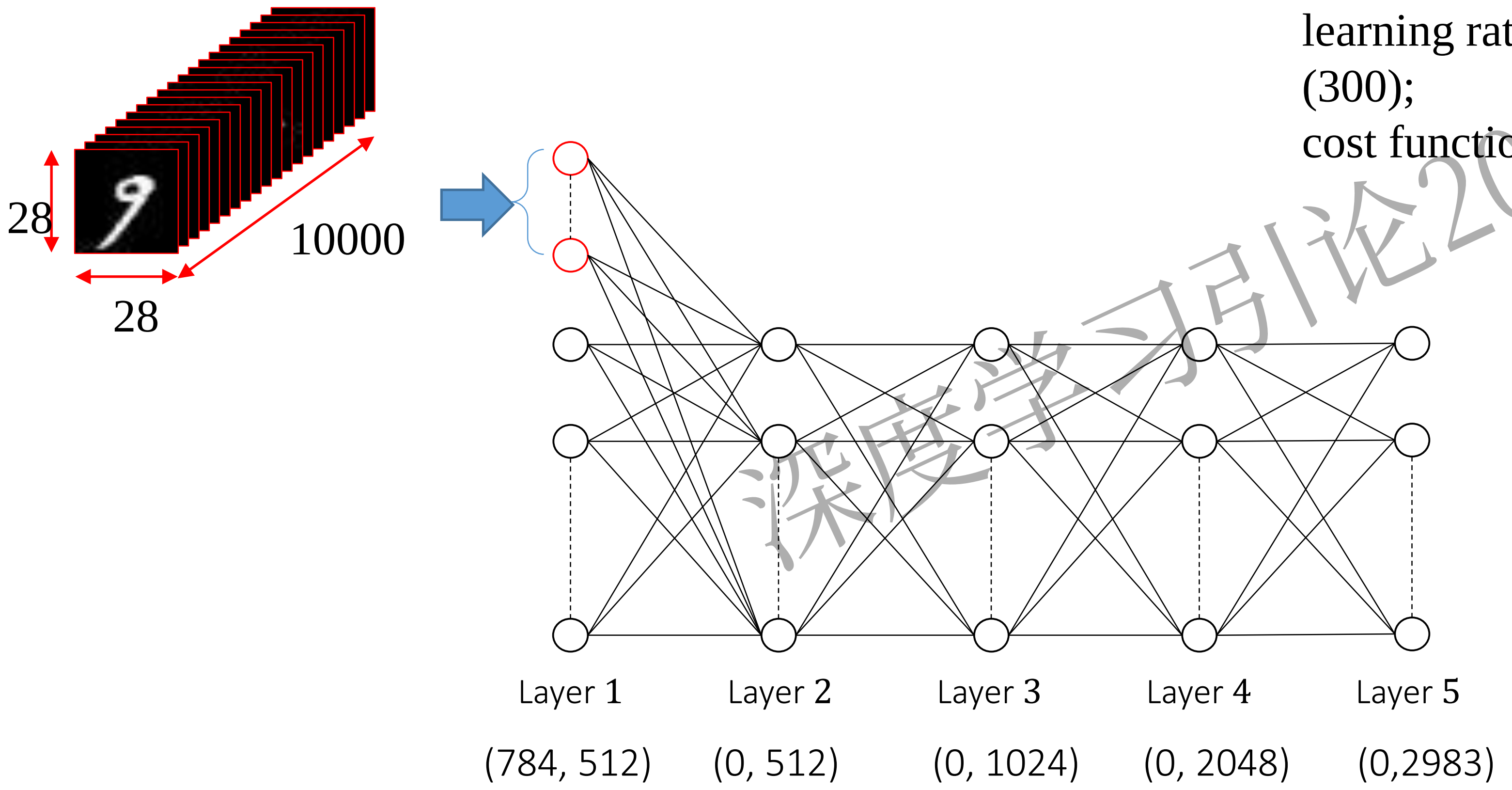


Thanks

Assignment: an example

Hint: One of my student used the following parameters for the network and successfully trained the network.

learning rate (0.1); mini batch (100); iteration (300);
cost function (mean square error).



Assignment: codes

fc.m %%forward computation file

```
function [a_next, z_next] = fc(w, a, x, f)

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code BELOW
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % forward computing (either component or vector form)
    z_next = w * [x; a];
    a_next = f(z_next);
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code ABOVE
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

Assignment: codes

bc.m %%backward computation file

```
function delta = bc(w, z, delta_next, df)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code BELOW
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % backward computing (either component or vector form)
    delta = df(z) .* (w(:, end-size(z,1)+1:end)' * delta_next);
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code ABOVE
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

Assignment: codes

get_audio.m `%%the function to get audio file`

```
function [ audio_y ] = get_audio( y, audio )

audio_y = zeros(size(audio,2), size(y,2));
for i = 1:size(y,2)
    audio_y(:,i) = audio(find(y(:,i)==1),:);
end
```

Assignment: codes

lab5.m `%%the main training function`

```
% clear workspace and close plot windows
clear;
close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Your code BELOW
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% prepare the data set
load mnist_small_matlab.mat
input_size = 28 * 28; % size of each patch
% prepare training data
train_size = size(trainLabels,2);
X_train{1} = reshape(trainData,[],train_size);% top-left
X_train{2} = zeros(0, train_size);
X_train{3} = zeros(0, train_size);
X_train{4} = zeros(0, train_size);
X_train{5} = zeros(0, train_size);
```

```
% prepare testing data
test_size = size(testLabels,2);
X_test{1} = reshape(trainData,[],test_size);% top-left
X_test{2} = zeros(0, test_size);
X_test{3} = zeros(0, test_size);
X_test{4} = zeros(0, test_size);
X_test{5} = zeros(0, test_size);

% prepare standard speech audio
sample_rate = 4000; % shall assert they all have a same sample rate
audio = zeros(2983, 10); % we checked with the audio file and know its 2983-dim input
for i = 1:10
    [audio(:,i), sample_rate] = audioread(fullfile('audio',sprintf('%d.wav',i-1)));
    soundsc(audio(:,i), sample_rate);
    pause(1)
end
audio = (audio+1)/2;

% choose parameters
alpha = 0.1; % learning rate
max_iter = 300;
mini_batch = 100;
```

```
layer_size = [input_size 512     % layer 1
                         0 512     % layer 2
                         0 1024     % layer 3
                         0 2048     % layer 4
                         0 2983];   % layer 5
L = size(layer_size, 1);
% define function
sigm = @(s) 1 ./ (1 + exp(-s));
dsigm = @(s) sigm(s) .* (1 - sigm(s));
lin = @(s) s;
dlin = @(s) 1;
fs  = {[],  sigm,  sigm,  sigm,  sigm,  sigm,  sigm,  sigm};
dfs = {[], dsigm, dsigm, dsigm, dsigm, dsigm, dsigm, dsigm};
% initialize weights
w = cell(L-1, 1);
for l = 1:L-1
    %w{l} = randn(layer_size(l+1,2), sum(layer_size(l,:)));
    % a tricky, but effective, initialization
    w{l} = (rand(layer_size(l+1,2), sum(layer_size(l,:))) * 2 -1) *
sqrt(6/(layer_size(l+1,2)+sum(layer_size(l,:))));
end

% train
J = [];
x = cell(L, 1);
a = cell(L, 1);
z = cell(L, 1);
delta = cell(L, 1);
```

```
for iter = 1:max_iter
    ind = randperm(train_size);
    % for each mini-batch
    for k = 1:ceil(train_size/mini_batch)
        % prepare internal inputs
        a{1} = zeros(layer_size(1,2),mini_batch);
        % prepare external inputs
        for l=1:L
            x{l} = X_train{l}(:,ind((k-1)*mini_batch+1:min(k*mini_batch, train_size)));
        end
        % prepare labels
        [~, ind_label] = max(trainLabels(:,ind((k-1)*mini_batch+1:min(k*mini_batch, train_size))));
        % prepare targets
        y = audio(:,ind_label);

        % batch forward computation
        for l=1:L-1
            [a{l+1}, z{l+1}] = fc(w{l}, a{l}, x{l}, fs{l+1});
        end

        % cost function and error
        J = [J 1/2/mini_batch*sum((a{L}(:)-y(:)).^2)];
        delta{L} = (a{L} - y).* dfs{L}(z{L});

        % batch backward computation
        for l=L-1:-1:2
            delta{l} = bc(w{l}, z{l}, delta{l+1}, dfs{l});
        end
        % update weight
        for l=1:L-1
            gw = delta{l+1} * [x{l};a{l}]' / mini_batch;
            w{l} = w{l} - alpha * gw;
        end
    end
```

```
 % end loop
    if mod(iter,1) == 0
        fprintf('%i/%i epochs: J=%.4f\n', iter, 
max_iter, J(end));
    end
end

% save model
save model.mat w layer_size J
```