MECE 5397

Project A – Poisson Equation

Write a computer code to solve the two-dimensional Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \;=\; -F(x,y) \tag{1}$$

The domain of interest is the rectangle

$$a_x < x < b_x\,, \qquad a_y < y < b_y \tag{2}$$

and the boundary conditions

$$u(x = a_x, y) \;=\; \phi_{ab}(y)\,, \qquad u(x = b_x, y) \;=\; \psi_{ab}(y)\,, \tag{3}$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=a_y} \;=\; 0\,, \qquad \left.\frac{\partial u}{\partial y}\right|_{y=b_y} \;=\; 0\,, \tag{4}$$

$$a_x \;= a_y \;=\; -\pi\,, \qquad b_x \;=\; b_y \;=\; \pi \tag{5}$$

$$\phi_{ab}(y) \;=\; (y - a_y)^2 \,\sin\frac{\pi(y-a_y)}{2(b_y - a_y)}\,, \qquad \psi_{ab}(y) \;=\; \{\cos[\pi(y-a_y)] - 1\}\,\cosh(b_y - y) \tag{6}$$

$$F(x,y) \;=\; \sin\left[\pi\frac{x - a_x}{b_x - a_x}\right]\,\cos\left[\frac{\pi}{2}\left(2\frac{y - a_y}{b_y - a_y} + 1\right)\right] \tag{7}$$

Use ghost node(s) for Neumann condition(s).
After carrying out all the simulations needed for the report, run one last simulation with $F = 0$ and include the results in the report.